COMP 550
Algorithms and Analysis
Spring 2020
Second Mid Semester Exam
Thursday, April 2, 2020
Open Book - Open Notes
Don't forget to write your name or ID and pledge on the exam sheet.
This exam has five pages.

1. (5 points) For hashing by the multiplication method, which of the following values for A is best: (a) 0.45 (b) .618033988 (c) 0.6 (d) .618 Fill in the blank with the letter of the correct choice. _B_

2. (5 points) For hashing by the division method, which of the following is the best value for the modulus m? Fill in the blank with the letter of the correct choice. (a) 256 (b) 100 (c) 199 (d) 257 _C_

3. (10 points) Suppose one is using hashing by the division method, the table size m is 53, and the key k is 230. Which bin will this key hash to? Fill in the blank with an integer. _18_

4. (10 points) Suppose one is using hashing by the multiplication method and A is .528 and the table size m is 20 and the key k is 18. Which bin will this key hash to? Fill in the blank with an integer. _10_

5. (10 points) Suppose one is doing universal hashing and the table size m is 2000 and there are 60000 hash functions in all in the set H. Let x and y be two distinct keys. What is the maximum number of hash functions h in H such that $h(x) = h(y)$, according to universal hashing? Fill in the blank with an integer. _30_

6. (6 points) If the hash table size is 150 and there are 100 elements in the table, what is the load factor? Fill in the blank with an integer or a fraction. 100/150

For the following questions, let the following asymptotic growth rates be labeled with the letters A,B,C,D,E, and F as follows:

A) Constant ($\Theta(1)$) B) $\Theta(\log n)$ C) $\Theta(\sqrt{n})$ D) $\Theta(n)$ E) $\Theta(n \log n)$ F) $\Theta(n^2)$ Recall that $f(n) = \Theta(g(n))$ means that f and g asymptotically grow at the same rate.

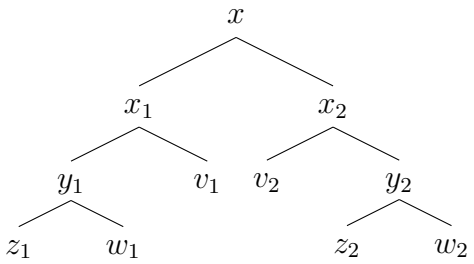
7. (12 points)
(a) Give an upper bound on the height of a red black tree having n internal nodes. Fill in the blank with A,B,C,D,E, or F as appropriate to indicate the asymptotic growth rate of this upper bound. B

(b) In a red black tree, what is the maximum number of red children that a black node can have? Fill in the blank with an integer. 2

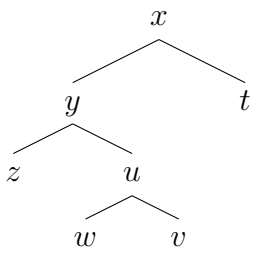
(c) Give an asymptotic bound on the worst case time to delete an element from a red-black tree as a function of n, the number of nodes in the tree. Fill in the blank with A,B,C,D,E, or F as appropriate. B

(d) In a red black tree, what is the maximum number of red children that a red node can have? Fill in the blank with an integer. 0

8. (4 points) (a) Assume that the following tree is a binary search tree, where x, x_1, y_1, z_1, w_1, v_1, x_2, y_2, z_2, w_2, and v_2 are nodes. Is this tree height balanced? Fill in the blank with "yes" or "no". yes



(b) Assume that the following tree is a binary search tree, where x, y, z, u, w, v, and t are nodes. Is the tree height balanced? Fill in the blank with "yes" or "no". no



9. (10 points) Suppose you insert the seven elements in the set $\{1, 5, 7, 20, 26, 28, 40\}$ into an initially empty binary search tree, to produce a perfectly balanced tree. Which four elements would end up at the leaves? List them smallest first. ___1, 7, 26, 40___

10. (10 points) Suppose a hash table has a load factor of 29/30. (a) If open addressing is being used, assuming simple uniform hashing, what is the expected number of probes in an unsuccessful search? _____30_____ (b) If hashing with chaining is being used, assuming simple uniform hashing, and the load factor is 29/30, what is the approximate expected number of elements examined during an unsuccessful search? Fill in the blank with the nearest integer. _____1_____

11. (10 points) Suppose one has $n = 1000$ real numbers uniformly distributed between 0 and 1. What is a good way to sort them? Consider the followng methods: a) Merge sort b) Radix sort c) Quick sort d) Bucket sort e) Counting sort

11a) Fill in the blank with the letter of the correct choice. __d__

11b) What is the asymptotic expected time for this sorting method, as a function of the number n of elements sorted? Fill in the blank with A,B,C,D,E, or F as appropriate. __D__

12. (10 points)
What is a good method to sort n integers in the range $\{0, 1, \ldots, n-1\}$ using $O(n)$ storage? Consider the following methods:
a) Merge sort b) Radix sort c) Quick sort d) Bucket sort e) Counting sort

12a) Fill in the blank with the letter of the correct sorting method. __e__

12b) Fill in the blank with A,B,C,D,E, or F indicating the asymptotic time bound of this method as a function of n. __D__

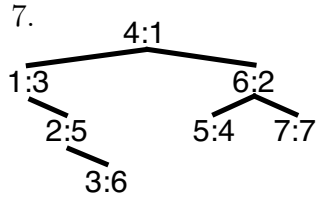
What is a good method to sort n integers in the range $\{0, 1, \ldots, n^2 - 1\}$ using $O(n)$ storage? Consider again the following methods:

a) Merge sort b) Radix sort c) Quick sort d) Bucket sort e) Counting sort

12c). Fill in the blank with the letter of the correct sorting method. __b__

12d) Fill in the blank with A,B,C,D,E, or F indicating the asymptotic time bound of this method as a function of n. __D__

13. (10 points) Essay Question. Consider the following set of elements, where $a : b$ denotes an element with key a and priority b and small numbered priorities are nearest the root. Construct a treap from these elements. $1 : 3, 2 : 5, 3 : 6, 4 : 1, 5 : 4, 6 : 2, 7 : 7$.



14. (5 points)
How many comparisons are needed to find the median of n elements? Fill in the blank with the letter A,B,C,D,E, or F indicating the asymptotic time bound of the best known method to do this. __D__

15. (10 points) Suppose one sorts by building a treap with randomly assigned priorities and then repeatedly extracting the smallest element from it.

a) What is the expected time to sort n elements in this way? Fill in the blank with the letter A,B,C,D,E, or F indicating the asymptotic time bound for this. __E__

b) What is the worst case time to sort n elements in this way? Fill in the blank with the letter A,B,C,D,E, or F indicating the asymptotic time bound for this. __F__

16. (10 points) Suppose one creates a hash table using external chaining and multiplicitive hashing with a good value for A. Suppose all the elements that has to the same slot are stored in an AVL tree, a binary search tree, instead

of a linked list, to make access faster. What is the worst case time to search for an element using this data structure? B

17. EXTRA CREDIT: (5 points) (a) How many comparisons are needed to find the second largest element of a set of 8 elements? 9

18. EXTRA CREDIT: (5 points) Essay Question. What is the smallest number of nodes in an AVL tree of height 7? Recall that the height of a single node AVL tree is 1. 33