Exam December 14th, 2023, ACIT4710

Problem 1

a) fs = 2B = 6kHz

b) fs = 2B = 8kHz

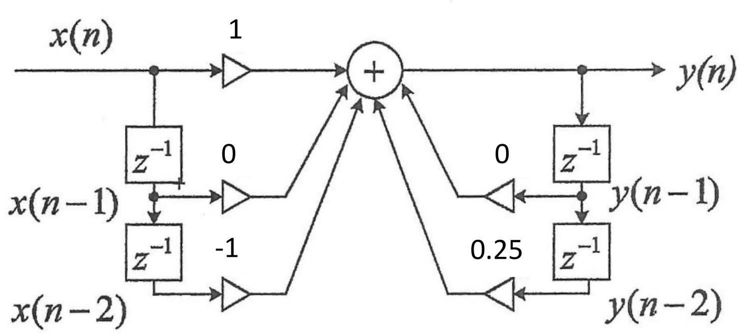
c) The system is linear and time-invariant, but non-causal and marginally stable.

d) The image is 3-dimensional and therefore an RGB-image

Size = 240x320x3 = 230400bytes = 230 kbytes

Problem 2

A time discrete circuit is shown below:



a) Determine the difference equation of the circuit.

b) Determine the transfer function of the circuit, H(z) = Y(z)/X(z).

c) Determine the zeros and the poles of the circuit.

d) Determine the impulse response, h(n), of the circuit.

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a) y(n) = 0.25y(n-2) + x(n) - x(n-2)

b) $Y(z)(1-0.25z^{-2}) = X(z)(1-z^{-2})$

$$H(z) = \frac{z^2-1}{z^2-0.25} = \frac{(z-1)(z+1)}{(z-0.5)(z+0.5)}$$

c) Zeros at z=1 and z=-1 and poles at z=0.5 and z=-0.5

d)

$$\frac{H(z)}{z} = \frac{(z-1)(z+1)}{z(z-0.5)(z+0.5)} = \frac{a}{z} + \frac{b}{z-0.5} + \frac{c}{z+0.5}$$

a = -1/-0.25 = 4,   b = (-0.5*1.5)/(0.5*1) = -1.5,   c = -1.5*0.5/(-0.5*(-1))= -1.5

$$H(z) = 4 - \frac{1.5z}{z-0.5} - \frac{1.5z}{z+0.5}$$

$$h(n) = 4\delta(n) - 1.5[(0.5)^n + (-0.5)^n]u(n)$$

Problem 3

Design a FIR high pass filter using the window design method with the following specifications:

Sampling frequency = 8000Hz

Pass band: 2000 – 4000Hz with maximum pass band ripple, δp = 0.5dB

Stop band: 0 – 1000Hz with minimum stop band attenuation, δs = 40dB

a) Determine the required window function, estimate the required order, and calculate the cut-off frequency.

b) Use MATLAB to calculate the frequency response of the filter.

c) Check the gain at 1000Hz and 2000Hz to find out if the filter fulfills all design specification.

d) Calculate the time-delay of this filter.

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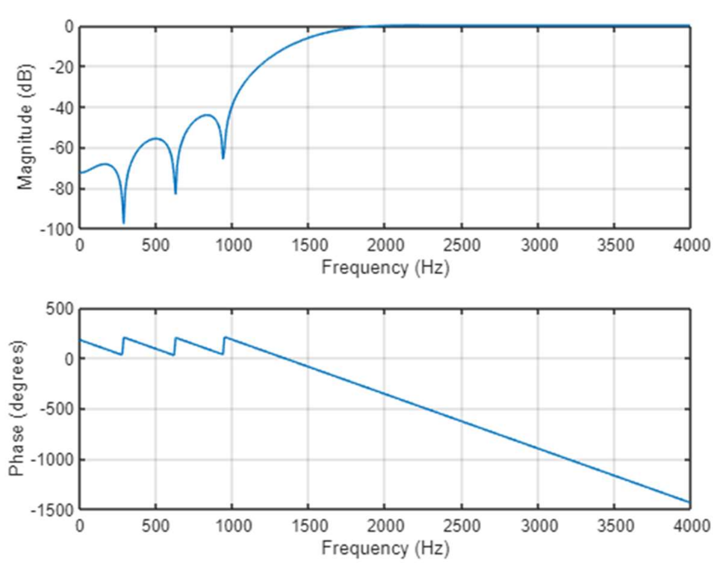
a) Hanning window, Δf=1000/8000, taps=3.1*8=24.8, N=24, fc=1500Hz

```
% Problem 3 b+c
fs=8000;    % sampling frequency
fc=1500;    % cut-off frequency
N=24;       % order
B=fir1(N,15/40,"high",hann(N+1));
freqz(B,1,401,fs)
```




```
% c magnitude at 1 and 2kHz
[H,f]=freqz(B,1,401,fs);
H1000=20*log10(abs(H(101))); H1000
```

```
H1000 = -39.5596
```

```
H2000=20*log10(abs(H(201))); H2000
```

```
H2000 = -0.1084
```

c) The attenuation at 1000Hz is -39.6dB and the design criteria are therefore marginally violated.

d) Time-delay = (N/2)*T = 12/8000Hz = 1.5ms.

