

# Comparison of sustainability assessment methods for electricity generation technologies

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# 1 Introduction

To assess the sustainability of electricity generating technologies one can use multi-criteria decision methods. These methods support the right choice of implementing a technology based on specified criteria with different weights. The outputs from these methods are scores and ranks of the various technologies. There exists many types of multi-criteria decision methods addressing this topic, all having different characteristics and approaches for ranking the technologies.

## 2 Models

In this study the methods used are collected and replicated from scientific papers. These methods and respective papers are; weighted sum multi-attribute utility method [1], TOPSIS [2] and AHP [3]. All methods can be used for ranking different types of electricity generation technologies. The criteria is based on four main categories; economic, technological, environmental and socio-political.

### 2.1 Weighted sum multi-attribute utility

The first applied approach is the multi-attribute utility method, which is widely applicable as it is a simple method. In this method the ranking of the different technologies is based on the sum of their criteria scores, and the weighting of the criteria. For better comparability the scores are normalized such that each technology has an utility value between 0 to 1. The normalization is performed by using two equations, as the criteria can be direct correlated with utility and inverse correlated with utility.

Criteria values that are directly correlated with utility is calculated by

$$u(x_k) = \frac{x_k - x_{min}}{x_{max} - x_{min}}, \quad (2.1)$$

and criteria values inversely correlated with utility is calculated by

$$u(x_k) = \frac{x_{max} - x_k}{x_{max} - x_{min}}. \quad (2.2)$$

In the equations  $x_k$  is the criteria score of the chosen technology. The smallest score of the criteria is  $x_{min}$ , and  $x_{max}$  represents the largest score in the respective criteria.

Subsequently, the utility values are multiplied with the criteria weighting, and summed to one total score for each technology. Based on the total score the various technologies can be compared regarding their sustainability, where the largest value is ranked as the best option and smallest value as the worst option.

## 2.2 TOPSIS

TOPSIS ranks the technologies based on the closeness from the ideal solution, and how far away it is from the negative ideal solution. Criteria are defined as  $n$ , with  $m$  alternatives. The matrix with criteria and alternatives is normalized to value  $r_{ij}$ , where  $f$  is the value for alternative in criteria  $i_j$

$$r_{ij} = \frac{f_{ij}}{\sqrt{\sum_{j=1}^m f_{ij}^2}}. \quad (2.3)$$

The normalized weighted value  $v_{ij}$  is calculated by multiplication of the weights and the normalized values,

$$v_{ij} = w_i r_{ij}. \quad (2.4)$$

The positive ideal solution  $A^+$  and negative ideal solution  $A^-$  are from the equations below. Where  $I'$  and  $I''$  is the benefit and cost criteria, which indicate if the value is a positive or a negative variable,

$$A^+ = \{v_1^+, \dots, v_n^+\} = \{(MAX_j v_{ij} | i \in I'), (MIN_j v_{ij} | i \in I'')\} \quad (2.5)$$

$$A^- = \{v_1^-, \dots, v_n^-\} = \{(MIN_j v_{ij} | i \in I'), (MAX_j v_{ij} | i \in I'')\}. \quad (2.6)$$

$D_j^+$  and  $D_j^-$  represent the distance to the positive ideal solution and the negative ideal solution for each alternative respectively,

$$D_j^+ = \sqrt{\sum_{i=1}^n (v_{ij} - v_i^+)^2} \quad (2.7)$$

$$D_j^- = \sqrt{\sum_{i=1}^n (v_{ij} - v_i^-)^2}. \quad (2.8)$$

Finally, the relative closeness to the ideal solution is calculated by,

$$C_j = \frac{D_j^-}{(D_j^+ + D_j^-)}. \quad (2.9)$$

## 2.3 Analytic hierarchy process

The contents in this section are referenced and integrating from a lecture materials [4], the lecture slides[5] and the textbook[3].

