Evaluation of Asset Pricing Models on NASDAQ100

▶ **Question :** To what extent can the variance of NASDAQ 100 be explained by one of the four models? Which model can explain the variance the best?

▶ **Overview :**

1.	2.	3.
Data Collection	**Portfolio Construction**	**Model Validation and Performance Measure**

2. Portfolio Construction

- CAPM
- Fama-French 3 Factor model
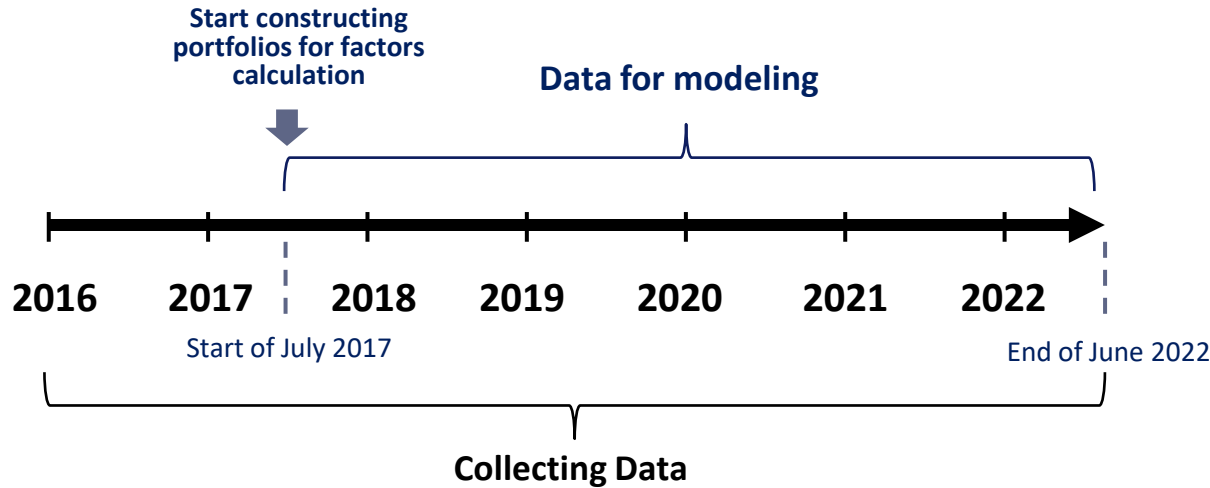- Cahart 4 Factor model
- Fama-French 5 Factor model

3. Model Validation and Performance Measure

- Variance Inflation Factor (VIF)
- Studentized Breusch-Pegan
- Breusch-Godfrey test
- ACF – Autocorrelation Function

- Bayesian Information Criterion (BIC)
- Adjusted R^2
- F-Statistics

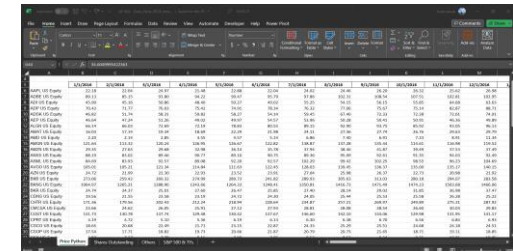| 1. Data Collection	2. Portfolio Construction	3. Model Validation and Performance measure

---

▶ **Models**

1.) Capital market model (CAPM) : $R_i - R_f = \alpha_i + \beta(R_m - R_f) + \epsilon_{i_t}$

2.) Fama-French 3 factor model (FF3) : $R_i - R_f = \alpha_i + \beta_1(R_m - R_f) + \beta_2(SMB_t) + \beta_3(HML) + \epsilon_t$

3.) Carhart 4 factor model (CH4) : $R_i - R_f = \alpha_i + \beta_1(R_m - R_f) + \beta_2(SMB_t) + \beta_3(HML) + \beta_4(MOM) + \epsilon_t$

