Usability of Deterministic and Stochastic Petri Nets in the Wood Industry: A Case Study

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Abstract. Deterministic and stochastic Petri nets (DSPNs) are commonly used for modeling processes having either deterministically, or exponentially distributed delays. However, DSPNs are not widespread in the wood industry, where the leaders of a company try to make decisions based on common sense instead of using high-level performance evaluation methods.

In this paper, we present a case study, in which we demonstrate the usability of DSPN models in the wood industry. In the case study, we model the production of wooden windows of a Hungarian company [2]. Using the model, we can simply determine the bottleneck of the manufacturing process and show how to eliminate it.

Keywords: wooden window manufacturing, DSPN, modeling, simulation.

1 Introduction

In many manufacturing processes, the work can be divided into different disjoint phases having deterministic holding times. Besides, there are some exponentially distributed variables even in these cases, like the inter-arrival times of the orders and the revealing of the material defects.

A process, in which the delays of the transitions are either exponentially, or deterministically distributed, can be appropriately described by a DSPN [4]. DSPNs are similar to stochastic Petri nets (SPNs [5]), except that deterministically delayed transitions are also allowed in the Petri Net model.

Although DSPNs have greater modeling strength than SPNs, there are some restrictions in the model evaluation phase. Namely, analytical solution using the underlying stochastic behavior can be obtained only if there is only *one* enabled deterministic transition in each marking. Otherwise, simulation must be used to get the steady state or the transient solution of the DSPN. In this paper, we model a window manufacturing process, where the production phases are deterministic, and work in parallel. Therefore, the traditional analytical approach cannot be used, so we will present some simulation results.

The rest of the paper is organized as follows. In Section 2, the formalism of DSPNs is described. In Section 3, we provide a short summary of the manufacturing process of wooden windows to help the understanding of the latter

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sections. Section 1 describes our model for the wooden window manufacturing process. In Section 5, we present the evaluation of the model. Finally, Section 6 concludes the paper.

2 DSPN Formalism

In this section, we give a short introduction to DSPNs, while a detailed description can be found, e.g. in [1] or [4].

DSPNs are bipartite directed graphs consisting two types of nodes: places and transitions. The places correspond to the state variables of the system, while the transitions correspond to the events that can induce a state change. The arcs are connected to places and vice versa expressing the relation between states and event occurrence. Places can contain tokens, and the state of a DSPN, called marking, is defined by the number of tokens in each place. We use the notation M to indicate a marking in general, and we denote by $M(p)$ the number of tokens in place p in marking M. Now we recall the basic definitions related to DSPNs.

Definition 1. *A DSPN system is a tuple*

$$(P, T, I, O, H, M_0, \tau, w),$$

where:

- P *is the finite set of* places. *A marking* $M \in \mathbb{N}^{|P|}$ *defines the number of tokens in each place* $p \in P$.
- T *is the set of* transitions. T *can be partitioned into the following disjoint sets:* T^Z *is for the set of immediate transitions,* T^E *is for the set of exponential transitions, and* T^D *is for the set of deterministic transitions. Note that* $P \cap T = \emptyset$.
- $I, O, H : \mathbb{N}^{|P|} \to \mathbb{N}$ *are the multiplicities of the* input arc *from* p *to* t*, the* output arc *from* t *to* p*, and the* inhibitor arc *from* p *to* t*, respectively.*
- $M_0 \in \mathbb{N}^{|P|}$ *is the* initial marking *of the net.*
- $\tau : \mathbb{N}^{|P|} \to \mathbb{R}^+$ *is the mean delay for* $\forall t \in T^E \cup T^D$ *(note that* τ *may be marking-dependent).*
- $w : \mathbb{N}^{|P|} \to \mathbb{R}^+$ *is the firing weight for* $\forall t \in T^Z$ *(note that* w *may be marking-dependent).*

The graphical representation of a place p is a circle, with the number of tokens in it is written inside (or illustrated with the corresponding number of black dots). A transition t is drawn as a box, a thin bar for the transitions in T^Z, an empty box for transitions in T^E, and a filled box for transitions in T^D. Input and output arcs end with an arrowhead on their destination, while inhibitor arcs have a small circle on the transition end. The multiplicity of an arc is written on the arc (the default value of multiplicity is one).