The Analytic hierarchy process (AHP), introduced by Saaty, T.L. (1980) [6], is a powerful method to deal with the multi-criteria decision analysis. In addition, the AHP allows to rank different alternatives based on expert opinions.

The AHP can be implemented in three steps [4]:

1. Computing the vector of criteria weights.
2. Computing the matrix of option scores.
3. Ranking the options.

Each step will be discussed in detail in the following sections. It is assumed that there are  $m$  criteria and  $n$  alternatives. As seen on the figure 2.1, this is the whole structure for this study with AHP method. There are three different layers which will be instituted in the following. First of all, it is a objective layer where the research objective could be established. Following, it is a criteria layer where the criteria could be introduced in this layer. For example, there are 10 different criteria in this study. The criterion could be seen as an index to evaluate different alternatives. Finally, it is an alternatives layer where the option could be set up in this layer. In other words, all the options in this layer are the target which the researcher wants to rank. For instance, there are 11 various technologies in this study.

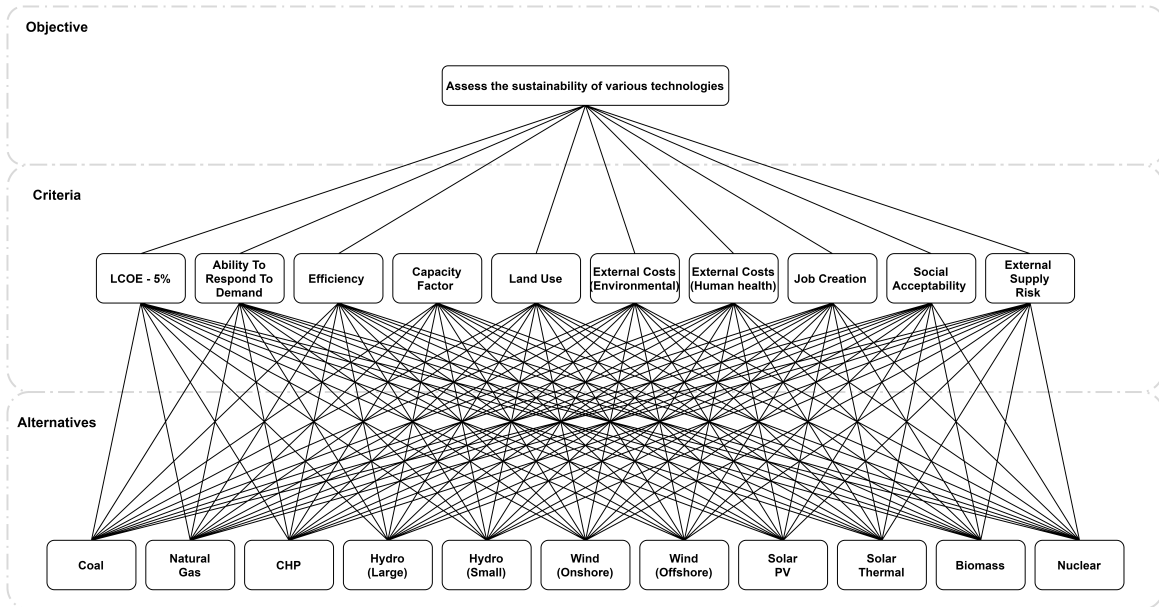


Figure 2.1: AHP tree with objective layer, criteria layer and alternatives layer.

### 2.3.1 Computing the vector of criteria weights

In order to compute the weights for the different criteria, the AHP starts from a pairwise comparison matrix  $\mathbf{A}$ . The matrix  $\mathbf{A}$ , which can be seen in the equation 2.10, is a  $m \times m$  real matrix, where  $m$  is the number of evaluation criteria.

$$A_{m,m} = \begin{bmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,m} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m,1} & a_{m,2} & \cdots & a_{m,m} \end{bmatrix} \quad (2.10)$$

Concerning the expression of the elements in the matrix,  $a_{i,j}$  means it is located in the  $i$ th row and the  $j$ th column.