4.) Fama-French 5 factor model (FF5) : $R_i - R_f = \alpha_i + \beta_1(R_m - R_f) + \beta_2(SMB_{ff5}) + \beta_3(HML) + \beta_4(RMW)$
$$+ \beta_5(CMA) + \epsilon_t$$

▶ **Factors**

- $SMB_t = \dfrac{s \backslash l + s \backslash m + s \backslash h}{3} - \dfrac{b \backslash l + b \backslash m + b \backslash h}{3}$

- $HML = \dfrac{s \backslash h + b \backslash h}{2} - \dfrac{s \backslash l + b \backslash l}{2}$

- $SMB_{INV} = \dfrac{s \backslash a + s \backslash n + s \backslash c}{3} - \dfrac{b \backslash a + b \backslash n + b \backslash c}{3}$

- $CMA = \dfrac{s \backslash c + b \backslash c}{2} - \dfrac{s \backslash a + b \backslash a}{2}$

- $RMW = \dfrac{s \backslash r + b \backslash r}{2} - \dfrac{s \backslash w + b \backslash w}{2}$

- $MOM = \dfrac{s \backslash w + b \backslash w}{2} - \dfrac{s \backslash lo + b \backslash lo}{2}$

- $SMB_{FF5} = \dfrac{SMB_{B/M} + SMB_{OP} + SMB_{INV}}{3}$

- $SMB_{op} = \dfrac{s \backslash w + s \backslash n + s \backslash r}{3} - \dfrac{b \backslash w + b \backslash n + b \backslash r}{3}$

The portfolios were reconstructed at the beginning of July of every year and was used from July of year T to the end of June of year T+1.

Out of 22 portfolios, we selected 18 portfolios that were constructed by fundamental criteria, and used them as a dependend variable in asset pricing models.

Companies in NASDAQ 100

50% Small
Median of Market capitalization

50% Big

Percentile of Book value by Market capitalization

Percentile of Operational Profit

Percentile of Growth in Total asset

Percentile of Yearly return of stock

Percentile of Yearly return of stock

Percentile of Growth in Total asset

Percentile of Operational Profit

Percentile of Book value by Market capitalization

Portfolio	Code	Description
30% Low	s/l	Small stock / Low book to market value
40% Neutral	s/n	Small stock / Neutral book to market value
30% High	s/h	Small stock / High book to market value
30% Weak	s/w	Small stock / Weak operational profitability
40% Neutral	s/m	Small stock / Neutral operational profitability
30% Robust	s/r	Small stock / Robust operational profitability
30% Conservative	s/c	Small stock / Conservative Investment
40% Medium	s/n	Small stock / Medium Investment
30% Aggressive	s/a	Small stock / Aggressive Investment
30% Losers	s/lo	Small stock / Loser Total Return Index
40%		
30% Winners	s/wi	Small stock / Winner Total Return Index
30% Losers	b/lo	Big stock / Loser Total Return Index
40%		
30% Winners	b/wi	Big stock / Winner Total Return Index
30% Conservative	b/c	Big stock / Conservative Investment
40% Medium	b/n	Big stock / Medium Investment
30% Aggressive	b/a	Big stock / Aggressive Investment
30% Weak	b/w	Big stock / Weak operational profitability
40% Neutral	b/m	Big stock / Neutral operational profitability
30% Robust	b/r	Big stock / Robust operational profitability
30% Low	b/l	Big stock / Low book to market value
40% Neutral	b/n	Big stock / Neutral book to market value
30% High	b/h	Big stock / High book to market value
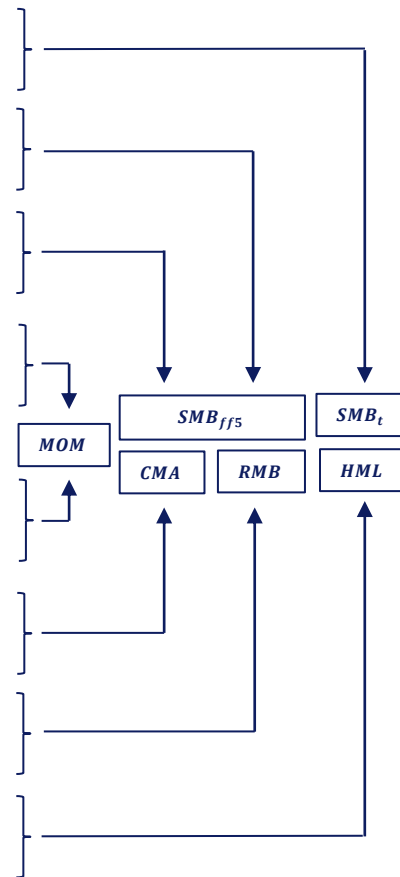
MOM