Problem 4

a) Design a lowpass Butterworth-filter with the following specifications:

Sampling frequency: fs=10kHz

Passband: 0 – 2kHz with attenuation less than 1dB

Stopband: 4 – 5kHz with attenuation at least 40dB.

b) Design also a lowpass Chebyshev type 1 filter with the same specifications as above.

c) What is the order of the Butterworth filter and the Chebyshev filter?

d) Compare the two filters. What is the main advantage and disadvantage of the Chebyshev filter compared to the Butterworth filter?

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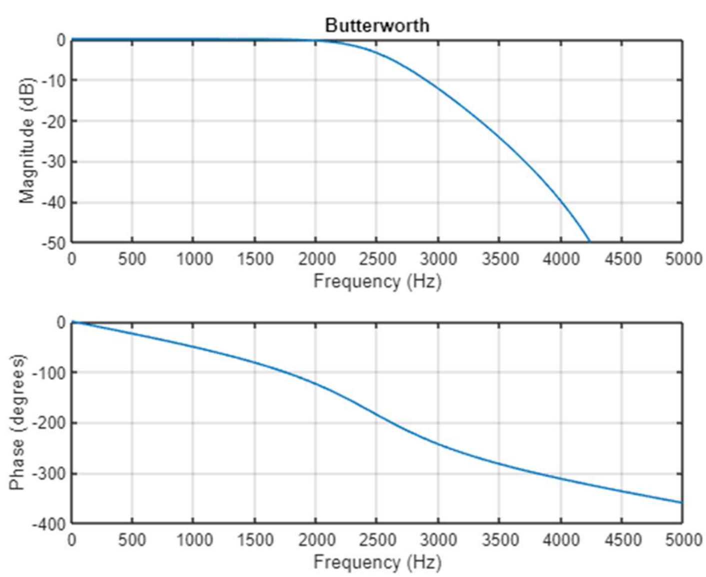
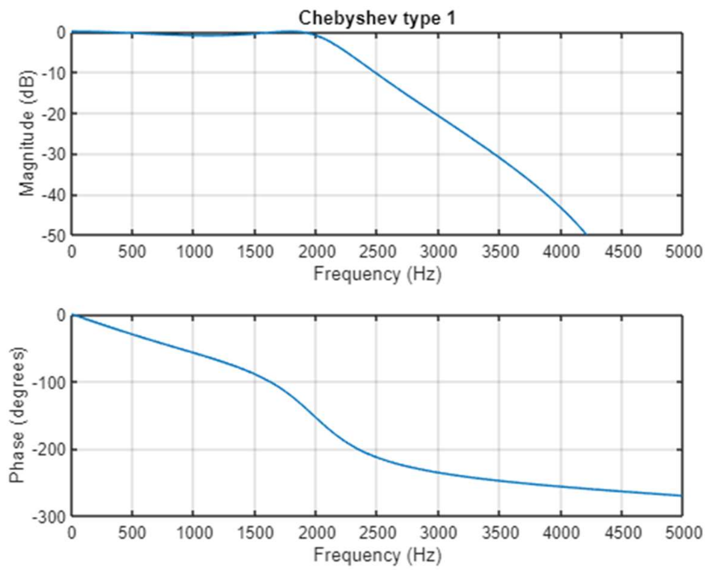
```
% Problem 4 Butter vs. Cheby
% a) Butter
fs=10000;
[NB,WB]=buttord(2/5,4/5,1,40);  % prototype
[BB,AB]=butter(NB,WB);          % Butterworth design
freqz(BB,AB,501,fs); ylim([-50 0]); title('Butterworth')
```



```
% b) Cheby1
[NC,WC]=cheb1ord(2/5,4/5,1,40); % prototype
[BC,AC]=cheby1(NC,1,WC);        % Chebyshev design
freqz(BC,AC,501,fs); ylim([-50 0]); title('Chebyshev type 1')
```



c) The order of the Butterworth filter is NB=4 and the order of the Chebyshev filter is NC=3.

d) The main advantage of the Chebyshev filter compared to the Butterworth filter is that it has a narrower transition band, and in the example above therefore lower order.

The main disadvantage of the Chebyshev filter is the ripples in the passband, while the Butterworth filter is monotonically decreasing and is known as maximally flat in the passband.

