1. Suppose you have a fair 6-sided die with the numbers 1 through 6 on the sides and a fair 5-sided die with the numbers 1 through 5 on the sides. What is the probability that a roll of the six-sided die will produce a value larger than the roll of the five-sided die?

   Answer. The total number of samples in this (probability) space is $6 * 5 = 30$.

   The number of samples where "the six-sided die will produce a value larger than the roll of the five-sided die" is $5 + 4 + 3 + 2 + 1 = 15$.

   Therefore, the chance of the event is $15/30 = 0.5$.

2. What is the expected number of rolls until a fair five-sided die rolls a 3? Justify your answer briefly.

   Answer. Consider some finite cases:

   The probability of rolling a 3 the 1st time is $1/5$.

   The probability of rolling a 3 only at the 2nd time is $4/5 \cdot 1/5$.

   The probability of rolling a 3 only at the 3nd time is $4/5 \cdot 4/5 \cdot 1/5$.

   ...

   So on and so on.

   Hence, the expectation of the number of rolls is

   $\frac{1}{5} \cdot 1 + \frac{4}{5} \cdot \frac{1}{5} \cdot 2 + (\frac{4}{5})^2 \cdot \frac{1}{5} \cdot 3 + \dots$

   $= \frac{1}{5}(1 + \frac{4}{5} \cdot 2 + (\frac{4}{5})^2 \cdot 3 + \dots)$

   $= \frac{1}{5} + \frac{1}{5} \sum_{k=1}^{\infty} (k+1)(\frac{4}{5})^k$

   $= \frac{1}{5} + \frac{1}{5} \sum_{k=1}^{\infty} k(\frac{4}{5})^k + \frac{1}{5} \sum_{k=1}^{\infty} (\frac{4}{5})^k$

   $= \frac{1}{5} + 4 + \frac{4}{5}$

   $= 5$