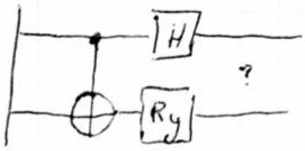
Policzyć stan *wyjściowy* układu $|x_1 x_0\rangle$:

zad 0.

$$R_{y\phi} = \begin{bmatrix} \cos\frac{\phi}{2} & -\sin\frac{\phi}{2} \\ \sin\frac{\phi}{2} & \cos\frac{\phi}{2} \end{bmatrix} \quad \phi = \frac{\pi}{3} \quad \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2} \quad H = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

$|x_1\rangle = |1\rangle$

$|x_0\rangle = |1\rangle$



$|x_1\rangle = |1\rangle \to |1\rangle \to \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$

$|x_0\rangle = |1\rangle \to |0\rangle \to z$

$$z = \begin{bmatrix} \cos\frac{\pi}{6} & -\sin\frac{\pi}{6} \\ \sin\frac{\pi}{6} & \cos\frac{\pi}{6} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos\frac{\pi}{6} \\ \sin\frac{\pi}{6} \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{bmatrix}$$

$$|x\rangle = |x_1 x_0\rangle = \left(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle\right) \otimes \left(\frac{\sqrt{3}}{2}|0\rangle + \frac{1}{2}|1\rangle\right) = \frac{\sqrt{3}}{\sqrt{2}\cdot 2}|00\rangle + \frac{1}{\sqrt{2}\cdot 2}|01\rangle - \frac{\sqrt{3}}{\sqrt{2}\cdot 2}|10\rangle - \frac{1}{\sqrt{2}\cdot 2}|11\rangle =$$

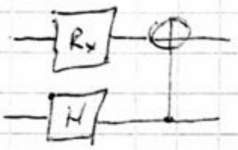
$$= \frac{\sqrt{6}}{4}|00\rangle + \frac{\sqrt{2}}{4}|01\rangle - \frac{\sqrt{6}}{4}|10\rangle - \frac{\sqrt{2}}{4}|11\rangle$$

zad 1.

$$R_{x,\phi} = \begin{bmatrix} \cos\frac{\phi}{2} & -i\sin\frac{\phi}{2} \\ -i\sin\frac{\phi}{2} & \cos\frac{\phi}{2} \end{bmatrix} \quad \phi = \frac{2\pi}{3} \quad |x_1 x_0\rangle = ?$$

$|x_0\rangle = |0\rangle$

$|x_1\rangle = |0\rangle$



$|x_1\rangle = |0\rangle \to \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$ $\cancel{- \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle}$

$|x_0\rangle = |0\rangle \to z$

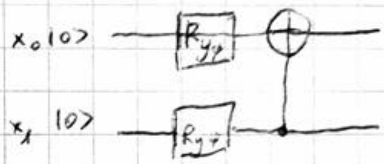
$$z = \begin{bmatrix} \cos\frac{\pi}{3} & -i\sin\frac{\pi}{3} \\ -i\sin\frac{\pi}{3} & -\cos\frac{\pi}{3} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ -i\frac{\sqrt{3}}{2} \end{bmatrix} = \frac{1}{2}|0\rangle - i\frac{\sqrt{3}}{2}|1\rangle$$

$$|x\rangle = |x_1 x_0\rangle = \left(\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\right) \otimes \frac{1}{2}|0\rangle - i\frac{\sqrt{3}}{2}|1\rangle = \frac{1}{2\sqrt{2}}|00\rangle - i\frac{\sqrt{3}}{2\sqrt{2}}|01\rangle + \frac{1}{2\sqrt{2}}|10\rangle - i\frac{\sqrt{3}}{\sqrt{2}2}|11\rangle$$

Po CNOT

$$|x\rangle = \frac{1}{\sqrt{2}\cdot 2}|00\rangle - i\frac{\sqrt{3}}{2\sqrt{2}}|01\rangle + \frac{1}{2\sqrt{2}}|11\rangle - i\frac{\sqrt{3}}{2\sqrt{2}}|10\rangle$$

$$R_{y,\varphi} = \begin{bmatrix} \cos\frac{\phi}{2} & -\sin\frac{\phi}{2} \\ \sin\frac{\phi}{2} & \cos\frac{\phi}{2} \end{bmatrix} \qquad \phi = \frac{\pi}{3} \qquad \psi = \frac{\pi}{2}$$



$$|x_0\rangle |0\rangle \to R_{y\frac{\pi}{3}}|0\rangle = \begin{bmatrix} \cos\frac{\pi}{6} & -\sin\frac{\pi}{6} \\ \sin\frac{\pi}{6} & \cos\frac{\pi}{6} \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos\frac{\pi}{6} \\ \sin\frac{\pi}{6} \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{bmatrix} = \frac{\sqrt{3}}{2}|0\rangle + \frac{1}{2}|1\rangle$$

$$|x_1\rangle = |0\rangle \Rightarrow R_{y\frac{\pi}{2}}|0\rangle = \begin{bmatrix} \cos\frac{\pi}{4} & -\sin\frac{\pi}{4} \\ \sin\frac{\pi}{4} & \cos\frac{\pi}{4} \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos\frac{\pi}{4} \\ \sin\frac{\pi}{4} \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{bmatrix} = \frac{\sqrt{2}}{2}|0\rangle + \frac{\sqrt{2}}{2}|1\rangle$$

$$|x_1 x_0\rangle = \frac{\sqrt{6}}{4}|00\rangle + \frac{\sqrt{6}}{4}|10\rangle + \frac{\sqrt{2}}{4}|01\rangle + \frac{\sqrt{2}}{4}|11\rangle$$

