

**Topics of the week** Compute with matrices:

1. do elimination and back substitution on square systems of linear equations, explain when and why this works or fails;
2. define the inverse of a matrix, compute inverses of  $2 \times 2$  matrices, characterize when the inverse exists (The Inverse Theorem), invert a product of matrices and the transpose of a matrix.

**CR decomposition**  $A = CR$ , where  $C$  are the independent columns of  $A$ .

**Elementary operations**

- Subtract one row from *another*, multiplied by any constant;
- Multiply any row by a *non-zero* scalar;
- Swap any two rows;

Can be represented by multiplying with a matrix on the left (see week 3).

**Systems of linear equations**  $Ax = b$ . Solve by applying row operations to  $A$  and  $b$  until  $A = I$ .

**Inverse matrix**  $A^{-1}$  s.t.  $AA^{-1} = A^{-1}A = I$ . Unique if exists. How to find?

- Solve  $A(A^{-1}e_k) = e_k$  for all  $e_k$  to find the  $k$ -th column of  $A^{-1}$ .
- Do elementary row operations on  $I$  and  $A$  simultaneously  $\implies$  turns  $A$  into  $I$  and  $I$  into  $A^{-1}$ .

**Determinant** The volume of a hypercube spanned on  $(v_1, \dots, v_n)$ . Properties:

1.  $V(e_1, \dots, e_n) = 1$ .
2.  $V(a + b, \dots) = V(a, \dots) + V(b, \dots)$ .
3.  $V(\alpha v, \dots) = \alpha V(\dots)$ .
4.  $V(a, b, \dots) = -V(b, a, \dots)$ .

**Inverse of  $2 \times 2$**  Try to derive Kramer's rule.

**In-class exercises**

1. Let  $p : \mathbb{R} \rightarrow \mathbb{R}$  be a polynomial of degree at most 2, i.e.  $p(x) = ax^2 + bx + c$ . Find  $a$ ,  $b$  and  $c$  s.t.  $p(-1) = 0$ ,  $p(0) = 2$  and  $p(1) = 2$ .

$$\begin{pmatrix} (-1)^2 & -1 & 1 \\ 0^2 & 0 & 1 \\ 1^2 & 1 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix}$$

- 2.