

# MATH WRITEUP

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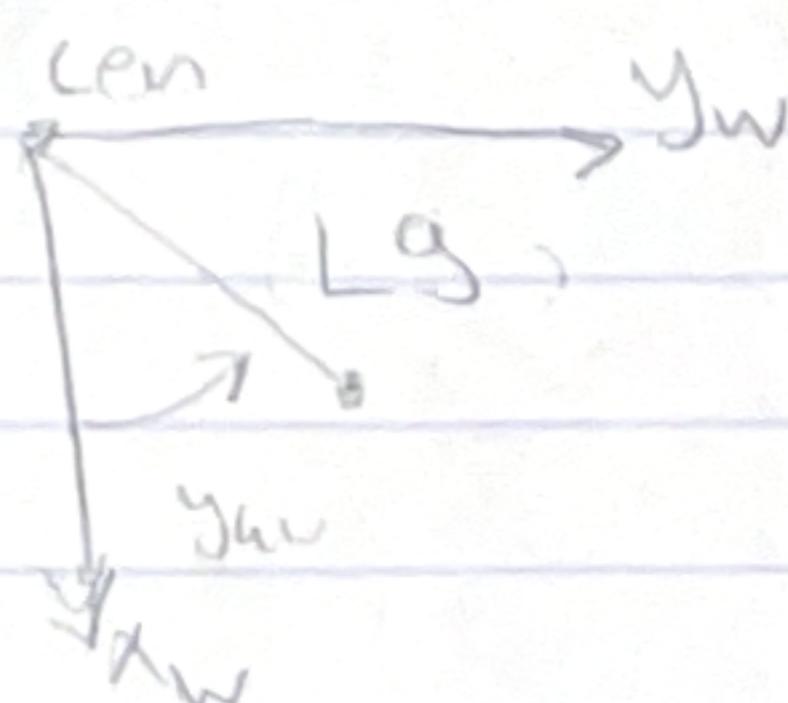
STEP 1:

$$H_w^b d^b = d^w \leftarrow \text{know this}$$

$$(H_w^b)^{-1} d^w = d^b$$

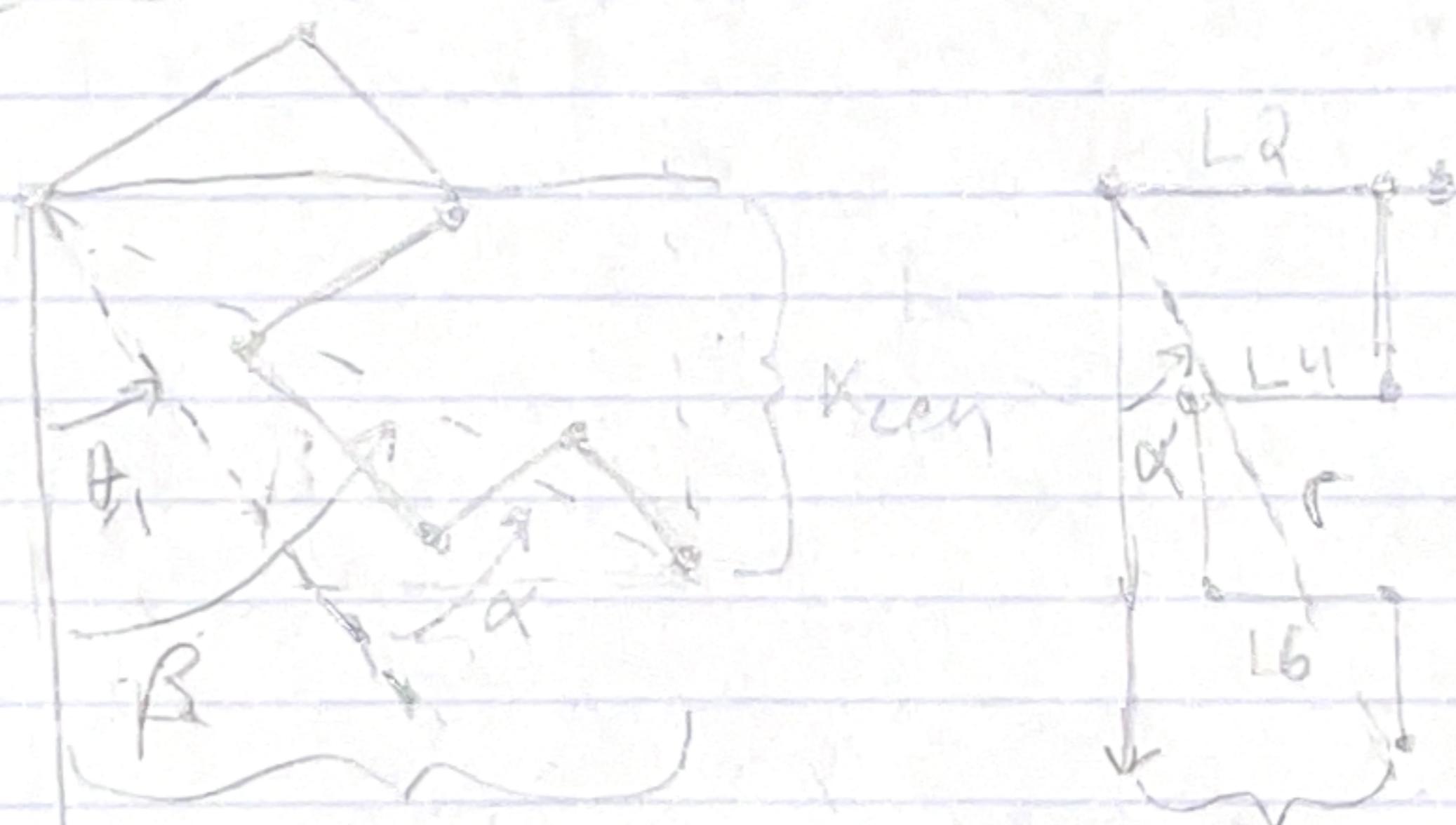
$$\text{Therefore } d^b = H_w^b + \begin{bmatrix} 0,15 \\ -0,15 \\ -0,01 \end{bmatrix}$$

