Tools for Planning and Analyzing Randomized Controlled Trials and A/B Tests

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WPI

While we are getting settled...

tinyurl.com/
sree-drct

Follow the steps at:

`https://tinyurl.com/sree-drct`

to get everything ready to follow along in *`RStudio`*!

Today's Plan

tinyurl.com/sree-drct

- 9:00–9:15 – Part I: Conceptual Overview
- 9:15–10:30 – Part II: Estimating Effects with RCT Data
- 10:30–11:00 – Part III: Incorporating Auxiliary Data
- 11:00–11:15 – Break 15 min
- 11:15–11:45 – Part IV: Treatment Effect Heterogeneity
- 11:45–12:15 – Part V: Planning Experiments

What You Need

tinyurl.com/sree-drct

- Tutorial website: *https://tinyurl.com/edmrct*
- RStudio
- Clone repo from Github: *https://github.com/mannczz/edm-rct-tutorial/*

Tutorial Structure

tinyurl.com/sree-drct

We will be alternating between:

- Conceptual descriptions of the methods
- Detailed walk-throughs of the software
- Opportunities for you to run analyses yourself, with our help

Please feel free to ask questions at any time!

- Calling out (unmute yourself if on Zoom)
- Zoom chat
- Any other way you can think of to get our attention

Conceptual Overview

Estimating Effects with RCT Data

Incorporating Auxiliary Data

Treatment Effect Heterogeneity

Planning Experiments

Experiments in Education Research

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"Experiment" = "RCT" = "Randomized Controlled Trial"

- Randomize subjects (students? teachers? schools?) between condition
- Expose subjects to their randomized conditions
- Measure outcome(s) of interest

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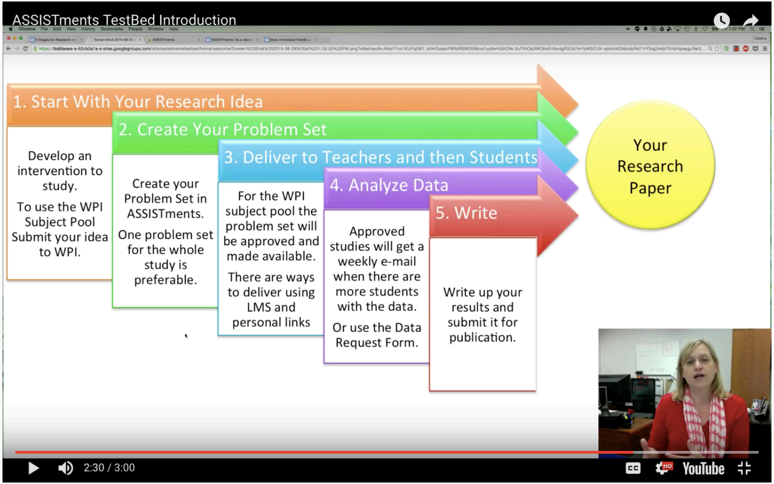
- Typical examples:
  - A/B tests in online learning
  - Field trials of (say) new curriculum vs. business as usual

Example 1: ASSISTments ETrials



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Integrated A/B test platform

Example 1: ASSISTments E-trials

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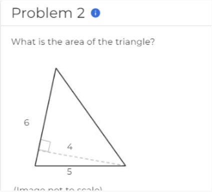
- Question: Text or video hints?
- Outcome: Complete skill builder?
- $n = 683$ middle school students



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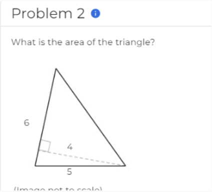

Results,

- Video: 205/337 (61%) completed
- Text: 193/346 (56%) completed

Example II: Cognitive Tutor Effectiveness Trial

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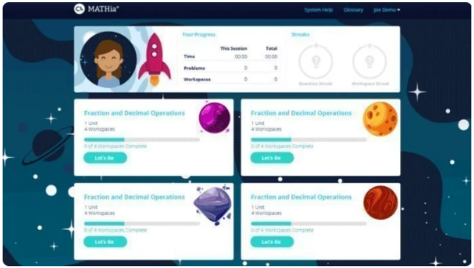
- 73 High Schools & 74 Middle Schools in 7 states
- Similar schools paired
- In each pair, one school randomized to treatment, one to control
- Algebra I students in Trt school used CTAI, Control school used business as usual
- All students took a posttest at the end of the year



Example II: Cognitive Tutor Effectiveness Trial

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Results

	Average Posttest			
	Middle		High	
	Year 1	Year 2	Year 1	Year 2
Control	17.4	16.9	10.3	9.7
Treatment	14.3	15.2	10.1	10.6

Scientific Goals

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1. What is the average effect of [intervention] on [outcome]?

