Homework 7

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MATH 4650

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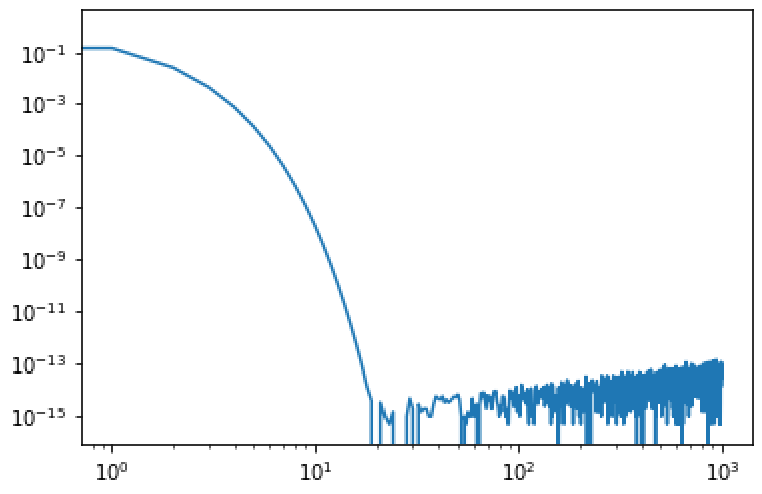
1)

In [6]:
```
from scipy.special import roots_laguerre
import numpy as np
```

a)

In [242]:
```
import numpy as np
import math
import matplotlib.pyplot as plt


def f(x):
    return 1/(1+x**2)

def integrate(deg):
    sum = 0
    x, w = np.polynomial.legendre.leggauss(deg)
    for i in range(deg):
        sum += w[i]*f(x[i])
    return sum;

N = 1000
I = []
err = []
for i in range(1,N):
    q = 2*integrate(i)
    er = abs(q-np.pi)
    I.append(q)
    err.append(er)
```

```
In [243]: plt.loglog(err)

Out[243]: [<matplotlib.lines.Line2D at 0x114eaa240>]
```



```
In [248]: print('The min error is ', min(err), 'and occurs when d = ', err.index(m
          in(err)))

          The min error is  0.0 and occurs when d =  20
```

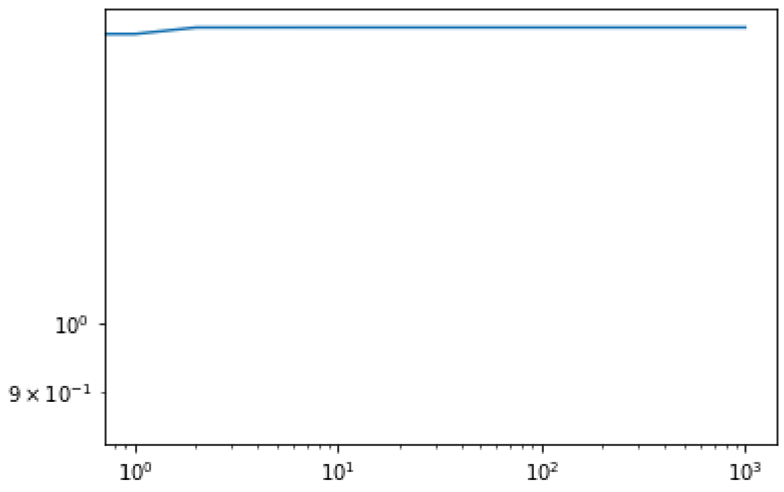
It looks like as the number of degrees increases we quickly become more accurate and we we hover around an error of 10^{-15}. That being said Because π is transendiential when we have an error of 0 im sure we are just at machine precision.

b)

Note, with $x = e^t - 1$, our function is now e^{-t}

```
In [283]: f = lambda x: math.exp(-x)
          def integrate(deg):
              sum = 0
              x, w = np.polynomial.legendre.leggauss(deg)
              for i in range(deg):
                  sum += w[i]*f(x[i])
              return sum;

          N = 1000
          I = []
          err = []
          for i in range(1,N):
              q = 2*integrate(i)
              er = abs(q-np.pi)
              I.append(q)
              err.append(er)
```

```
In [284]: plt.loglog(err)

Out[284]: [<matplotlib.lines.Line2D at 0x1515b82b70>]
```



Well this looks very odd. It appears that our error is fixed at a rather large value no matter what d is. I will talk more about what I think is happeneing in part c.

c)

So for part b I think we are some weird things happening with the weights from Leggauss polynomial and our function that is now just e^{-x} I think this has to do with the fact that the weights are orthogonal to the weight function, which just so happens to be e^{-x} so I think we are getting some weird cancelation. So for this problem I dont thing the change of variables would be the best and I would just use part a instead.

