## Topics of the week

- 1. derive and explain the LU factorization from elimination;
- 2. compute the REF and RREF of a given  $m \times n$  matrix A, explain why it equals R in A = CR;

**LUP factorization** Do Gaussian elimination and reduce A to upper-triangular form U:

$$M_n \dots M_1 A = U \iff A = (M_1^{-1} \dots M_n^{-1})U$$

To recover L, keep a matrix B = I. Whenever applying  $M_i$  to A, apply  $M_i^{-\top}$  to B. In the end,

$$B = M_n^{-\top} \dots M_1^{-\top} \iff B^{\top} = M_1^{-1} \dots M_n^{-1}$$

So, we get the decomposition  $A = B^{\top}U$ . If B is not lower-triangular, its rows can be sorted such that  $L = PB^{\top}$  is lower-triangular. Thus, PA = LU.

REF and RREF Gauss-Jordan elimination turns a matrix into the row-echelon form:

$$\begin{bmatrix} 1 & 0 & a_1 & 0 & b_1 \\ 0 & 1 & a_2 & 0 & b_2 \\ 0 & 0 & 0 & 1 & b_3 \end{bmatrix}$$

Reduced row-echelon form: Remove zero rows. Other conventions:

- 1. REF: Not necessary to keep pivots 1 and elements above them 0;
- 2. RREF: What we call REF in this course.

## In-class exercises

- 1. Elimination, back substitution, LU factorization:
  - (a) Compute an LU factorization of the matrix

$$A = \begin{bmatrix} 2 & -12 & 6 \\ 1 & -4 & 1 \\ 2 & -11 & -5 \end{bmatrix}$$

(b) For the factorization above, solve Ly = b with

$$b = \begin{bmatrix} 4 \\ 4 \\ 25 \end{bmatrix}$$

- (c) For the y from the previous exercise, solve Ux = y.
- (d) Prove that Ax = b.
- 2. Consider the matrix

$$A = \begin{bmatrix} 1 & 2 & 0 & 3 \\ 2 & 4 & 1 & 2 \\ 3 & 6 & 2 & 5 \end{bmatrix}$$

with CR-decomposition

$$A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & -2 \end{bmatrix} = CR$$

Let  $R_0$  be the matrix in row echelon form obtained by performing Gauss-Jordan elimination on A. Determine  $R_0$  by performing the elimination and verify Theorem 3.24 by checking that R corresponds to the non-zero rows of  $R_0$ .