This document includes code to produce all of the results and run all of the models reported in "The Role of Mastery Learning in Intelligent Tutoring Systems: Principal Stratification on a Latent Variable."

The auxilliary files sourced here are available at our github repository, `https://github.com/adamSales/ctaiAdvance`.

First, load in and transform the (pre-imputed) data:

```
load('data/HSdata.RData')
load('data/advanceData.RData')
```

We'll use the `R` package `rstan` to run the models:

```
library(rstan)
rstan_options(auto_write = TRUE)
options(mc.cores = parallel::detectCores())
```

1 Data Description (Section ??)

This code produces the missigness information from Table 1, summarizing the student level data:

```
miss <- NULL
for(i in c('race','sex','spec','xirt')) miss <- rbind(miss,
 c(sum(is.na(covs[[i]])),mean(is.na(covs[[i]])),error[i,'error']))
miss <- as.data.frame(miss)
miss$`Error Type` <- c('PFC','PFC','PFC','SRMSE')
rownames(miss) <- c('Race/Ethnicity','Sex','Special Education','Pretest')
names(miss)[1:3] <- c('# Missing','% Missing','Imputation Error')
miss[,2] <- as.integer(round(miss[,2]*100))
miss[,1] <- as.integer(miss[,1])
miss['Pretest','Imputation Error'] <- sqrt(miss['Pretest','Imputation Error'])/sd(covs$xirt


print(xtable::xtable(miss))
```

	# Missing	% Missing	Imputation Error	Error Type
Race/Ethnicity	1071	8	0.23	PFC
Sex	526	4	0.35	PFC
Special Education	199	1	0.11	PFC
Pretest	2367	18	0.20	SRMSE

This code produces the covariate balance information:

```r
covBal <- NULL
for(i in c('race','sex','spec')){
    covBal <- rbind(covBal,c(i,NA,NA,NA,NA))

    for(ll in levels(dat[[i]])){
        covBal <- rbind(covBal,c(NA,ll,round(c(mean(dat[[i]]==ll),mean(dat[[i]][dat$treatmen

    }
}
colnames(covBal) <- c('Covariate','Category','Overall Percent','Percent of Treated','Percent
print(xtable(covBal),floating=FALSE,include.rownames=FALSE)
```

Covariate	Category	Overall Percent	Percent of Treated	Percent of Control
race				
	WhiteAsian	0.49	0.52	0.47
	BlackMulti	0.3	0.26	0.32
	HispAIAN	0.21	0.22	0.21
sex				
	F	0.5	0.49	0.51
	M	0.5	0.51	0.49
spec				
	typical	0.87	0.86	0.87
	speced	0.08	0.08	0.08
	gifted	0.05	0.06	0.05

The overall p-value for balance is:

```r
library(RItools) ## using development version
balMod <- balanceTest(treatment~poly(xirt,2)+spec+race+sex+strata(pair)+cluster(schoolid2),c
print(balMod$overall['pair',])

## chisquare        df   p.value
##    8.4464    7.0000    0.2949
```

2 PS Model with $\bar{m}_T$

Here we estimate the model in Section **??** stratifying on $\bar{m}_T$.

First, we create the datasets:

```r
source('R/prelimMbar.r')
```

The model is encoded in the file `psmodObs.stan`. It may be summarized as follows. The model for $\bar{m}_T$ is:

$$\bar{m}_{Ti} = \alpha_s^U + \boldsymbol{x}_i^T \boldsymbol{\beta}^{\boldsymbol{U}} + \epsilon_i^{Ui} + \epsilon_{t[i]}^{Ut} \tag{1}$$

where α_s^U is a separate intercept for each state, and $\boldsymbol{x}_i$ is a vector of covariates: dummy variables for racial/ethnic category, a dummy variable for sex, dummy variables for special education category, and linear and quadratic terms for pretest. The normally-distributed errors ϵ^{Ui} and $\epsilon^{Ut}_{t[i]}$ vary at the individual and teacher levels.

The model for Y is

$$Y_i = \alpha_p^Y + \boldsymbol{x}_i^T \boldsymbol{\beta^Y} + a_1 \bar{m}_{Ti} + Z_i(b_0 + b_1 * \bar{m}_{Ti}) + \epsilon_i^{Yi} + \epsilon_{t[i]}^{Yt} + \epsilon_{s[i]}^{Ys} \qquad (2)$$

where α_p^Y is a separate intercept for each randomization block p, Z_i is a dummy variable for treatment status, $\epsilon_{s[i]}^{Ys}$ is a normally distributed error at the school level, and the rest of the variables are analogous to those in (1). We run the model with the `stan` command from `rstan`:

