Adam Sanchez
Math 4650
Mid term 2.

Problem 1

a) 1) Given 3 nodes polynomial interpolation will result in a 2^{nd} degree polynomial.

2) For both Lagrange and Barycentric Interpolation we need only know the values A, B, and C.

b) 1) if we have piecewise linear then each polynomial would be degree 1
if we have piecewise quadratic then each polynomial would be degree 2
if we have piecewise cubic then each polynomial would be degree 3

2) For all 3 of them we need to only A, B, C because the rest of the conditions only depend on the polynomials themselves.

c) 1) Each polynomial would be degree 3.

2) We only need to know A, B, C becase natural boundry conditions only depend on $S''(X_0)$ and $S''(X_2)$ in this case.

d) 1) We would get a $2n+1$ degree polynomial, which is a degree 5 polynomial.

2) For the scope of this class we need A, B, C, D, E, F

But if you want to do other fancy stuff with more conditions I believe you can use G, H, & I as well.

Problem 2.

Since the error for both C_1 & C_2 looks like it converges sublinearly,

I believe C_2 must converge quadratically.

So $N_2(h) = M + ch^2 + O(h^2)$

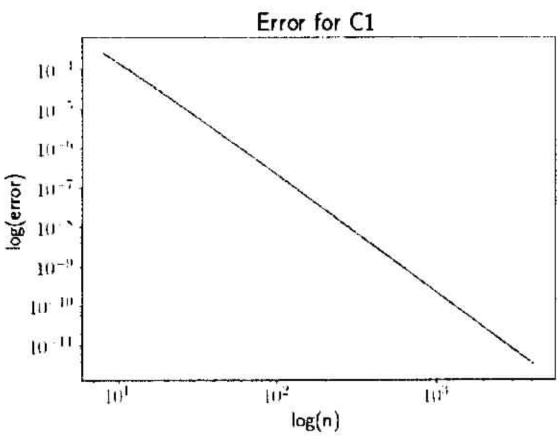
if we take $q=2$ we get

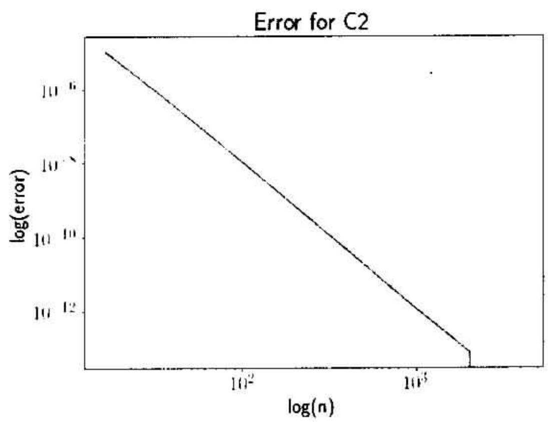
$$N_2(h) = N_1(h/2) + \frac{N_1(h/2) - N_1(h)}{2^2 - 1}$$

Note $N_1(h/2) = 3.1415934786 3 6688$

$N_1(h) = 3.1416028818 06524$

So $N_2(h) = 3.14158407546 6852.$





Problem 3

a) $f = 10y^2$ so we know f is continuous on $D = \{(t,y): 0 \le t \le 1, -\infty \le y \le \infty\}$

But f is not Lipschitz because f is unbounded wrt y so its not possible for it to be less than some constant L.

Thus the ODE does not satisfy the revelant Lipschitz continuity condition

b) I dont think we can gaurntee existance or Uniqueness because f is not Lipschitz Continuous wrt y, so we cannot apply theorem 9.4.

c) I don't belive we do because f is not Lipschitz.

d) $w_0 = .1$

$w_1 = .1 + .25(10(.1^2)) = .125$

$w_2 = .125 + .25(10(.125^2)) = .164$

$w_3 = .164 + .25(10(.164^2)) = .231$

Not that f does not depend on t.

t	$w_i \approx y(t)$
0	.1
.25	.125
.5	.164
1	.231

Problem 4

a) By theorem 3.3 in the book

$$f(x) = P(x) + \frac{f^{(4)}(\xi(x))}{4!}(x-x_0)(x-x_1)(x-x_2)$$

Without loss of generality we let $h = b-a$ and then let $a=0$. So $x_0 = 0, x_1 = \frac{h}{4}, x_2 = h$.

so

$$f(x) = P(x) + \frac{f^{(4)}\{\xi(x)\}}{4!} x(x-\tfrac{h}{4})(x-h)$$

So $$\underbrace{\int_0^h f(x)}_{I} = \underbrace{\int_0^h P(x)dx}_{Q} + \int_0^h \frac{f^{(4)}\{\xi(x)\}}{4!} x(x-\tfrac{h}{4})(x-h)\,dx$$

So $$|I-Q| = \left|\int_0^h \frac{f^{(4)}\{\xi(x)\}}{4!} x(x-\tfrac{h}{4})(x-h)\,dx\right| = \int_0^h \left|\frac{f^{(4)}\{\xi(x)\}}{4!} x(x-\tfrac{h}{4})(x-h)\,dx\right|$$

$$\leq \int_0^h \left|\frac{M_4}{4!} x(x-\tfrac{h}{4})(x-h)\,dx\right|$$

$$= \frac{M_4}{4!}\int_0^h \left|x(x-\tfrac{h}{4})(x-h)\right|dx$$

$$= \frac{M_4}{4!}\cdot\frac{71}{1536}h^4$$

$$= \frac{M_4}{4!}\cdot\frac{71}{1536}(b-a)^4$$

$$\therefore\ |I-Q| \leq \frac{M_4}{4!}\cdot\frac{71}{1536}(b-a)^4$$

b) Because we only have 3 nodes we know p is a 2nd degree polynomial.
Further, because we interpolated w/ p if f is f is a 2^{nd} degree polynomial
then we know ar formula is exact of order 2.

If we choose to make h very small our error becomes very small as well
at a rate of $O(h^4)$. So I belive we can say the order of exactness
is 3.