2. How Does the effect vary?

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2. How Does the effect vary?
   - From one (type of) student to the next
   - From one context to the next

Statistical Goals

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1. Get the most out of your data: more data $\rightarrow$ better estimation!!
2. ...Without making unnecessary assumptions
3. Easily
4. Design better experiments to start with

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3. Easily
   - i.e. without a PhD in statistics
   - Use our software package :)
4. Design better experiments to start with

Types of Variables: Baseline Covariates

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Uses:

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- Log data. For each previous skillbuilder,
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  - Time to mastery
  - ...
- Demographic data

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Don’t use post-treatment variables!

Auxiliary Data

- Covariate and outcome data from *other* subjects
- Often: historical data

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  - Separate sample from RCT
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Reiterate Goals

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Estimate treatment effects

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Using all our data

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Without bias or extra assumptions

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Potential Outcomes (Neyman-Rubin)

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Consider a randomized experiment with:

- N participants
- One treatment group, one control group

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  The other is a counterfactual.

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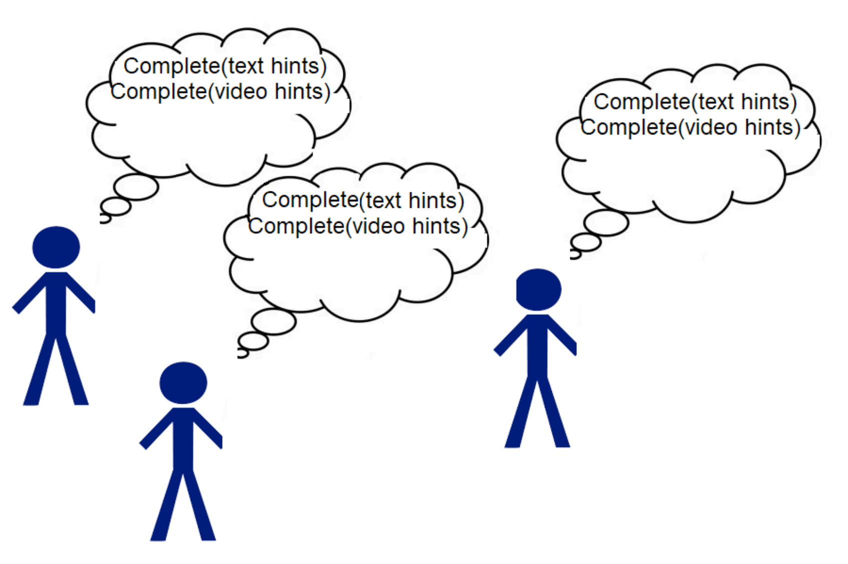

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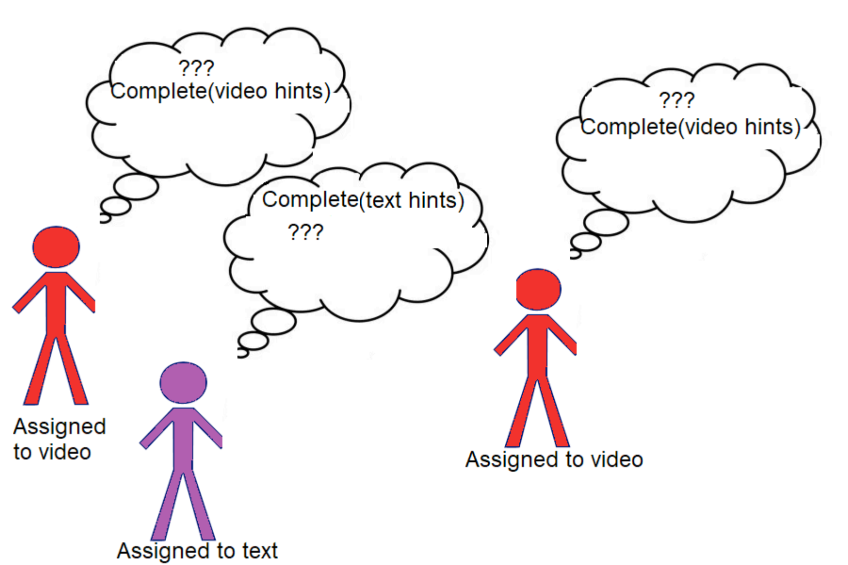