Each element in matrix  $\mathbf{A}$  such as  $a_{j,k}$  represents the importance of the  $j$ th criterion relative to the  $k$ th criterion. If  $a_{j,k}$  is greater than 1, then the  $j$ th criterion is more important than the  $k$ th criterion. On the other hand,  $a_{j,k}$  is less than 1, then the  $j$ th criterion is less important than the  $k$ th criterion. If two criteria are equal important, then the element  $a_{j,k}$  will be given as 1. Furthermore, the elements  $a_{j,k}$  and  $a_{k,j}$  should satisfy the following relation:

$$a_{j,k} \cdot a_{k,j} = 1 \quad (2.11)$$

For the diagonal elements such as  $a_{j,j}$ , where  $j = 1, 2, \dots, m$ . The importance relative to itself should be one, i.e.  $a_{j,j} = 1$ . The relative importance can be between 1 and 9. Moreover, how to give the score in the pairwise comparison matrix to the relative elements is shown in the table 2.1.

Table 2.1: The fundamental scale of pairwise comparison matrix [6].

*Intensity of importance	1	3	5	7	9
Definition	Equal importance	Moderate importance	Strong importance	Very strong importance	Extreme importance
Explanation	Two elements contribute equally to the objective	Experience and judgment slightly favor one element over another	Experience and judgment strongly favor one element over another	One element is favored very strongly over another, its dominance is demonstrated in practice	The evidence favoring one element over another is of the highest possible order of affirmation

\*Intensities of 2, 4, 6, and 8 could be used to express intermediate values.

\*Intensities could be decimal such as 1.1, 1.2, 1.3, etc. The decimal expression can be used for elements that are very close in importance.

After building matrix  $\mathbf{A}$ , the next step is to normalize the pairwise comparison matrix  $\mathbf{A}_{norm}$  for having the sum of each element in each column of matrix  $\mathbf{A}$  equal to 1. The normalized element label as  $\overline{a_{j,k}}$  is illustrated in equation 2.12.

$$\overline{a_{j,k}} = \frac{a_{j,k}}{\sum_{l=1}^m a_{l,k}} \quad (2.12)$$

Eventually, the *criteria weight vector*  $\mathbf{w}$ , which is a  $m$ -dimensional column vector, can be obtained by doing arithmetic mean of the elements in each row of  $\mathbf{A}_{norm}$ , which is shown in the equation 2.13.

$$\mathbf{w}_j = \frac{\sum_{l=1}^m \overline{a_{j,l}}}{m} \quad (2.13)$$

### 2.3.2 Computing the matrix of option scores

The option scores matrix is a  $n \times m$  real matrix  $\mathbf{S}$ . Each element  $s_{i,j}$  in the  $\mathbf{S}$  represents the score of the  $i$ th option respects to the  $j$ th criterion.

To obtain the scores, a *pairwise comparison matrix*  $\mathbf{B}^{(j)}$  is first created for each of the  $m$  criteria, where  $j$  is represented from 1 to  $m$  criterion.

The matrix  $\mathbf{B}^{(j)}$  is a  $n \times n$  real matrix, where  $n$  is the number of options. Each element  $b_{i,h}^{(j)}$  of the matrix  $\mathbf{B}^{(j)}$  describes the evaluation of the  $i$ th option in comparison to the  $h$ th option respects to the  $j$ th criterion. As long as  $b_{i,h}^{(j)}$  is greater than 1, then the  $i$ th option is better than the  $h$ th option. If  $b_{i,h}^{(j)}$  is smaller than 1, then the  $i$ th option is worse than the  $h$ th option. If two options are equally important with respect to the  $j$ th criterion, then the element should be 1. Besides, the elements  $b_{i,h}^{(j)}$  and  $b_{h,i}^{(j)}$  should respect the relation in the following:

$$b_{i,h}^{(j)} \cdot b_{h,i}^{(j)} = 1 \quad (2.14)$$

An evaluation scale is as same as the one which is introduced in table 2.1 already.