$\downarrow$ po CNOT

$$|x_1 x_0\rangle = \frac{\sqrt{6}}{4}|00\rangle + \frac{\sqrt{6}}{4}|11\rangle + \frac{\sqrt{2}}{4}|01\rangle + \frac{\sqrt{2}}{4}|10\rangle$$

Wyjasnienie: kontrolowana bramka Hadamarda włącza działanie bramki H na qbicie docelowym (tutaj młodszy qbit) jesli qbit kontrolny (tutaj starszy qbit) jest ustawiony na $|1\rangle$. Czyli działanie na stany bazowe wygląda tak:

$$\text{Ctrl-H}\,|00\rangle = |00\rangle$$

$$\text{Ctrl-H}\,|01\rangle = |01\rangle$$

$$\text{Ctrl-H}\,|10\rangle = |1\rangle \otimes H\,|0\rangle$$

$$\text{Ctrl-H}\,|11\rangle = |1\rangle \otimes H\,|1\rangle$$

Zadanie znaleźć **jednoqbitowe** bramki (t.j. macierze unitarne 2×2 działajace na poszczególne qbity) potrzebne do wyprodukowania stanu $|\psi\rangle = \frac{1}{\sqrt{3}}|00\rangle + \frac{1}{\sqrt{3}}|01\rangle + \frac{1}{\sqrt{3}}|10\rangle$, zakładając, ze:

- startujemy ze stanu $|00\rangle$
- na końcu, po znalezionych przez nas bramkach, zostanie użyta bramka Ctrl-H.

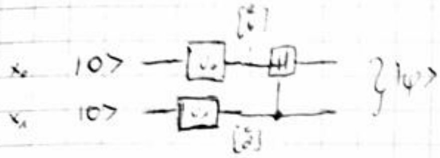
$CTRL\text{-}H\ |00\rangle = |00\rangle$

$CTRL\text{-}H\ |01\rangle = |01\rangle$

$CTRL\text{-}H\ |10\rangle = |1\rangle \otimes H|0\rangle$

$CTRL\text{-}H\ |11\rangle = |1\rangle \otimes H|1\rangle$

$|\psi\rangle\ \frac{1}{\sqrt{3}}|00\rangle + \frac{1}{\sqrt{3}}|01\rangle + \frac{1}{\sqrt{3}}|10\rangle$



$|\psi\rangle = CTRL\text{-}H\,(U_1 \otimes U_0)\,|00\rangle$

$U_0|0\rangle = a|0\rangle + b|1\rangle$

$U_1|0\rangle = c|0\rangle + d|1\rangle$

$U_1 \otimes U_0 = ca|00\rangle + cb|01\rangle + da|10\rangle + bd|11\rangle = *$

$\begin{cases} ca = \frac{1}{\sqrt{3}} \\ \cancel{cb} = \frac{1}{\sqrt{3}} \\ \cancel{da} = \frac{1}{\sqrt{3}} \\ \cancel{db} = 0 \end{cases}$

$\cancel{CTRL\text{-}H|00\rangle}$: $|\psi\rangle = CTRL\text{-}H(*) = ac|00\rangle + bc|01\rangle + \frac{1}{\sqrt{2}}bd|10\rangle + \frac{1}{\sqrt{2}}ad|11\rangle + \frac{1}{\sqrt{2}}bd|10\rangle - \frac{1}{\sqrt{2}}bd|11\rangle$

$\left[\ |a|^2 + |b|^2 = 1 \Rightarrow a = \frac{1}{\sqrt{2}} = b\right.$

$\begin{cases} ac = \frac{1}{\sqrt{3}} \\ bc = \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{2}}(ad + bd) = \frac{1}{\sqrt{3}} \iff \frac{1}{\sqrt{2}}d(a+b) = \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{2}}(ad - bd) = 0 \iff \frac{1}{\sqrt{2}}d(a-b) = 0 \end{cases}$

$d \neq 0$

$\Rightarrow \begin{cases} bc = \frac{1}{\sqrt{3}} \\ \frac{2}{\sqrt{2}}db = \frac{1}{\sqrt{3}} \\ a = b \end{cases}$

$\begin{cases} c = \sqrt{\frac{2}{3}} \\ d = \frac{1}{\sqrt{3}} \\ b = \frac{1}{\sqrt{2}} \\ a = \frac{1}{\sqrt{2}} \end{cases} \Rightarrow U_0 = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$

$U_1 = \begin{bmatrix} \sqrt{\frac{2}{3}} & e \\ \sqrt{\frac{1}{3}} & f \end{bmatrix}$ $\quad U_1 U_1^{\dagger} = \mathbb{1}$

$\Downarrow$

$\begin{cases} f = \sqrt{\frac{2}{3}} \\ e = \sqrt{\frac{1}{3}} \end{cases}$