A transition is "enabled" in a marking if each of its input places contains the "necessary" amount of tokens where "necessary" is defined by the input functions

I and H. Formally, transition t is enabled in marking M if for all places p of the net we have $M(p) \geq I(t,p)$ and $M(p) < H(t,p)$. An enabled transition can fire and the firing removes tokens from the input places of the transition and puts tokens into the output places of the transition. The new marking M' after the firing of transition t is formally given $M'(p) = M(p) + O(t,p) - I(t,p)$, $\forall p \in P$. Note that function H does not influence $M'(p)$.

The firing of a transition in T^E occurs after a random delay, while the transitions in T^D after a deterministic delay. The random delay associated with a transition in T^E has exponential distribution whose parameter depends on the firing intensity of the transition and on the current marking. In this paper, we assume that the number of tokens in a place does not influence the firing delay. This concept is called single server model, which practically corresponds to the real operation of the manufacturing process (other server policies also exist, for details see [5]).

When a marking is entered, a random delay is chosen for all enabled transitions by sampling the associated delay distribution. The transition with the lowest delay fires and the system changes the marking.

3 The Manufacturing Process of Wooden Windows

In this section, we describe the manufacturing process of wooden windows in detail.

From the company's point of view, the wooden window manufacturing process lasts from the arriving of an order to the delivery of the accomplished windows. After an order is accepted, the company starts manufacturing the sufficient number of windows. Each single window can be considered unique, since a building typically contains windows in several various sizes and types, so the quantity production cannot be carried out in this area.

In the first part of the production of a single window, the frame members are produced separately. Typically, a wooden window consists of 8 frame members: 4-4 frame members compose the frame and the casement frame, respectively. Since the manufacturing process cannot be started in the absence of raw material, the companies try to appropriately manage the quantity of the raw material in their store. The raw material typically means 6 meter long lumber.

The first work phase is cutting the raw material to size using circular saw. After cutting to size, the frame members must be planed with a planing machine. The third work phase is molding. In this work phase, a computer numerical control (CNC) machine is applied to bring the frame members into their final form. CNC machines are fully automated and programmable, and their use for molding ensures high level of precision and speed.

After molding, the frame members are glued into a frame. From this point, the basic unit of the manufacturing is the frame. The next work phase is dipping, which is a surface treatment for protecting the wooden material from damages caused by insects and fungi. Then, the frames are sanded with a sanding machine to obtain a smooth surface. After sanding, priming and final coating give the final color of the frame and ensure further protection for the wooden material. The last work phase is fixing the hardware parts. In this phase, the frame and the casement frame is supplied with the required metal components and the glass. Finally, the frame and the casement frame is joined, and the window is ready for transportation.

The peculiarity of the manufacturing process is that material defects can be revealed in any work phases before the final coating. If a defect is revealed before gluing, the single frame member can easily be re-produced, while after gluing, the glued frame must be decomposed, and all work phases must be repeated with the frame, when the re-produced single frame member is available. In both cases, the re-production has priority over the other works.

4 An Operation Model for the Manufacturing Process of Wooden Windows

In this section, we describe the operation model of a Hungarian wooden window manufacturing company.

From modeling point of view, the events of the production can be divided into two categories, based on the distribution of the transitions modeling the given event. Accordingly, we model the orders and the occurrence of material defects with exponentially delayed transitions, while we describe the different production phases (cutting, planing, molding, gluing, dipping, sanding, priming, final coating and fixing the hardware parts) with deterministic transitions (Fig. 1). Since the number of waiting window frames and frame members do not influence the intensity of a work phase in the model, we apply the exclusive server model [5] in this work.