Next, the AHP applies to each matrix  $\mathbf{B}^{(j)}$  with the same procedure, which is described in how to achieve the pairwise comparison matrix  $\mathbf{A}$ . That divide each element by the sum of the all elements in the same column, and making the averages the elements on each row, finally, the score vectors  $\mathbf{s}^{(j)}$ ,  $j = 1, \dots, m$  can be obtained. The score vector  $\mathbf{s}^{(j)}$  consists of the scores of the evaluated options with corresponding to the  $j$ th criterion.

Eventually, the score matrix  $\mathbf{S}$  is achieved in the equation 2.15, where the  $j$ th column of  $\mathbf{S}$  represents to  $\mathbf{s}^{(j)}$ .

$$\mathbf{S} = [\mathbf{s}^{(1)} \mathbf{s}^{(2)} \mathbf{s}^{(3)} \dots \mathbf{s}^{(m)}] \quad (2.15)$$

### 2.3.3 Ranking the options

Now, the *weight vector*  $\mathbf{w}$  and the *score matrix*  $\mathbf{S}$  are calculated already, the AHP has a vector  $\mathbf{v}$  of global scores by multiplying  $\mathbf{S}$  and  $\mathbf{w}$ , the result is shown in the equation 2.16.

$$\mathbf{v} = \mathbf{S} \cdot \mathbf{w} \quad (2.16)$$

The  $i$ th element  $v_i$  of  $\mathbf{v}$  represents the global score which is given by the AHP method to the  $i$ th option. As the last step, the option ranking is achieved by ordering the global scores in decreasing order. In other words, the highest score means it is the best option in comparison to the others.

### 2.3.4 Consistency Check

With pairwise comparison method, inconsistency may arise. We can look into the following example to explain what is inconsistency:

- A is more important than B
- B is more important than C
- **C is more important than A**

In the third statement, the inconsistency occur between A and C. Therefore, checking inconsistency in the pairwise comparison matrix should be introduced. Raising matrix  $\mathbf{A}$  for consistency check as an example, as seen in the equation 2.17.

$$u_j = \frac{(\mathbf{A} \cdot \mathbf{w})_j}{w_j} \quad (2.17)$$

Furthermore, calculating  $x$  from doing the average in the vector  $u$  in the above equation 2.17, the  $x$  and  $u$  relation is shown in equation 2.18 ,where  $n$  is the number of all elements in the vector  $u$ .

$$x = \frac{\sum_{k=1}^n u_k}{n} \quad (2.18)$$

Finally, the consistency index is defined in the equation 2.19.

$$C.I. = \frac{x - m}{m - 1} \quad (2.19)$$

In the consistency index, smaller value is better, consistent value = 0 which is under perfect condition.



Evaluation of  $C.I.$  is usually done concerning the consistency that would be obtained for completely random orders of  $\mathbf{A}$  which is called random index ( $R.I.$ ). The relation for *consistency ratio* is shown in equation 2.20. Besides,  $C.R. < 1$  is a common tolerable magnitude.

$$C.R. = \frac{C.I.}{R.I.} \quad (2.20)$$

Since the  $R.I.$  values from *T. L. Saaty* is not supported the matrix order higher than 10 [7], new random index ( $R.I.$ ) in table 2.2 is provided by Alonso and Lamata which is published in 2006 [8]. The table 2.2 can support the order of matrix up to 15 which can cover the order of matrix which is implemented in this research.

Table 2.2: Random Index for consistency check [8].

