HW1_AdamSanchez

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1 Homework 1

1.1 Adam Sanchez

1.1.1 MATH 4650

Importing all the Libraries

```
In [2]: import matplotlib
        matplotlib.rcParams['text.usetex'] = True
        import matplotlib.pyplot as plt
        %matplotlib inline
        import numpy as np
        import sympy as sym
        from sympy import init_printing
        init_printing()
        import math
```

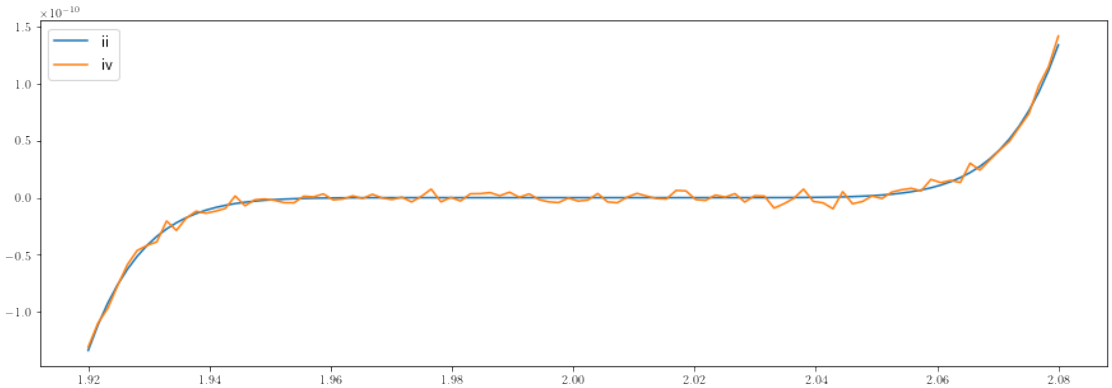
Problem 1

```
In [12]: x_vals = np.linspace(1.92,2.08,100)
         seq1 = (2,2,2,2,2,2,2,2,2)
         y=0
         coeff = np.poly(seq1)
         poly = np.poly1d(coeff)
         def MakeAPoly(coeffs, x):
             n = len(coeffs)
             y=0
             for i in range(n):
                 y += (x**(i+1))*coeffs[1]
             return y
         def Horner (x, coeffs):
             y=coeffs[-1]
             i=len(coeffs)-2
             while i >= 0:
                 y = y * x + coeffs[i]
                 i -= 1
             return y
```

```
i = MakeAPoly(coeff, x_vals)
ii = (x_vals-2)**9
iii = Horner(x_vals, coeff)
iv = np.polyval(poly,x_vals)
```

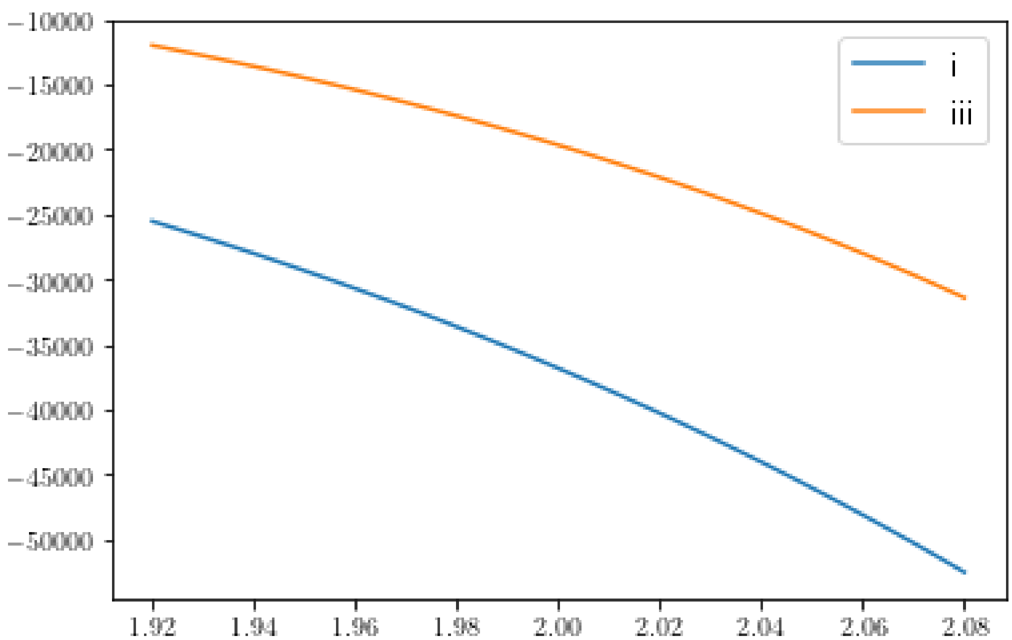
In [13]:
```
plt.figure(figsize = (15,5))
plt.plot(x_vals, ii, x_vals, iv)
plt.legend(['ii', 'iv'], fontsize = 13)
```