In this example, there are three typical types of orders:

- for specific single window,
- for all windows of a family house, a windows on the average,
- for windows of a big building, b windows on the average.

According to the types of orders, the arrival process consists of transitions *arriveSingle*, *arriveAverage* and *arriveBig* in Fig. 1. In this case study, a and b are considered 18 and 65, respectively.

After the order was accepted[1], the 8 frame members of each window must be cut to size (each frame has 4 members and each casement frame has also 4 members in most cases[2]). The cut frame members must be planed, then molded (see transitions *cut*, *plane* and *mold* in Fig. 1). Until this point is reached, the frame members are individual entities each belonging to a specific window. Therefore, if a material defect is discovered, the given single member can be re-cut, re-planed and re-molded without causing bigger delay in the process (see transitions *md1*, *md2* and *md3* in Fig. 1).

[1] In Fig. 1, we took only the accepted orders into account
[2] Having the typical case is a modeling simplification.

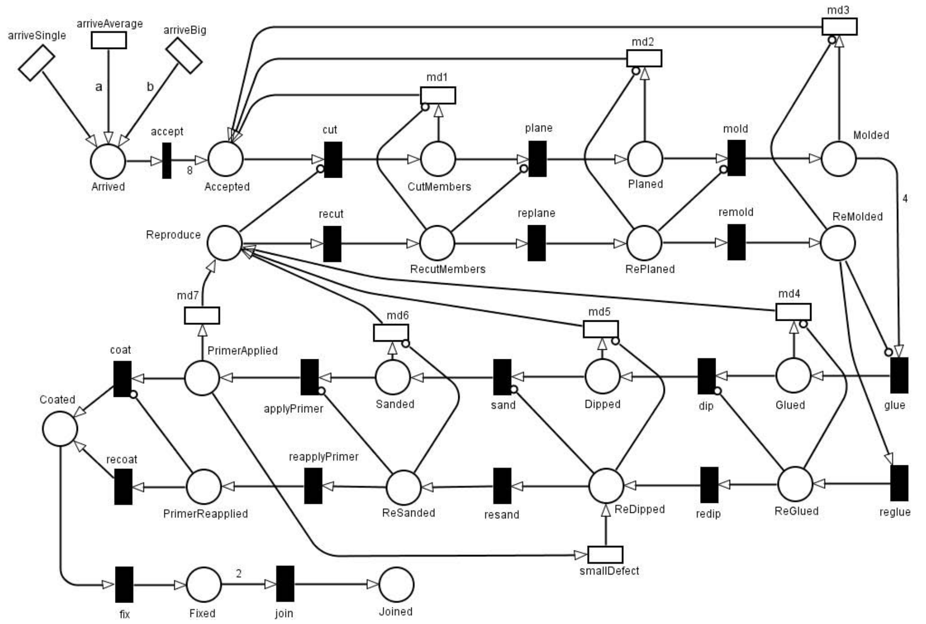

Fig. 1. The proposed operation model for the manufacturing process of wooden windows

After gluing the 4 frame members, one token denotes a frame in the model. The next operations after gluing are the dipping of the frame, then the sanding, the priming and the final coating (see transitions *glue*, *dip*, *sand*, *applyPrimer* and *coating* in Fig. 1). Note that a material defect can be discovered in *any* production phase before the final coating. However, when the members compose a frame, the delay is much bigger, since the frame must be recomposed, and the single member must be re-produced (see transitions *md4*, *md5*, *md6* and *md7* in Fig. 1). Therefore, we assume that if a material defect is discovered after the members compose a frame, further defects will not be discovered. This assumption is based on the thorough investigation, which follows when a frame must be decomposed to prevent further delay.

In our model, there is a dedicated path for this incident: since 3 frame members wait for the re-production of the fourth one, the transitions on the dedicated path have higher priority than the concurrent ones (see transitions *recut*, *replane*, *remold*, *reglue*, *redip*, *resand*, *reapplyPrimer* and *recoat*). In the DSPN model, inhibitor arcs ensure the priority. After priming, the smaller defects can be corrected by re-sanding, what is denoted by transition *smallDefect* in Fig. 1.