Order	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$R.I.$	0	0	0.5245	0.8815	1.1086	1.2479	1.3417	1.4056	1.4499	1.4857	1.5141	1.5365	1.5551	1.5713	1.5838

### 2.3.5 Input matrices

Feeding data to create the matrix  $\mathbf{A}$ , firstly, used the weighting of different technologies for the normalization, the weighting is given in table 3.2. The weighting will distribute from 0 to 1. For building the matrices  $\mathbf{B}$ , the feeding data is shown in table 3.1. For the normalization equation 2.1 and 2.2, in order to convert all the data into direct correlation. The last step for both matrix  $\mathbf{A}$  and matrices  $\mathbf{B}$  are the same. Discretizing the normalized data into 1 to 9 scale, in order to respect the score system in AHP.

## 3 Data Input

The technologies investigated and their respective sustainability criteria scores are shown in table 3.1. There are 14 technologies included in the study, and 10 criteria. The technologies and criteria scores are collected from [1], but three technologies are excluded in this study, as there was not sufficient information about the criteria scores in these options.

Table 3.1: The technologies investigated in this study, and their criteria scores.

	Coal	Natural gas	CHP	Hydro (large)	Hydro (small)	Wind (onshore)	Wind (offshore)	Solar PV	Solar thermal	Biomass	Nuclear
LCOE	64.37	78.06	62.81	26.35	124.97	76.28	128.68	202.94	177.80	72	53.79
Ability to respond to demand	0.5	1	0.5	1	0	0	0	0	0.5	0.5	0.5
Efficiency	48	59	79	100	100	100	100	100	40	40	33
Capacity factor	85	85	85	54	50	27	27	20	45	70	85
Land use	0.39	0.31	0.35	4.1	0.02	1.57	2.76	0.33	0.46	12.65	0.12
External costs (environmental)	360.15	256	305.95	10.89	10.89	2.74	2.74	4.37	4.37	11.25	0.65
External costs (human health)	390.15	16	173.23	44.89	44.89	11.26	11.26	31.97	31.97	361.25	47.07
Job creation	0.11	0.11	0.11	0.55	0.27	0.17	0.17	0.87	0.23	0.21	0.14
Social acceptability	0	0.5	0	1	1	1	1	1	1	0.5	0
External supply risk	1.6	9.8	5.7	0	0	0	0	0	0	0	1.8

Sustainability criteria and their respective weights are shown in table 3.2. The selection of the appropriate criteria is done in the perspective of electricity generation technologies, and not general energy systems. Weighting of the criteria is decided by a questionnaire of academics.

Table 3.2: Sustainability criteria and their respective weighting applied in the assessment.

Indicator	Weight
LCOE	0.114
Ability to respond to demand	0.112
Efficiency	0.114
Capacity factor	0.097
Land use	0.088
External costs (environmental)	0.109
External costs (human health)	0.110
Job creation	0.082
Social acceptability	0.076
External supply risk	0.099

## 4 Results

A comparison of the three different methods investigated in this study is shown in figure 4.1. It is important to point out that for the AHP method, the scores are reverse. Consequently, the smallest score represent the most desired option. This is in constrast to the other two methods, where the largest score gives the best technology option. The ranks can also be seen in table 4.1. All of the methods, weighted sum multi-attribute utility, TOPSIS and AHP rank large hydro technology as the best option based on the applied sustainability criteria. The least desired option for the weighted sum multi-attribute utility and AHP methods is coal, while for TOPSIS it is natural gas.

It should be noted that only natural gas technology gives approximately the same score for weighted sum multi-attribute utility and TOPSIS, but not the same rank.

