

CS5200 Homework 5 Theory

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Question 1

$$\text{Calories: } 300 = 60X + 60Y$$

$$\text{Vitamin A: } 36 = 12X + 6Y$$

$$\text{Vitamin C: } 90 = 6X + 30Y$$

$$\text{Cost} = 1.2X + 1.5Y \text{ Solution:}$$

$$X = 2.5 \text{ cups}$$

$$Y = 2.5 \text{ cups}$$

Question 2

$$P = 20x_1 + 10x_2 + 15x_3$$

$$3x_1 + 2x_2 + 5x_3 \leq 55$$

$$2x_1 + x_2 + x_3 \leq 26$$

$$x_1 + x_2 + 3x_3 \leq 30$$

$$5x_1 + 2x_2 + 4x_3 \leq 57$$

$$x_1, x_2, x_3 \geq 0$$

$$P = 20x_1 + 10x_2 + 15x_3$$

$$3x_1 + 2x_2 + 5x_3 + s_1 = 55$$

$$2x_1 + x_2 + x_3 + s_2 = 26$$

$$x_1 + x_2 + 3x_3 + s_3 = 30$$

$$5x_1 + 2x_2 + 4x_3 + s_4 = 57$$

$$x_1, x_2, x_3, s_1, s_2, s_3, s_4 \geq 0$$

$$x_1 = 0, x_2 = 0, x_3 = 0, s_1 = 55, s_2 = 26, s_3 = 30, s_4 = 57$$

$$P = 0$$

Non-basic set: $\{x_1, x_2, x_3\}$

Basic set: $\{s_1, s_2, s_3, s_4\}$

$$3x_1 + 2x_2 + 5x_3 + s_1 = 55 \Rightarrow x_1 = 55/3$$

$$2x_1 + x_2 + x_3 + s_2 = 26 \Rightarrow x_1 = 26/2$$

$$x_1 + x_2 + 3x_3 + s_3 = 30 \Rightarrow x_1 = 30$$

$$5x_1 + 2x_2 + 4x_3 + s_4 = 57 \Rightarrow x_1 = 57/5 < -tightest$$

$$x_1 = 11.4, x_2 = 0, x_3 = 0, s_1 = 55, s_2 = 26, s_3 = 30, s_4 = 0$$

$$P = 260$$

Non-basic set: $\{x_2, x_3, s_4\}$

Basic set: $\{x_1, s_1, s_2, s_3\}$

$$3x_1 + 2x_2 + 5x_3 + s_1 = 55 \Rightarrow x_2 = 10.4$$

$$2x_1 + x_2 + x_3 + s_2 = 26 \Rightarrow x_2 = 3.2 < -tightest$$

$$x_1 + x_2 + 3x_3 + s_3 = 30 \Rightarrow x_2 = 18.6$$

$$5x_1 + 2x_2 + 4x_3 + s_4 = 57 \Rightarrow x_2 = \text{no restriction}$$

$$x_1 = 13, x_2 = 0, x_3 = 0, s_1 = 55, s_2 = 0, s_3 = 30, s_4 = 57$$

$$P = 260$$

Non-basic set: $\{s_2, x_2, x_3\}$

Basic set: $\{x_1, s_1, s_3, s_4\}$

Question 3

Since we are given the nodes one at a time we can sort the nodes as we get them. It would only take n time to insert a node into a sorted list. Since it only takes n manipulations after that, for each new node we could find the convex hull in n^2 steps.

Question 4

Yes A is in P.

Yes A is in NP.

He should not get the million dollar reward because all he showed was that P is a subset of NP problems. In order to get the prize he would need to show that a np complete problem could be reduced to a problem in P.