# MATH/COMP 562 ASSIGNMENT 5
# NOT DUE
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**Instructions.** Submit your solutions on MyCourses course page. You can get help from other students, but you should do the write up yourself. Code solutions can be a PDF of a workbook, you can use any programming language, but NumPy is recommended.

5.1. **Solving the Bellman equation.**

*Exercise* 5.1 (Grid world from AI book). Consider the grid world problem `https://aimacode.github.io/aima-exercises/figures/sequential-decision-world-figure.png` along with the solutions for different values of the rewards. `https://aimacode.github.io/aima-exercises/figures/sequential-decision-policies-figure.png` Verify that the solutions in the figure satisfy the Bellman equation.

5.2. **Contraction mappings in the maximum norm.**

*Exercise* 5.2. Let $T : \mathbb{R}^d \to \mathbb{R}^d$ be a strict $\gamma$-contraction, with fixed point $x^*$. Given $x^0$, define $x^{n+1} = T(x^n)$. Prove that $\|x^n - x^*\| \leq \gamma^n \|x^0 - x^*\|$.

*Exercise* 5.3. Suppose $g : \mathbb{R}^d \to \mathbb{R}$ is a $\gamma$-contraction, and $g$ is monotone increasing. Prove that $g$ satisfies

$$\gamma \min_i (x_i - y_i, 0)\mathbf{1} \leq g(x) - g(y)$$

*Exercise* 5.4. Let $w_1, w_2 \in \mathbb{R}^d$. Suppose $w_1, w_2 \geq 0$ and $\sum_{i=1}^d (w_1)_i = \sum_{i=1}^d (w_2)_i = \gamma < 1$.

(a) Prove that $\gamma \min_i x_i \leq w_1 \cdot x \leq \gamma \max_i x_i$
(b) Let $r_1, r_2 \in R$. Prove that $\max(w_1 x + r_1, w_2 x + r_2)$ is a contraction in the max norm.
(c) Let $W$ be a $d \times d$, matrix, with nonnegative coefficients. Let $\gamma < 1$. Suppose that $\sum_{i=1}^d w_{ji} \leq \gamma$ for each $j = 1, \ldots, d$. Let $r \in \mathbb{R}^d$. Define $g(x) = Wx + r$. Prove that $g$ is a $\gamma$ contraction, and conclude that $g$ has a unique fixed point.