Exercises for 5.5–5.6

1. Consider the following formula:

   $\forall x\big[(Px \vee Sx) \supset Cx\big]$

   (a) Construct an interpretation that makes the above formula true.

   (b) Construct an interpretation that makes the above formula false.

2. Consider the following formula:

   $Pa \supset \forall x(Rx \supset \neg Pa)$

   (a) Construct an interpretation that makes the formula true.

   (b) Construct an interpretation that makes the formula false.

3. Consider the following formula:

   $\forall x \forall y(Rxy \supset Ryx)$

   (a) Construct an interpretation that makes the formula true.

   (b) Construct an interpretation that makes the formula false.

4. Consider the following formula:

   $\exists xFx \supset \neg \forall x \neg Fx$

   Explain why it is not possible to construct an interpretation that makes this formula false.

5. Consider:

   $\exists xFx \wedge \forall x \neg Fx$

   Explain why it is not possible to construct an interpretation that makes this formula true.

6. Which of the following are logical truths? For those that are not, provide counterexamples. (A counterexample is a quick of providing an interpretation that makes the sentence false. For instance, consider $\forall x(Mx \wedge Hx) \supset (\forall xMx \vee \forall xHx)$. Here is a counterexample: suppose everyone in class is a CMC or a HMC student. It doesn't follow that everyone is a CMC student or everyone is a HMC student—maybe there's a mix of the two groups).

   (a) $\forall x(Fx \supset Gx) \supset (\forall xFx \supset \forall xGx)$

   (b) $\exists x(Fx \supset Gx) \supset (\exists xFx \supset \exists xGx)$

   (c) $(\exists xFx \wedge \exists xGx) \supset \exists x(Fx \wedge Gx)$

   (d) $\exists x(Fx \wedge Gx) \supset (\exists xFx \wedge \exists xGx)$