Mixed Reality Enabled Robot Assisted Telesurgery

# Introduction

In war zones across the world children are suffering and dying from injuries. Many require instant surgery. Complicated surgery who requires highly specialized surgeries and medical specialists is not commonly available to people in war zones. To

Person wearing MR headset.

Robot in front of patient

# Background

## Literature Review

Ingredients of a litreuature review needs to be defined. A system must be chosen.

What needs to be researched?

Mixed Reality -> Headsets, applications,

Telesurgery -> State of field, eyesurgery, required precsicion, required delay,

Robot surgery -> State, safety, tolerances

## Mixed Reality

Headsets

Meta Quest 3

Hololens 2

## Telesurgery

What is telesurgery

## Robot Assisted Surgery

Robots that are plasusable to use: ABB Robot :O

## Examples

Find examples like the pitched project.

Medivis

Hololens apps

Telesurgery Implementations

## Conformal Geometric Algebra 101

Defining an orthonormal coordinate space. The coordinate spaces are defined from a set of basis vectors, as follows:

A non-orthogonal coordinate space is shown in Figure 1. Orthogonal Figure 2. Orthonormal coordinate space is shown in Figure 3.

A graph of equations and equations

Description automatically generated with medium confidence

Figure 1 Non-orthogonal Cartesian coordinate space.

A black and white image of a musical note

Description automatically generated

Figure 2 Orthogonal Cartesian coordinate space.

A grid with lines and arrows

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Figure 3 Orthonormal Cartesian coordinate space.

Consider the two following vectors, and , as shown in Figure 4.

: for every two units moved along , also move one unit along .

: for every three units moved along , also move four unit along .

A drawing of a graph

Description automatically generated

Figure 4 Vectors (a) and (b) in orthonormal space.

### Addition

This means, the sign does not mean addition. Thus, scalars of and cannot be added. However, vectors can be added together.

### Magnitude

Follows Pythagorean theorem, and is defined:

### Inner Product

= dot product. Defined:

Using this we can find: if we move along , how much do we move along . This is shown in Figure 5.

Notice, the inner product is commutative () because the cosine function is symmetric .

Notice, if , moving along does not move along . This means that orthogonal vectors must have an inner product equal to zero, because . This applies to our basis vectors as follows:

Notice, the inner product is related to magnitudes (). This means, “What do we get if we move in the direction of by several units equal to the length of ”.

What is using the above values of and ?

A black and white drawing of a triangle

Description automatically generated

Figure 5 Inner product of (a) and (b).

### Outer Product

“If you start pointed in the direction of , how much do you need to rotate to point in the direction of ?”

Notice, the outer product is not commutative because . This also means that .

Notice, the outer product is related to rotation of . We assign to a counterclockwise rotation. Thus, smaller and larger rotations are multiplited by a scalar. A rotation is then . A clockwise rotation is then .

Notice, the outer product of two orthonormal vectors must be either or . This is shown in Figure 6.

A group of math equations

Description automatically generated

Figure 6 Rotating basis vectors with (R).

### Geometric Product

“How do you transform into ?”

“How much do you need to rescale and rotate so that it is equal to ?”

To build the intuition for this, lets multiply and with regular algebraic rules:

We then have 4 transformations: and .

* means turning into itself, thus scaling is 1 and rotation is 0. Recall, and . Then .
* is also 1, for the same reasons.
* means turning into , however, we know that our basis vectors are orthonormal. Then this must be equal to a clockwise rotation . The inner product must be 0 because , thus .
* is , for similar reasons.

Finishing the geometric product from the previous example:

Now, this means that to to transform into we need to rescale it by 10 and rotate it 5 quarter turns counterclockwise. This is very much incorrect, however the intuition of this is crucial. We will know go over how to correct this.

Then

Simplifying Geometric Products

Simplify the following

Meaning of the Geometric Product

Let us look once more at and . Now with four basis vectors: .

These new vectors are called “Multivectors”. Multivector has four components:

0 scalar, 2 in , 1 in , 0 bivector direction.

### Examples

Example ab vs ba

Where \* is the complex conjugate. This then means rotating clockwise instead of counterclockwise.

Move from to :

If ab represents a transformation from a to b, ohw do we show it? Keeping in mind that geometric product operations are ot commutative, should we pre- or post- multiply a with ab to transform it into b?

Pre:

This is not equal to b… Try post multiplying:

Then b is multiplied by a factor of 5. Where does the factor 5 come from? If we normalized a and b to have unit length ( a / |a| and b / |b| ), the 5 disappears.

