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Research Statement

Over the course of my academic career I have had the good fortune to study a wide variety of mathematical subjects. I have always found myself drawn to operations research (OR) and mathematical modeling due to the sheer variety of problems from science and engineering that they may be used to study. OR in particular is home to an endless supply of interesting practical problems related to planning and analyzing large-scale infrastructure networks. It is a field rich in real-world applications and is extremely important in an increasingly urbanized and digitized world. In addition to the theoretical challenges posed by classic problems from OR, many are also computationally difficult to solve, resulting in a need for efficient solution algorithms. Given my background in programming and the algorithmic focus of my previous research, these are exactly the sorts of puzzles that I find interesting to work on.

The following sections will describe a selection of topics that I would be interested in researching. A number of common threads run through them, including mathematical models from civil infrastructure network optimization, which was the focus of my doctoral research [18, 19, 27]. All of these topics also involve the study of networks, including some problems from pure graph theory and mathematical biology, the latter of which was the subject of my undergraduate research [2].

Transportation Network Design

Transportation network design problems include a large family of network optimization models related to road network and public transit design. Common applications include planning road construction, lane orientation, traffic signals, public transit routes, and public transit scheduling. Although the specific models used for each application differ significantly, they share a number of similarities that make them computationally expensive to solve. This includes a tendency to contain a large number of variables and constraints due to the size of the network, a large number of integer variables due to scheduling or decision constraints, and a bilevel structure due to the need to consider user behavior separately from operator decisions. For these reasons exact solution algorithms are generally not possible or computationally tractable, and so it is commonplace to instead aim for a near-optimal solution through use of metaheuristics such as tabu search, simulated annealing, and genetic algorithms (see Guihaire and Hao 2008 [17] and Farahani et al. 2013 [13] for reviews of the subject). Part of my doctoral research required the development of exactly such a solution algorithm for a public design problem [27].

The bilevel aspect of transportation design makes these problems extremely unique in the field of network design. The upper level consists of the designer's objective and constraints, which generally involve a combination of budgetary constraints and a minimization of operator and user costs, while the lower level consists of a user assignment model to determine the traffic flows that result from the design decisions. In the case of public transit design this is generally described using a strategy-based model wherein each user selects a travel strategy that minimizes their expected travel time given the strategies of all other users. This model was introduced in Spiess and Florian 1989 [32] but has been modified extensively in the decades since (see Fu et al. 2012 [15] for an overview). In particular it has proven difficult to develop a public transit user assignment model that allows for strict enforcement of capacity restrictions. Recent attempts include the use of variational inequalities by Marcotte et al. 2004 [24] and fail-to-board probabilities by Schmöcker et al. 2008 [30], but these models are also prohibitively expensive to solve and must generally be solved many times over during the course of a design solution algorithm. For this reason there is plenty of room for the development of new transit assignment models and efficient solution methods for them.

It is also important to note that the objectives generally considered in transportation network design problems have thus far been fairly limited [13]. Common objectives include various combinations of minimizing user travel time, user travel distance, user vehicle transfers, unsatisfied user demand, vehicle operation costs, vehicle travel distance, vehicle load, construction costs, and other operator and user costs. While these are certainly important goals, other objectives important for civil society have gone largely ignored. For example, environmental objectives are becoming an increasingly relevant topic, and part of my work with H. Kaul [27] focused on achieving social equity while still respecting user and operator costs. I would like to extend this approach to include other social and environmental objectives of interest.

Interdependent Networks

The subject of network interdependence has become increasingly important in recent years due to the high degree of interconnectedness of civil infrastructure systems. A relatively straightforward model for network interdependence is *input dependence*, wherein components of one network require delivery of material from other networks in order to function. For example, a subway train in the transportation network requires delivery of power from the electrical power network in order to function. These types of interdependencies introduce the possibility of *cascading failures*, wherein damage to one system can cause loss of functionality that extends far beyond the initial damage. A particularly well-studied example of this is the phenomenon of rolling blackouts within the electrical power grid, for example in Kinney et al. 2005 [22].

There are many interesting questions that can be asked about interdependent networks. A large portion of the research into the subject focuses on qualitative descriptions of how different types of network topology interact with each other as cascading failures proliferate, for example Gao et al. 2012 [16] which uses percolation theory to study the phenomenon. However, a natural application is to bring network interdependencies into existing types of network model to see how the presence of such interdependence affects their behavior (see Ouyang 2014 [25] for an overview of some such studies).

Cavdaroglu et al. 2013 [9] studied the problem of scheduling repairs to restore a damaged system of interdependent networks using a network flows framework. The presence of interdependencies significantly complicates the restoration process by changing the relative importance of certain components, as a component cannot become fully functional until its input is also functional. This study served as the motivation for part of my work with H. Kaul [18] in which we applied an input dependence model to the minimum-cost network flows problem and developed a generalized network simplex algorithm for its solution. We also used this model as part of a larger trilevel program [19] to examine its effects on the resulting fortification design decisions and solution process. I would like to continue to develop such models for use in different generalized problems from OR and civil infrastructure design, in particular focusing on efficient solution algorithms for them.