The last two steps of the production are fixing the hardware parts and join the casement frame and the frame (transitions *fix* and *join* in 1). The transitions with the corresponding delays are collected in Table 1[3] (in case of the exponentially delayed transitions, the expected delay is shown).

[3] The values of Table 1 were set based on a personal discussion with the representative of the company.

Table 1. The transitions of the DSPN model

Transitions	Description	Delay
$arriveSingle$	A single wooden window is ordered.	16.64
$arriveAverage$	18 wooden windows are ordered.	12.07
$arriveBig$	65 wooden windows are ordered.	1392
$md1$	A flaw is discovered after cutting.	3.125
$md2$	A flaw is discovered after planing.	15
$md3$	A flaw is discovered after molding.	50
$md4$	A flaw is discovered after gluing.	10
$md5$	A flaw is discovered after dipping.	60
$md6$	A flaw is discovered after sanding.	33.33
$md7$	A flaw is discovered after priming.	22.22
$smallDefect$	A frame is sent back to sand.	33.33
$cut/recut$	Cutting/re-cutting of a frame member.	0.0556
$plane/replane$	Planing/re-planing of a frame member.	0.0104
$mold/remold$	Molding/re-molding of a frame member.	0.05
$glue/reglue$	Gluing/re-gluing of the frame members.	0.1667
$dip/redip$	Dipping/re-dipping of a frame.	0.0333
$sand/resand$	Sanding/re-sanding of a frame.	0.2
$applyPrimer/reapplyPrimer$	Priming/re-priming of a frame.	0.1111
$coat/recoat$	Coating/re-coating of a frame.	0.1111
fix	Fixing hardware parts.	0.3333
$join$	Join the casement frame and the frame.	0.01

5 Evaluation of the Production Process

In this section, we investigate the production process using transient simulation. So, we can determine the throughput of the system for one year, and the bottlenecks of the production process. Moreover, we can investigate the effects of the material defects. However, several other aspects could be investigated, too.

For the performance evaluation of DSPNs, several tools exist, such as the ones presented in [3,6]. Since the analytical approach cannot be applied in our case, we used TimeNet [6] to obtain simulation results, which reflect 99% confidence level with 1% maximal error rating.

Table 2 shows the expected token distribution after one year. We can observe that the company can produce more than 3000 windows a year, while the bottleneck of the system is the fixing of the hardware parts, since there are about 585 frames on the *Coated* place (as a matter of fact, the employees in this session must work overtime to accomplish the orders until their deadline).

Table 2. The token distribution of the DSPN model after one year

Places	Expected number of tokens
$Accepted$	189.81933
$CutMembers + RecutMembers$	0.13467
$Planed + RePlaned$	0.63745
$Molded + ReMolded$	3.25834
$Glued + ReGlued$	0.10666
$Dipped + ReDipped$	0.70611
$Sanded + ReSanded$	0.35967
$PrimerApplied + PrimerReapplied$	0.35056
$Coated$	584.55711
$Fixed$	0.52422
$Joined$	3021.378

The bottleneck (fixing the hardware parts) is a limiting factor in the production if the company wants to increase the number of accepted orders. Fig. 2 shows that as we increase the intensity of the *arriveAverage* transition, the production is throttled by the fixing of the hardware parts work phase. In Fig. 2, we also show the case when the company detects the material defects always *before* gluing (e.g. by applying quality control) to prevent the major overhead caused by these defects. Technically, we deleted the transitions *md4*, *md5*, *md6* and *md7* from the DSPN model. We can observe that eliminating the late detection of the material defects does not affect the total throughput. However, it can obviously cause a delay of certain jobs. On the other hand, the elimination of the mentioned transitions increased the number of tokens in the bottleneck place when the system was heavily loaded.