STEP 2:



$$\begin{bmatrix} x_{enx} \\ x_{eny} \\ x_{enz} \end{bmatrix} = \begin{bmatrix} \text{grip}_x - L_g \cos(\text{yaw}) \\ \text{grip}_y - L_g \sin(\text{yaw}) \\ \text{grip}_z \end{bmatrix}$$

STEP 3:



$$\beta_1 = \beta - \alpha$$

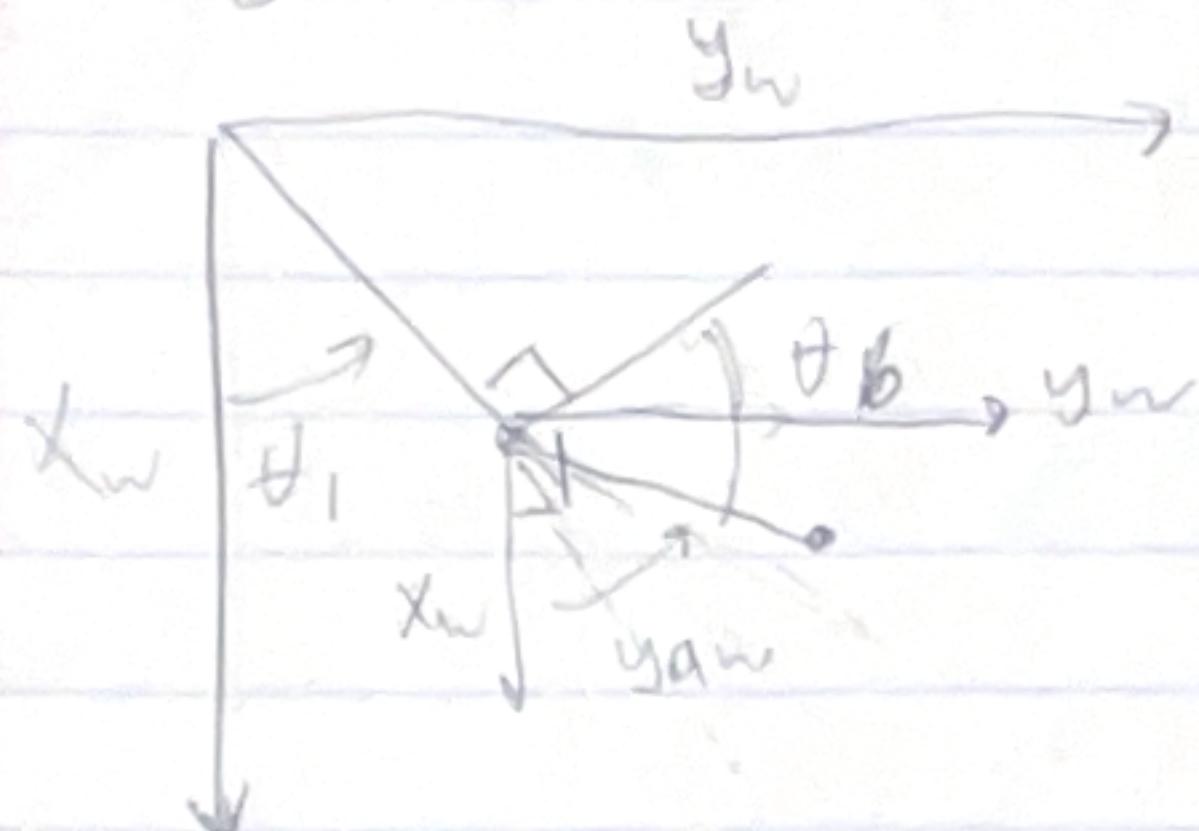
$$\beta = \text{Atan2}(x_{en}, y_{en})$$