If you can see that r represents a pure rotation, then the transform from a to b could be described as: “Normalize a, rotate by r and then scale by |b| to reach b.”. Thus, the true way to go from a to b is:

# Methods

## Project development

System Links:

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Automatisk generert beskrivelse

Hololens -> My Computer data communication system, how?

<https://learn.microsoft.com/en-us/windows/mixed-reality/develop/advanced-concepts/using-the-hololens-emulator#deploying-apps-to-the-hololens-emulator>

<https://learn.microsoft.com/en-us/windows/mixed-reality/develop/unity/unity-development-overview?tabs=arr%2CD365%2Chl2>

My Computer to Robot data communication system, how? Microcontroller, Raspberry Pi,

MR Computer to Robot Computer communication system, how? Cloud?

## Robot Kinematics

|  |  |  |
| --- | --- | --- |
| Type | Representation 1 | Representation 2 |
| Point |  | None |
| Sphere |  |  |
| Plane |  |  |
| Circle |  |  |
| Line |  |  |
| Point Pair |  |  |

A multivector in Euclidean space is a linear combination of the basis elements:

The geometric product of two multivectors is given:

The outer product for any multivector .

In Euclidean space , the pseudoscalar is . The Euclidean dual of multivector is:

In conformal space, the pseudoscalar is

The conformal dual of multivector A is:

In conformal Euclidean space we define a point:

It is easy to form primitives out of conformal objects, such as points. Let points in conformal space, then a circle is defined:

A line:

Sphere:

Plane:

Point Pair:

In 3D Geometric algebra, the trivector is the pseudoscalar. In 2D the bivectors are pseudoscalars. The unit pseudoscalar is , and . Meaning the trivectors can be thought of as the imaginary numbers. For trivector pseudoscalars for all multivectors . The trivector consists of a  , meaning  , because the x-es cancels.

To rotate a vector V by angle Theta in the plane I, where I is the unit bivector in that plane, a rotation is defined:

A screenshot of a computer

Description automatically generated

The inverse kinematics derivation for a 6-DOF robot is quite complex. Therefore the method used follows Fu et al. closely (Fu et al., 2013).

Consider 3D Euclidean space with orthonormal basis , a set of linearly independent combinations of these basis elements using geometric product is given, as shown in Figure 1:

These elements are the basic elements of , thus:

In the comformal model, we extend the space by adding two additonal orthognal basis elements , and form Conformal space :

, ,

Another basis ;

,

Can be defiend:

These are null vectors

Then dual unit is defined:

where

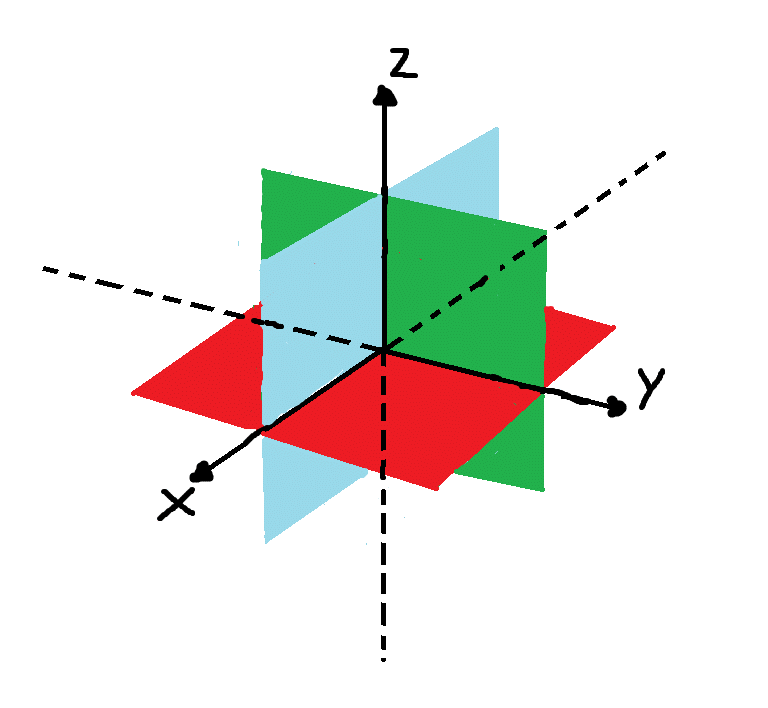


Figure 7 3D Euclidean Space.

A dual number is referred to as a dual angle between two lines and in space, as shown in Figure 2. Given by:

Where is the projected angle and is the length of the common perpendicular between lines and . It is also represented as:

Where is referred to as the pitch of dual angle . If is zero, it is a pure rotation; if is infinity, it is a pure translation. From Taylor Series Expansion, we get equations:

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Automatisk generert beskrivelse

Figure 8 Dual Angle.