<u>Problem 5</u>

Simulate a signal with frequency = 1Hz that is contaminated with random noise. In the simulation use an amplitude of 1 for the 1Hz signal and an amplitude of 1 also for the random noise (randn(n)). Use a sampling frequency of 500Hz.

a) Plot the signal with noise for 2 seconds.

b) Plot also, the frequency spectrum of the signal between 0 and 250Hz.

c) Design a FIR lowpass filter using the Parks-McClellen algorithm (optimal design). The filter specifications are:

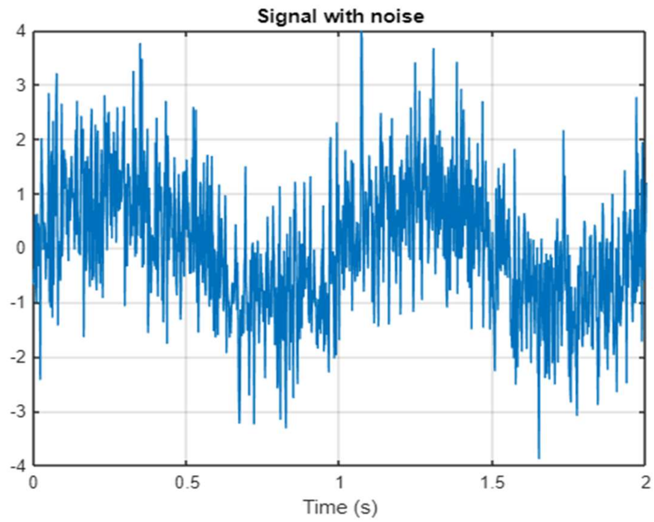
Passband: 0 – 2Hz with ripple less than 1dB

Stopband: 20 – 250Hz and attenuation better than 40dB

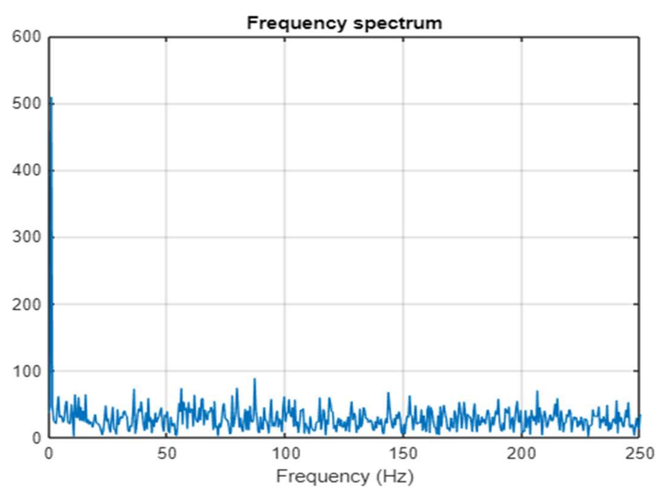
Find the required order of the filter that meet the specifications and plot the frequency response.

d) Use the filter to clean the above simulated signal and plot the filtered signal for 2 seconds.

```
% Problem 5a
fs=500;
t= 0:1/fs:2;    % time axis
s=sin(2*pi*t)+randn(size(t));   % signal with noise
plot(t,s); grid
title('Signal with noise'); xlabel('Time (s)')
```



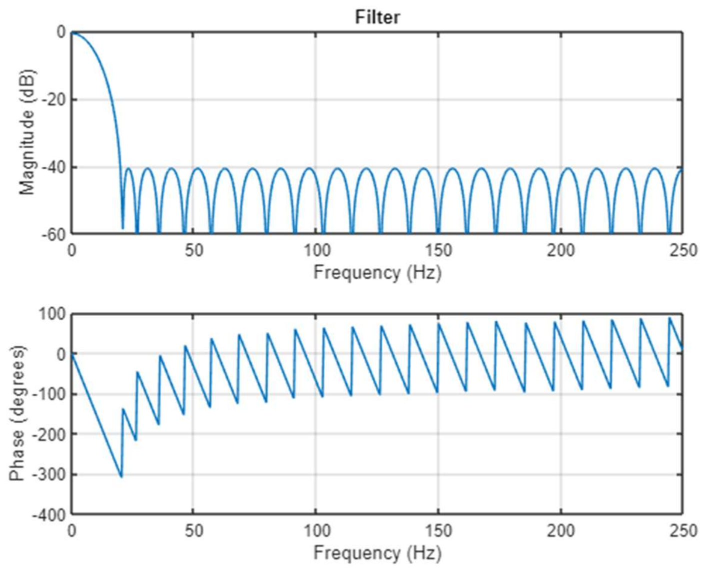

```
% 5b
S=fft(s);
f=0:.5:fs/2;    % frequency axis
plot(f,abs(S(1:501))); grid
title('Frequency spectrum'); xlabel('Frequency (Hz)')
```




```
% 5c
F=[0 2/250 20/250 1];   % edge frequencies
M=[1 1 0 0];            % ideal magnitude
W=[1 12];               % weight factors
N=42                    % order
```

N = 42

```
B=firpm(N,F,M,W);       % design filter
freqz(B,1,512,fs); grid
title('Filter')
axis([0 fs/2 -60 0]); grid
```




```
% 5d
y=filter(B,1,s);        % apply filter
plot(t,y); grid
title('Filtered signal'); xlabel('Time (s)')
```