COMP 550-001
Algorithms and Analysis
Spring 2020
Mid Semester Exam
Tuesday, February 18, 2020
Closed Book - Closed Notes
Don't forget to write your name or ID and pledge on the exam sheet.
This exam has four pages.

1. (8 points) A fair *die* when tossed will give each of the values 1 through 6 with equal probability. The plural of die is *dice*.
a). What is the expected value for the sum of three tosses of a fair die? ____________________
b). Suppose two fair dice are tossed. What is the probability that the sum of their values will equal 4 or more? ____________________
c). What is the expected value for a single toss of a fair die? ____________________
d). Suppose a fair die is tossed twice. What is the probability that the first toss will produce a larger value than the second toss? ____________________

For the following questions consider the following asymptotic growth rates:

A) Constant ($\Theta(1)$) B) $\Theta(\log n)$ C) $\Theta(\sqrt{n})$ D) $\Theta(n)$ E) $\Theta(n \log n)$ F) $\Theta(n^2)$ Recall that $f(n) = \Theta(g(n))$ means that f and g asymptotically grow at the same rate.

2. (10 points) Solve the recurrence $T(n) = 3T(n/3) + \Theta(\sqrt{n})$. Fill in the blank with the letter A,B,C,D,E, or F indicating the asymptotic growth rate of $T(n)$ as a function of n. _____

3. (10 points) Solve the recurrence $T(n) = T(n/2) + 2T(n/2) + n^2 + n \log n$. Fill in the blank with the letter A,B,C,D,E, or F indicating the asymptotic growth rate of $T(n)$ as a function of n. _____

4. (4 points) Let $f(n)$ be the time required to build a min-heap of n elements. Fill in the blank with the letter A,B,C,D,E, or F indicating the asymptotic growth rate of $f(n)$ as a function of n. _____

5. (4 points)

a) Fill in the blank with A,B,C,D,E, or F, whichever indicates the asymptotic worst case time bound for quicksort as a function of n. _____

b) Fill in the blank with A,B,C,D,E, or F, whichever indicates te asymptotic average time bound for quicksort as a function of n. _____

6. (4 points) What is the asymptotic worst case time bound for heapsort? Fill in the blank with A,B,C,D,E, or F as appropriate. _____

7. (10 points) Give an asymptotic estimate for the sum $\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \ldots + \frac{1}{2n}$ as a function of n. Fill in the blank with A,B,C,D,E, or F as appropriate. _____

8. (10 points) What is the sum of the series $1 + \frac{1}{3} + (\frac{1}{3})^2 + (\frac{1}{3})^3 + \ldots$? Fill in the blank with an integer or fraction giving the answer. _____

9. (4 points) What is the height of a max heap having n elements? Fill in the blank with A,B,C,D,E, or F indicating the asymptotic growth rate of this height as a function of n. _____

10. (4 points) What is the asymptotic worst case time bound for merge sort as a function of n, the number of elements in the list? Fill in the blank with A,B,C,D,E, or F as appropriate. _____

11. (10 points) Suppose Algorithm X operates on linear arrays. Suppose that if the array has length one, then Algorithm X returns an answer with a constant amount of work. Otherwise, Algorithm X calls itself recursively three times on linear arrays that are 1/4 as long, and in doing so performs a linear amount of work creating the subproblems and combining their solutions. That is, the work performed in creating the subproblems and combining their solutions is proportional to the number of elements in the array. Let $T(n) = aT(n/b) + f(n)$ be a recurrence for Algorithm X as a function of the number n of elements in the array.

a) Fill in the blank with the value of a. _____

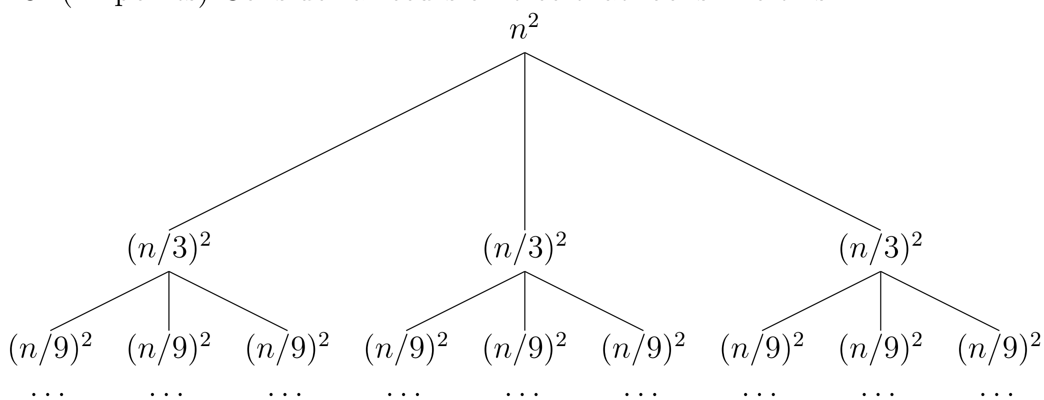
b) Fill in the blank with the value of b. _____

c) Fill in the blank with the letter A,B,C,D,E, or F indicating the asymptotic growth rate of $f(n)$ as a function of n. _____

12. (12 points) For each problem, choose all elements F of the set $\{\Theta, O, o, \Omega, \omega\}$ such that the statement $f(x) = F(g(x))$ is a correct statement of the asymptotic relationship between f and g. Thus if $f(x) = \Omega(g(x))$ and $f(x) = \Theta(g(x))$ and $f(x) = O(g(x))$ are the only three valid asymptotic relationships between f and g, choose Ω, Θ, O.

a). $f(x) = 2\log_2 x, g(x) = 3\log_3(2x)$. ____________________
b). $f(x) = \sqrt{x}, g(x) = 4\log x$. ____________________
c). $f(x) = 3^x + 2x, g(x) = 2^x + 3x + 1$. ____________________
d). $f(x) = 5\log^2 x + 2, g(x) = 2x + 1$. ____________________
e). $f(x) = x^2 + x, g(x) = 3x^2 + 5x$. ____________________
f). $f(x) = x^2 + 1, g(x) = 4x - 2$. ____________________

13. (12 points) Consider a recursion tree that looks like this:



Let $T(n) = aT(n/b) + f(n)$ be the recurrence relation used to generate this recursion tree.
a) Fill in the blank with the value of a. _____
b) Fill in the blank with the value of b. _____
c) Fill in the blank with the letter giving the asymptotic growth rate of $f(n)$.
_____

d). How many levels would there be in this tree, as a function of n? Fill in the blank with the letter A,B,C,D,E, or F corresponding to the number of levels in this tree as a function of n. Thus if the number of levels grows asymptotically at the same rate as n, put the letter D in the blank. _____

e). How many leaves would there be in this tree, as a function of n? Fill in the blank with the letter A,B,C,D,E,F giving the asymptotic growth rate of this number of leaves. _____

f). Solve the recurrence to obtain an asymptotic expression for $T(n)$ as a function of n. Fill in the blank with the letter A,B,C,D,E,F giving the asymptotic growth rate of $T(n)$ as a function of n. _____

14. (10 points) EXTRA CREDIT Essay Question: Solve the recurrence relation $T(n) = 2T(\sqrt{n}) + \Theta(\log n)$ ____________________

15. (5 points) EXTRA CREDIT: Fill in the blank with a value c such that $\sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} (i + j + k)$ is $\Theta(n^c)$. ____________________