APPM 4650 — Extra problems Exam 2

The problems provided here are on the material from the week before the exam. **You should not think of this document as a comprehensive review of the material on the exam.**

1. Show that $f(x) = \sqrt{x^2+3}$ Lipschitz in $\mathbb{R}$.
2. (a) Show that $f(x) = e^x$ is not Lipschitz on $\mathbb{R}$.
   (b) Show that $f(x) = e^x$ is Lipschitz on $[-10, 10]$.
3. Show that $y^3t + yt = 2$ implicitly defines a solution to the following initial value problem.

$$\begin{cases} y' &= \frac{y^3+y}{(3y^2+1)t} \quad 1 \le t \le 2 \\ y(1) &= 1 \end{cases}$$

4. Let $f(t, y) = \frac{1+y}{1+t}$.
   (a) Does f satisfy a Lipschitz condition on $D = \{(t, y) : 0 \le t \le 1, y \in \mathbb{R}\}$?
   (b) Is the problem

$$\begin{cases} y' &= f(t, y) \quad 0 \le t \le 1 \\ y(0) &= 1 \end{cases}$$

   well-posed (using the theroems from this class)? Justify your answer.