Exercises for 4.1–4.4

1. Fill in missing items:

1.	Γ	$\vdash (P \vee Q) \supset R$	premise
2.	P	$\vdash P$	A
3.	$\underline{P}$	$\vdash \underline{P \vee Q}$	2,$\vee$I
4.	$\underline{\Gamma, P}$	$\vdash \underline{R}$	1,3,$\supset$E
5.	Γ	$\vdash \underline{P \supset R}$	4,$\supset$I
6.	Q	$\vdash Q$	A
7.	$\underline{Q}$	$\vdash \underline{P \vee Q}$	6,$\vee$I
8.	$\underline{\Gamma, Q}$	$\vdash \underline{R}$	1,7,$\supset$E
9.	Γ	$\vdash Q \supset R$	8,$\supset$I
10.	Γ	$\vdash \underline{(P \supset R) \wedge (Q \supset R)}$	5,9,$\wedge$I

2. Add missing annotations for the following proof of $\vdash (\neg P \vee \neg Q) \supset \neg(P \wedge Q)$

1.	$\neg P \vee \neg Q$	$\vdash \neg P \vee \neg Q$	A
2.	$P \wedge Q$	$\vdash P \wedge Q$	$\underline{\text{A}}$
3.	$\neg P$	$\vdash \neg P$	$\underline{\text{A}}$
4.	$P \wedge Q$	$\vdash P$	$\underline{2,\wedge\text{E}}$
5.	$\neg P, P \wedge Q$	$\vdash \neg P$	$\underline{3}$
6.	$\neg P$	$\vdash \neg(P \wedge Q)$	$\underline{4,5,\neg\text{I}}$
7.	$\neg Q$	$\vdash \neg Q$	$\underline{\text{A}}$
8.	$P \wedge Q$	$\vdash Q$	$\underline{2,\wedge\text{E}}$
9.	$\neg Q, P \wedge Q$	$\vdash \neg Q$	$\underline{7}$
10.	$\neg Q$	$\vdash \neg(P \wedge Q)$	$\underline{8,9,\neg\text{I}}$
11.	$\neg P \vee \neg Q$	$\vdash \neg(P \wedge Q)$	$\underline{1,6,10,\vee\text{E}}$
12.		$\vdash (\neg P \vee \neg Q) \supset \neg(P \wedge Q)$	$\underline{11,\supset\text{I}}$

3. Prove $\vdash \neg(P \supset Q) \supset \neg Q$. (Hint: assume $\neg(P \supset Q)$ as well as Q .)

1.	$\neg(P \supset Q)$	$\vdash \neg(P \supset Q)$	A
2.	Q	$\vdash Q$	A
3.	Q, P	$\vdash Q$	2
4.	Q	$\vdash P \supset Q$	3,$\supset$I
5.	$\neg(P \supset Q), Q$	$\vdash \neg(P \supset Q)$	1
6.	$\neg(P \supset Q)$	$\vdash \neg Q$	4,5,$\neg$I
7.		$\vdash \neg(P \supset Q) \supset \neg Q$	6,$\supset$I

4. Here is part of a proof of $\neg(P \vee Q) \vdash \neg P \wedge \neg Q$. Complete the rest.

1.	$\neg(P \vee Q)$	$\vdash \neg(P \vee Q)$	A
2.	P	$\vdash P$	A
3.	P	$\vdash P \vee Q$	2,$\vee$I
4.	$\neg(P \vee Q), P$	$\vdash \neg(P \vee Q)$	1
5.	$\neg(P \vee Q)$	$\vdash \neg P$	3,4,$\neg$I

6.	Q	$\vdash Q$	A
7.	Q	$\vdash P \vee Q$	6,$\vee$I
8.	$\neg(P \vee Q), Q$	$\vdash \neg(P \vee Q)$	1
9.	$\neg(P \vee Q)$	$\vdash \neg Q$	7,8,$\neg$I
10.	$\neg(P \vee Q)$	$\vdash \neg P \wedge \neg Q$	5,9,$\wedge$I

5. Add missing items.

1.	$\neg(P \supset Q)$	$\vdash \neg(P \supset Q)$	A
2.	$\neg P$	$\vdash \neg P$	A
3.	$\underline{\neg P, \neg Q}$	$\vdash \underline{\neg P}$	2
4.	$\underline{P}$	$\vdash \underline{P}$	A
5.	$\underline{P, \neg Q}$	$\vdash \underline{P}$	4
6.	$\neg P, P$	$\vdash \neg\neg Q$	3,5,¬I
7.	$\neg P, P$	$\vdash Q$	6,¬E
8.	$\underline{\neg P}$	$\vdash \underline{P \supset Q}$	7,⊃I
9.	$\neg(P \supset Q), \underline{\neg P}$	$\vdash \neg(P \supset Q)$	1
10.	$\neg(P \supset Q)$	$\vdash \neg\neg P$	8,9,¬I
11.	$\neg(P \supset Q)$	$\vdash P$	10,¬E
12.		$\vdash \neg(P \supset Q) \supset P$	11,⊃I

6. Prove $P \supset Q, P \vee Q \vdash Q$. Hint: Assume $P \supset Q$ and assume Q. Use $\vee$E.

1.	$P \supset Q$	$\vdash P \supset Q$	A
2.	$P \vee Q$	$\vdash P \vee Q$	A
3.	P	$\vdash P$	A
4.	$P \supset Q, P$	$\vdash Q$	1,3,⊃E
5.	Q	$\vdash Q$	A
6.	$P \supset Q, P \vee Q$	$\vdash Q$	2,4,5,∨E

7. Recall the Prisoner's Dilemma (exercise for 3.1-4). Let P mean that Jerry will confess, and let Q mean that Ben is better off confessing. Turn the reasoning in the standardized form provided in the answer key into a derivation that utilizes EM (hint: the derivation will have two premises).

1.	Γ	$\vdash P \supset Q$	..... premise
2.	Δ	$\vdash \neg P \supset Q$	..... premise
3.		$\vdash P \vee \neg P$	..... EM
4.	P	$\vdash P$	..... A
5.	Γ, P	$\vdash Q$	..... 1,4,$\supset$E
6.	$\neg P$	$\vdash \neg P$	..... A
7.	$\Delta, \neg P$	$\vdash Q$	..... 2,6,$\supset$E
8.	Γ, Δ	$\vdash Q$	..... 3,5,7,$\vee$E