SMB_{ff5}

SMB_t

CMA

RMB

HML

3.1) Variance Inflation Factor (VIF)

$$VIF = \frac{1}{1 - R_i^2}$$

- Fama French 3 Factor model :

$$R_m - R_f \ / \ SMB \ / \ HML$$

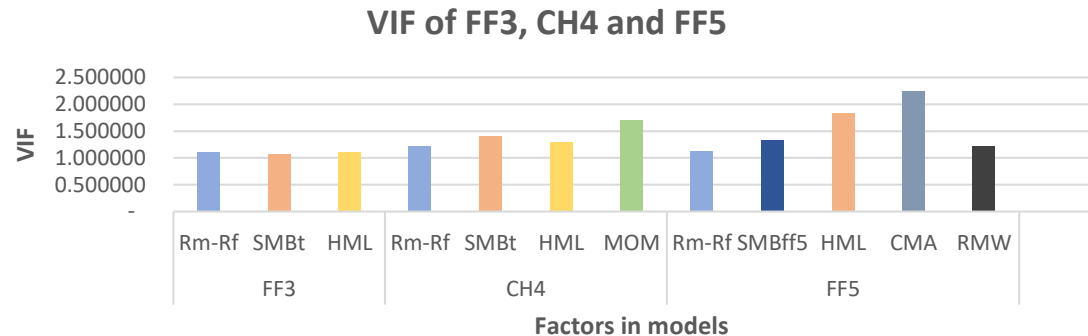- Carhart 4 Factor model :

$$R_m - R_f \ / \ SMB \ / \ HML \ / \ MOM$$

- Fama French 5 Factor model :

$$R_m - R_f \ / \ SMB_{ff5} \ / \ HML \ / \ CMA \ / \ RMW$$

Result of VIF:

VIF of FF3, CH4 and FF5

- One of the methods we used to validate the model is the VIF.
- The VIF measures the severity of multicollinearity in the ordinary least square regression analysis.
- Generally, a VIF above 4 indicates that multicollinearity might exist and further investigation is required.
- Based on the results we have achieved, the VIF is generally less than 2.5, which indicates moderate multicollinearity.
- Hence, it suggests that multicollinearity is not a substantial concern among the variables.

3.2) Breusch–Pagan test

$$H_0 = \sigma_{\epsilon 1}^2 = \sigma_{\epsilon 2}^2 = \sigma_{\epsilon 3}^2 = \ldots = \sigma_{\epsilon}^2$$

Result of BP test

Heatmap of BP Test Results

	s/h	s/n	s/l	b/h	b/n	b/l	s/w	s/m	s/r	b/w	b/m	b/r	s/c	s/n.1	s/a	b/c	b/n.1	b/a
CAPM	0.752	0.713	0.405	0.910	0.004	0.385	0.077	0.631	0.823	0.891	0.001	0.444	0.643	0.231	0.746	0.422	0.977	0.083
FF3	0.000	0.221	0.437	0.217	0.048	0.001	0.084	0.139	0.000	0.199	0.026	0.235	0.001	0.885	0.099	0.887	0.625	0.018
FF4	0.000	0.060	0.114	0.043	0.021	0.002	0.049	0.085	0.000	0.281	0.002	0.321	0.015	0.748	0.042	0.487	0.667	0.033
FF5	0.000	0.187	0.153	0.611	0.111	0.278	0.870	0.568	0.006	0.010	0.028	0.482	0.001	0.348	0.325	0.716	0.641	0.002