In the Conformal space the Euclidean transformation is decomposed into a rotation followed by a translation, via a ***rotor*** and a ***translator****.*

A rotor, , satisfies , where is the conjugate of . Then:

And rotation of vector over rotor is given:

Where the rotation axis is an orthogonal to the plane that is spanned by the normalized bivector and is the rotation angle around the axis, as shown in Figure 3. Rotors combines easily, followed by is defined .

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Figure 9 Rotation of X.

Translator is a special rotation acting at infinity by using the the null vector and is defined:

Where is a vector, defined:

Motor describes a rigid transformation as a composition of a rotation and a translation, both related to the same rotation axis, as shown in Figure 4:

Transformation of vector over motor (screw motion) is given by:

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Figure 10 Transformation of X.

Introducing dual angles:

Thus, a rigid transformation of object can be carried out by multiplying the motor for the left and its reverse from the right, as follows:

### Denavit-Hartenberg Model

To solve the IK equations, the equation must be simplified. The Denavit-Hartenberg (DH) method is a common method to accomplish the simplification. The DH method introduced mathematical constraints on the robot. The constraints are defined:

1. The -axis must be perpendicular to both the and axes.
2. The -axis intersects both the and axes.
3. The origin of joint is the intersaction of and .
4. The axis must be aligned to complete a right-handed reference frame based on and .

Where denotes the index of the link.

A DH model is defined by a set of 4 parameters per link in the robotic chain. represents the rotation along the z-axis (degrees). is the rotation along the x-axis (degrees). and represents the translation along the z and x axes, respectively. Adhering to the constraints the DH parameters were calculated. Tabell 1 shows the parameters for each link.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  |  |  |
| 0 |  | 90 | 7.15 | 0 |
| 1 |  | 0 | 0 | 12.5 |
| 2 |  | 0 | 0 | 12.5 |
| 3 |  | 0 | 0 | 12.5 |
| 4 |  | 0 | 0 | 4.8 |

Tabell Denavit-Harternberg parameters.

Where is the rotation along the z-axis at joint index n. Using Tabell 1, both forward and inverse kinematics can be calculated.

A DH-transformation matrix is defined as:

Where, and is respectively and . and is respecticly and . Using this definition, the forward kinematics chain can be calculated, defined as:

Where denotes the value for the link index 0 from Tabell 1. Doing this calculation results in a transformation matrix, where P is the location of the end-effector. The same convention applies for the rest of the parameters.

### Kinematic Analysis

Transformation from frame to consists of a sequence of two motors, one variable and another constant , as shown in Figure 5. Note that dual angles are used.

Because , we find the inverse transformation:

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Figure Denavit Hartenberg transformation.

### Kinematic Modeling

Where   and  , for . Then:

Where and are functions of DH-parameters of the robot. This satisfies:

Hence, the rotation and location of the end effector with respect to the base frame can be expressed as 4 x 4 homogenous transformation matrix in 3D Euclidean space with in conformal space **.** The transformation from to is defined:

From these eqautions, the transformation from to can be computed. For the Inverse Kinematics, both position and orientation of the end effector, i.e., is known. The previous equation can be rewritten:

Let the left ahnd and right hand of the equation be expressed as and :

Where and for are functions of the DH-parameters.

### Solutions

Substituting DH-parameters into the previous equations, in terms of for , we get:

Where and for .

The elements of and for rely on the DH-parameters.

Let be divided by and introduce for .

Where:

Extract from previous equation yields:

Where .

### Solving t1, t3 and t4

Construct the following determinant, according to principle of resultant elimination:

Then:

Where are new variables.

Each element of is only the fourth degree polynomial of variable . By the corresponding column transformation and extracting the common factor , we get a 16th degree equation of variable . However, ill-condition problem will appear when solving the 16th degree equation in some cases, and the requirement ofr accuracy cannot be guaranteed. Therefore, we can reduce the problem of roots to the eigenvalue algorithm of the resulting matrix.

We express the matrix as:

Where and are 6x6 constant matrices. Then, when the matrix is well-conditioned, use previous equation to construct 24x24 matrix :

Where are 6x6 null and identity matrices, respectively. The eigenvalues of **E** correspond exactly to the solutions of (including eight imaginery roots). Furthermore, the eigenvectors of **E** corresponding to eigenvalues have the form:

Where is the 6x1 vector whose elements are not equal but proprotianl to the corresponding ones of vector . Thus, the eigenvectors of can be used to compute amd . Finally, we can get the values of and using the formula for .

