Exercises for 3.13–3.14

1. Here is something obvious. If we have evidence that $P \vee Q$ and we have evidence that $\neg P$, we have evidence that Q. Our proof system confirms this. Add the missing datums in the following derivation:

1.	Γ	$\vdash P \vee Q$	premise
2.	Δ	$\vdash \neg P$	premise
3.	P	$\vdash P$	A
4.	__	$\vdash \neg P$	2
5.	__	$\vdash P$	3
6.	__	$\vdash \neg\neg Q$	4,5,¬I
7.	Δ, P	$\vdash Q$	6,¬E
8.	Q	$\vdash Q$	A
9.	__	$\vdash Q$	1,7,8,∨E

2. Derivations can often be adapted to prove something similar. For instance, the above can be adapted easily to derive from $\Gamma \vdash \neg P \vee Q$ and $\Delta \vdash P$ to $\Gamma, \Delta \vdash Q$. Fill in the missing datums and annotations.

1.	Γ	$\vdash \neg P \vee Q$	premise
2.	Δ	$\vdash P$	premise
3.	__	$\vdash \neg P$	A
4.	__	$\vdash P$	2
5.	__	$\vdash \neg P$	3
6.	__	$\vdash \neg\neg Q$	4,5,¬I
7.	__	$\vdash Q$	6,¬E
8.	Q	$\vdash Q$	A
9.	__	$\vdash Q$	1,7,8,∨E

3. The Greek capital letters on the datum side are place-holders. You can plug anything you want into them. Take the derivation in Problem 2 above. You can plug P into Δ and add one more step at the end to show that you can infer from $\Gamma \vdash \neg P \vee Q$ to $\Gamma \vdash P \supset Q$ (which we should expect given the way the conditional is defined). Construct such a derivation.

4. The following is a derivation from $\Gamma \vdash P \supset Q$ to $\Gamma \vdash \neg P \vee Q$. Again, we should expect that there is such a derivation given the way the conditional was defined. Fill in the missing parts of the derivation:

1.	Γ	$\vdash P \supset Q$	premise
2.	$\neg(\neg P \vee Q)$	$\vdash$ __	A
3.	__	$\vdash \neg P$	A
4.	$\neg P$	$\vdash \neg P \vee Q$	__
5.	__	$\vdash$ __	2
6.	$\neg(\neg P \vee Q)$	$\vdash \neg\neg P$	4,5,¬I
7.	__	$\vdash$ __	6,¬E
8.	__	$\vdash$ __	1,7,⊃E
9.	$\Gamma, \neg(\neg P \vee Q)$	$\vdash \neg P \vee Q$	8,∨I
10.	Γ	$\vdash \neg\neg(\neg P \vee Q)$	__
11.	Γ	$\vdash \neg P \vee Q$	__

5. Suppose there is evidence that $P \supset Q$. In that case, there is is evidence that $\neg Q \supset \neg P$. Our proof system confirms this. Construct a derivation from $\Gamma \vdash P \supset Q$ to $\Gamma \vdash \neg Q \supset \neg P$. Hint: you can adapt and modify one the derivations given in Section 3.13.

6. Correct any errors in the annotations of the following derivation from $\Gamma \vdash P \supset Q$ and $\Delta \vdash R \vee \neg Q$ to $\Gamma, \Delta \vdash P \supset R$ (the only errors are in the annotations):

1.	Γ	$\vdash P \supset Q$	premise
2.	Δ	$\vdash R \vee \neg Q$	A
3.	R	$\vdash R$	A
4.	R, P	$\vdash R$	3
5.	R	$\vdash P \supset R$	$\supset$I
6.	$\neg Q$	$\vdash \neg Q$	2,4,$\vee$E
7.	P	$\vdash P$	A
8.	Γ, P	$\vdash Q$	1,7,$\wedge$E
9.	$\neg Q, P$	$\vdash \neg Q$	6
10.	$\Gamma, \neg Q$	$\vdash \neg P$	6,8,$\neg$I
11.	$\Gamma, \neg Q$	$\vdash \neg P \vee R$	7,10,$\vee$I
12.	$\neg P$	$\vdash \neg P$	A
13.	$\neg P, \neg R$	$\vdash \neg P$	11,3,$\supset$E
14.	$P, \neg R$	$\vdash P$	10
15.	$\neg P, P$	$\vdash \neg\neg R$	13,14,$\neg$E
16.	$\neg P, P$	$\vdash R$	15,$\neg$I
17.	$\neg P$	$\vdash P \supset R$	16,$\wedge$I
18.	$\Gamma, \neg Q$	$\vdash P \supset R$	11,17,5,$\vee$E
19.	Γ, Δ	$\vdash P \supset R$	2,5,17,18,$\vee$E

7. If you have evidence that $(W \vee Y) \supset Z$, you have evidence that $W \supset Z$. Construct a derivation from $\Gamma \vdash (W \vee Y) \supset Z$ to $\Gamma \vdash W \supset Z$.