Exam 15dec2022

Problem 1

a) Marginally stable

b) Stable

c) 8kHz

d) 2kHz

e) The matrix elements (pixels) are indices pointing to a row in a colormap. The colormap has tree columns, one for each color red-green-blue. The index therefore defines the color of the pixel.

Problem 2

Given the difference equation: $y(n) = x(n) + 0.25y(n-2)$

Where x(n) is the input and y(n) is the output of a time discrete system.

a) Determine the z-domain transfer function of the system, H(z)=Y(z)/X(z).
b) Determine the zeros and poles of the system.
c) Determine the impulse response of the system, h(n), the response when x(n)=δ(n).
d) Determine the magnitude of the frequency response of the system, |H(exp(jΩ))|.

a) $y(n) - 0.25y(n-2) = x(n)$

$$Y(z)(1 - 0.25\,z^{-2}) = X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 - 0.25\,z^{-2}} = \frac{z^2}{z^2 - 0.25} = \frac{z^2}{(z-0.5)(z+0.5)}$$

b) The system has two zeros both in z=0 and two poles in z=0.5 and z=-0.5.

c) x(n)=δ(n) → X(z)=1:

$$\frac{Y(z)}{z} = \frac{z}{(z-0.5)(z+0.5)} = \frac{a}{z-0.5} + \frac{b}{z+0.5}$$

$$a = \frac{0.5}{0.5+0.5} = 0.5 \quad ; \quad b = \frac{-0.5}{-0.5-0.5} = 0.5$$

$$Y(z) = \frac{0.5z}{z-0.5} + \frac{0.5z}{z+0.5}$$

$$h(n) = Z^{-1}\{Y(z)\} = 0.5[(0.5)^n + (-0.5)^n]u(n)$$

d) z → exp(jΩ):

$$|H(j\Omega)| = \frac{|\exp(j2\Omega)|}{|\exp(j2\Omega) - 0.25|} = \frac{1}{|\cos(2\Omega) + j\sin(2\Omega) - 0.25|}$$

$$= \frac{1}{\sqrt{(\cos(2\Omega) - 0.25)^2 + (\sin(2\Omega))^2}} = \frac{1}{\sqrt{1.0625 - 0.5\cos(2\Omega)}}$$

Problem 3

Design a FIR lowpass antialiasing filter for an audio signal with bandwidth 0-18kHz. The sampling frequency is 88kHz, but the signal should be down sampled to 44kHz and therefore we need an antialiasing filter. We require a stopband attenuation better than 50dB. Use the window design method.

a) Determine the required passband frequency, stopband frequency, window type, the order of the filter and the cut-off frequency.

b) Use Matlab to design the filter and plot its frequency response. Does the filter meet the requirements?

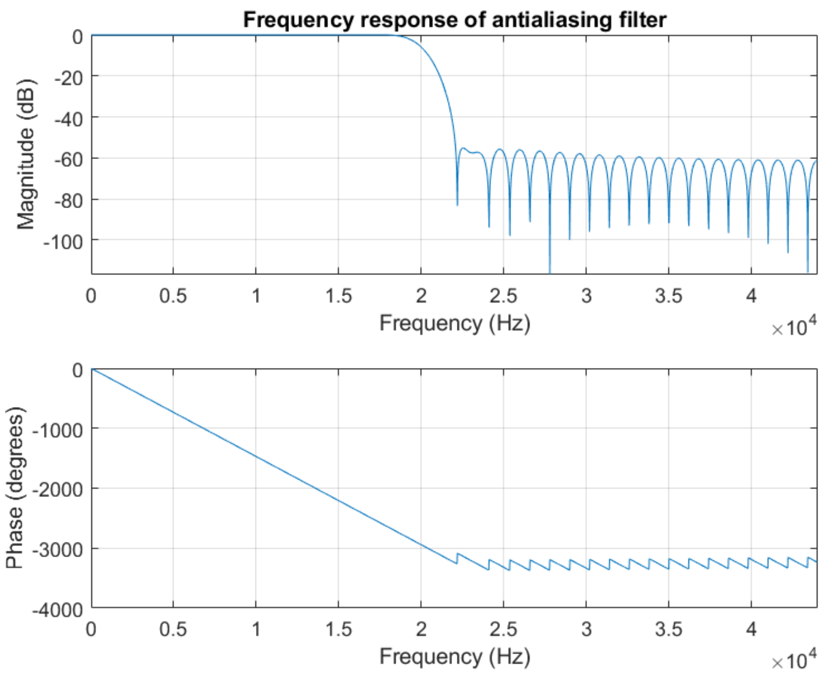
c) How many zeros and poles does this filter have?

d) What is the time delay of this filter?

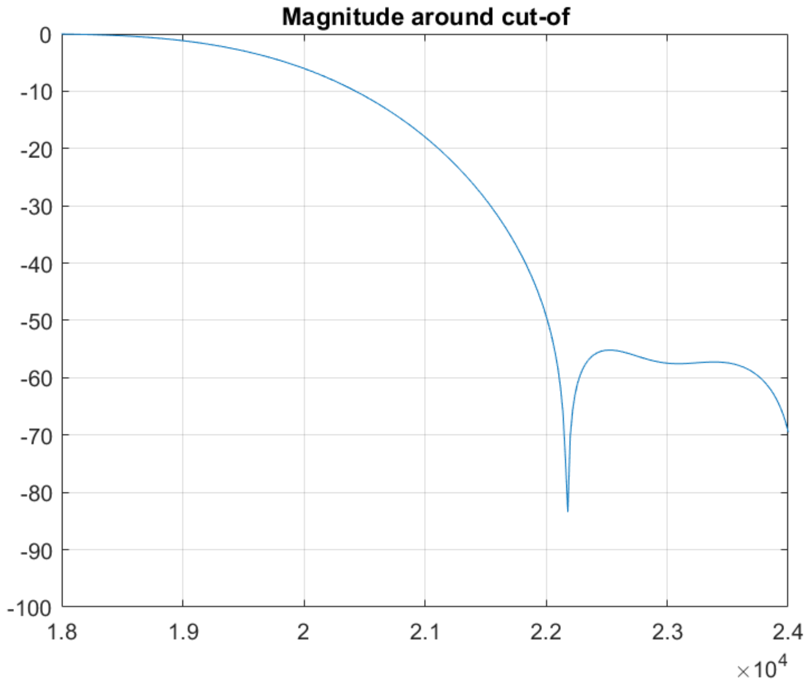
a) fpass=18kHz, fstop=22kHz, Hamming window, Δf=(fstop-fpass)/fs=4/88=1/22, N=3.3x22=72, fc=(fpass+fstop)/2=20kHz

b)

```
% Antialiasing filter
N=72;
fs=88000;
fn=fs/2;
fc=20000;
Wn=fc/fn;
[B,A]=fir1(N,Wn);
freqz(B,A,2200,fs);
title('Frequency response of antialiasing filter')
```



```
[H,f]=freqz(B,A,2200,fs);
plot(f,20*log10(abs(H))); axis([18000 24000 -100 0]);
title('Magnitude around cut-of'); grid
```



The filter attenuation is -50dB at 22kHz and fulfills the design criteria.

c) The order is 72 and the filter has 72 zeros and 72 poles at z=0.

d) The time delay is N/2*1/fs = 72/2*1/88kHz = 0.409ms

Problem 4

Design a digital Butterworth bandpass filter with the following specifications:

Sampling frequency = 20kHz

Passband: 2 – 3kHz with amplification above -3dB

Stopbands: 0 – 1kHz and 5 – 10kHz with amplification less than -30dB.

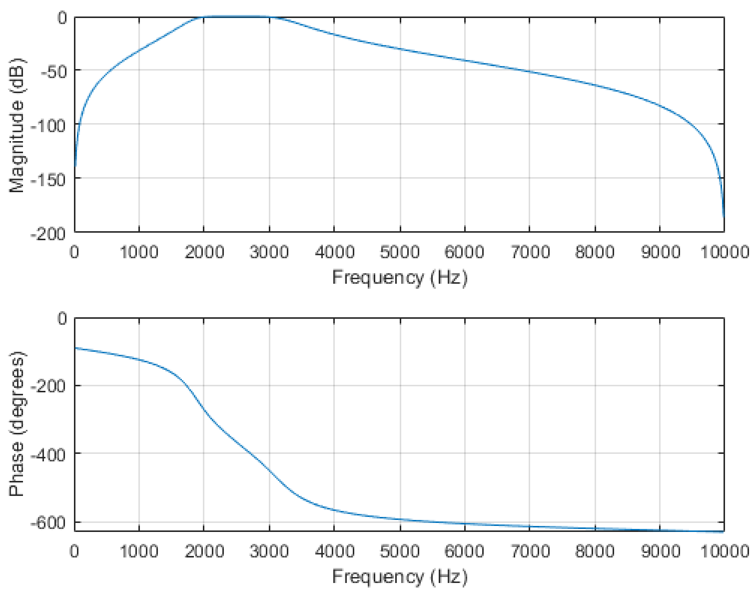
a) (10p) Use Matlab to design the filter and plot its frequency response.

b) (5p) How many zeros and poles does the filter have?

c) (5p) Write down the transfer function, H(z).

a)

```
% Butterworth bandpass filter
fs=20000;
Wp=[2000 3000]*2/fs; Rp=3;
Ws=[1000 5000]*2/fs; Rs=30;
[n,Wn]=buttord(Wp,Ws,Rp,Rs);
[B,A]=butter(n,Wn);
freqz(B,A,512,fs)
```



```
B
```

```
B = 1×7

    0.0074         0    -0.0222         0    0.0222         0    -0.0074
```

```
A
```

```
A = 1×7

    1.0000   -3.6625    6.6333   -7.1684    4.9147   -2.0073    0.4072
```

b) The filter has 6 zeros and 6 poles.

c)

$$H(z) = \frac{0.0074z^6 - 0.0222z^4 + 0.0222z^2 - 0.0074}{z^6 - 3.66z^5 + 6.63z^4 - 7.17z^3 + 4.91z^2 - 2.01z + 0.407}$$

Problem 5

A sine signal of frequency 10Hz and an amplitude of 5 is contaminated with noise. The noise can be generated with the Matlab-function randn(N) where N is the number of samples with a Gaussian distributed random value.

