Homework 4

Problem 1

Consider the two-parameter linear model

$$Y \sim N(\theta_1 + \theta_2, 1),$$

with prior distributions $\theta_1 \sim N(a_1, b_1^2)$ and $\theta_2 \sim N(a_2, b_2^2)$, with θ_1 and θ_2 independent.

1. Clearly θ_1 and θ_2 are individually identified only by the prior; the likelihood provides information only on $\mu = \theta_1 + \theta_2$. Still, the full conditional distributions, $p(\theta_1 \mid \theta_2, y)$ and $p(\theta_2 \mid \theta_1, y)$ for available in closed form. Derive these distributions.

2. Now derive the marginal posterior distributions: $p(\theta_1 \mid y)$ and $p(\theta_2 \mid y)$. Do the data update the prior distributions for these parameters?

3. Set $a_1 = a_2 = 50, b_1 = b_2 = 1000$,and suppose we observe $y = 0$. Run the Gibbs sampler defined in part (a) for $t = 100$ iterations, starting your chains near the prior mean (say, between 40 and 50), and monitoring progress of θ_1, θ_2 and μ. Does this algorithm “converge” in any sense?. Estimate the posterior mean of μ. Does your answer change using $t = 1000$ iterations.

4. Now keep the same values for a_1 and a_2, but set $b_1 = b_2 = 10$. Again run 100 iterations using the same starting values as in part (c). What is the effect on convergence?. Repeat for $t = 1000$ iterations; is your estimate for $E(\mu \mid y)$ unchanged?

Problems 2

Problem 9.2 from Hoff

Problem 3

Problem 9.3 from Hoff

Problem 4

Prove the identity in equation 9.9 (Page 165) in Hoff’s book