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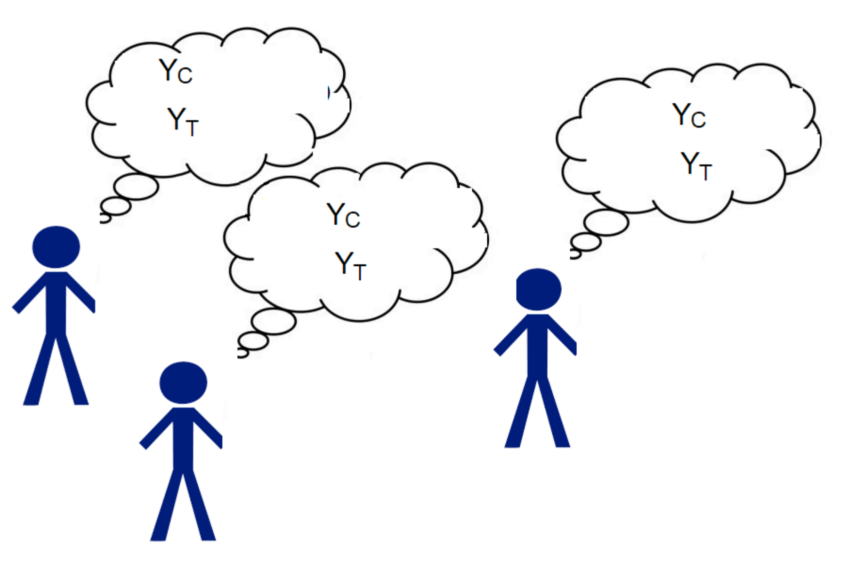

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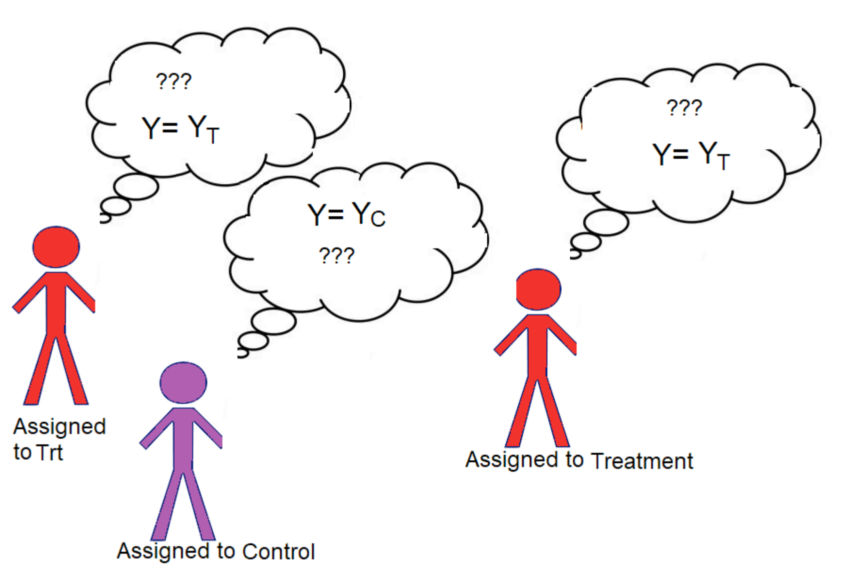

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- Let Y_i be the observed outcome for unit i. If unit i is assigned to treatment, we observe y_i^t; otherwise, we observe y_i^c:

$$Y_i = \begin{cases} y_i^c & \text{if } T_i = 0 \\ y_i^t & \text{if } T_i = 1 \end{cases}$$

Individual and Average Treatment Effects

- The individual treatment effect is

$$\tau_i = y_i^t - y_i^c$$

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- Also: average effects for subgroups of subjects (more later)

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tinyurl.com/sree-drct

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- We only observe one potential outcome for each subject
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  2. Use covariates & and model: biased, but precise
  3. Our approach: use both!

