HW4_AdamSanchez

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1 Homework 4

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1.1.1 MATH 4650

Problem 1

a)

i)

$$K_f^{rel}(x) = \left| \frac{x}{\frac{1}{1-x}} \frac{1}{(1-x)^2} \right| = \left| \frac{x - x^2}{(1-x)^2} \right| = \left| \frac{x}{1-x} \right|$$

Now:

$$\lim_{x \to \infty} \left| \frac{x}{1-x} \right| = \infty$$

So when $x \approx 1$ the relative condition number is very bad!

ii)

```
In [53]: import math
         f = lambda x: 1/(1-x)
         x = 1-(10**(-13))
         print(f(x))
         error = 10**(13)-f(x)
         print('The error is:', error)
```

```
9996891514695.885
The error is: 3108485304.1152344
The numer of correct digits are: 3.5074511814908544
```

iii). There are 3 correct digits, but this expected because we know the relative condition number of $f(x)$ is bad at $x \approx 1$

```
In [54]: print('The numer of correct digits are:',-math.log10(error / (10**(13))))
```

```
The numer of correct digits are: 3.5074511814908544
```

iv) All the digits are correct as we can see below:

```
In [56]: x = 1-(2**(-43))
         print(f(x))
         error = 2**(43)-f(x)
         print('The error is:', error)
```

```
8796093022208.0
The error is: 0.0
```

b) First lets find $f'(x)$ because that looks like a headache, honestly I plugged this into to a calcultor because computers are faster at ths:

$$f'(x) = -\frac{d-c}{(x-2)^2}$$

Now lets plug it in:

$$K_f^{rel}(x) = \left| \frac{\frac{x}{\frac{c}{1-x}+d}}{\frac{1}{1-x}+1} \frac{-(c-d)}{(x-2)^2} \right| = \left| \frac{\left(\frac{1}{1-x}+1\right)x}{\frac{c}{1-x}+d} \frac{-(c-d)}{(x-2)^2} \right| = \left| \frac{x(2-x)}{c-dx+d} \frac{-(c-d)}{(x-2)^2} \right| = \left| -\frac{(2x-x^2)(c-d)}{(c-dx+d)(x-2)^2} \right|$$

$$K_f^{rel}(1) = \left| -\frac{(2-1)(c-d)}{(c-d+d)(1-2)^2} \right| = \left| \frac{c-d}{c} \right|$$

Problem 2

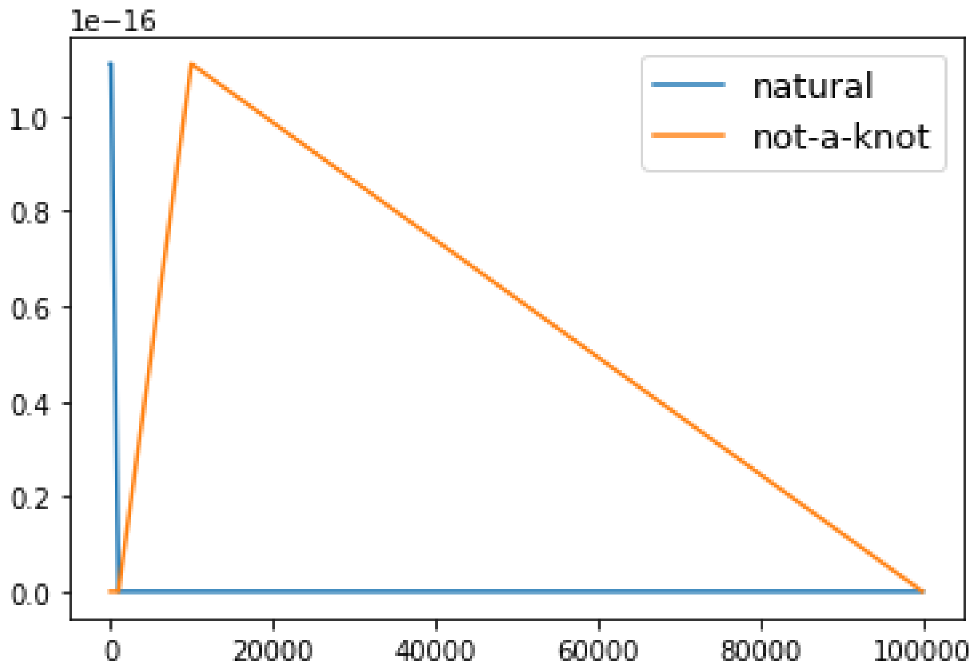
```
In [13]: import matplotlib.pyplot as plt
         import numpy as np
         import scipy.interpolate as interp
```