1. Data Collection	2. Portfolio Construction	3. Model Validation and Performance measure

3.2) Breusch–Pagan test

$$H_0 = \sigma_{\epsilon 1}^2 = \sigma_{\epsilon 2}^2 = \sigma_{\epsilon 3}^2 = \ ... = \sigma_{\epsilon}^2$$

Explanation for BP test and result

- One of the assumptions of the classical linear regression model is the constant variance in the error terms. Breaking this assumption means estimators are no longer the best Linear Unbiased Estimators because their variances are not the lowest. If we assume there is no heteroskedasticity but in reality there exists, we will get unbiased but inefficient estimates and biased standard errors.

- And BP test is used to test the heteroskedasticity in a linear regression model. The test is derived from Lagrange multiplier, and approximately follows the χ 2 distribution with the null hypothesis of a constant variance. So, if the p-value is below than 0.05 then we reject the null hypothesis.

- From our result, the variable b/m in all 4 models is less than 0.05, which means "b/m" has a significant impact on the heteroskedasticity . We may consider transforming this variable. Like non-linear transformations.
And s/h, s/r, s/c and b/a show the similar situation in other models except CAPM. We could consider some correlation methods like weighted least squares model or heteroskedasticity -robust method to address the effect.

- Overall, CAPM is more tolerant of the heteroskedasticity , while other models could be more sensitive. And for our dataset, the Fama-French 5-factor model is also an acceptable model that provides relatively stable results.
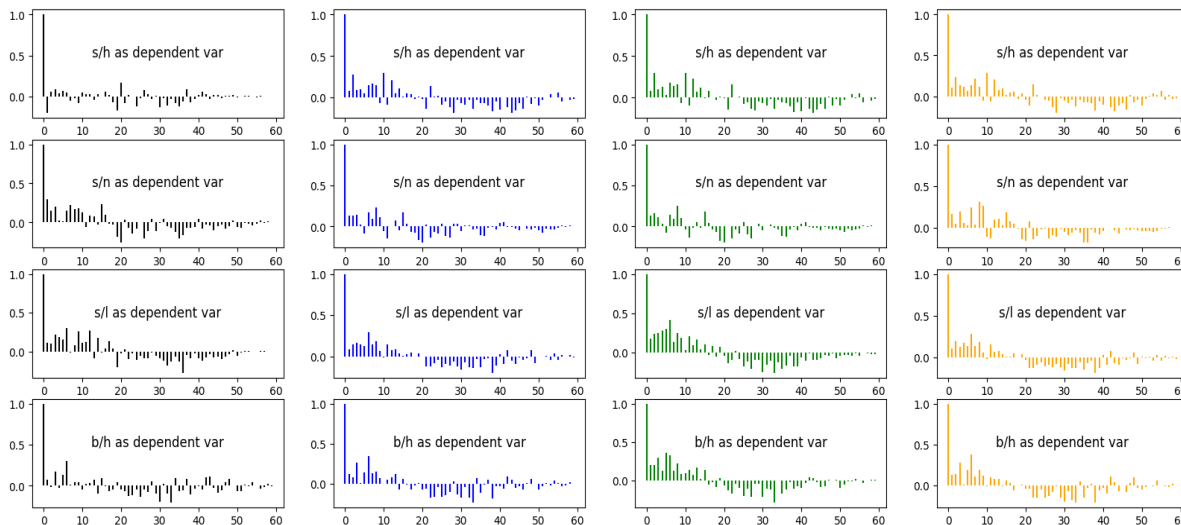
3.3) Autocorrelation Function

$$r_k = \frac{\sum_{t=k+1}^{T}(y_t - \overline{y})(y_{t-k} - \overline{y})}{\sum_{t=1}^{T}(y_t - \overline{y})^2}$$

ACF of an error term:

18 plots per model

- We use Autocorrelation Function (ACF) with the error to check if there's any hidden pattern or correlation in the residuals

- Overall, for all of the models, there is no significant autocorrelation between the error term and the lag value of itself.

