

Inference Rules for Predicate Logic

Fill in blanks.

1. 1. $\neg \exists x Fx$ $\vdash \neg Fk$ premise
 2. $\neg \exists x Fx$ $\vdash \underline{\hspace{1cm}}$ 1, $\forall I$

2. 1. $Fk \supset Gk$ $\vdash Hc$ premise
 2. $\underline{\hspace{1cm}}$ $\vdash \forall x Hx$ 1, $\forall I$

3. 1. $\exists x (Gx \vee Hx)$ $\vdash Fa$ premise
 2. $\underline{\hspace{1cm}}$ $\vdash \underline{\hspace{1cm}}$ 1, $\forall I$

4. 1. Fa $\vdash \forall x (Hx \supset Px)$ premise
 2. Fa $\vdash \underline{\hspace{1cm}}$ 1, $\forall E$
 3. Ha $\vdash Ha$ A
 4. Fa, Ha $\vdash Pa$ 2,3, $\supset E$

5. 1. Fa $\vdash \forall x (Hx \supset Px)$ premise
 2. $\underline{\hspace{1cm}}$ $\vdash \underline{\hspace{1cm}}$ 1, $\forall E$
 3. Hb $\vdash Hb$ A
 4. Fa, Hb $\vdash Pb$ 2,3, $\supset E$

6. 1. $\exists x Qx$ $\vdash \exists x Qx$ A
 2. Qa $\vdash \exists x Rx$ premise
 3. $\underline{\hspace{1cm}}$ $\vdash \underline{\hspace{1cm}}$ 1,2, $\exists E$

7. 1. Qb $\vdash Qb$ A
 2. $\underline{\hspace{1cm}}$ $\vdash \underline{\hspace{1cm}}$ 1, $\exists I$

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| 8. | 1. Γ | $\vdash \exists zWz \supset \exists zMz$ | premise |
| | 2. Δ | $\vdash Wa$ | premise |
| | 3. Δ | $\vdash _\$ | 2, $\exists I$ |
| | 4. Γ, Δ | $\vdash \exists zMz$ | 1, 3, $\supset E$ |
| 9. | 1. $\exists xFx$ | $\vdash \exists xFx$ | premise |
| | 2. Fa | $\vdash Fa$ | A |
| | 3. $_\$ | $\vdash _\$ | 2, $\exists I$ |
| | 4. $\exists xFx$ | $\vdash \exists yFy$ | 1, 3, $\exists E$ |
| 10. | 1. $\exists x(Rx \vee Fx)$ | $\vdash \exists x(Rx \vee Fx)$ | A |
| | 2. $Ra \vee Fa$ | $\vdash Ra \vee Fa$ | A |
| | 3. $\forall x[(Rx \vee Fx) \supset Wx]$ | $\vdash \forall x[(Rx \vee Fx) \supset Wx]$ | A |
| | 4. $\forall x[(Rx \vee Fx) \supset Wx]$ | $\vdash _\$ | 3, $\forall E$ |
| | 5. $\forall x[(Rx \vee Fx) \supset Wx], Ra \vee Fa$ | $\vdash Wa$ | 2, 4, $\supset E$ |
| | 6. $\forall x[(Rx \vee Fx) \supset Wx], Ra \vee Fa$ | $\vdash \exists xWx$ | 5, $\exists I$ |
| | 7. $_\$ | $\vdash _\$ | 1, 6, $\exists E$ |
| 11. | 1. $\exists x\forall yLyx$ | $\vdash \exists x\forall yLyx$ | A |
| | 2. $\forall yLyr$ | $\vdash \forall yLyr$ | A |
| | 3. $\forall yLyr$ | $\vdash Ldr$ | 2, $\forall E$ |
| | 4. $\forall yLyr$ | $\vdash \exists xLdx$ | 3, $\exists I$ |
| | 5. $\forall yLyr$ | $\vdash _\$ | 4, $\forall I$ |
| | 6. $_\$ | $\vdash \forall y\exists xLyx$ | 1, 5, $\exists E$ |