APPM 4515-001: Homework #8

Due on November 25, 2020 at 11:59pm

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Problem 1

$$y' = \lambda y$$
$$k_1 = h\lambda w_i$$
$$k_2 = h\lambda\left(w_i + \frac{h\lambda w_i}{2}\right) = h\lambda w_i + \frac{h^2\lambda^2 w_i}{2}$$
$$k_3 = h\lambda\left(wi + \frac{h\lambda w_i + \frac{h^2\lambda^2 w_i}{2}}{2}\right) = h\lambda w_i + \frac{h^2\lambda^2 w_i}{2} + \frac{h^3\lambda^3 w_i}{4}$$
$$k_4 = h\lambda\left(h\lambda w_i + \frac{h^2\lambda^2 w_i}{2} + \frac{h^3\lambda^3 w_i}{4}\right)$$

$$
\begin{aligned}
w_{i+1} &= w_i + \frac{1}{6}\left[h\lambda w_i + 2h\lambda w_i + \frac{2h^2\lambda^2 w_i}{2} + 2h\lambda w_i + \frac{2h^2\lambda^2 w_i}{2} + \frac{2h^3\lambda^3 w_i}{4} + h\lambda w_i + h^2\lambda^2 w_i + \frac{h^3\lambda^3 w_i}{2} + \frac{h^4\lambda^4 w_i}{4}\right] \\
&= w_i + \frac{w_i}{6}\left[h\lambda + 2h\lambda + h^2\lambda^2 + 2h\lambda + h^2\lambda^2 + \frac{h^3\lambda^3}{2} + h\lambda + h^2\lambda^2 + \frac{h^3\lambda^3}{2} + \frac{h^4\lambda^4}{4}\right] \\
&= \left[1 + h\lambda + \frac{h^2\lambda^2}{2} + \frac{h^3\lambda^3}{6} + \frac{h^4\lambda^4}{24}\right]w_i
\end{aligned}
$$

$$y'' + 4y' + 13y = 0$$
$$\text{ansatz } y = e^{rx}$$
$$r^2 + 4r + 13 = 0$$
$$r = -2 \pm 3i$$
$$\implies y(x) = C_1 e^{2x}\cos 3x + C_2 e^{-2x}\sin 3x$$

$$
\begin{aligned}
|Q(h\lambda)| &= \left|1 - 2\left(\frac{3}{4}\right) + \frac{1}{2}\left(-2\frac{3}{4}\right)^2 + \frac{1}{6}\left(-2\frac{3}{4}\right)^3 + \frac{1}{24}\left(-2\frac{3}{4}\right)^4\right| \\
&= \left|\frac{35}{128}\right| < 1
\end{aligned}
$$