However, the matrix may be ill-conditioned, so we use equations and reduce it to a generalized eigenvalue problem by contrsuting two 24x24 matrices, and as follows:

Where are 6x6 null and identity matrices. The solutions of correspond exactly to the eigenvalues of the generalized eigenvalue problem and the eigenvectors have the same structure as Eq 33. Finally, the solution of and are obtained with the same procedure.

### Solving for t2, t5 and t6

Substituting each solution of and into Eq.30 correspondingly, we can work out the corresping 16 solutions of variable . The, the values of can be worked out using the formula .

Similarly, substituting the solutions of into Eq.28, we can work out the correspoing 16 solutions of linearly.

## Inverse Kinematics 2

Source (Kleppe & Egeland, 2016).

The following rotors are defined for each joint I:

Where denotes the three basis alements of the i-th joint frame. Then, two main rotors can be constructed from the previous one.

Notice, rotor contains both joint variables, while is a constant rotor. represent as screw motion around the i-th joint axis, , while represents a screw motion around the axis of the (i-1)-th joint frame.

Now we can formulate forward kinematics

Where is the null vector representation of either the end-effector position fector or the fectors defining the end-effector orientation, in relation to the reference frame.

### Pose Representation

The desired end-effector pose in matrix form is given by:

Then we can construct a rotor that represents the same pose.

We can compute the rotor that transform into . First, the position vector defines the rotor:

Applied to gives , the null vector representation of . Now, to compute the rotor that relates the oritatinos determined by the rotation matrcies , we notice that both rotation matrices can be seen as two sets of orthogonal cectors in , namely and . Then, the problem is reduced to find a rotor transforming one set of orthonogal vectors into another. Associated with any arbirty set of orthogonal vectors, there exists another set of orthogonal vectors, denoted by and defined:

for all

Where denotes the Kronecker . Such a set is said to be the reciprocal frame of .

Then following the formula:

Where, again, is the orientation of the reference frame, while is the desired orientation of the end-effector. Finally, the rotor M that transforms is the product of rotors

As stated earlier, for any serial robot with a spherical wrist, the target position can be moved to the wrist center point by a fixed transformation. Then, the new target position is and thus, it can be assured that the first three joints contribute to the position and orientation, while the last three oly contribute to the orientation. Tis allows us to decoulple the inverse kinematics into the position and the orientation subproblems.

The inverse kinematics for a 6Dof robot will now be calculated. It is divided into the position and orientatin subproblems. Four different combinations of prismatic and revolute joints that constute the position part of the robot are analysed. For each of them a geometric strateg is developed. It consists of computing the null vector representation of some auxiliary points placed at each joint to recoer the corresponding joint variables as the angle or displacement between two geometric objects defined by such points. For each case, the point at the origin, , is denoted by and is placed at the base of the robot. The target position is expressed as a null vector , obtained through Hestenes embedding. Finally, depending on the information available, for a given geometric object it could be convenient to use its inner representation or its outer representation . Clearly, a geometric object described with one of these two representations can be expressed in the other representation using the dual operator

Three prismatic joints

First of all, the case without offsets between consecutive joints is considered, meaning the axes of each pair of consecutive joints intersect. Given the first tralsation axis , the two rotors defined can be used to obtain the joint axes and as follows:

The joint axis together with defines a line whose inner representation is

Now, a plane contating the joint axis and passing thought the point is also defined so its inner representation is

Where denotes the cross product between three-dimensional vectors.

Sine the end-effector position is not restricted to a fixed plane, there are not parallel prismatic joint axes. Moreover, and this the intersection of the line and plane represented by is non-empty:

Where is a bivector. Hence, it represents a pair of points in conformal geometric algebra. However, since the intersection between a line and a plane is a single point, the bivector is of the form:

For a null point that can be eextraced from B. With this nullpoint, the following lines are defined thourgh their inner representations:

Where is the vector whose null vector representation is and that is recovered with the projection , that is, the inverse of the Hestenes embedding. In addition if are the outer representations of the two lines and plane defined before, L1 ^ TT1 = 0 and L2 ^TT1 = 0, since the joint axes z1 and z2 are not parallel, they have non-empty intersection. This can b e see

## Inverse Kinematics 3

Source (Zaplana et al., 2022).

Source (Zhang et al., 2022)

Source (Long et al., 2020)

# Discussion

Telesurgery-> opportunities, further research, shortcomings,

# Conclusion

(Hopefully)-> solutions for a 6-DOF robot inverse kinematics have been developed, implemented, and tested.

# References

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