Exercises for 5.7 to 5.10

1. Consider: derivation: everyone gets grumpy when they are hungry; Erika is hungry; so someone is grumpy. Add the missing annotations in the formalization of this (e is a constant referring to Erika; Hx means 'x is hungry', Gx that 'x is grumpy'; e does not occur in Γ, Δ):

   1. Γ $\vdash \forall x(Hx \supset Gx)$ .... premise
   2. Δ $\vdash He$ .... premise
   3. Γ $\vdash He \supset Ge$ ....__
   4. Γ, Δ $\vdash Ge$ ....__
   5. Γ, Δ $\vdash \exists xGx$ ....__

2. Consider: All philosophy majors take logic. So if everyone majors in philosophy, everyone takes logic. Below is a formalization this. Add the missing annotations: (Px means 'x majors in philosophy', Lx means 'x takes logic'):

   1. Γ $\vdash \forall x(Px \supset Lx)$ .... premise
   2. $\forall xPx$ $\vdash \forall xPx$ ....__
   3. $\forall xPx$ $\vdash Pa$ ....__
   4. Γ $\vdash Pa \supset La$ ....__
   5. $\Gamma, \forall xPx$ $\vdash La$ ....__
   6. $\Gamma, \forall xPx$ $\vdash \forall xLx$ ....__
   7. Γ $\vdash \forall xPx \supset \forall xLx$ ....__

3. Add missing annotations:

   1. Γ $\vdash \exists x(Fx \wedge Gx)$ .... premise
   2. $Fa \wedge Ga$ $\vdash Fa \wedge Ga$ ....__
   3. $Fa \wedge Ga$ $\vdash Fa$ ....__
   4. $Fa \wedge Ga$ $\vdash \exists xFx$ ....__
   5. $Fa \wedge Ga$ $\vdash Ga$ ....__
   6. $Fa \wedge Ga$ $\vdash \exists xGx$ ....__
   7. $Fa \wedge Ga$ $\vdash \exists xFx \wedge \exists xGx$ ....__

8. Γ $\vdash \exists x F x \wedge \exists x G x$ .... __

4. What is wrong with the following attempted derivation?

1. Γ $\vdash \forall x (F x \vee G x)$ .... premise
2. Γ $\vdash F c \vee G c$ .... 1,$\forall$E
3. $F c$ $\vdash F c$ .... A
4. $F c$ $\vdash \forall x F x$ .... 3,$\forall$I
5. $F c$ $\vdash \forall x F x \vee \forall x G x$ .... 4,$\vee$I
6. $G c$ $\vdash G c$ .... A
7. $G c$ $\vdash \forall x G x$ .... 6,$\forall$I
8. $G c$ $\vdash \forall x F x \vee \forall x G x$ .... 7,$\vee$I
9. Γ $\vdash \forall x F x \vee \forall x G x$ .... 2,5,8,$\vee$E

5. Add missing annotations:

1. $\exists x (F x \vee G x)$ $\vdash \exists x (F x \vee G x)$ .... __
2. $F a \vee G a$ $\vdash F a \vee G a$ .... __
3. $F a$ $\vdash F a$ .... __
4. $F a$ $\vdash \exists x F x$ .... __
5. $F a$ $\vdash \exists x F x \vee \exists x G x$ .... __
6. $G a$ $\vdash G a$ .... __
7. $G a$ $\vdash \exists x G x$ .... __
8. $G a$ $\vdash \exists x F x \vee \exists x G x$ .... __
9. $F a \vee G a$ $\vdash \exists x F x \vee \exists x G x$ .... __
10. $\exists x (F x \vee G x)$ $\vdash \exists x F x \vee \exists x G x$ .... __

6. Add missing items.

1. $\forall x(Fx \supset Gx)$ $\vdash \forall x(Fx \supset Gx)$ .... A

2. $\forall xFx$ $\vdash \forall xFx$ ....A

3. $\forall xFx$ $\vdash Fa$ ....__

4. __ $\vdash$ __   .... 1,∀E

5. $\forall x(Fx \supset Gx), \forall xFx$ $\vdash Ga$ .... 3,4,⊃E

6. __ $\vdash$ __   .... 5, ∀I

7. __ $\vdash$ __   .... 6,⊃I

8. $\vdash \forall x(Fx \supset Gx) \supset (\forall xFx \supset \forall xGx)$
     7,⊃I

7. What's wrong with the following derivation (a is a constant not occurring in Γ):

1. Γ $\vdash \exists xFx \supset \exists xGx$ .... premise

2. $\exists xFx$ $\vdash \exists xFx$ ....A

3. $Γ, \exists xFx$ $\vdash \exists xGx$ ....1,2,⊃E

4. Ga $\vdash Ga$ ....A

5. Fa, Ga $\vdash Ga$ 4

6. Ga $\vdash Fa \supset Ga$ ....5,⊃I

7. Ga $\vdash \exists x(Fx \supset Gx)$ ....6,∃I

8. $Γ, Ga$ $\vdash \exists x(Fx \supset Gx)$ 7

9. Γ $\vdash \exists x(Fx \supset Gx)$ .... 3,8,∃E

8. Give an example illustrating why line 9 in the previous problem does not follow from line 1.

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9. Here is something you might be tempted to do: from $\Gamma \vdash \forall x (Fx \lor Gx)$ derive $\Gamma \vdash \forall x Fx \lor \forall x Gx$. Explain using an example why this would be a fallacious inference.

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