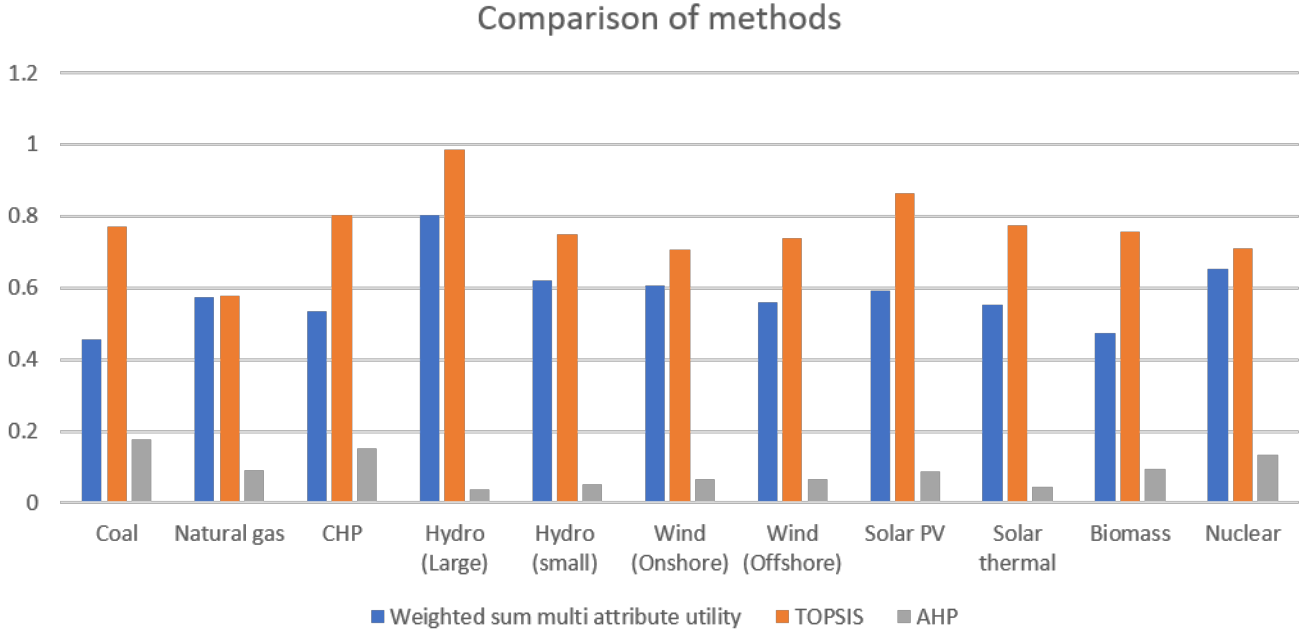


Figure 4.1: Ranking based on the weighted sum multi-attribute utility model. For AHP the smallest score represents the most desired technology, while for weighted sum multi-attribute utility and TOPSIS the largest score is the best option.

Table 4.1: The ranking of the various technologies with the studied methods.

	Coal	Natural gas	CHP	Hydro (large)	Hydro (small)	Wind (onshore)	Wind (offshore)	Solar PV	Solar thermal	Biomass	Nuclear
Weighted sum multi attribute utility	11	6	9	1	3	4	7	5	8	10	2
TOPSIS	5	11	3	1	7	10	8	2	4	6	9
AHP	11	7	10	1	3	4	5	6	2	8	9

## 5 Discussion & Conclusion

As seen on figure 4.1, there is a big variation in the scores among the methods. The comparison of scores given by the methods is complicated, as the methods use inconsistent scoring. This applies especially for AHP, which has a reverse scoring. AHP has been criticized for its possible rank reversal phenomenon [9][10][11]. The comparison of ranks is therefore better seen in table 4.1, where only the final ranks are shown. However, the variation of ranks can also be observed here. The only technology which has the same ranking in all three methods is large hydro. This can be due to the different nature of the methods and their complexity, additional to their use of different scales

for scoring. Deciding the best method is difficult, as the selection of a method must be seen in perspective and there are many factors having an impact on the selection. This can for example be availability of data and how detailed and complex the analysis should be. All methods have their weaknesses and strengths which should be known before selecting the most appropriate method.

Further work would find a more precise approach to compare the methods, independent from their scale of scoring. It would also be desirable to investigate more methods.

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