$$dy_2 = L_2 - L_4 + L_6$$

$$\sin(\alpha) = \frac{dy}{r}$$

$$\alpha = \sin^{-1}\left(\frac{dy}{r}\right)$$

STEP 4:



$$\begin{aligned} \text{yaw} + \theta_b &= \theta_1 + \alpha_0 \\ \theta_b &= \theta_1 + \alpha_0 - \text{yaw} \end{aligned}$$

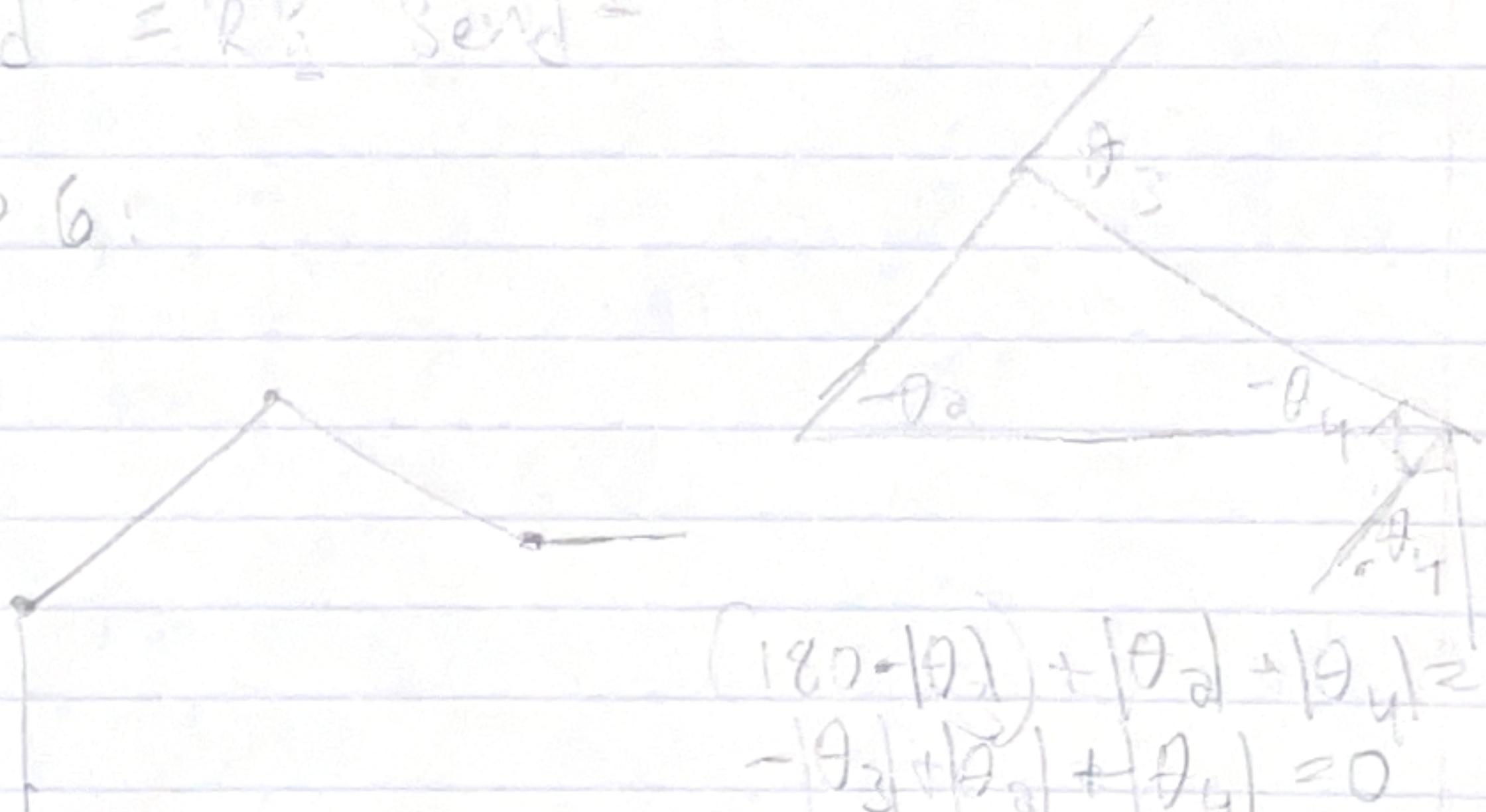
STEP 5:

$R_i^0 = R_{z, \theta_1}$ , where frame 0 is base frame  
and frame 1 is after first joint  
easier to calculate 3rd M frame 1

$$\begin{aligned} \text{3end}^1 &= \text{cen}^1 + [ \begin{matrix} 0 & 0 & 1 \\ -L_6 & 0 & 0 \\ 0 & L_8 & 0 \end{matrix} ] \\ \text{cen}^1 &= R_s^1 \text{cen} = [R_i^0]^T \text{cen} \end{aligned}$$

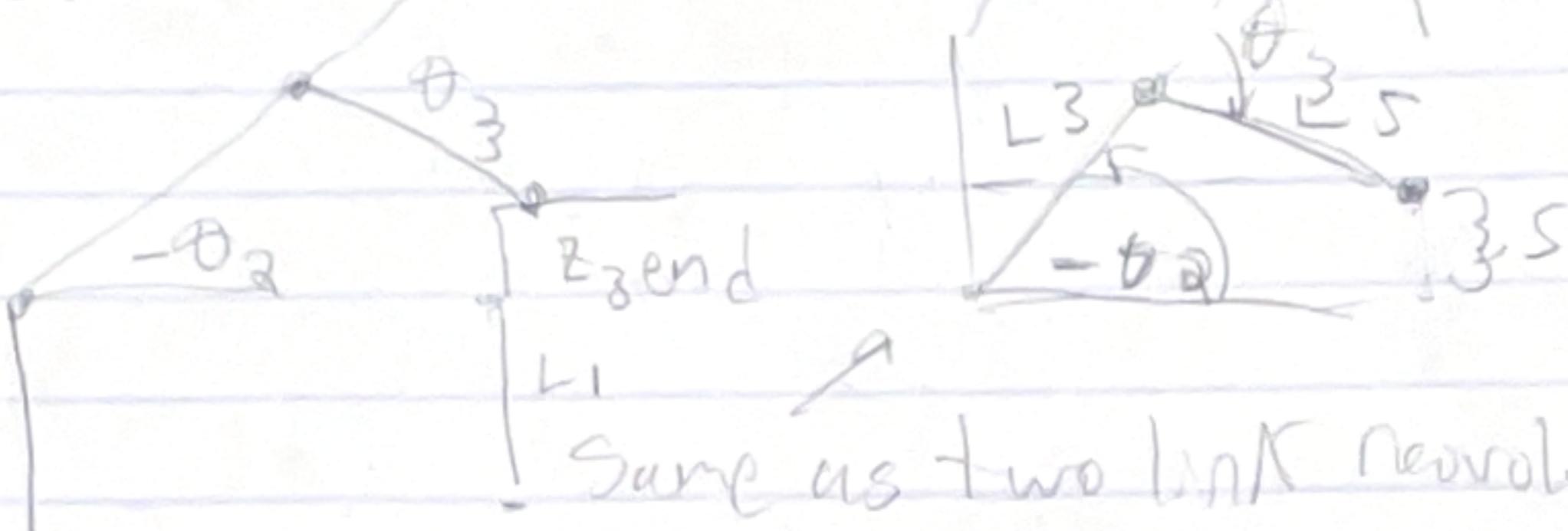
$$\text{3end}^0 = R_i^0 \text{3end}^1$$

STEP 6:



$$|\theta_4| = |\theta_3| - |\theta_2|$$

STEP 6 cont.



Same as two link revolute robot

$$r^2 = z_{\text{end}x}^2 + z_{\text{end}y}^2$$

$$s = z_{\text{end}z} - L_1$$

$$D^2 = r^2 + s^2 - L_3^2 - L_5^2$$

$$\theta_{32} = \text{atan}_2(D, \sqrt{1 - D^2})$$

Negative link elbow up solution

$$\theta_{32} = \text{atan}_2(r, s) - \text{atan}_2(L_3 + L_5 \cdot \cos(\theta_3), L_5 \cdot \sin(\theta_3))$$

however both angles need to be multiplied  
by  $-1$  to account for configuration  
of robot.