2

a)

```
In [202]: from scipy.integrate import quadrature, quad, dblquad
          import time
```

In [183]:
```
def baseQuadratureRule(f,a,b):
    Q, errEstimate = quad(f,a,b,epsabs=1.5e-8,epsrel=1.5e-8,limit=50)
    return Q

def my2D_quadrature(f,a,b,c,d, baseQuadRule ):
    @np.vectorize
    def g(x):

        def f_as_fcn_of_y(y):
            return f(x,y)


        Q = baseQuadRule( f_as_fcn_of_y, c, d )
        return Q

    Q = baseQuadRule( g, a, b )
    return Q

def my3D_quadrature(f,a1,b1,a2,b2,a3,b3, my2D_quadrature ):
    @np.vectorize
    def g(x):

        def f_as_fcn_of_yz(y,z):
            return f(x,y,z)


        Q = my2D_quadrature( f_as_fcn_of_yz, a2, b2, a3, b3, baseQuadrat
ureRule )
        return Q

    Q = baseQuadratureRule( g, a1, b1 )
    return Q
def my4D_quadrature(f,a1,b1,a2,b2,a3,b3,a4,b4, my3D_quadrature ):
    @np.vectorize
    def g(x):

        def f_as_fcn_of_yzw(y,z,w):
            return f(x,y,z,w)


        Q = my3D_quadrature( f_as_fcn_of_yzw, a2, b2, a3, b3, a4, b4,  m
y2D_quadrature )
        return Q

    Q = baseQuadratureRule( g, a1, b1 )
    return Q
```

b)

In [147]:
```
from scipy.linalg import hilbert
d=2
A = hilbert(d)
def f(x,y):
    args = np.array([x,y])
    return args.T.dot(args.dot(A))
def p(x,y):
    return((1/(2*np.pi)**(2/2))*np.exp(-(x**2+y**2)/2))
def g(x,y):
    return(f(x,y)*p(x,y))
#my2D_quadrature(g,-5,5,-5,5, baseQuadratureRule(g, -5,5))
```

In [148]:
```
my2D_quadrature(g,-5,5,-5,5, baseQuadratureRule)
```

Out[148]:
```
1.3333119816098895
```

In [174]:
```
d=3
A = hilbert(d)
def f(x,y,z):
    args = np.array([x,y,z])
    return args.T.dot(args.dot(A))
def p(x,y,z):
    return((1/(2*np.pi)**(3/2))*np.exp(-(x**2+y**2+z**2)/2))
def g(x,y,z):
    return(f(x,y,z)*p(x,y,z))
```

In [179]:
```
my3D_quadrature(g,-5,5,-5,5,-5,5, my2D_quadrature )
```

Out[179]:
```
1.533307899800628
```

In [184]:
```
d=4
A = hilbert(d)
def f(x,y,z,w):
    args = np.array([x,y,z,w])
    return args.T.dot(args.dot(A))
def p(x,y,z,w):
    return((1/(2*np.pi)**(4/2))*np.exp(-(x**2+y**2+z**2+w**2)/2))
def g4(x,y,z,w):
    return(f(x,y,z,w)*p(x,y,z,w))
```

In [185]:
```
my4D_quadrature(g4,-5,5,-5,5,-5,5,-5,5,my3D_quadrature)
```

Out[185]:
```
1.6761617121244665
```

i)

As we can see from the line of code above, $E\left[f(x)\right] \approx 1.6761676$, when $d = 4$.

ii)

My error tollerence (both absolute annd relative) for 1D is very low so it doesnt surprise that 4D took a very long time to run (it was approx. 5 min). For both 2D and 3D the functions were evulated much quicker as would be expected in a problem like this.

c)

In [219]:
```
def MonteCarlo(d, N, Verbose = True):
    start_time = time.time()
    A = hilbert(d)
    I = 0.0
    for i in range(N):
        x = np.random.randn(d)
        f = x.T.dot(x.dot(A))
        I += f
    I = (1/N)*I
    end_time = time.time()
    if Verbose == True:
        print('Excution time is:',end_time-start_time, ' seconds')
    return I
```

Running our MonteCarlo for d = 2,3,4:

In [218]:
```
I2 = MonteCarlo(2,10**6, Verbose = False)
I3 = MonteCarlo(3,10**6, Verbose = False)
I4 = MonteCarlo(4,10**6, Verbose = False)

print("For d=2: ", I2)
print("For d=3: ", I3)
print("For d=4: ", I4)
```

```
For d=2:  1.3342270837817078
For d=3:  1.53219163268725
For d=4:  1.678155905224539
```

Now finding the runtime and error for d = 4,12,100

```
In [221]: #runtime and error for d=4
          I4 = MonteCarlo(4,10**6, Verbose = True)
          print('Error for d=4 is:', abs(I4 - 1.6761617121244665))

          Excution time is: 2.024691104888916  seconds
          Error for d=4 is: 0.00175656939114055
```

```
In [222]: #runtime and error for d=12
          I12 = MonteCarlo(12,10**6, Verbose = True)
          print('Error for d=12 is:', abs(I12 - 2.224352838648))

          Excution time is: 2.243312120437622  seconds
          Error for d=12 is: 0.0021304304003479935
```

```
In [224]: #runtime and error for d=100
          I100 = MonteCarlo(100,10**6, Verbose = True)
          print('Error for d=100 is:', abs(I100 - 3.284342189302))

          Excution time is: 5.517652988433838  seconds
          Error for d=100 is: 0.002000626990464216
```

d)

It seems like quadature is probably more accurate than Monte Carlo is expically in low levels of d. However quadature would probably be almost impossible to run on my compute for d =100 ao I think when we have igh dimensions Monte Carlo is the way to go!