Regularization of Contractive Autoencoders using the Schatten Norm

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Abstract

Contractive autoencoders have been used in unsupervised learning to learn a useful representation of data that minimizes reconstruction error and captures manifold structure through regularization by the Frobenius norm. We study a more general contractive autoencoder by replacing the Frobenius norm with the Schatten norm. The Schatten norm is a matrix norm that is equivalent to the p-norm of the singular values of a matrix while the Forbenius norm is a special case of the Schatten Norm with p = 2. We show that using this penalty with p = 1 results in greater contraction in the hidden layer of the autoencoder but tends to produce less accurate reconstruction.

1 Introduction

Algorithms that learn useful low-dimensional representations of high-dimensional data automatically have recently garnered attention because of their application in deep learning [Bengio 2009], manifold learning [Tenenbaum 2000, Roweis 2000], initialization of state-of-art classifiers [Bengio 2007], Deep Belief Networks [Hinton 2006 - Neural Comp], and Deep Boltzmann Machines [Salakhutdinov 2009].

Autoencoders are multilayer neural networks that can be used to learn low-dimensional codes of data [Hinton 1995, Hinton 2006]. Autoencoder applications include hand-written character and image recognition [Tan 2010], novelty assessment [Thompson 2002], and data visualization [Nadjarpoorsiyahkaly 2011]. Other types of autoencoders have been introduced that focus on smaller weights and robustness to noise. To encourage smaller weights and weight decay, a penalty term is added to the loss function that requires the sum of the weight values to be as small as possible [Kavukcuoglu 2009, Lee 2008]. Denoising autoencoders encourage invariance by first corrupting the input with noise and then trying to learn the uncorrupted form [Vincent 2010].

The traditional autoencoder with h hidden nodes trained on n points with d dimensions has the following encoding function, $e(x) = f_1(Wx+b_1)$ and decoding function, $d(x) = f_2(W^T e(x)+b_2)$, where the activation functions $f_1(x)$, $f_2(x)$ take the form of the sigmoid function $\frac{1}{1+e^{-x}}$ in this study. $f_2(x)$ may also be a linear activation function depending on the application. W is a matrix of real numbers of size h x d, b_1 and b_2 are bias vectors of real numbers of size h and d, respectively.

2 Contractive Autoencoders (CAE)

Contractive autoencoders introduced by Rifai et al. [Rifai 2011] are similar in spirit to autoencoders with weight decay. The contractive autoencoder objective function is $L(W, b_1, b_2) = \sum_i^n ||x_i - d(e(x_i))||^2 + \lambda||J_e(x_i)||_{S_p}$ where $||Y||_{S_p}$ represents the Schatten norm of Y with a value

Table 1: Results of KNN classifiers on first 1,000 samples of MNIST

CAE Encoding	**Error**
Schatten $p = 1$	23.50%
Schatten $p = 2$	0.0%
Schatten $p = \infty$	0.0%
KNN with no encoding	27.25%

of p. Rifai et al. use the Schatten norm with $p = 2$ also known as the Frobenius norm of the Jacobian of the nonlinear encoding function: $J_e(x_i) = \partial e(x_i)/\partial x_i$. Through this regularization, contractive autoencoders encourage a sparse represenation of the data that is also locally invariant. The goal of CAEs is to minimize the reconstruction error as well as the contraction around each of the training points. The contraction is maximized in directions orthogonal to the manifold so representations change very little in these directions while parallel directions have the most change in representation.

CAEs learn deep representations of the data that are also robust to variations in the data by using an overcomplete representation and including the Jacobian penalty term. An overcomplete representation can be illustrated by choosing more hidden nodes than dimensions allowing an autoencoder multiple complete and perfect reconstructions. In a CAE, the additional Jacobian penalty term chooses reconstructions that are robust to small changes to the data and learns the manifold's tangent directions. It is through this overcomplete representation that a deeper understanding of the data can be found than in a traditional autoencoder. It should also be noted, that the jacobian penalty term is not limited to autoencoders and other models could benefit by minimizing the contractive penalty term.

2.1 Schatten Norm

3 Experiments

Training contractive autoencoders involves minimizing the $L(W, b_1, b_2)$. For these results, a gradient descent method was used to train the weights, W, b_1, and b_2.

We compare the performance of the CAE with varying p using the basic MNIST dataset. After training each CAE with 1,000 hidden nodes on the first 400 samples taken from the dataset, the CAE was tested and compared by using a k nearest neighbors classifier on the MNIST test data. The classifier was set up such that the trained CAE encoded each training point and each test point. For each encoded test point, the Euclidean distance was computed between it and each training point. The k closest neighbors voted on the label for the test point with ties were broken randomly using a uniform distribution. Results can be seen in Table **??**.

One way to measure the invariance of in training data is to study how the encoding changes as you vary the data. Figure **??** shows the contraction curves for each of the CAEs with 1,000 hidden nodes trained on 400 samples from the MNIST data set.

4 Conclusions

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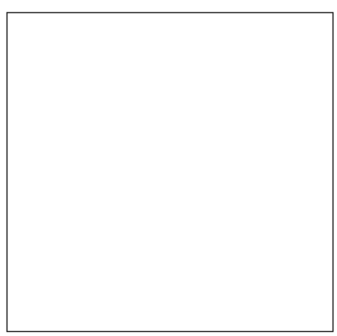

Figure 1: Sample figure caption.

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[1]Sample of the first footnote

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Table 2: Sample table title

PART	DESCRIPTION
Dendrite	Input terminal
Axon	Output terminal
Soma	Cell body (contains cell nucleus)

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Acknowledgments

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[1] Alexander, J.A. & Mozer, M.C. (1995) Template-based algorithms for connectionist rule extraction. In G. Tesauro, D. S. Touretzky and T.K. Leen (eds.), *Advances in Neural Information Processing Systems 7*, pp. 609-616. Cambridge, MA: MIT Press.

[2] Bower, J.M. & Beeman, D. (1995) *The Book of GENESIS: Exploring Realistic Neural Models with the GEneral NEural SImulation System.* New York: TELOS/Springer-Verlag.

[3] Hasselmo, M.E., Schnell, E. & Barkai, E. (1995) Dynamics of learning and recall at excitatory recurrent synapses and cholinergic modulation in rat hippocampal region CA3. *Journal of Neuroscience* **15**(7):5249-5262.