Regularization of Contractive Autoencoders using the Schatten Norm

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Abstract

Contractive autoencoders have been used in unsupervised learning to learn a useful representation of data that minimizes reconstruction error and captures manifold structure through regularization of the hidden layer by the Frobenius norm. We study a more general contractive autoencoder by replacing the Frobenius norm penalty term with the Schatten norm. The Schatten norm is a matrix norm that is equivalent to the p-norm of the singular values of a matrix with the Forbenius norm being a special case of the Schatten Norm with p=2. We study the effect of the choice of p in classification when the autoencoder is used to project data into a overcomplete feature space. We also look at the autoencoder's sensitivity to changes in the input data and learned manifold structure. Lastly, we study how classification can be improved by using the autoencoder to perform dimensionality reduction.

1 Introduction

Autoencoders are unsupervised artificial neural networks used to learn new representations of data [Hinton 1995, Hinton 2006] with applications in hand-written character recognition, image recognition [Tan 2010], novelty assessment [Thompson 2002], and data visualization [Nadjarpoorsiyahkaly 2011]. Traditionally, autoencoders have been used to learn low-dimensional codes of data but new types of autoencoders have been introduced that learn useful high-dimensional representations of data with applications in deep learning [Bengio 2009], manifold learning [Tenenbaum 2000, Roweis 2000], initialization of state-of-art classifiers [Bengio 2007], Deep Belief Networks [Hinton 2006 - Neural Comp], and Deep Boltzmann Machines [Salakhutdinov 2009]. Autoencoders of this nature focus more on learning a sparse network structure and invariance to noisy data. To encourage sparsity and weight decay, a penalty term is added to the loss function that requires the sum of the weight values to be as small as possible [Kavukcuoglu 2009, Lee 2008]. Denoising autoencoders encourage invariance by first corrupting the input with noise and then try to learn the uncorrupted form of the data [Vincent 2010].

The basic autoencoder maps data of dimension d into an h dimensional space. The autoencoder encodes data, x, using $e(x) = f_1(Wx + b_1)$ and decodes it to the original input space with $d(x) = f_2(W^T e(x) + b_2)$, where the activation functions $f_1(x)$, $f_2(x)$ take the form of the sigmoid function $\frac{1}{1+e^{-x}}$. $f_2(x)$ may also be a linear activation function depending on the application. W is a matrix of real numbers of size $h \times d$, h_1 and h_2 are bias vectors of real numbers of size h and h_2 and is trained through gradient descent while trying to minimize the objective function $h_2(W, h_1, h_2) = \sum_{i=1}^n ||x_i - h_2(x_i)||^2$. Autoencoders that encourage weight decay add the following term $h_2(x_i) = \frac{1}{2} ||x_i - h_2(x_i)||^2$. Autoencoders that encourage weight

Table 1: Results of KNN classifiers on first 1,000 samples of MNIST

CAE Encoding	Error
Schatten $p = 1$	23.50%
Schatten $p=2$	0.0%
Schatten $p = \infty$	0.0%
KNN with no encoding	27.25%

2 Contractive Autoencoders (CAE)

Contractive autoencoders introduced by Rifai et al. [Rifai 2011] are similar in spirit to autoencoders with weight decay however instead of trying to minimize the weights, the contractive autoencoder objective function is $L(W,b_1,b_2) = \sum_i^n ||x_i - d(e(x_i))||^2 + \lambda ||J_e(x_i)||_{S_p}$ where $||Y||_{S_p}$ represents the Schatten norm of Y with a value of p. Rifai et al. use the Schatten norm with p=2 also known as the Frobenius norm of the Jacobian of the nonlinear encoding function, $J_e(x_i) = \partial e(x_i)/\partial x_i$. Through this regularization, contractive autoencoders encourage a sparse representation of the data that is also locally invariant and maximizes contraction around each of the training points. The contraction is maximized in directions orthogonal to the manifold so representations change very little in these directions while parallel directions have the most change in representation.

In addition to the regularization, CAEs learn deep representations of the data by using an over-complete representation of the data. An overcomplete representation is produced by choosing more hidden nodes than dimensions allowing the autoencoder to have multiple perfect reconstructions. By also including the regularization, the autoencoder must choose reconstructions that are robust to small changes to the data with the added benefit that the CAE learns the manifold's tangent directions. It should be noted, that this regularization is not limited to autoencoders and other models could also benefit by maximizing the contraction around each data point.

3 Experiments

Training contractive autoencoders involves minimizing the $L(W, b_1, b_2)$. For these results, a gradient descent method was used to train the weights, W, b_1 , and b_2 .

We compare the performance of the CAE with varying p using the basic MNIST dataset. After training each CAE with 1,000 hidden nodes on the first 400 samples taken from the dataset, the CAE was tested and compared by using a k nearest neighbors classifier on the MNIST test data. The classifier was set up such that the trained CAE encoded each training point and each test point. For each encoded test point, the Euclidean distance was computed between it and each training point. The k closest neighbors voted on the label for the test point with ties were broken randomly using a uniform distribution. Results can be seen in Table $\ref{Table 2}$?

One way to measure the invariance of in training data is to study how the encoding changes as you vary the data. Figure ?? shows the contraction curves for each of the CAEs with 1,000 hidden nodes trained on 400 samples from the MNIST data set.

Additionally, a contraction curve can be calculated after each epoch during the CAE training.

Lastly, the average singular values for each trained CAE can be plotted as shown in Figure ??. The most sensitive directions have high singular values with values that quickly decrease due to the contractive nature of the autoencoder.

4 Conclusions

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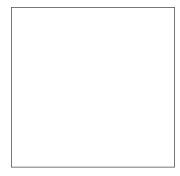


Figure 1: Sample figure caption.

Table 2: Sample table title

PART	DESCRIPTION
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- [1] Alexander, J.A. & Mozer, M.C. (1995) Template-based algorithms for connectionist rule extraction. In G. Tesauro, D. S. Touretzky and T.K. Leen (eds.), *Advances in Neural Information Processing Systems* 7, pp. 609-616. Cambridge, MA: MIT Press.
- [2] Bower, J.M. & Beeman, D. (1995) *The Book of GENESIS: Exploring Realistic Neural Models with the GEneral NEural SImulation System.* New York: TELOS/Springer-Verlag.
- [3] Hasselmo, M.E., Schnell, E. & Barkai, E. (1995) Dynamics of learning and recall at excitatory recurrent synapses and cholinergic modulation in rat hippocampal region CA3. *Journal of Neuroscience* **15**(7):5249-5262.