One interdependent network modeling paradigm of particular interest is *multilayer network design*, a generalization of the standard network design problem. As with standard network design, the goal is to select arcs to construct and to set the flows through the arcs with the objective of minimizing design and flow costs, but rather than having a single network there are several interdependent layers. A natural application for this model is a telecommunications network, for example in Dahl et al. 1999 [11], which used a virtual network layer to represent routing and a physical network layer to represent the physical components. The specific nature of the interdependencies can vary widely by application and may include combinations of flow and design variables from the different layers. Multilayer network design is a relatively recent area of OR, and there are still a lot of open questions in the field. In their 2019 literature review [20], Kazemzadeh et al. noted that almost all existing studies are limited to only two network layers, that almost no studies consider both design and flow constraints in the same program, and that there is still a lot of work to be done in developing efficient solution algorithms. These problems are directly related to my doctoral research and are of great interest to me.

Network Interdiction Games

In order to more effectively design civil infrastructure networks that are resilient to damage, a common modeling technique involves optimizing the performance of the network after some damage has occurred. For modeling natural disasters this may be done through use of stochastic programming, as in Döyen et al. 2011 [12]. This type of model corresponds to optimizing the network for what is in some sense the "average case" damage. In other cases, for example modeling a targeted attack, it may be of more interest to optimize the network for a "worst case" damage scenario, and this is accomplished through use of a *network interdiction game* (see Smith and Lim 2008 [31] for an overview). This type of model was the subject of part of my doctoral research [19].

A network interdiction game is a Stackelberg game in which two players, the *attacker* and the *defender*, take turns modifying a network within specified bounds, for example by reinforcing or destroying a limited number of arcs. The objective is the final performance of the network, which might take the form of a shortest path length, a maximum flow, or a minimum-cost flow. The goal of the defender is to minimize the cost of the final network while the goal of the attacker is to maximize it. Each player has complete knowledge of what has occurred so far, but must consider the future actions of themself and their opponent when selecting their move. Similar modeling paradigms exist for other common OR design problems (see, for example, Zheng and Albert 2018 [36] which studied a facility interdiction game).

Network interdiction games take the form of multilevel programs, and because even general bilevel programs are NP-hard [10], network interdiction games can be computationally difficult as well. Several recent studies have utilized a trilevel (min-max-min) framework for modeling the three stages of network design, network damage, and network recovery. For example, Sarhadi et al. 2015 [29] and Sadeghi et al. 2017 [28] used binary and fractional models for protection and damage, respectively, to fortify a minimum-cost flow network against an attack, and part of my work with H. Kaul [19] involved formulating and solving a similar network interdiction game in the case of interdependent networks. I am interested in exploring interdiction games based on different types of modified OR models, expanding on my previous work and the solution techniques involved to tackle different types of robust design problem.

Vehicle Routing Problems

The *vehicle routing problem* (VRP) is a fundamental combinatorial optimization problem in OR that generalizes the well-known *traveling salesman problem* (TSP). As with the TSP, the goal of the VRP is to find a minimum-cost tour of a given network, however in the VRP there is more than one available vehicle and thus the tour can be composed of more than one circuit. Like the TSP, finding an optimal set of tours for the VRP is NP-hard, and the additional layers of complexity that can be added from more advanced modeling techniques make the problem extremely computationally difficult to solve.

The VRP has long been a staple of OR, and has huge economic significance due to its use in large-scale transportation systems. Much of the recent work on the VRP involves modifying the basic model to incorporate recent developments and interests in transportation engineering (see Vidal et al. 2020 [33] for a review of such work). For example, Bruglieri et al. 2019 [7] considers the routing of alternative fuel vehicles in order to incorporate an environmental objective, while Wang and Sheu 2019 [34] considers package delivery by trucks assisted by delivery drones. There are many more fascinating developments that could be incorporated into the classical VRP, and I would like to build on my public transit design work with H. Kaul [19] to include environmental and social equity objectives which have until recently gone largely ignored in the literature.

Graph Decomposition

Given a graph G and a subgraph H, an *H-decomposition* of G consists of a partition of the edge set of G into isomorphic copies of H. Graph decompositions have important applications in combinatorial design. A well-known example is the problem of finding a Steiner triple system of order n, which is equivalent to finding a triangle decomposition of K_n.