Our Method, in a Nutshell

Step 1:

Train algorithms to predict y^c, y^t as a function of covariates

$$f^c : \mathbf{X} \to y^c \text{ (use data from ctl group)}$$

$$f^t : \mathbf{X} \to y^t \text{ (use data from trt group)}$$

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Step 4:

Use randomization-based method to estimate effects on $Y - \hat{m}$ instead of Y

Important Caveat

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For this to be *strictly* unbiased, we need:

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"Leave-One-Out Potential Outcomes" or "LOOP"

Sticks and Stones May Break my Bones, but Bad Models Won't Hurt Me

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- Estimate will still be unbiased!
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- (core of inference is based on randomization)
- Covariate adjustment won't help much
- In moderate/large samples, it won't hurt either!

Digression: What about old-fashioned regression?

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Regression method:

Fit model:

$$Y_i = \beta_0 + \beta_1 T_i + \beta_2 X_{1i} + \beta_3 X_{2i} + \ldots$$

Estimated effect: $\hat{\beta}_2$

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Good news: $\hat{\beta}$ is *approximately* unbiased *in large samples*

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1. *Exactly* unbiased *in any sample*
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   - Flexible for non-linearity, interactions

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   - $\Rightarrow$ better effect estimates
   - We recommend random forest

What You Need for Our Method

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4. What is the experimental design?

One last digression[1]: experimental designs

What You Need for Our Method

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1. Randomized treatment variable
2. Outcome variable
3. Covariates
4. What is the experimental design?

One last digression[1]: experimental designs

[1] This is not a promise.

The Two Questions of Experimental Design

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1. Who or What is being randomized?
2. How are they being randomized?

Who/What is being randomized?

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- Individual randomization
- Cluster or Group randomization

How are they being randomized?

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- What's the probability each unit is assigned to treatment?
- How does one unit's assignment affect other units?

Examples We'll Cover

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- Individual randomization
  - Bernoulli
  - Paired
- Cluster randomization
  - Paired

Examples

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- ASSISTments E-Trials A/B test
  - Students are randomized individually
  - Students are randomized independently
  - $\Rightarrow$ Bernoullli

Examples

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- ASSISTments E-Trials A/B test
  - Students are randomized individually
  - Students are randomized independently
  - $\Rightarrow$ Bernoullli
- Cognitive Tutor Effectiveness Study
  - *Schools* are randomized
  - Randomization is within pairs
  - (if your school is randomized to treatment, its pair *must* be randomized to control)
  - $\Rightarrow$ paired cluster design

Other Designs

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To be implemented (hopefully) soon:

- "Completely randomized design"
    - At the outset, fix # randomized to treatment, # randomized to control
    - Now T_i and T_j are dependent!
- Block-randomized design
    - e.g. a separate completely randomized experiment in each classroom
    - Paired designs are a special case

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Probably won’t get to for a while:

- Bandit designs
  - Probability i is assigned to treatment depends on previous subjects’ outcomes

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Estimating Effects in Practice

dRCT Package Overview

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Installation:

- You will need to install the package from Github using the `devtools` package in *R*
- e.g. `install_github("manncz/dRCT")`

Primary Functions:

```
loop(Y, Tr, Z, pred, p, ...)
p_loop(Y, Tr, Z, pred, P, n, ...)
```

Covariate Adjustment with Bernoulli Randomized Trails (LOOP)

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`loop(Y, Tr, Z, pred, p, ...)`

- `Y`: outcome vector
- `Tr`: treatment assignment vector
- `Z`: matrix of covariates
- `pred`: interpolation algorithm
- p: probability of treatment
- `...`: optional inputs for interpolation algorithm

LOOP interpolation algorithms

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pred

- *loop_rf*
- *loop_ols*
- *loop_glm*

Covariate Adjustment with Paired Trails (P-LOOP)