a) It looks like the decay of the natual spline error decays at a very fastrate (quadric rate) as n get bigger, and not-a-knot error decays at a slower rate, probably linear.

```
In [41]: error_natural=[]
         error_nak = []
         n=[]
         mr = [10, 10**2, 10**3, 10**4, 10**5]
         for i in mr:
             x = np.linspace(1.01,1.99, i)
             y = np.sin(20*x)
             s1 = interp.CubicSpline(x, y, bc_type = 'natural')
             s2 = interp.CubicSpline(x, y, bc_type = 'not-a-knot')
             error_natural.append(max(abs(s1(x)-y)))
             error_nak.append(max(abs(s2(x)-y)))
             n.append(i)
```

```
In [43]: plt.plot(n, error_natural, n, error_nak)
         plt.legend(['natural', 'not-a-knot'], fontsize = 13)
```

Out[43]: <matplotlib.legend.Legend at 0x110097c88>



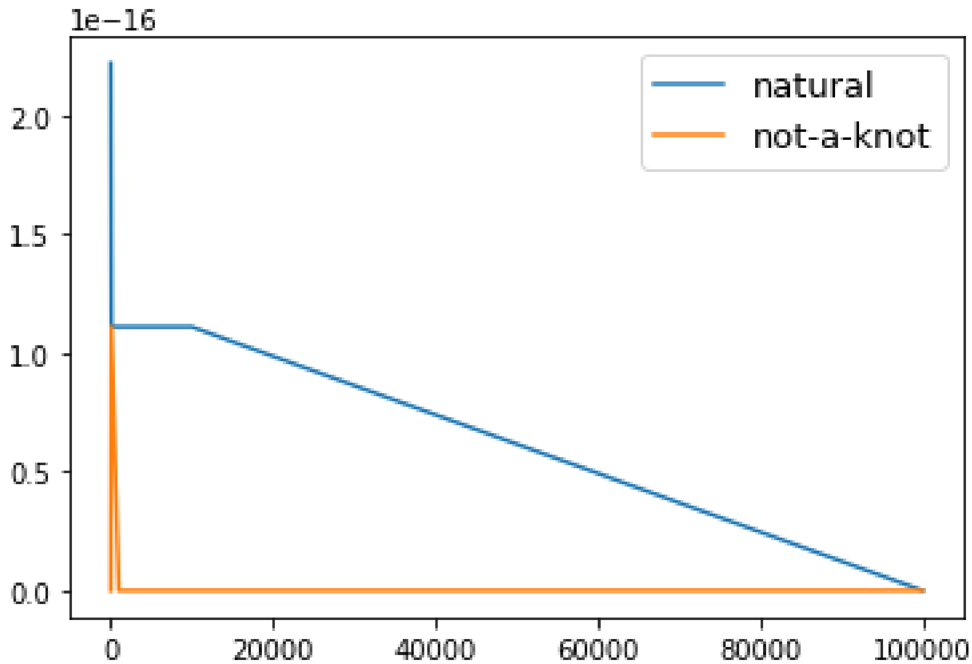
b) This sort of makes sense to me. Becasue not-a-knot requires s‴ to be continious and natural only has to calculate s″ it would make sense that the error decays faster in natural

c) The error decay looks very diffent. Natural looks like it decays at a fast rate then slows to liner, while not-a-knot looks like it decays very fast. I assume this is because we are now including the endpoints.