3.4) Breusch-Godfrey test

$$H_0 = \rho_1 = \rho_2 = \rho_3 = \dots = \rho_p = 0$$

Result of BG test

Breusch-Godfrey(BG)
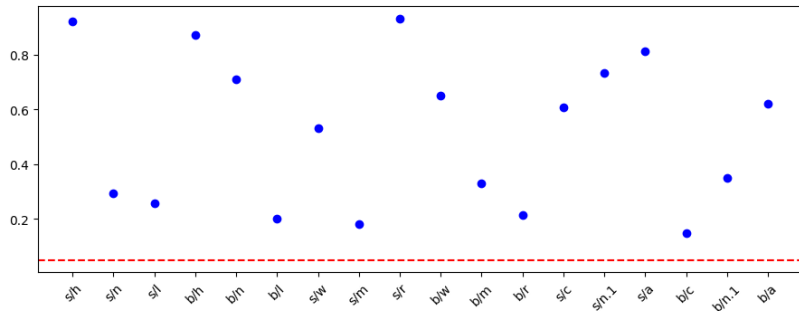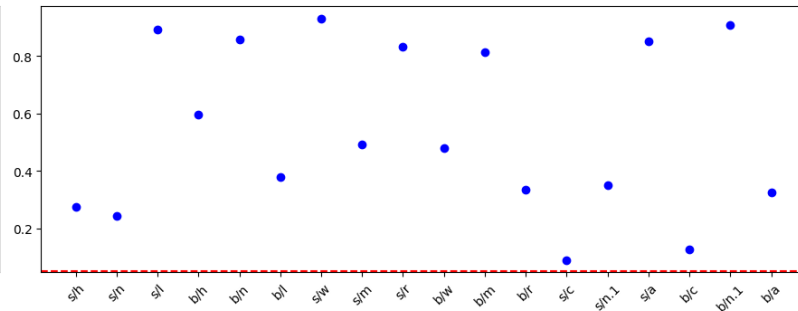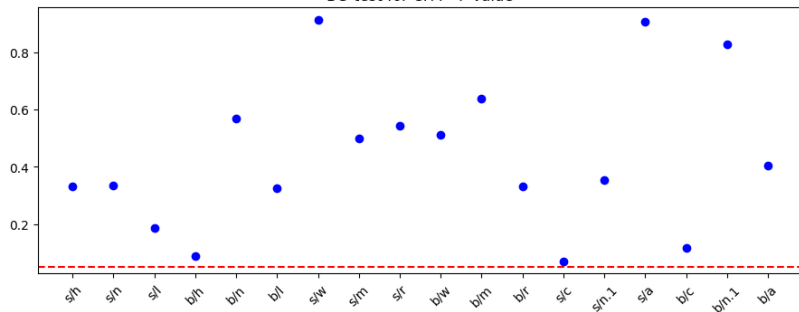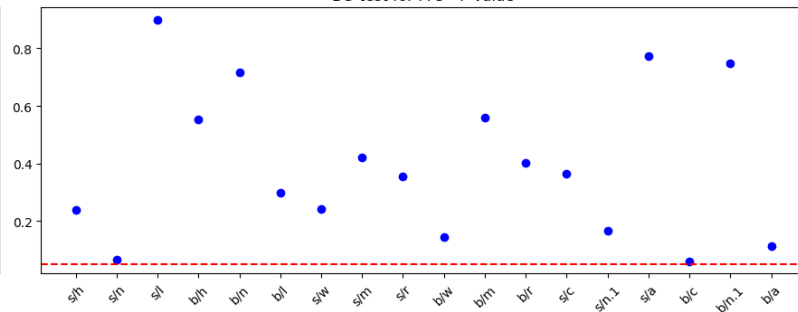
1. Data Collection	**2.** **Portfolio Construction**	**3.** **Model Validation and Performance measure**

3.4) Breusch-Godfrey test

$$H_0 = \rho_1 = \rho_2 = \rho_3 = \dots = \rho_p = 0$$

Explanation for BG test and result

The Breusch-Godfrey test, often abbreviated as BG test, is a diagnostic tool used to detect the presence of autocorrelation in the residuals of a regression model. If present, autocorrelation can invalidate the results of our regression model, as it violates the assumption that the residuals are not autocorrelated.