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```
p_loop(Y, Tr, Z, pred, P, n, ...)
```

- `Y`: outcome vector
- `Tr`: treatment assignment vector
- `Z`: matrix of covariates
- `pred`: interpolation algorithm
- P: vector of pair assignments
- n: optional vector of cluster sizes
- `...`: optional inputs for interpolation algorithm

P-LOOP interpolation algorithms

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pred

- *p_ols_po*
- *p_ols_v12*
- **p_ols_interp**
- *p_rf_po*
- *p_rf_v12*
- **p_rf_interp**

Real Data Example: Texas School Data

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- AEIS: School-level data from Texas Education Agency from 2003-2011
- > 3,000 schools
- TAKS (standardized test) passing rates
- Thousands of additional possible covariates

TEA Texas Education Agency

Real Data Example: Synthetic School-Level RCT

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- **RCT Sample:** 50 Texas middle schools
- **Treatment:** Alternative 8th grade mathematics curriculum
- **Design:** Schools randomly assigned to implement new curriculum or continue standard in the 2007/8 school year

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- **Outcome:** 2008 8th grade math TAKS passing rate
- **Pretest:** 2007 8th grade math TAKS passing rate

Your Turn!

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1. Follow along while we talk through *01-explore-aeis-data.Rmd*
2. Work through *02-effect-est.Rmd*
   - Effect estimate for Bernoilli randomized trial
   - Effect estimate for paired randomed trial
   - Effect esitmate for paired cluster randomed trial
3. Flag any of us down as you have questions!

Auxiliary Data

By “Auxiliary Data” we mean a dataset that meets these criteria:

1. Doesn’t include data from RCT participants
2. Includes covariate data
3. Includes outcome data

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Examples:

- A/B test: historical log data from users who worked on similar modules before the experiment started
- Field trial: Administrative (e.g. SLDS) data from students in schools that were not part of the RCT

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- Field trial: Administrative (e.g. SLDS) data from students in schools that were not part of the RCT

Note: we have sometimes called this the "remnant"

What use is more data??

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- Already imputing potential outcomes with f^c and f^t in LOOP
- f^c and f^t can be flexible, high dimensional
- They are fit to representative data

What use is more data??

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- Already imputing potential outcomes with f^c and f^t in LOOP
- f^c and f^t can be flexible, high dimensional
- They are fit to representative data

Limits on f^c and f^t

- RCT sample size might be too small to fit *really* good models
- Human-adaptive modeling: no good!

Example 1: ASSISTments

Covariates:

- Log data. For each previous skillbuilder,
  - Completed skill builder?
  - # problems attempted / completed?
  - Time to mastery
- Demographic data

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Example 1: ASSISTments

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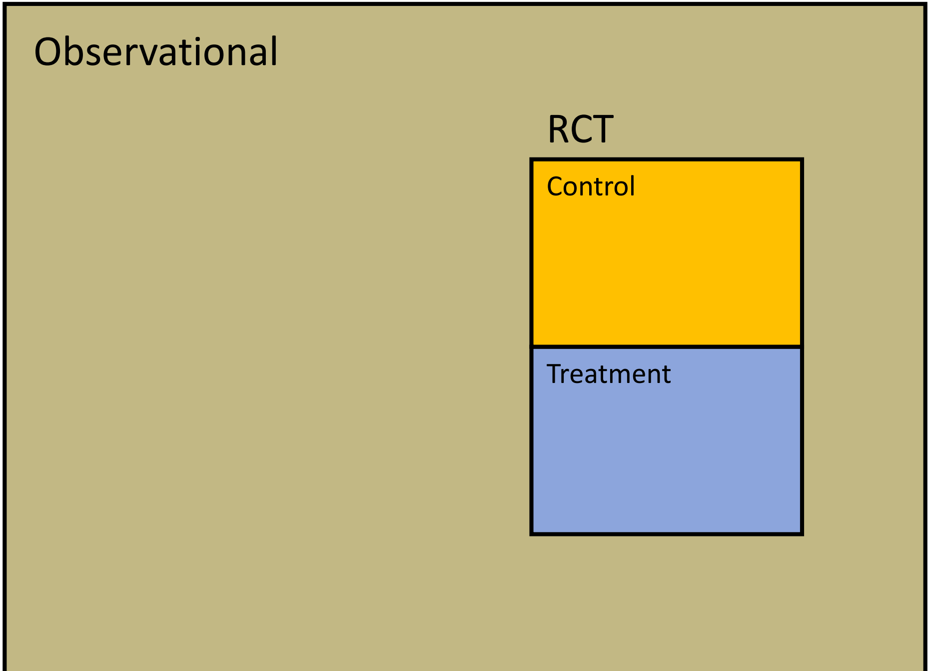
Auxiliary Data:

- Observational
- Students who were *not* randomzied
    - Previous users
    - Current users not assigned to that skillbuilder
- Same covariates available



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Observational

Step 1:

Train Model $\hat{y}(\cdot) : \boldsymbol{x} \to Y$

With auxiliary data

RCT

Control

Treatment

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Observational

Step 1:
Train Model $\hat{y}(\cdot) : \boldsymbol{x} \rightarrow Y$
With auxiliary data

Step 2:
Extrapolate
With fitted model & RCT data

RCT

Control $\hat{y}(\boldsymbol{x}_i)$
Treatment $\hat{y}(\boldsymbol{x}_j)$

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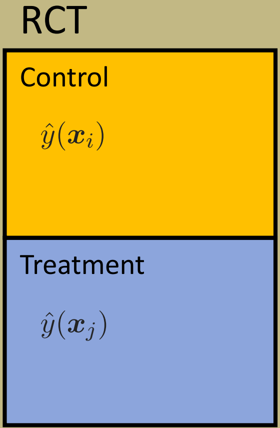
Observational

Step 1:
Train Model $\hat{y}(\cdot) : \boldsymbol{x} \to Y$