Out[13]: <matplotlib.legend.Legend at 0x102536cf8>



In [85]:
```
plt.plot(x_vals, i, x_vals, iii)
plt.legend(['i', 'iii'], fontsize = 13)
```

Out[85]: <matplotlib.legend.Legend at 0x11b047f28>



b) Im not sure whats going on here. I know you asked to graph all 4 on the same plot but I can't figure out whats going on with my fuctions for i and iii. My only thought is that I am getting awful rounding errors. Clearly I think ii or iv are the most accurate.

Problem 2

a) In this problem we would run into issues with underflow when x is very close to 0 (any decimal with more than approximatly 15 digits). So I think the best way to evaluate it is with the Taylor Series of the function. The first few terms of the Taylor series of this function are:

$$\frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3$$

we can see that we will not get cancellation now.

b) Again we would run into the same issues so we should try and change the equation. Note that

$$\sin 2a = 2\sin a \cos a$$

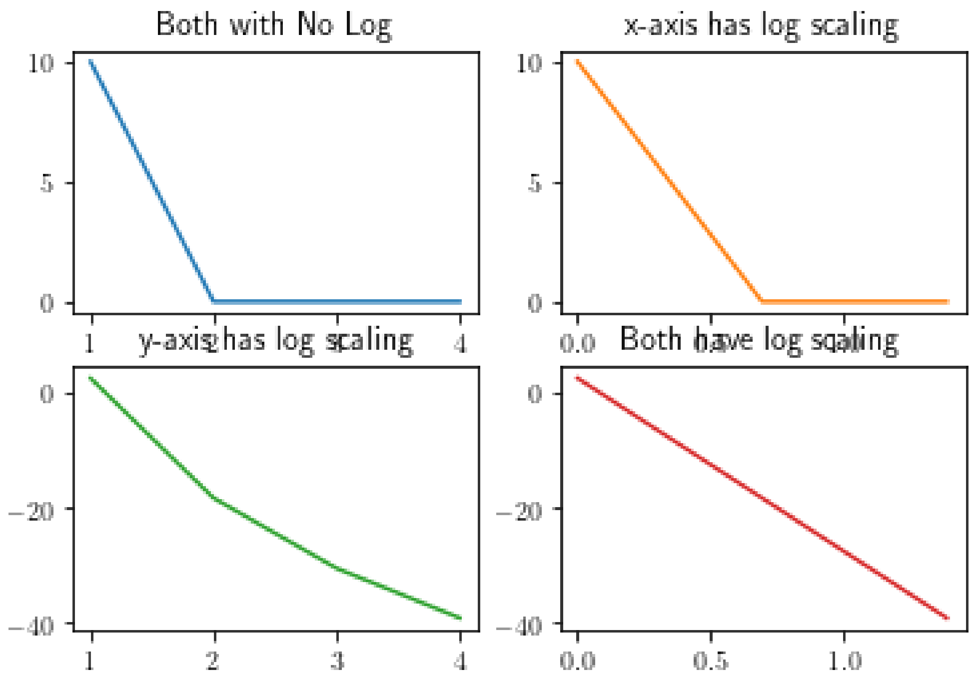
Thus our equation turns into:

$$2\sin(x+a)\cos a + x - 2\sin a \cos a = 2(\sin a \cos x + \cos a \sin x) - 2\sin a \cos a$$

