List of Case Studies

P1 Minimization of the Rosenbrock test function ($n = 2$ and $n = 5$)

$$f(\mathbf{x}) = \sum_{i=1}^{n-1} 100(x_{i+1} - x_i^2)^2 + (1 - x_i)^2$$

subject to the bound constraints $-5 \leq x_i \leq 5, i = 1, 2, \ldots, n$.

P2 Constrained optimization problem (algebraic in 2 variables)

$$\begin{aligned} \textbf{Minimize}: &\quad x_1^2 + 0.5x_1 + 3x_1x_2 + 5x_2^2 \\ \textbf{Subject to}: &\quad 3x_1 + 2x_2 + 2 \leq 0 \\ &\quad 15x_1 - 3x_2 - 1 \leq 0 \\ &\quad -1 \leq x_1 \leq 1 \\ &\quad -1 \leq x_2 \leq 1 \end{aligned}$$

P3 Constrained bump test function ($n = 2$ and $n = 10$)

$$\begin{aligned} \textbf{Minimize}: &\quad -\text{abs}\left(\textstyle\sum_{i=1}^n \cos^4(x_i) - 2\prod_{i=1}^n \cos^2(x_i)\right) / \sqrt{\textstyle\sum_{i=1}^n ix_i^2} \\ \textbf{Subject to}: &\quad \textstyle\prod_{i=1}^n x_i > 0.75 \quad \text{and} \quad \sum_{i=1}^n x_i < 15n/2 \\ &\quad 0 \leq x_i \leq 10, i = 1, 2, \ldots, n, \end{aligned}$$

P4 Brachistochrone Problem ($n = 15$ and $n = 30$): This is a classic test problem that involves finding the optimal path connecting two points such that the time taken by a particle traversing this path, subject only to the force of gravity is minimized. A detailed description of this test problem can be found in §D.1.7 of Martins and Ning, *Engineering Design Optimization*, Cambridge University Press, 2021, download link: `https://mdobook.github.io`. Set the starting point to $(0, 1)$, the end point to $(1, 0)$ and the friction coefficient to 0 in your numerical studies. Use n uniformly spaced points to parametrize the trajectory as described in the book to formulate an unconstrained optimization problem with $n - 2$ variables. You can compare your approximate solution to Equation (D.27) in the book which is the analytical solution obtained for this setting using calculus of variations.

P5 **Inverse problem:** The dynamic response of a viscously damped linear oscillator can be modeled by the ordinary differential equation $m\ddot{u} + ku + c\dot{u} = f(t)$,

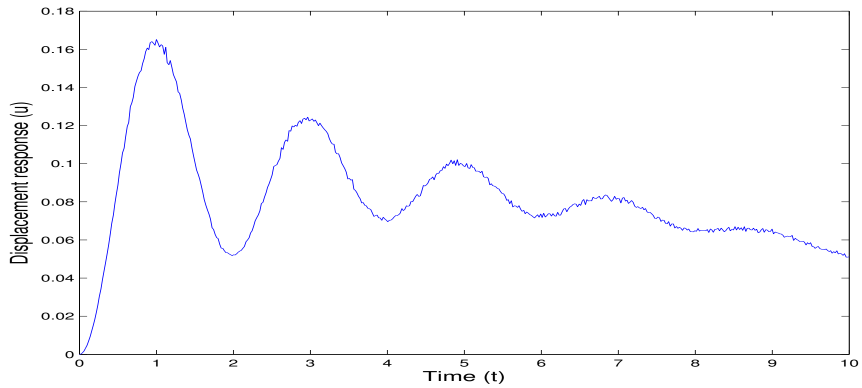

Figure 1: Measured displacement of the linear oscillator as a function of time.

subject to initial conditions on the displacement and velocity at the time instant $t = 0$. The mass of the oscillator is known ($m = 1.0$) but the values of the damping factor (c) and stiffness (k) are unknown.

In order to estimate the values of c and k an experiment was carried out by applying the forcing function $f(t) = F_0 \cos(\omega t)$ (with $F_0 = 1$, $\omega = 0.1$, and the initial conditions $u(0) = \dot{u}(0) = 0$) to the oscillator. The measured reponse over the time interval $[0, 10]$ seconds is shown in Figure 1 and also provided in the datafile MeasuredResponse.dat on Qercus. The first column of this data file contains the time instant and the second column contains the measured value of u at that time instant.

Formulate an optimization problem statement that can be solved to estimate the values of c and k.

- While formulating the optimization problem, you can use the fact that the response of the oscillator to the forcing $f(t) = F_0 \cos(\omega t)$ with the intial conditions $u(0) = \dot{u}(0) = 0$ can be computed analytically as follows

$$u(t) = (A\cos(\omega_d t) + B\sin(\omega_d t))\exp(-\frac{ct}{2m}) + \frac{F_0}{C}\cos(\omega t - \alpha),$$

  where

$$A = -\frac{F_0}{C}\cos(\alpha), B = -\frac{F_0}{C\omega_d}\left(\omega\sin(\alpha) + \frac{c}{2m}\cos(\alpha)\right),$$

$$C = \sqrt{(k - m\omega^2)^2 + (c\omega)^2}, \omega_d = \sqrt{\frac{k}{m} - \left(\frac{c}{2m}\right)^2}, \alpha = tan^{-1}\left(\frac{c\omega}{k - m\omega^2}\right)$$

- The file P5.m on Quercus contains a MATLAB routine for evaluating the analytical solution. You will need to modify this routine while solving this inverse problem.