With auxiliary data

Step 2:
Extrapolate
With fitted model & RCT data

Step 3:
Use $\hat{y}(\boldsymbol{x})$ as a "super-covariate"



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Auxiliary Data

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Basic idea: Use auxiliary-based predictions $\hat{y}(\boldsymbol{x}_i)$ as a *covariate* in the RCT.

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- The covariates $\boldsymbol{x}$ are pre-treatment
- $\Rightarrow \hat{y}(\boldsymbol{x})$ is invariant to treatment assignment

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- $\Rightarrow \hat{y}(\boldsymbol{x})$ is invariant to treatment assignment
- $\hat{y}(\boldsymbol{x})$ might be an amazing covariate
- ...or it might not

Special Prediction Algorithm for LOOP

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- If $\hat{y}(x)$ predicts Y really well, we would expect a linear relationship
  - $\Rightarrow$ fit OLS models within LOOP

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Special Prediction Algorithm for LOOP

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  - $\Rightarrow$ fit OLS models within LOOP
- Maybe $\hat{y}(x)$ isn't that much better than other covariates (or, maybe it's useless)
  - $\Rightarrow$ use random forest within LOOP
- Let the data decide!
  - `pred=reloop`

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Incorporating Auxiliary Data in Practice

Incorporating Auxiliary Information

`loop( Y , Tr , Z , pred = reloop , p, yhat , ...)`

- `Y`: outcome vector
- `Tr`: treatment assignment vector
- `Z`: matrix of covariates
- `pred = reloop`: specify auxiliary data interpolation algorithm
- *p*: probability of treatment
- `yhat`: vector of auxiliary predictions
- `...`: optional inputs for interpolation algorithm

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Real Data Example: Texas Schools

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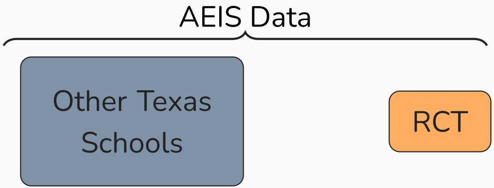
- AEIS data includes thousands of schools not in our RCT
- A great setting for integrating auxiliary and RCT data



Your Turn!

tinyurl.com/sree-drct

1. Work through *03-integrate-aux.Rmd*
   - We fit an auxiliary model and generate predictions to input as *yhat*
2. Apply what you learned in *04-effect-estABtest.Rmd*
3. Flag any of us down as you have questions!

Conceptual Overview

Estimating Effects with RCT Data

Incorporating Auxiliary Data

Treatment Effect Heterogeneity

Planning Experiments

Heterogeneous Treatment Effects

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Recall: The individual treatment effect is $\tau_i = y_i^t - y_i^c$

- Until now, our goal has been the average treatment effect (ATE)

$$\bar{\tau} = \frac{1}{N} \sum_{i=1}^{N} \tau_i$$

Heterogeneous Treatment Effects

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- Until now, our goal has been the average treatment effect (ATE)

$$\bar{\tau} = \frac{1}{N}\sum_{i=1}^{N} \tau_i$$

- We can use the same tools for other models of τ_i:
  - Averages for subgroups (subgroup effects)
  - Moderation: look for patterns in effects$\leftrightarrow$covariates $\tau_i|\boldsymbol{x}_i$
  - Predict an individual's treatment effect $\hat{\tau}_i$

Example

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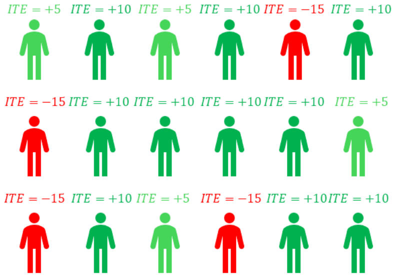
Example

tinyurl.com/sree-drct



Conditional Average Treatment Effect

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- The conditional average treatment effect (CATE) is

$$\tau(x) = \mathbb{E}[\tau_i | \boldsymbol{X}_i = \boldsymbol{x}] = \mathbb{E}[y_i^t - y_i^c | \boldsymbol{X}_i = \boldsymbol{x}]$$

- The expected treatment effect **conditional on having a specific set of covariate values**.

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- The expected treatment effect **conditional on having a specific set of covariate values**.
- “iCATE”: expected effect based on i’s covariates,

$$\tau(\boldsymbol{x}_i) = \mathbb{E}[\tau_i | \boldsymbol{X}_i = \boldsymbol{x}_i]$$

We're already almost there