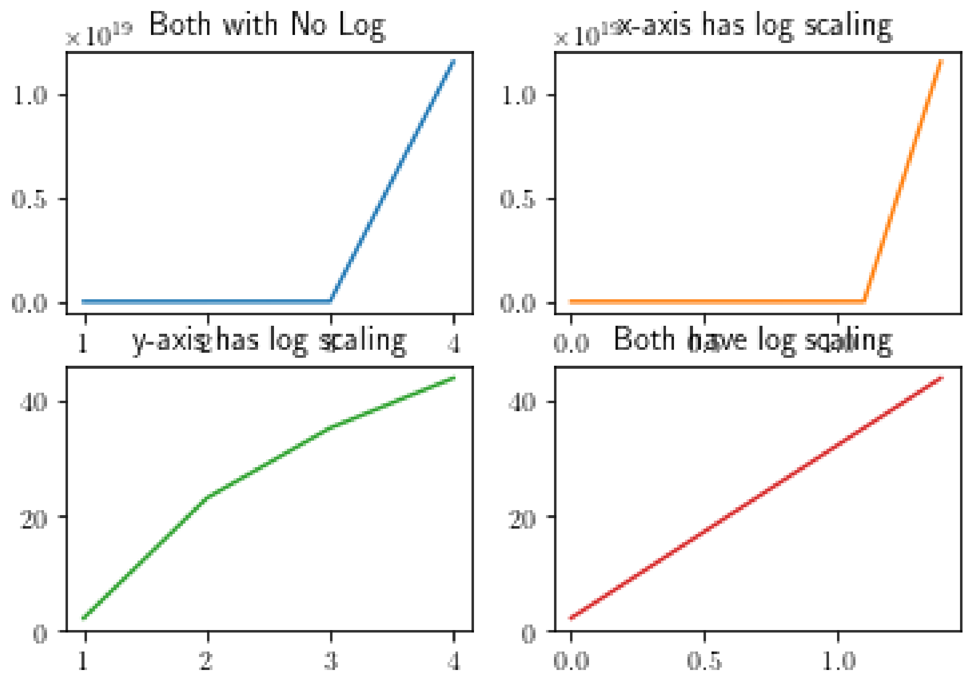
which wouldn't be an issue.

Problem 3

```
In [131]: # 3a) Based on the plots bellow we should use a plot where both axes are logarithmic
          C = 10
          a = 30

          x = np.linspace(1,4,4)
          xlog = np.log(x)
          xn = (1/(x**a))*C
          xnlog = np.log(xn)
          fig, axs = plt.subplots(2, 2)
          axs[0, 0].plot(x, xn)
          axs[0, 0].set_title('Both with No Log')
          axs[0, 1].plot(xlog, xn, 'tab:orange')
          axs[0, 1].set_title('x-axis has log scaling')
          axs[1, 0].plot(x, xnlog, 'tab:green')
          axs[1, 0].set_title('y-axis has log scaling')
          axs[1, 1].plot(xlog, xnlog, 'tab:red')
          axs[1, 1].set_title('Both have log scaling')

Out[131]: Text(0.5,1,'Both have log scaling')
```

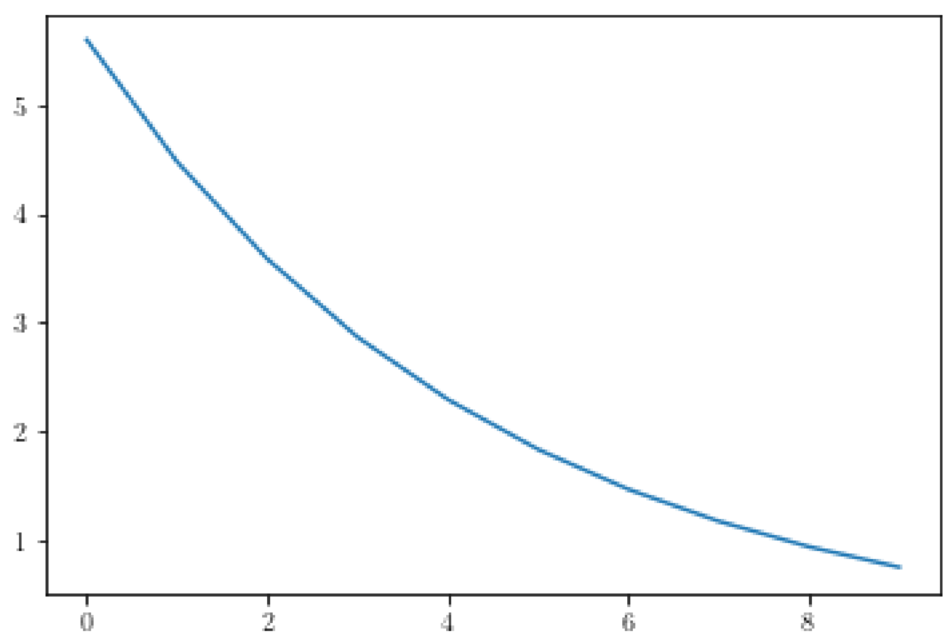



```
In [135]: # 3b) Based on the plots bellow we should use a plot where both axes are logarithmic
          D = 10
          p = 30

          x = np.linspace(1,4,4)
          xlog = np.log(x)
          xn = C*x**p
          xnlog = np.log(xn)
          xntry = C*xlog**p
          fig, axs = plt.subplots(2, 2)
          axs[0, 0].plot(x, xn)
          axs[0, 0].set_title('Both with No Log')
          axs[0, 1].plot(xlog, xn, 'tab:orange')
          axs[0, 1].set_title('x-axis has log scaling')
          axs[1, 0].plot(x, xnlog, 'tab:green')
          axs[1, 0].set_title('y-axis has log scaling')
          axs[1, 1].plot(xlog, xnlog, 'tab:red')
          axs[1, 1].set_title('Both have log scaling')

Out[135]: Text(0.5,1,'Both have log scaling')
```