We first estimate our regression model and obtain the residuals. Next, we regress these residuals on the original independent variable and the lagged values of the residuals. The test statistic is then computed, which follows a chi-squared distribution. We compare the test statistic to the critical value from the chi-squared distribution to determine the presence of autocorrelation. The null hypothesis, Ho, states that there is no autocorrelation up to the p-th order. If the p-value is below a 0.05 significance level, we reject the null hypothesis, indicating the presence of autocorrelation.

we've applied the BG test for each of four models: CAPM, FF3, CH4 and FF5. The dotted red line represents the significance level at 0.05. Looking at the p-values, any value below this line suggests the presence of autocorrelation in the residuals of that model.

From our BG test results, it show that all model consistently shows p-values above the 0.05 significance line, indicating no significant autocorrelation in its residuals across all the time frames considered. However, the absence of autocorrelation is just one aspect of model reliability. Other factors should also be considered when determining the overall best model.
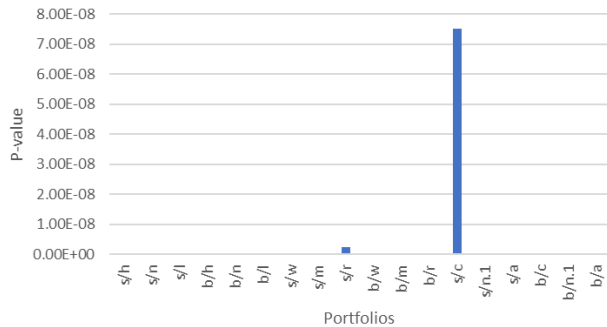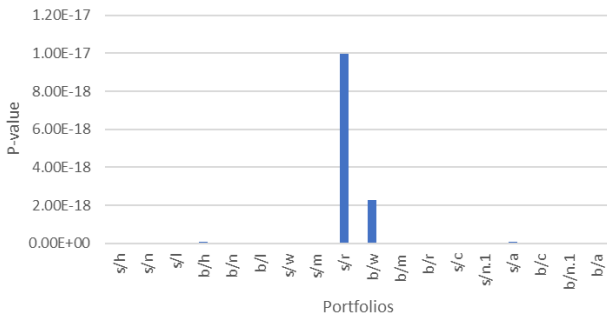
3.5) Performance measurement – F-Statistic

P-value of F-statistic - CAPM

P-value of F-statistic - FF3
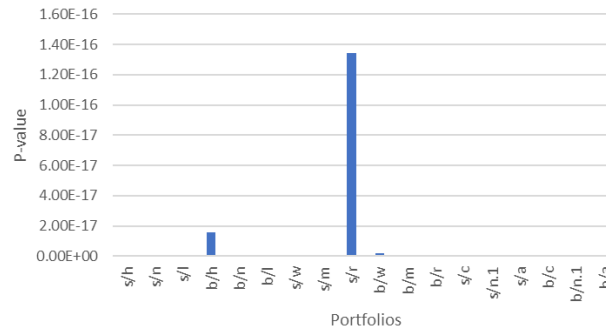
P-value of F-statistic - CH4

P-value of F-statistic - FF5
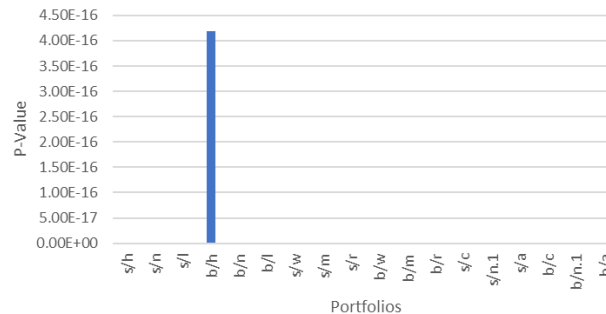
Model	Average p-val
CAPM	4.32e-09
FF3	8.43e-18
CH4	6.93e-19
FF5	2.33e-17