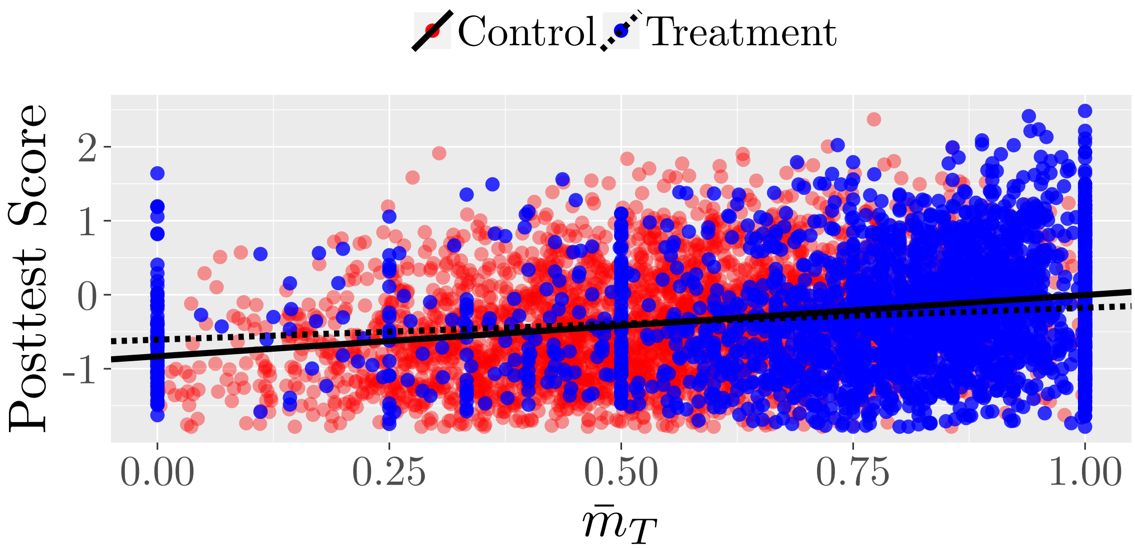
```
mbarMod <- stan('R/psmodObs.stan',data=sdat,seed=613)
```

Figure **??** can be replicated with the following code:

```
library(tikzDevice) ## allows latex code in figure
options( tikzLatexPackages = c(
getOption( "tikzLatexPackages" ),
"\\usepackage{amsmath,amsfonts}"
))

draw <- 1000

samps <- extract(mbarMod)
plotDatObs <- with(sdatObs,data.frame(Y=c(YtO,YtM,Yc),mbar=c(MbarTO,samps$MbarTM[draw,],samp
plotDatObs$treat <- ifelse(plotDatObs$Z==1,'Treatment','Control')
plotDatObs$slope <- ifelse(plotDatObs$treat=='Control',samps$a1[draw],samps$a1[draw]+samps$b
plotDatObs$int <- ifelse(plotDatObs$treat=='Control',samps$a0[draw],samps$a0[draw]+samps$b0[

plotDatObs <- within(plotDatObs, int <- int-( mean(int+slope*mbar)-mean(plotDatObs$Y)))
plotDatObs <- plotDatObs[order(plotDatObs$treat),]
plotDatObs$treat2 <- plotDatObs$treat

tikz(file = "mbarModel.tex",
  standAlone = T,
  width  = 6, height  = 3)

ggplot(plotDatObs,aes(mbar,Y,fill=treat,group=treat,alpha=treat,color=treat))+geom_point(siz
    geom_abline(aes(intercept=int,slope=slope,linetype=treat2),color='black',size=2,alpha=1)
    scale_colour_manual(values=c('red','blue'))+
    labs(group=NULL,fill=NULL,alpha=NULL)+xlab('$\\bar{m}_T$')+
    ylab('Posttest Score')+theme(legend.position='top',text=element_text(size=20))+
    guides(color = guide_legend(title=NULL,override.aes=list(alpha=1),keywidth=3),linetype=g
dev.off()
```

pdf 2

```
tools::texi2dvi('mbarModel.tex', pdf = T, clean = T)
```



To save memory, save and delete the $\bar{m}_T$ model:

```
save(mbarMod,file='fittedModels/mbarMod.RData')
```

```
rm(mbarMod); gc()
```

3 The Main PS Model

This section reproduces our paper's main model, described in Section **??**.