The obvious solution for the bottleneck problem is to expand the critical production contingent (i.e. by employing more workers). Therefore, we investigated the throughput of the system as a function of the delay of transition *fix*, while the delay of transition *arriveAverage* remains on the lowest value of the previous simulation (the exact value is 7.606709638). Fig. 3 shows that decreasing the delay of transition *fix* from 0.33 hours to 0.22 hours is enough to move the

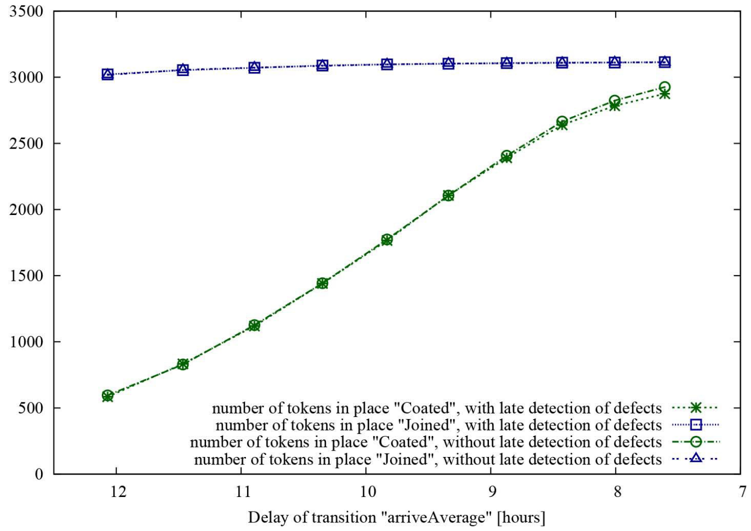

Fig. 2. The number of tokens in places “Coated” and “Joined”

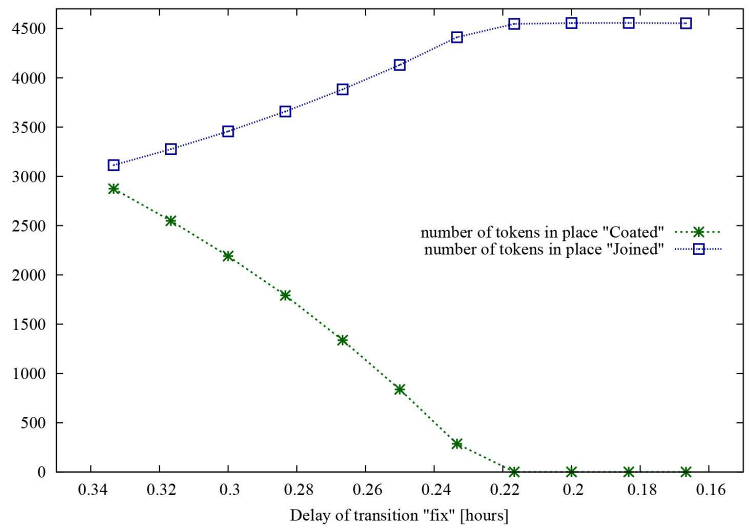

Fig. 3. The number of tokens in places “Coated” and “Joined”

bottleneck in the system, and gives the chance to increase the throughput by about 50%. In other words, the maximal throughput can be increased by 50%, if the delay of fixing the hardware parts onto one frame could be reduced from 20 minutes to 13.2 minutes.

6 Summary

In a production process, where the different work phases are deterministic processes, a DSPN is an appropriate modeling tool. In this paper, we presented the operation model of a wooden window production company. Obtaining the solution of the model, we identified the main bottleneck of the production process. Moreover, we determined the measure of improving the given work phase in order to eliminate the main bottleneck from the process. With this example, we demonstrated that the stochastic models can be applied in the wood industry.

Since our work is a pioneer one in the wood industry, several other investigations can be made. As a result, the whole manufacturing process can be optimized, what offers us a good opportunity in this area in the future.

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