MATH/COMP 562 ASSIGNMENT 2
TODO DUE: FEB 2ND (THURSDAY)
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Instructions. Submit your solutions on MyCourses course page. You can get help from other students, but you should do the write up yourself. Code solutions can be a PDF of a workbook, you can use any programming language, but NumPy is recommended.

Rademacher Complexity.

Exercise 1.1 (Rademacher complexity of linear hypothesis). Prove the following theorem.

Theorem 1.1 (Theorem 5.10 of Mohri). *Define* $B_r = \{x \in \mathbb{R}^d \mid \|x\| \leq r\}$. *Let* $S = \{x_1, \dots x_m\} \subset X \subset \mathbb{R}^d$ *with* $X = B_r$. *Consider the linear hypotheses* $h(w, x) = w \cdot x$ *and set* $\mathcal{H} = \{h(x, w) = w \cdot x \mid x \in X, w \in B_\lambda\}$. *Prove that the empirical Rademacher complexity is bounded as follows,*

$$\widehat{\mathfrak{R}}_S(\mathcal{H}) \leq \frac{r\lambda}{\sqrt{m}}$$

Hint: refer to class notes, Mohri textbook Theorem 5.10.

Exercise 1.2. Given a hypothesis class $\mathcal{H}$, of functions $h : X \to \mathbb{R}$, and a dataset $S = \{x_1, \dots, x_m\} \subset X$. Define $\Phi(S) = \sup_{h\in\mathcal{H}} \left(\mathbb{E}[h] - \widehat{\mathbb{E}}_S[h]\right)$ to be the least upper bound for the generalization gap of a function in $\mathcal{H}$. Prove that $\mathbb{E}_S[\Phi(S)] \leq 2\mathfrak{R}_m(\mathcal{H})$

Exercise 1.3 (Rademacher Identities). Mohri 3.8(a) and 3.8(b).

Convex Learning Problems. Refer to Ch 12 of Understanding Machine Learning (Shalev-Shwartz).

Exercise 1.4 (non convexity of 0-1 loss). Problem 12.1. Hint: Consider samples in a checkerboard pattern.

Exercise 1.5 (Convexity, Lipschitz, and Smoothness of logistic regression loss.). Problem 12.2.

Exercise 1.6 (Lipschitz continuity of the hinge loss). Problem 12.3.

Coding exercises on Rademacher Complexity. Sample Code for exercises (from Viet). Instructions: clone a copy into you own Google Drive using "File – Save a Copy in Drive" `https://colab.research.google.com/drive/1S2IcFWPExpRgSLT0An9Uz3KStKov2x` `usp=sharing`