Zeros of Polynomials and Muller's Method

Sunday, September 13, 2020 9:05 AM

Somewhat specialized (doesn't give insight into other problems)

Use specialized algorithms (like roots or numpy.roots)

Basic idea: evaluate polynomial via Horner's Method as we already discussed

Apply Newton, $f(x)$ and $f'(x)$ both polynomials

Issue: complex roots

$f(x) = x^2+1$ has no "roots", ie, it has no real root.

But if we want to know the complex roots?

- Could start w/ $x_0 \in \mathbb{C}$ (complex) and use complex arithmetic

- or... Müller's Method (1956)

which reduces it to a sequence of quadratic problems

for which we can use the quadratic formula

Refresher: Polynomials

"Fundamental Theorem of Algebra"

(1) Every polynomial of degree ≥ 1 has at least 1 (possibly complex) root

(2) (Corollary) A n^{th} degree polynomial has n (possibly complex) roots

if you count with multiplicity *

* Except 0 polynomial has ∞ roots

Def "0 polynomial" is $0(x) = 0$ ie., all coefficients are zero.

⚠ zero of a polynomial $\neq$ the zero polynomial

Ex: $f(x) = (x-1)^2(x-4)$ has "3" roots: $\{1, 1, 4\}$

if we count w/ multiplicity.

In particular, $f(x) = a_n(x-x_1)^{m_1} \cdot (x-x_2)^{m_2} \cdot \ldots \cdot (x-x_k)^{m_k}$

m_i unique, $\sum_{i=1}^{k} m_i = n$

Corollary (2.18)

If $P(x)$ and $Q(x)$ are polynomials, both of degree n or less, then if we have a set of k distinct points $\{x_1, x_2, \ldots x_k\}$,

then if

$$P(x_i) = Q(x_i) \quad \forall\, i=1,\ldots,k$$

then $k > n \Rightarrow P = Q$.

Proof: If $k > n$, this means $P-Q$ is a n degree polynomial with more than n roots, which is impossible unless $P-Q=0$. $\square$