In [44]:
```
error_natural=[]
error_nak = []
n=[]
mr = [10, 10**2, 10**3, 10**4, 10**5]
for i in mr:
    x = np.linspace(1,2, i)
    y = np.sin(20*x)
    s1 = interp.CubicSpline(x, y, bc_type = 'natural')
    s2 = interp.CubicSpline(x, y, bc_type = 'not-a-knot')
    error_natural.append(max(abs(s1(x)-y)))
    error_nak.append(max(abs(s2(x)-y)))
    n.append(i)
```

In [45]:
```
plt.plot(n, error_natural, n, error_nak)
plt.legend(['natural', 'not-a-knot'], fontsize = 13)
```

Out[45]: <matplotlib.legend.Legend at 0x10a28a1d0>

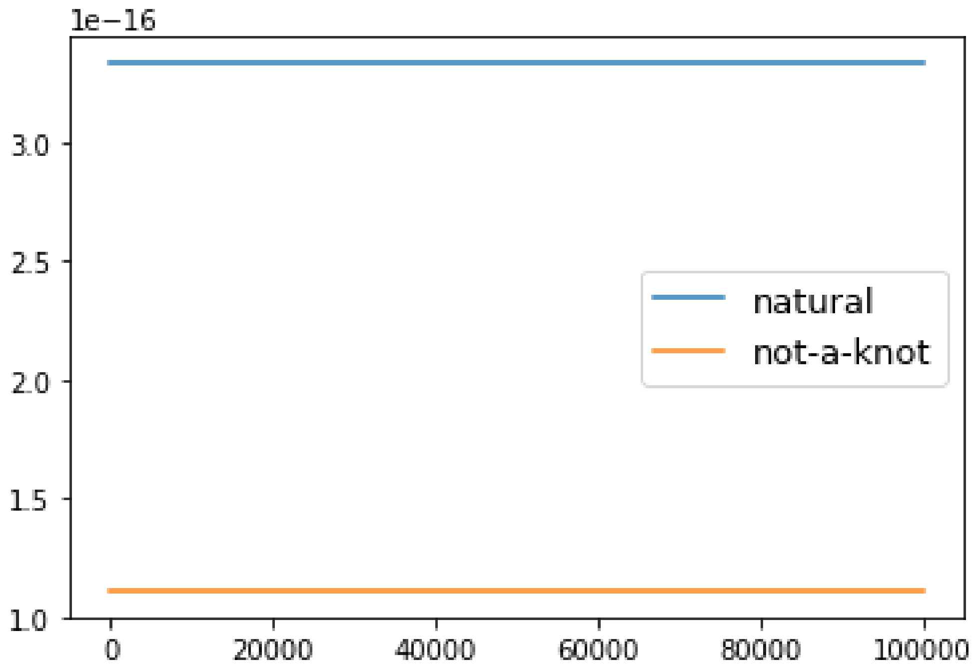


d) It looks like our error for natural is just constat at value greater than 0 and the error for not-a-knot is also constant but for a lower value. I dont know why this is. Maybe because its a piecewise so its jumping around?

In [59]:
```
error_natural=[]
error_nak = []
n=[]
mr = [10, 10**2, 10**3, 10**4, 10**5]
for i in mr:
    g = np.piecewise(x, [x < 1.3, x >= 1.3], [lambda x: np.sin(20*x), lambda x: np.si
    s1 = interp.CubicSpline(x, g, bc_type = 'natural')
    s2 = interp.CubicSpline(x, g, bc_type = 'not-a-knot')
    error_natural.append(max(abs(s1(x)-g)))
    error_nak.append(max(abs(s2(x)-g)))
    n.append(i)
```

In [60]:
```
plt.plot(n, error_natural, n, error_nak)
plt.legend(['natural', 'not-a-knot'], fontsize = 13)
```

Out[60]:
```
<matplotlib.legend.Legend at 0x110032b38>
```


1e−16
3.0
2.5
2.0
1.5
1.0
0
20000
40000
60000
80000
100000
natural
not-a-knot