The data for the main model (similar to the $\bar{m}$ model but including student-section level mastery data) relies on a secondary file (available at github):

```
source('R/prelimStan.r')
```

Since this is the main model, we will include full stan code in this online supplement:

```
data{
//Sample sizes
 int<lower=1> nsecWorked;
 int<lower=1> ncov;
 int<lower=1> nstud;
 int<lower=1> nteacher;
 int<lower=1> nsec;
 int<lower=1> nschool;
 int<lower=1> npair;

// indices
 int<lower=1,upper=nteacher> teacher[nstud];
 int<lower=1,upper=npair> pair[nstud];
 int<lower=1,upper=nschool> school[nstud];
 int<lower=1,upper=nstud> studentM[nsecWorked];
 int<lower=1,upper=nsec> section[nsecWorked];

// data data
 int<lower=0,upper=1> grad[nsecWorked];
 matrix[nstud,ncov] X;
 int<lower=0,upper=1> Z[nstud];
 real Y[nstud];

}
parameters{

 vector[nstud] studEff;

 vector[ncov] betaU;
 vector[ncov] betaY;

 real a0;
 real a1;
 real b0;
 real b1;

 real teacherEffY[nteacher];
 real teacherEffU[nteacher];
 real pairEffect[npair];
 real schoolEffU[nschool];
 real schoolEffY[nschool];
 real secEff[nsec];
```

```
 real<lower=0> sigTchY;
 real<lower=0> sigSclY;
 real<lower=0> sigY[2];
 real<lower=0> sigTchU;
 real<lower=0> sigSclU;
 real<lower=0> sigU;
}

model{
 real linPred[nsecWorked];
 vector[nstud] muY;
 vector[nstud] muU;
 real useEff[nstud];
 real trtEff[nstud];
 real sigYI[nstud];


// grad model
 for(i in 1:nsecWorked)
  linPred[i]= secEff[section[i]]+studEff[studentM[i]];

 for(i in 1:nstud){
  useEff[i]=a0+a1*studEff[i];
  trtEff[i]=b0+b1*studEff[i];
  muU[i]=teacherEffU[teacher[i]]+schoolEffU[school[i]];
  muY[i]=teacherEffY[teacher[i]]+schoolEffY[school[i]]+pairEffect[pair[i]]+useEff[i]+Z[i]*tr
  sigYI[i]=sigY[Z[i]+1];
 }

 //priors
 betaY~normal(0,2);
 betaU~normal(0,2);
 pairEffect~normal(0,2);

 a0~normal(0,1);
 a1~normal(0,1);
 b0~normal(0,1);
 b1~normal(0,1);


 schoolEffY~normal(0,sigSclY);
 schoolEffU~normal(0,sigSclU);
 teacherEffU~normal(0,sigTchU);
 teacherEffY~normal(0,sigTchY);

 grad~bernoulli_logit(linPred);
```

```
 studEff~normal(muU+X*betaU,sigU);
 Y~normal(muY+X*betaY,sigYI);
}
```

This code runs the model:

```
main <- stan('src/psmod.stan',data =sdat,warmup=1500,chains=10,iter=5000)
```

4 Figures Comparing PS with $\bar{m}_T$ to PS with η

Figure **??**:

```
## smart jittering:
datObs$mbarJ <- datObs$mbar
datObs$nsecJ <- datObs$nsec
tab <- with(datObs,table(mbar,nsec))
mult <- which(tab>1,arr.ind=TRUE)
ms <- sort(unique(datObs$mbar))
ns <- sort(unique(datObs$nsec))
for(i in 1:nrow(mult)){
    w <- which(datObs$mbar==ms[mult[i,'mbar']] & datObs$nsec==ns[mult[i,'nsec']])
    s <- length(w)
    if(s>1){
        width=min(s*0.002,0.01)
        height=min(s*0.2,2)
        datObs$nsecJ[w] <- datObs$nsecJ[w]+runif(s,-width,width)
        datObs$mbarJ[w] <- datObs$mbarJ[w]+runif(s,-width,width)
    }
}



tikz(file='mbarSampleSize.tex',
     standAlone=T,
     width=3,height=3)
ggplot(datObs,aes(mbarJ,nsecJ))+geom_point()+xlab('$\\bar{m}$')+ylab('$n_{sec}$')+theme(text
dev.off()
```