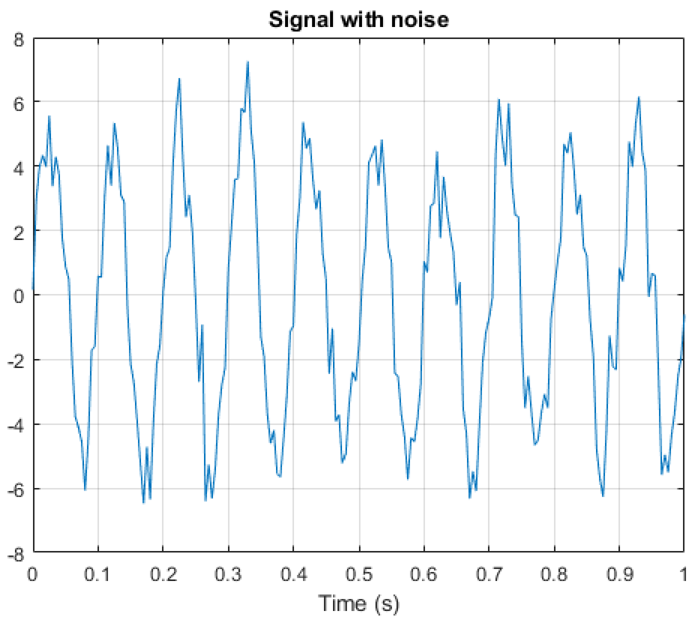
a) Generate and plot the signal for a length of 1s. Use a sampling interval of T=0.005s.

b) Design a narrow pass band filter by placing the zeros at $z=\pm 1$ and the poles at $z=r\cdot\exp(\pm j\theta)$. Determine r and θ for the filter when the 3dB-band width should be 2Hz and the center frequency is matched to the sine signal above. Plot the frequency response of the filter.

c) Filter the signal generated in a) with the filter designed in b) and plot the filtered signal as function of time for 1s.

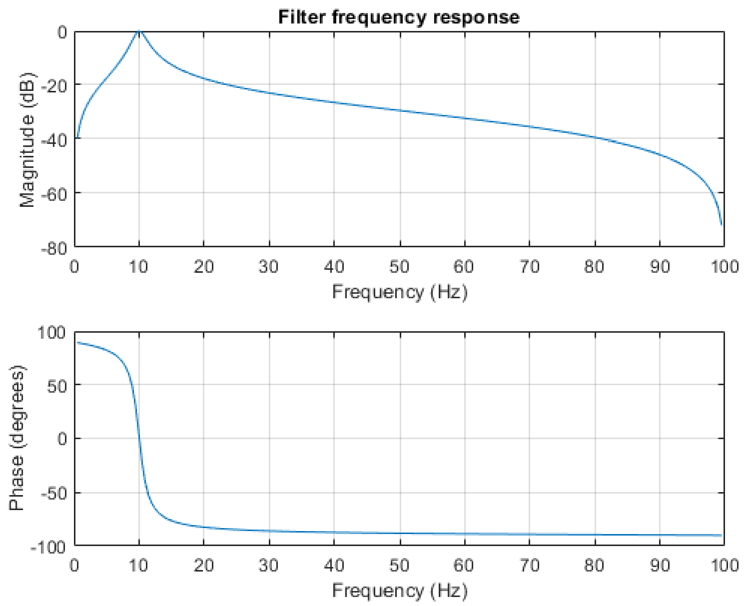
Solution 5

```
% Problem 5
% a) Signal with noise
T=0.005; fs=1/T;
f0=10;
t=0:T:1;
N=size(t);
x=5*sin(2*pi*f0*t)+randn(N);
plot(t,x); title('Signal with noise'); xlabel('Time (s)'); grid
```




```
% b) Filter design
BW=2;
theta=2*pi*f0/fs;
r=1-pi*BW/fs;
K=(1-r)*sqrt(1-2*r*cos(2*theta)+r^2)/(2*abs(sin(theta)));
```

```
B=[K 0 -K];
A=[1 -2*r*cos(theta) r^2];
freqz(B,A,201,fs); title('Filter frequency response')
```




```
% c) Filtered signal
y=filter(B,A,x);
plot(t,y); title('Filtered signal'); xlabel('Time (s)'); grid
```