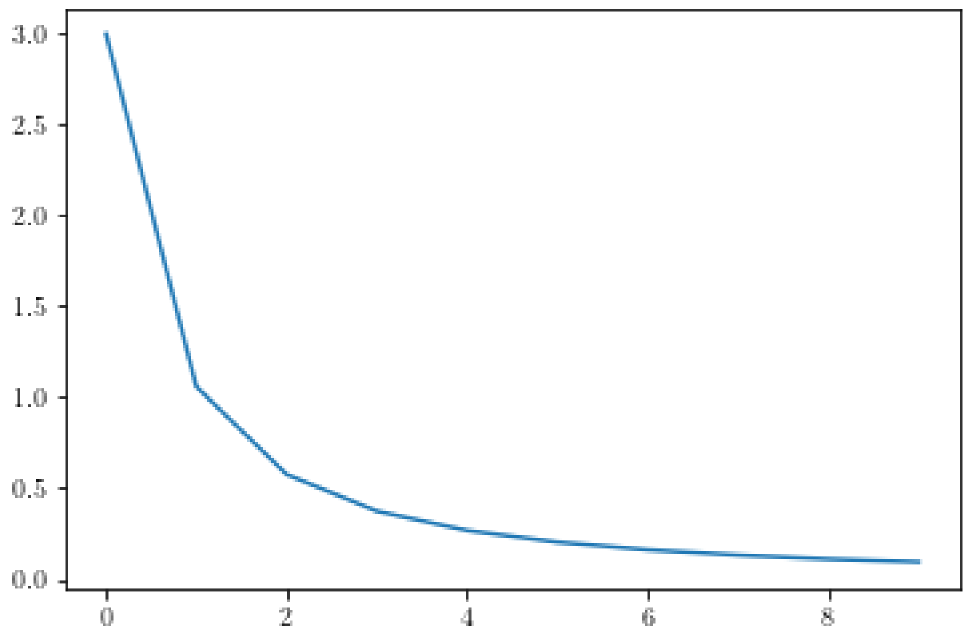
In [137]:
```
# 3c)
b = [5.6000, 4.4800, 3.5840, 2.8672, 2.2938, 1.8350, 1.4680, 1.1744, 0.9395, 0.7516]
plt.plot(b)
#based on the plot I would guess that the sequence converges superlinearly
#This has the form Dp^(n) where D=5.6 and p = .8
```

Out[137]:
```
[<matplotlib.lines.Line2D at 0x11fc1f048>]
```



```
In [3]: ##3d)
        q=[3.0000, 1.0607, 0.5774, 0.3750, 0.2683, 0.2041, 0.1620, 0.1326, 0.1111, 0.0949]
        plt.plot(q)
        #Based on the plot I would guess that the sequence converges quadratically
        #I dont think this has the form of either?
```

```
Out[3]: [<matplotlib.lines.Line2D at 0x10c5000f0>]
```



Problem 4
For this problem we should look at the at the Maclaurin series:

$$\frac{1}{1-h} - x - 1 = 0 + \frac{\frac{d}{dx}\left(\frac{1}{1-h} - h - 1\right)(0)}{1!}x + \frac{\frac{d}{dx}\left(\frac{1}{1-h} - h - 1\right)(0)}{2!}x^2 + ...$$

$$= 0 + \frac{0}{1!}x + \frac{2}{2!}x^2 + \frac{6}{3!}x^3 + \frac{24}{4!}x^4 + ...$$

$$= x^2 + x^3 + x^4 + x^5 + ...$$

$$= x^2 + O(x^3)$$

Problem 5

a)

$$K_f(x) = \left|\frac{x}{e^x - 1}e^x\right|$$

f(x) appears to be well conditioned for all x

b)

$$g(x) = e^x \rightarrow K_g x = |x|$$

$$K_g(x) < K_f(x)$$

$$h(x) = x - 1 \rightarrow K_h(x) = \left|\frac{x}{x-1}\right|$$

$$K_h(x) > K_f(x)$$

So the algorthim is unstable.