pdf 2

```
tools::texi2dvi('mbarSampleSize.tex', pdf = T, clean = T)
```

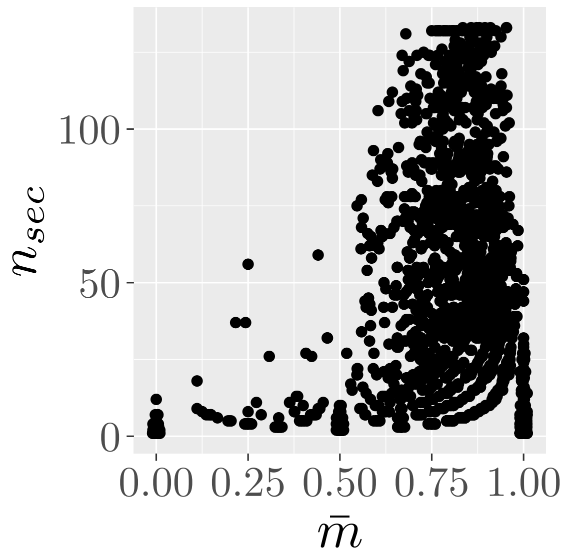


Figure **??**:

```
sdatLat <- sdat
nsec <- as.vector(table(sdatLat$studentM))
etaDraws <- extract(main,'studEff')[[1]][,sort(unique(sdatLat$studentM))]
eta <- colMeans(etaDraws)
eta <- etaDraws[draw,]
etasd <- apply(etaDraws,2,sd)

plotDat <- data.frame(nsec=nsec,eta=eta,etasd=etasd)

tikz(file='etaSampleSize.tex',
     standAlone=T,
     width=3,height=3)
ggplot(plotDat,aes(eta,nsec,size=1/etasd))+geom_point()+ylab(NULL)+#ylab('£n_{sec}£')+
    labs(size='$1/\\text{SE}(\\eta_T)$')+scale_size(range=c(.5,2))+guides(size=FALSE)+xlab('
    theme(text=element_text(size=20))+
    theme(axis.title.y=element_blank(),
        axis.text.y=element_blank(),
        axis.ticks.y=element_blank())#+ggtitle('One Posterior Draw')#+xlab('£\\mathbb{E}\\et
dev.off()
```

pdf 2

```
tools::texi2dvi('etaSampleSize.tex', pdf = T, clean = T)
```

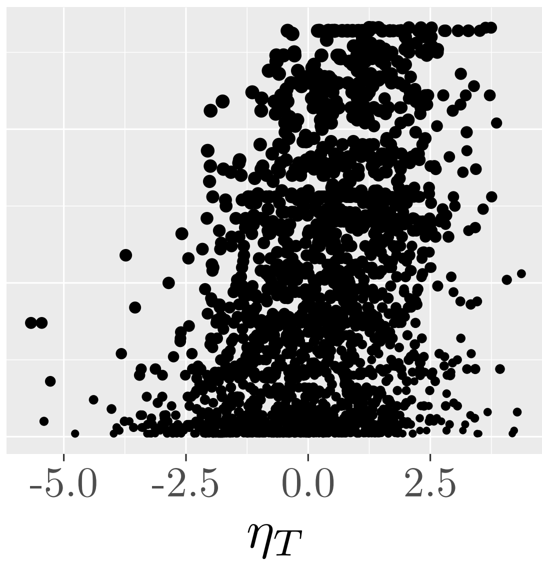


Figure ??:

```
secDiff <- -extract(main,'secEff')[[1]][draw,]
sss <- secDiff[sdatLat$sec]
mDiff <- aggregate(sss,list(stud=sdatLat$studentM),mean)
mbar <- aggregate(sdatLat$grad,list(sdatLat$studentM),mean)

mbarDiffDat <- data.frame(mbar=mbar$x,mDiff=mDiff$x)


tikz(file='mbarDiff.tex',
     standAlone=T,
     width=3,height=3)
ggplot(mbarDiffDat,aes(mbar,mDiff))+geom_point()+xlab('$\\bar{m}$')+ylab('Avg. Sec. Difficul
    theme(text=element_text(size=20))
dev.off()
```

pdf 2

```
tools::texi2dvi('mbarDiff.tex', pdf = T, clean = T)
```

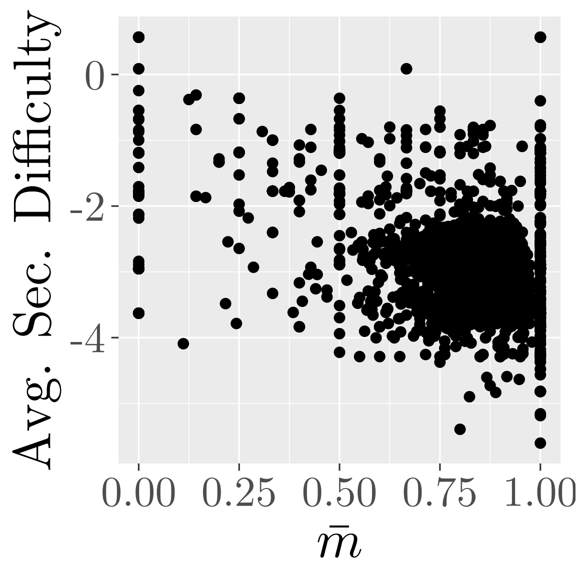


Figure ??: