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#5.6

$y_k = \sigma(a_k)$ logistic sigmoid function

$$\frac{d\sigma}{da} = \sigma(1-\sigma)$$

$$\therefore \frac{dE(w)}{da_k} = -t_k \frac{1}{y_k}[y_k(1-y_k)] + (1-t_k)\frac{1}{1-y_k}[y_k(1-y_k)]$$

$$= [y_k(1-y_k)]\left[-t_k \frac{1}{y_k} + (1-t_k)\frac{1}{1-y_k}\right]$$

$$= -t_k(1-y_k) + y_k(1-t_k)$$

$$= -t_k + \cancel{t_k y_k} + y_k - \cancel{t_k y_k} = y_k - t_k \quad \#$$

#5.24 根據(5.113)、(5.115)(5.116)(5.117)：

$$\tilde{a}_j = \sum_i \tilde{w}_{ji}\tilde{x}_i + \tilde{w}_{j0}$$

$$= \sum_i \frac{1}{a} w_{ji}(ax_i + \cancel{b}) + w_{j0} - \cancel{\frac{b}{a}\sum_i w_{ji}}$$

$$= \sum_i w_{ji}x_i + w_{j0} = a_j$$

(經過轉換後 hidden unit a_j 沒有改變，若作用在 a_j 上的 activation function 也沒變，則 $\tilde{z}_j = z_j$)

接著處理第2層 layer：

$$\tilde{y}_k = \sum_j \tilde{w}_{kj}\tilde{z}_j + \tilde{w}_{k0}$$

$$= \sum_j (cw_{kj}\cdot z_j) + cw_{k0} + d = c\left[\sum_j (w_{kj}\cdot z_j) + w_{k0}\right] + d$$

$$= cy_k + d$$

由此可証，$\tilde{y}_k$ 和 y_k 之間的線性轉換可由(5.119)與(5.120)達到 #

#6.16 Based on total derivative of function f

$$f((w+\Delta w)^T\phi_1, (w+\Delta w)^T\phi_2, \cdots, (w+\Delta w)^T\phi_N)$$

$$= \sum_{n=1}^{N} \frac{\partial f}{\partial(w^T\phi_n)} \cdot \Delta w^T\phi_n = \left[\sum_{n=1}^{N} \frac{\partial f}{\partial(w^T\phi_n)} \cdot \phi_n^T\right] \cdot \Delta w$$

$$\therefore \nabla_w f = \sum_{n=1}^{N} \frac{\partial f}{\partial(w^T\phi_n)} \cdot \phi_n^T$$

$$\nabla_w g = \frac{\partial g}{\partial(w^Tw)} \cdot 2w^T$$

為了找到w可使J最小，設J對w的偏微分為0

$$\nabla_w J = \nabla_w f + \nabla_w g = \sum_{n=1}^{N} \frac{\partial f}{\partial(w^T\phi_n)} \cdot \phi_n^T + \frac{\partial g}{\partial(w^Tw)} \cdot 2w^T = 0$$

$$\therefore w = -\frac{\partial(w^Tw)}{2\cdot\partial g} \cdot \sum_{n=1}^{N} \frac{\partial f}{\partial(w^T\phi_n)} \cdot \phi_n$$

$$= -\frac{1}{2a} \cdot \sum_{n=1}^{N} \frac{\partial f}{\partial(w^T\cdot\phi(x_n))} \phi(x_n)$$

(a 為 $\frac{\partial g}{\partial(w^Tw)}$，因為 g 為 monotonically increasing function，所以 $a>0$)

#6.26 $p(a_{N+1}|t_N) = \int p(a_{N+1}|a_N)p(a_N|t_N)da_N$ (式6.77)

式(6.78)
式(6.86) $\leftarrow = \int \mathcal{N}(a_{N+1}|k^TC_N^{-1}a_N, c-k^TC_N^{-1}k)\cdot\mathcal{N}(a_N|a_N^*, H^{-1})da_N$

By $p(y) = \int p(y|x)p(x)dx$

可得 $p(a_{N+1}|t_N) = \mathcal{N}(A\mu+b, L^{-1}+A\Lambda^{-1}A^T)$

其中 $A = k^TC_N^{-1}$, $b=0$, $L^{-1} = c-k^TC_N^{-1}k$, $\mu = a_N^*$, $\Lambda = H$.

因此，mean：(式6.84)

$$A\mu+b = k^TC_N^{-1}a_N^* = k^TC_N^{-1}C_N(t_N-\sigma_N) = k^T(t_N-\sigma_N)$$

Covariance matrix：式(6.85)

$$L^{-1} + AA^{-1}A^T = c - k^T C_N^{-1} k + k^T C_N^{-1} H^{-1} (k^T C_N^{-1})^T$$

$$= c - k^T (C_N^{-1} - C_N^{-1} H^{-1} C_N^{-1}) k$$

$$= c - k^T (C_N^{-1} - C_N^{-1} (W_N + C_N^{-1})^{-1} C_N^{-1}) k$$

$$= c - k^T (C_N^{-1} - (\underbrace{C_N}_{\text{symmetric}} W_N C_N + C_N^{-1})^{-1}) k$$

接著採用matrix identity(C.7)化簡上式，即可得式(6.88)#