ACIT4710 December 16th, 2021

Problem 1 (16%)

Classify the systems below, represented by their difference equation or transfer function, with respect to the following properties:

1. Stability: Stable, marginally stable, or unstable
2. Linearity: Linear or non-linear
3. Causality: Causal or non-causal
4. Time variance: Time invariant or time-variant

Classify all system according to all four properties. The systems are:

a) $y(n) = 1.5y(n-1) + nx(n) - x(n-1)$

b) $y(n) = 0.5y(n-1) + x(n) \cdot x(n+1)$

c) $H(z) = \frac{z-0.5}{z-1}$

d) $H(z) = z - z^{-1}$

Solutions 1

a) Unstable, linear, causal and time variant

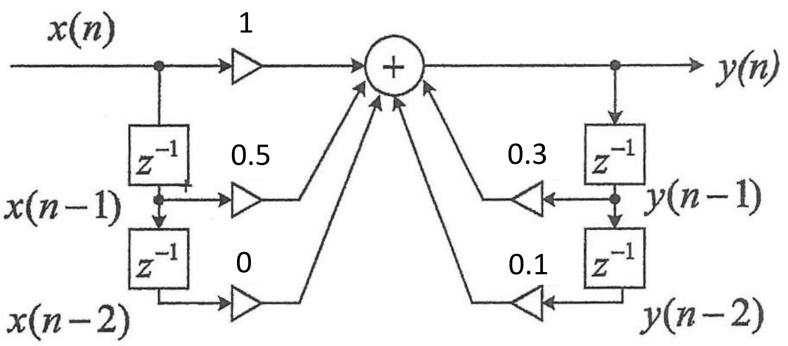
b) Stable, non-linear, non-causal and time invariant

c) Marginally stable, linear, causal, time invariant

d) Stable, linear, non-causal, time invariant.

Problem 2 (21%)

A time discrete circuit is shown below:



a) Determine the difference equation of the circuit.

b) Determine the transfer function of the circuit, H(z) = Y(z)/X(z).

c) Determine the zeros and the poles of the circuit.

d) Determine the impulse response, h(n), of the circuit, the response when x(t) = δ(t).

Solution 2

a) y(n)= 0.3y(n-1) + 0.1y(n-2) + x(n) + 0.5x(n-1)

b) $Y(z)[1-0.3z^{-1}-0.1z^{-2}] = X(z)[1+0.5z^{-1}]$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{z^2+0.5z}{z^2-0.3z-0.1} = \frac{z(z+0.5)}{(z-0.5)(z+0.2)}$$

c) Zeros: z = 0 and z = -0.5 ; Poles: z = 0.5 and z = -0.2

d) $h(n) = Z^{-1}\{H(z)\}$

$$\frac{H(z)}{z} = \frac{z+0.5}{(z-0.5)(z+0.2)} = \frac{a}{z-0.5} + \frac{b}{z+0.2} = \frac{1.43}{z-0.5} + \frac{-0.43}{z+0.2}$$

$$H(z) = \frac{1.43z}{z-0.5} - \frac{0.43z}{z+0.2}$$

$$h(n) = [1.43(0.5)^n - 0.43(-0.2)^n]u(n)$$

Problem 3 (21%)

Design a FIR high pass filter using the window design method with the following specifications:

Sampling frequency = 20kHz

Pass band: 5 – 10 kHz with maximum pass band ripple, δp = 1dB

Stop band: 0 – 2kHz with minimum stop band attenuation, δs = 50dB

a) Determine the required window function, estimate the required order, and calculate the cut-off frequency, using Table 7.7 in the textbook.

b) Use MATLAB to calculate the amplitude response of the filter.

c) Check the gain at 2000Hz and 5000Hz to find out if the filter fulfills all design specification.

Solution 3

a) Hamming window, Δf=3/20, N=3.3/3*20=22, fc=(2+5)/2=3.5kHz

b)
```
% Solution 3 b)
fs=20000;   % sampling frequency
fc=3500;    % cut-off frequency
wn=2*fc/fs; % normalized cut-off
N=22;       % filter order
B=fir1(N,wn,'high'); % filter coeffisients, Hamming window as default
[H,f]=freqz(B,1,1000,fs);    % frequency response
plot(f,20*log10(abs(H))); grid; xlabel('Frequency (Hz)'); ylabel('dB');
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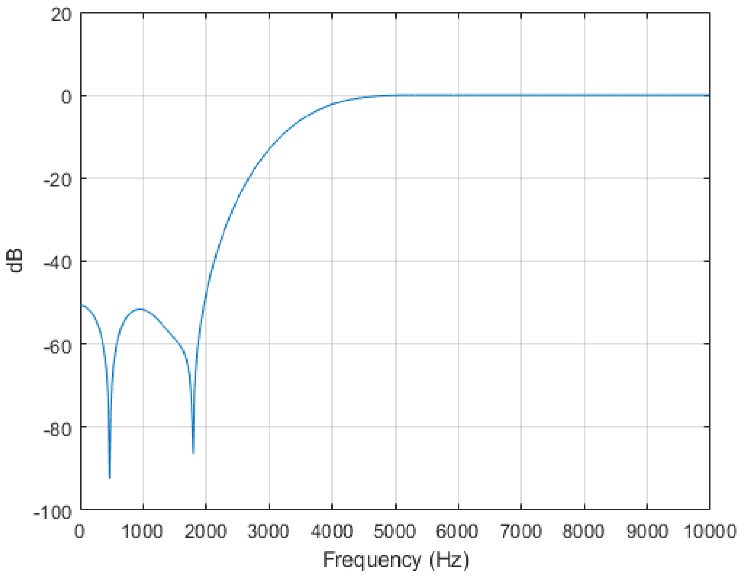

c)

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% c) Find gain at 2000 and 5000Hz
H1=20*log10(abs(H(201)))
H2=20*log10(abs(H(501)))
H1=-48
H2=-0.022
```

The frequency response is well above -1dB from 5000Hz and upwards, the passband criteria is fulfilled. The frequency response is -48dB at 2000Hz and the stopband criteria is therefore marginally violated.

Problem 4 (21%)

Design an IIR bandpass filter using the bilinear transform of a Butterworth filter with the following specifications:

Sampling frequency: 20kHz

Pass band: 4 – 6kHz, with maximum pass band ripple, δp = 1dB

Stop band: 0 – 2kHz and 8 – 10kHz, with minimum stop band attenuation, δs = 40dB

a) Determine the required order of the prototype filter and the design frequencies (-3dB-frequencies)

b) Determine the discrete transfer function of the filter, H(z).

c) Plot the amplitude response

Solution 4

a) N=4; Design frequencies: 3693 and 6307

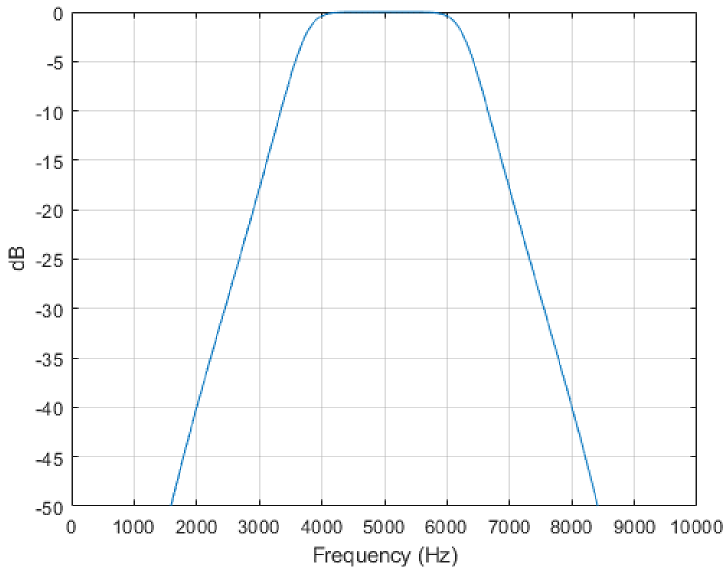
b) Order of bandpass filter is 8.

B=[0.011823318211291,0,-0.047293272845165,0,0.070939909267747,0,-
0.047293272845165,0,0.011823318211291]

A=[1,-2.109423746787797e-15,1.877826823427847,-1.346145417358002e-15,1.621355274174814,-2.498001805406602e-16,0.663018614948618,0,0.108663255582310]

$$H(z) = \frac{0.0118z^8 - 0.0473z^6 + 0.0709z^4 - 0.0473z^2 + 0.0118}{z^8 + 1.878z^6 + 1.621z^4 + 0.6630z^2 + 0.1087}$$

c)




```
% Solution 4
fs=20000;   % sampling frequency
wp=[4000 6000]*2/fs;    % passband
ws=[2000 8000]*2/fs;    % stopband
dp=1;   % passband rippel
ds=40;  % stopband attenuation
[N,wn]=buttord(wp,ws,dp,ds);    % prototype filter
[B,A]=butter(N,wn);     % Butterworth filter
[H,f]=freqz(B,A,512,fs);    % frequency response
plot(f,20*log10(abs(H))); grid; xlabel('Frequency (Hz)');
ylabel('dB');
axis([0 10000 -50 0])
```

Problem 5 (21%)

a) Generate a signal x(nT) = sin(2π5nT) + 0.5sin(2π50nT). The 5Hz signal is the signal you want to measure, but the signal is contaminated with a 50Hz that we regard as noise. Plot the signal with noise for a period of 1 sec. Use a sampling period of T=2ms.

b) Design a notch filter by placing the zeros at z= exp(±jθ) and the poles at z=r·exp(±jθ). Determine r and θ for the filter when the 3dB-band width should be 5Hz and the center frequency is matched to the 50Hz noise signal above. Plot the frequency response of the filter.

c) Filter the signal generated in a) with the filter designed in b) and plot the filtered signal as function of time for 1 sec.

Solution 5

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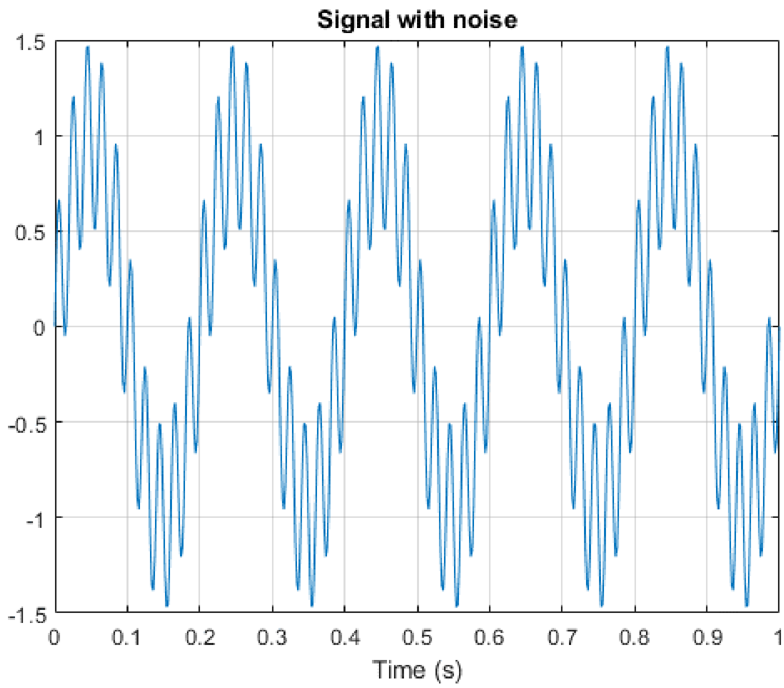
% Solution 5
% a) generate signal and noise
T=0.002;     % sampling period
t= 0:T:1;    % time axis
f1=5;        % signal frequency
f2=50;       % noise frequency
x=sin(2*pi*f1*t)+0.5*sin(2*pi*f2*t);    % signal + noise
plot(t,x); grid; title('Signal with noise'); xlabel('Time (s)');

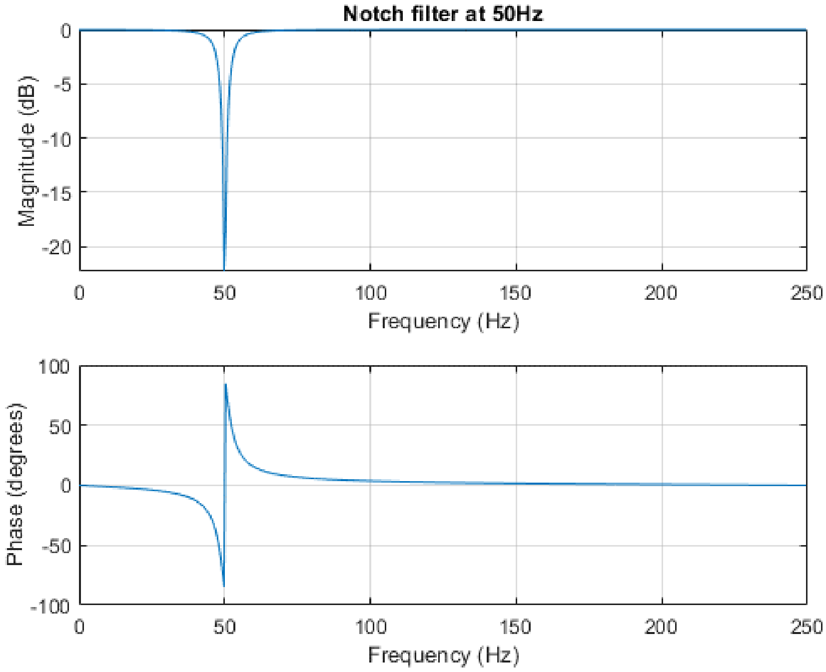
% b) Design of Notch filter
fs=1/T;      % sampling frequency
th=(50/fs)*2*pi;     % angle for 50Hz
r=1-(5/fs)*pi;       % radius of poles
K=(1-2*r*cos(th)+r^2)/(2-2*cos(th));     % gain
B=K*[1 -2*cos(th) 1];      % nummerator of H(z)
A=[1 -2*r*cos(th) r^2];       % denominator of H(z)
figure;
freqz(B,A,512,fs); title('Notch filter at 50Hz')

% c) Filtering of signal
y=filter(B,A,x);
figure;
plot(t,y); grid; title('Filtered signal'); xlabel('Time (s)')
```

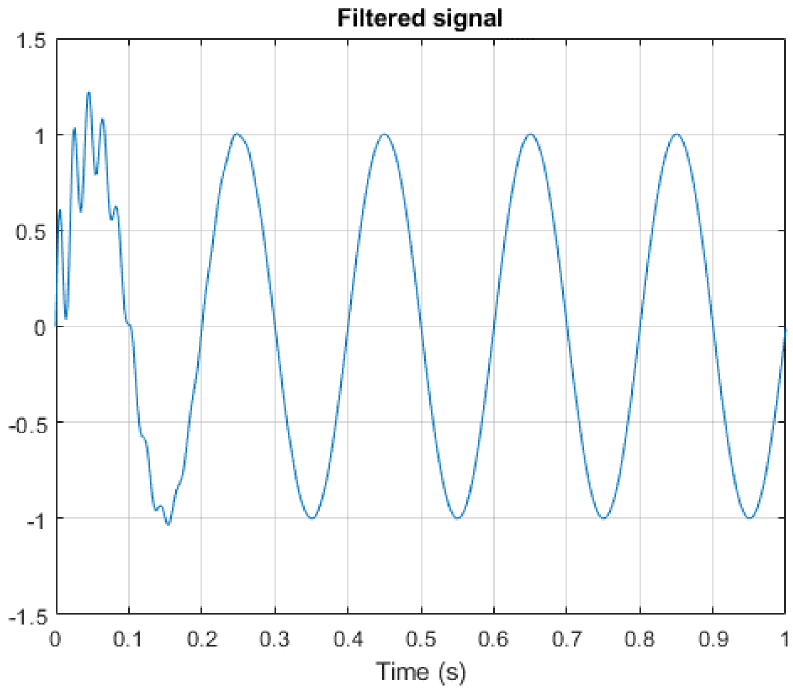
a)



b)


Notch filter at 50Hz
Magnitude (dB)
0
-5
-10
-15
-20
0
50
100
150
200
250
Frequency (Hz)
Phase (degrees)
100
50
0
-50
-100
0
50
100
150
200
250
Frequency (Hz)

c)


Filtered signal
1.5
1
0.5
0
-0.5
-1
-1.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Time (s)