- F-statistic is used to measure the performance of the models. If the p-value is lower that 0.05, it means that the overall of our regression model is significant
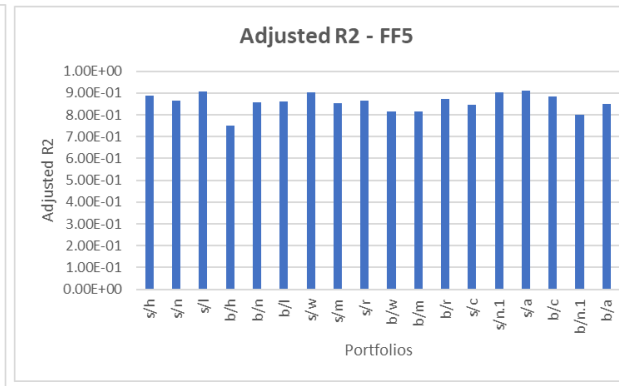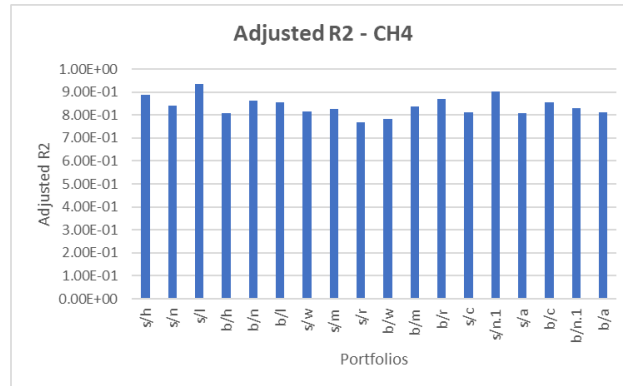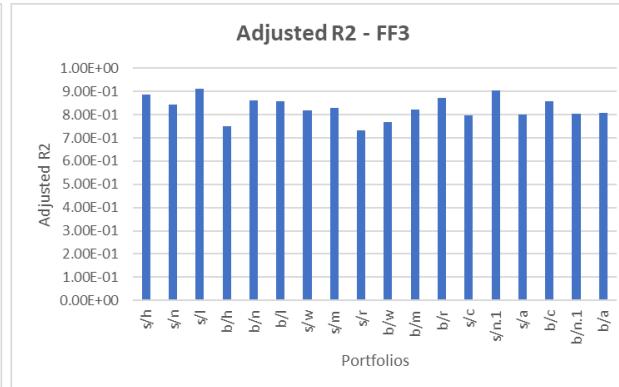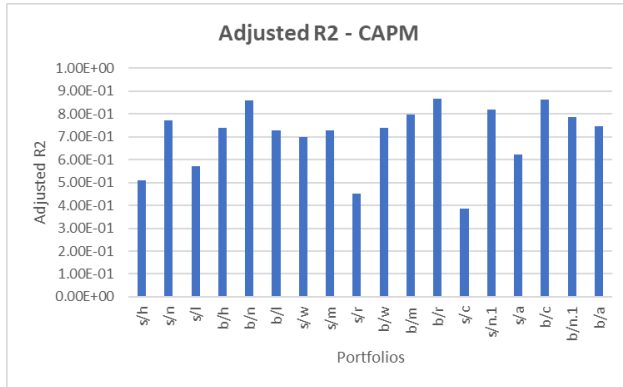
- Based on these results, all the models are statistically significant since their p-values are very close to zero.

