## Exercises for 2.1-2.3

- 1. Which of the following are wff?
  - (a) *P*
  - (b)  $W \vee S$
  - (c)  $\{[Y \land (\land T)] \land L\} \land L$ .....malformed
  - (d)  $(P \wedge M) \vee \neg [(W \wedge) \vee (R \vee Q)]$ .....malformed
  - (e)  $Z \vee Z$
  - (f)  $Z \wedge [(S \vee R) \wedge P \vee L]$ .....malformed
  - (g)  $Z \wedge [(S \vee R) \wedge (P \vee L)]$
  - (h)  $\lceil (Z \vee L) \wedge \neg Z \rceil \vee (Q \wedge S)$
  - (i)  $L \vee Q$
  - (j)  $L \vee Q \vee R$ .....malformed
  - (k) WR.....malformed
  - (1)  $S \vee (S \wedge Y)$
  - (m)  $(L \wedge T) \wedge \{ [T \vee (\{ \vee W)] \vee (Q \vee M) \} \dots$  malformed
  - (n)  $[(L \land M) \lor P] \land (Y \lor P \dots malformed)$
  - (o)  $[(L \land P) \lor T] \lor \lor (M \lor L) \ldots malformed$
  - (p)  $L \wedge \{(W \wedge W) \wedge [(W \wedge Z) \vee (Z \vee P]\}\dots$  malformed
- 2. Circle, highlight, or otherwise clearly mark the main connective of each of the following sentences.
  - (a)  $\{ [(Z \land R) \land T] \land [(Y \land Y) \lor (S \land Z)] \} \lor [S \lor (T \lor Y)] \}$
  - (b)  $T \vee \neg W$
  - (c)  $\{[(W \land R) \land (L \land T)] \lor (P \land Z)\} \lor (Y \lor S)$
  - (d)  $[(Q \wedge Q) \wedge (R \vee Q)] \wedge [(L \wedge W) \vee T]$
  - (e)  $\neg R \lor \{(T \lor S) \land [\neg W \land (W \lor P)]\}$
  - (f)  $\neg M \land M$
  - (g)  $\neg (M \land M)$
  - (h)  $(L \wedge M) \vee \{[(T \vee S) \vee (Z \wedge T)] \wedge (W \wedge Y)\}$

- (i)  $M \wedge (Q \wedge Z)$
- (j)  $Q \vee S$
- (k)  $\{ [(Q \lor P) \land (Y \land Y)] \lor [(M \lor S) \land T] \} \lor Q$
- (l) ¬Q

(m) 
$$(\{[T \land (R \lor \neg Z)] \land Y\} \land (Z \land \neg T)) \land (S \lor L)$$

(n) 
$$\neg P \land \{ [(L \lor Y) \land (Z \land Q)] \lor [P \lor (M \land M)] \}$$

- (o)  $(Q \wedge W) \vee (T \wedge P)$
- (p)  $(Q \wedge Q) \vee W$
- (q)  $[S \lor (W \land Y)] \land \{[Y \lor (Z \land Q)] \land Q\}$
- (r)  $\{(R \vee L) \wedge [(R \vee P) \wedge Y]\} \vee (Z \wedge Y)$
- (s)  $[(Z \lor M) \lor (S \land R)] \land \{[(S \land W) \land (L \lor Q)] \land Q\}$
- 3. Which of the sentences (a) through (h) in the previous question are of the form
  - (a)  $s_1 \vee s_2$ : a, b, c, e, h
  - (b)  $s_2 \wedge s_2$ : none. Notice  $s_2$  on both sides of the  $\wedge$  which means the sides must be identical
  - (c)  $\neg s : g$
- 4. A well-formed formula (wff) is a string of symbols (logical connectives, opening and closing brackets, letters of the Roman alphabet possibly with some subscripts). Let's think a bit about how a wff must look like. For each of the following claim, state whether it's true or false of wffs:
  - (a) A closing bracket cannot be the first symbol......True
  - (b) A closing bracket cannot be the last symbol. . . . . . False
  - (c) A logical connective cannot be the first symbol. . . . . . False
  - (d) The negation can be the first symbol. . . . . . . . . . True
  - (e)  $\neg$  cannot be followed immediately by  $\land$ . . . . . . . . . . True
  - (f) ∧ cannot be followed immediately by ¬. . . . . . . . . False

5. Take the truth table for conjunction. Plug  $(A \wedge B)$  into  $s_1$  and  $(A \vee B)$  into  $s_2$  and produce the resulting truth conditions for

	$A \wedge B$	$A \vee B$	$(A \land B) \land (A \lor B)$
	T	T	T
$(A \wedge B) \wedge (A \vee B).$	F	T	F
	T	F	F
	F	F	F