Fractional graph decomposition is a natural fractional generalization of classical (binary) graph decomposition. Rather than partitioning the edge set of G, we instead assign to each edge a weight between 0 and 1 associated with each copy of H. This has the effect of allowing some of the copies of H to "overlap", making it possible to generate fractional decompositions for which no analogous binary decomposition exists. For example, C_5 cannot be decomposed into copies of P_2, but it *can* be *fractionally* decomposed by assigning a weight of $\frac{1}{2}$ to each of the five instances of the P_2 subgraph. For this reason fractional decomposition can be a useful tool for studying binary decompositions, since the existence of a fractional H-decomposition is necessary for the existence of a binary one. The graph decomposition problem can be expressed as an integer program, using binary variables to define which edges of G are assigned to which copy of H and with constraints to ensure the validity of the decomposition. The linear relaxation of this problem is exactly the fractional H-decomposition problem, and because it is significantly easier to solve (though still difficult due to the large number of constraints), it provides a more attractive avenue of attack for many graph decomposition problems.

Much of the recent work in this field focuses on finding necessary conditions for the existence of particular types of fractional decomposition, for example Barber et al. 2017 [3] which provides minimum degree requirements for the existence of fractional K_r decompositions. Of particular interest to me is the approach used in Bowditch and Dukes 2019 [5], which makes use of the linear program formulation of the fractional triangle decomposition problem (specifically by characterizing geometric features of the cone of feasible solutions). This is similar to my work with H. Kaul on the linear relaxation of the binary network interdependence model [18]. I am interested in exploring linear relaxations of binary graph problems and then applying concepts from linear algebra and linear optimization to characterize their solutions. These techniques can potentially be applied to many open problems in the field of graph decomposition.

Graph Games

Similar to the network interdiction games discussed above, a variety of classical problems from graph theory can be modified into sequential games. There are several fundamental problems in graph theory used to model various notions of "coverage" (such as edge cover or dominating set) or "conflict avoidance" (such as independent set or proper coloring), and both of these have applications in common OR problems like facility location and scheduling problems. The game versions of these problems can be used to study worst-case behavior and to help with certain types of robust design problem. They can also help to model dynamic situations for which decisions must be made over time as more information is revealed.

A large number of *pursuit-evasion games*, such as *cops and robbers* (see, for example, Bonato et al. 2017 [4]), are played on graphs. In such games, the *robber* player and the *cop* player take turns moving their piece (or pieces) between vertices of a given graph, with the cops having the goal of catching the robber in as few moves as possible and the robber having the goal of evading capture for as long as possible. There are also various modifications of graph coloring problems, such as the *slow coloring game* (see, for example, Mahoney et al. 2018 [23] and Puleo and West 2019 [26]), in which the *lister* and *painter* players take turns specifying a nonempty subset of vertices and choosing a proper coloring of those vertices, respectively. The score of the slow coloring game is the total size of all of the lister's sets chosen before the graph becomes fully colored, and the goal of the lister is to maximize this score while the goal of the painter is to minimize it. In general the colorings resulting from coloring games are suboptimal as compared to the chromatic number due to the additional restrictions imposed by the second player. For both of these problems theoretical bounds exist regarding particular classes of graph, but there are many possible generalizations that could be explored. There has also been relatively little work done on developing solution algorithms for particular instances of the games. I would like to extend the techniques used in my previous work on network interdiction games [19] to examine these types of questions for other types of graph game.

Network Epidemiology

One of the most important areas of mathematical biology is epidemiology, the study of how diseases spread. A particularly common type of epidemiological model is the compartmental model, wherein a population is divided into several discrete categories whose interactions are modeled using systems of differential or difference equations. For example, in the susceptible-infectious-recovered (SIR) model [21], any given member of the population can be either susceptible to the disease, a carrier capable of spreading the disease, or recovered and therefore immune from the disease. Countless variants of this model exist that include different compartments and different interactions appropriate for modeling different diseases.

A more recent refinement of compartmental models treats individuals as nodes in a network rather than simply counting the totals in each compartment. This allows each individual to contact (and therefore potentially infect) only their neighbors in the contact network rather than the entire population. In the special case where this relationship graph can be arranged in a regular grid, the model can be thought of as a cellular automaton. For more general networks, most existing studies develop theoretical results for a particular class of random graph (such as Erdős-Rényi or scale-free) rather than a specific network (see Brauer 2008 [6] for an overview).

To give just a few examples of recent work in this field, Wu et al. 2013 [35] studied the dynamics of multiple diseases coexisting in a single population, Fotouhi and Shirkoohi 2016 [14] studied a temporal network that grows over time as the infection evolves, and Burch et al. 2017 [8] studied the problem of estimating the basic reproductive number (R_0) of a disease in a stochastic network epidemiological model. A particularly unique and interesting study was conducted by Al Marzooq et al. 2018 [1], which considers the phenomenon of bacteria spreading through a water supply network in the form of biofilm, and incorporates aspects of epidemiology and fluid dynamics. Due to the large number of potential formulations for both the network and the disease, each of which can lead to completely different dynamics, there is still a great amount of work to be done in this field, and my research background in both networks [18, 19, 27] and epidemiology [2] places me in a good position to study this type of model.

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