3.6) Performance measurement – Adjusted R^2

Adjusted R2 - CAPM

Adjusted R2 - FF3

Adjusted R2 - CH4

Adjusted R2 - FF5

Model	Average Adj. R2
CAPM	0.705
FF3	0.829
CH4	0.840
FF5	0.859

- We used adjusted R2 to measure the performance of our models. It refers to how much of total variation is explained, with a penalizing factor for more parameters.
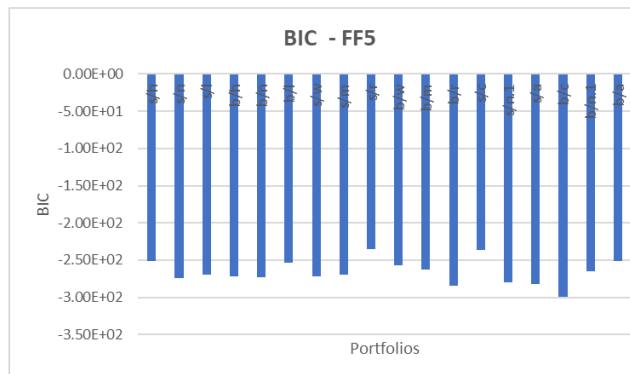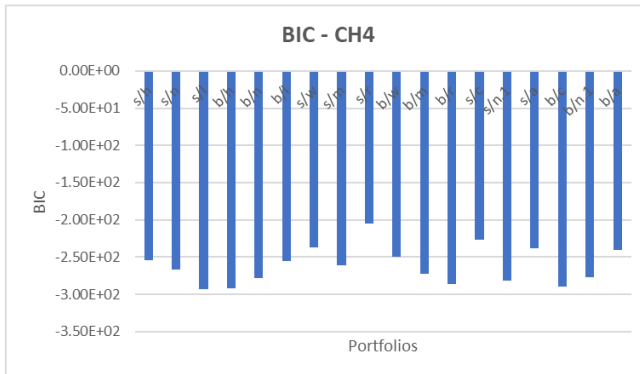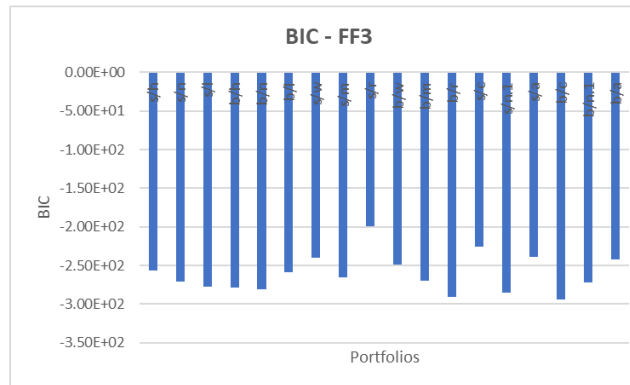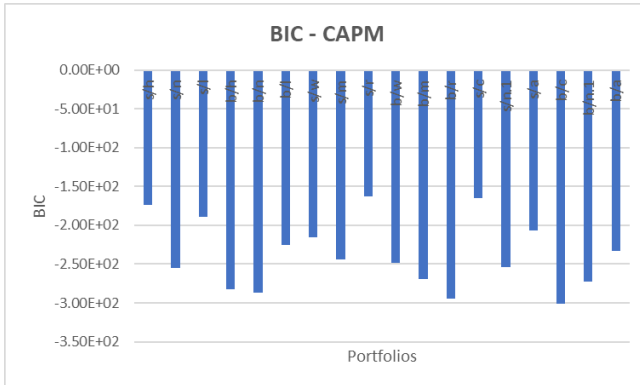- Based on the result on the adjusted R2, FF5 seems to have the best explanatory power among all the models, followed closely by CH4, then FF3, and lastly, CAPM.

3.7) Performance measurement – Bayesian Information Criterion (BIC)

BIC - CAPM

BIC - FF3

BIC - CH4

BIC - FF5

Model	Average BIC
CAPM	-238
FF3	-261
CH4	-261
FF5	-266

- We also used BIC to measure the performance of models. a lower BIC value indicates a better model fit.
- The result of BIC aligns in the same way as adjusted R2, that FF5 is the best fit model followed by CH4, FF3 and CAPM