c) As we can see from the code below the algorthim gives us 8 correct digits. This is expected because we know that the algorthim is not stable even though the function is well conditioned

```
In [6]: x = 9.999999995000000*10**(-10)
        kf= (x*math.exp(x))/(math.exp(x)-1)

        def j(h):
            y = math.exp(h)
            return y-1
        j(x)
```

```
Out[6]: 1.000000082740371e-09
```

d) First lets find the taylor expansion

```
In [31]: x = sym.Symbol('x')

         Taylor = sym.series( sym.exp(x)-1, x )
         Taylor.removeO()
```

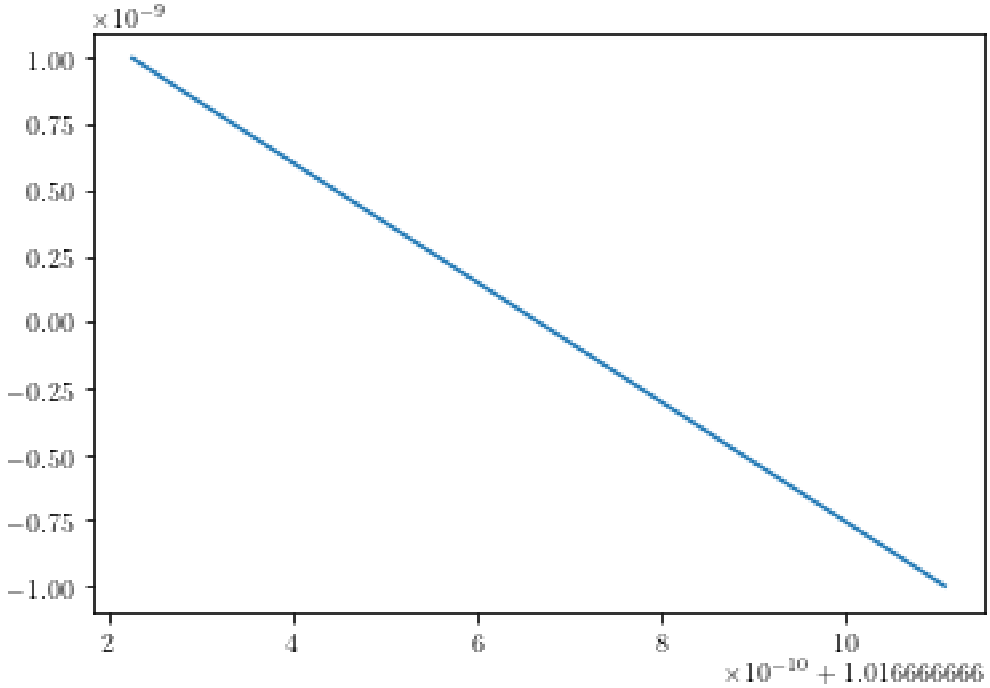
Out[31]:

$$\frac{x^5}{120} + \frac{x^4}{24} + \frac{x^3}{6} + \frac{x^2}{2} + x$$

$$K_f(x) = \left|\frac{x\left(\frac{x^4}{24} + \frac{x^3}{6} + \frac{x^2}{2} + 1\right)}{\frac{x^5}{120} + \frac{x^4}{24} + \frac{x^3}{6} + \frac{x^2}{2} + x}\right| = \left|\frac{5x^4 + 20x^3 + 60x^2 + 120}{x^4 + 5x^3 + 20x^2 + 60x + 120}\right|$$

```
In [10]: #Lets plot K for -10^(-9)<=x<=10^9
         x_vals = np.linspace(-10**(-9),10**(-9),100)
         k = ((5*x_vals**4)+(20*x_vals**3)+(60**x_vals*2)+120)/((x_vals**4)+(5*x_vals**3)+(20*
         plt.plot(k,x_vals)
```

```
Out[10]: [<matplotlib.lines.Line2D at 0x1025c2908>]
```



notice that we sould have accuracy for about 30 digits
e)

```
In [33]: guess = Taylor.removeO().subs(x,9.999999995000000*10**(-10)).evalf()
         print("My guess is:", guess)
```

```
My guess is: 1.00000000000000e-9
```

so we are accurate to atleast 16 digits