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In order to get the ATE, we already have imputations:

$$\hat{y}_i^c = f^c(X_i)$$
$$\hat{y}_i^t = f^t(X_i)$$

And weighted average: $\hat{m}_i = Pr(Z_i = 0)\hat{y}_i^t + Pr(Z_i = 1)\hat{y}_i^c$
(For each i, we use everyone *but* i to estimate functions $f^c(\cdot)$ and $f^t(\cdot)$.)

We're already almost there

tinyurl.com/sree-drct

Also: an unbiased estimator for τ_i (!):

$$\hat{\tau}_i = U_i \left(Y_i - \hat{m}_i \right)$$

where

$$U_i = \begin{cases} \frac{1}{p_i} \text{ if } T_i = 1 \\ \frac{-1}{1-p_i} \text{ if } T_i = 0 \end{cases}$$

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If

$$T_i \hat{m}_i$$

Then

$$\mathbb{E}[\hat{\tau}_i] = \tau_i$$

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Modeling with $\hat{\tau}$

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$\hat{\tau}$ will typically be too noisy to be of much use by itself

$\ldots$ but it can be used for modeling

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Modeling with $\hat{\tau}$

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$\hat{\tau}$ will typically be too noisy to be of much use by itself

... but it can be used for modeling

- Estimating subgroup effects
- Parametric moderation modeling
- Non-parametric (or ML) modeling for the iCATE

Subgroup Effects

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The sample mean of $\hat{\tau}$ for a subgroup is unbiased for the CATE (conditional average treatment effect) in that subgroup.

Moderation Analysis

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Example: OLS

Fit model:

$$\hat{\tau}_i = \beta_0 + \beta_1 X_{i1} + \cdots + \beta_k X_{ik} + \epsilon_i$$

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tinyurl.com/sree-drct

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$$\hat{\tau}_i = \beta_0 + \beta_1 X_{i1} + \cdots + \beta_k X_{ik} + \epsilon_i$$

- If τ is linear in $X_1, \ldots, X_k$ then estimated slopes $\hat{\beta}$ are unbiased for true slopes
- If not, estimated slopes $\hat{\beta}$ are unbiased for "populaiton regression"—slopes you would estimate if you had true τ instead of estimates

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Use heteroskedasticity-robust SEs

OLS is only one example

Estimating iCATEs

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iCATE: average treatment effect for subgroup with $\boldsymbol{x} = \boldsymbol{x}_i$

Use any old model for $\hat{\tau}(\boldsymbol{x})$, as long as it fits well

Advantages of This Approach

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- Can model y^C with any model–including ML
  - Regardless of heterogeneity research question
  - Model need not be correct
- Flexible with regards to model for τ as a function of $\boldsymbol{x}$
- Built off of unbiased $\hat{\tau}$
  - If model for $\tau|\boldsymbol{x}$ is wrong, may still get biased estimators
  - ... but probably less biased than methods built on *biased* $\hat{\tau}$

In the Package

tinyurl.com/sree-drct

You can use the function $getITE$ to retrieve the ITE estimates from a LOOP estimator.

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You can use the function $getITE$ to retrieve the ITE estimates from a LOOP estimator.

- This function will work for an estimator built with or without auxiliary data, which allows us to improve precision further.
- However, it is currently only for Bernoulli-randomized experiments.
- Once you have retrieve the estimates, choose your favorite model and do some regressing!

Your Turn!

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1. Work through *`05-heterogeneousEffects.Rmd`*
    - We fit retrieve ITE estimates from the model in *`04-effect-estABtest.Rmd`*.
    - We then estimate the CATE by regressing these estimates on the covariates.
2. Flag any of us down as you have questions!

What You Need

tinyurl.com/sree-drct

- We'll be using the *dRCTpower* package to plan experiments
- Main function is *run_app*
- You can download the package in R using the following commands:

```
install.packages("devtools")
devtools::install_github("jaylinlowe/dRCTpower")
```

- We will be using the *aux_dat_small.csv* file from the Github repo

Main Question

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How to choose a sample size for our experiment, particularly if auxiliary data will be incorporated?

Overview

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- Incorporating auxiliary data in our analysis can improve precision, meaning we can have a smaller sample size with the same power

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- Gain in precision is determined by how predictive a model fit on the auxiliary data is for the RCT

Overview

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- Incorporating auxiliary data in our analysis can improve precision, meaning we can have a smaller sample size with the same power
- Gain in precision is determined by how predictive a model fit on the auxiliary data is for the RCT
- But....we don't have the RCT data!

Method

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1. Break auxiliary dataset into subgroups

Method

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1. Break auxiliary dataset into subgroups
2. For each subgroup, treat it as the RCT and the rest of the data as the auxiliary data

Method

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1. Break auxiliary dataset into subgroups
2. For each subgroup, treat it as the RCT and the rest of the data as the auxiliary data
3. Calculate the required sample size under this framework

Requirements

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Large auxiliary dataset that:

- is substantially larger than the RCT will be

Requirements

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Large auxiliary dataset that:

- is substantially larger than the RCT will be
- has covariates

Requirements

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Large auxiliary dataset that:

- is substantially larger than the RCT will be
- has covariates
- has the same outcome of interest as the RCT

Limitations

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- Method is only plausible if it's reasonable to assume the RCT looks like some subgroup of the auxiliary data, even if we don't know what subgroup that is

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Limitations

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- Method is only plausible if it's reasonable to assume the RCT looks like some subgroup of the auxiliary data, even if we don't know what subgroup that is
- Dangerous to assume RCT looks like any one subgroup
- Dangerous to choose most optimistic option

General Power Calculations

$$n = 2\sigma^2 \frac{(\xi_{1-\alpha/2} + \xi_{1-\beta})^2}{\Delta_A^2}$$

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$$n = 2\sigma^2 \frac{(\xi_{1-\alpha/2} + \xi_{1-\beta})^2}{\Delta_A^2}$$

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- $\xi_{1-\beta}$ is the critical value from a normal distribution for Type II error rate β.

General Power Calculations

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- Δ_A is the effect size, typically 20% of the standard deviation of the outcome in the population

General Power Calculations

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- $\xi_{1-\alpha/2}$ is the critical value obtained from a normal distribution for Type I error equal to α.
- $\xi_{1-\beta}$ is the critical value from a normal distribution for Type II error rate β.
- Δ_A is the effect size, typically 20% of the standard deviation of the outcome in the population
- σ^2 is the true variance of the outcome in the population, typically replaced with an estimate from a sample

Our Modification

tinyurl.com/sree-drct

- We replace σ^2 with an estimate from each subgroup

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- ”Without auxiliary data” estimate is variance of outcome for that subgroup

Our Modification

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- We replace σ^2 with an estimate from each subgroup
- Shiny app gives two estimates, one if you were to use auxiliary data in analysis, and one without
- "Without auxiliary data" estimate is variance of outcome for that subgroup
- "With auxiliary data" estimate is variance of the residuals, $(y_i - \hat{y}_i)$, where $\hat{y}_i$ are out-of-bag predictions from model

Defining Subgroups

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Three options:

1. Categorical Variable
    - Divide based on levels of categorical variable
    - Can create your own categorical variables

Defining Subgroups

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Three options:

1. Categorical Variable
   - Divide based on levels of categorical variable
   - Can create your own categorical variables
2. Numerical Variable
   - Divide into 10 (adjustable) equally sized groups
   - May need to divide into fewer if there isn't enough variation

Defining Subgroups

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Three options:

1. Categorical Variable
   - Divide based on levels of categorical variable
   - Can create your own categorical variables
2. Numerical Variable
   - Divide into 10 (adjustable) equally sized groups
   - May need to divide into fewer if there isn't enough variation
3. Best-Worst Case Scenario
   - Divide based on how predictive we expect the auxiliary model to be for that group
   - Good starting point

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Shiny App Demo

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