

1. Add missing annotations.

1.  $\Gamma \quad \vdash \neg P \vee \neg Q$  ..... premise
2.  $P \wedge Q \quad \vdash P \wedge Q$  ..... \_
3.  $\neg P \quad \vdash \neg P$  ..... \_
4.  $P \wedge Q \quad \vdash P$  ..... \_
5.  $\neg P, P \wedge Q \quad \vdash \neg P$  ..... \_
6.  $\neg P \quad \vdash \neg(P \wedge Q)$  ..... \_
7.  $\neg Q \quad \vdash \neg Q$  ..... \_
8.  $P \wedge Q \quad \vdash Q$  ..... \_
9.  $\neg Q, P \wedge Q \quad \vdash \neg Q$  ..... \_
10.  $\neg Q \quad \vdash \neg(P \wedge Q)$  ..... \_
11.  $\Gamma \quad \vdash \neg(P \wedge Q)$  ..... \_

2. Add annotations.

1.  $\Gamma \quad \vdash \neg(P \wedge Q)$  ..... \_
2.  $\neg(\neg P \vee \neg Q) \quad \vdash \neg(\neg P \vee \neg Q)$  ..... \_
3.  $\neg P \quad \vdash \neg P$  ..... \_
4.  $\neg P \quad \vdash \neg P \vee \neg Q$  ..... \_
5.  $\neg(\neg P \vee \neg Q), \neg P \quad \vdash \neg(\neg P \vee \neg Q)$  ..... \_
6.  $\neg(\neg P \vee \neg Q) \quad \vdash \neg\neg P$  ..... \_
7.  $\neg(\neg P \vee \neg Q) \quad \vdash P$  ..... \_
8.  $\neg Q \quad \vdash \neg Q$  ..... \_
9.  $\neg Q \quad \vdash \neg P \vee \neg Q$  ..... \_
10.  $\neg(\neg P \vee \neg Q), \neg Q \quad \vdash \neg(\neg P \vee \neg Q)$  ..... \_
11.  $\neg(\neg P \vee \neg Q) \quad \vdash \neg\neg Q$  ..... \_
12.  $\neg(\neg P \vee \neg Q) \quad \vdash Q$  ..... \_
13.  $\neg(\neg P \vee \neg Q) \quad \vdash P \wedge Q$  ..... \_
14.  $\Gamma, \neg(\neg P \vee \neg Q) \quad \vdash \neg(P \wedge Q)$  ..... \_
15.  $\Gamma \quad \vdash \neg\neg(\neg P \vee \neg Q)$  ..... \_
16.  $\Gamma \quad \vdash \neg P \vee \neg Q$  ..... \_

3. Add missing datums.

1. $\Gamma$	$\vdash \neg(P \wedge Q)$	..... premise
2. $\underline{\quad}$	$\vdash \neg(\neg P \vee \neg Q)$	..... A
3. $\underline{\quad}$	$\vdash \neg P$	..... A
4. $\underline{\quad}$	$\vdash \neg P \vee \neg Q$	..... 3, $\vee I$
5. $\underline{\quad}$	$\vdash \neg(\neg P \vee \neg Q)$	..... 2
6. $\underline{\quad}$	$\vdash \neg\neg P$	..... 4, 5, $\neg I$
7. $\underline{\quad}$	$\vdash P$	..... 6, $\neg E$
8. $\underline{\quad}$	$\vdash \neg Q$	..... A
9. $\underline{\quad}$	$\vdash \neg P \vee \neg Q$	..... 8, $\vee I$
10. $\underline{\quad}$	$\vdash \neg(\neg P \vee \neg Q)$	..... 2
11. $\underline{\quad}$	$\vdash \neg\neg Q$	..... 9, 10, $\neg I$
12. $\underline{\quad}$	$\vdash Q$	..... 11, $\neg E$
13. $\underline{\quad}$	$\vdash P \wedge Q$	..... 7, 12, $\wedge I$
14. $\underline{\quad}$	$\vdash \neg(P \wedge Q)$	..... 1
15. $\underline{\quad}$	$\vdash \neg\neg(\neg P \vee \neg Q)$	..... 13, 14, $\neg I$
16. $\underline{\quad}$	$\vdash \neg P \vee \neg Q$	..... 15, $\neg E$

4. Add missing items.

1. $\Gamma$	$\vdash \neg(P \supset Q)$	..... premise
2. $\neg P$	$\vdash \neg P$	..... A
3. $\underline{\quad}$	$\vdash \underline{\quad}$	..... 2
4. $\underline{\quad}$	$\vdash \underline{\quad}$	..... A
5. $\underline{\quad}$	$\vdash \underline{\quad}$	..... 4
6. $\neg P, P$	$\vdash \neg\neg Q$	..... 3, 5, $\neg I$
7. $\neg P, P$	$\vdash Q$	..... 6, $\neg E$
8. $\underline{\quad}$	$\vdash \underline{\quad}$	..... $\underline{\quad}$
9. $\Gamma, \underline{\quad}$	$\vdash \neg(P \supset Q)$	..... 1
10. $\Gamma$	$\vdash \neg\neg P$	..... 8, 9, $\neg I$
11. $\Gamma$	$\vdash P$	..... $\underline{\quad}$

5. Add missing datums.

1. $\Gamma$	$\vdash P \vee Q$	.....premise
2. $\Delta$	$\vdash \neg Q \vee R$	.....premise
3. $\underline{\quad}$	$\vdash P$	.....A
4. $\underline{\quad}$	$\vdash Q$	.....A
5. $\underline{\quad}$	$\vdash R$	.....A
6. $\underline{\quad}$	$\vdash \neg Q$	.....A
7. $\underline{\quad}$	$\vdash Q$	.....4
8. $\underline{\quad}$	$\vdash \neg Q$	.....6
9. $\underline{\quad}$	$\vdash \neg\neg P$	.....7,8, $\neg$ I
10. $\underline{\quad}$	$\vdash P$	.....9, $\neg$ E
11. $\underline{\quad}$	$\vdash P \vee R$	.....10, $\vee$ I
12. $\underline{\quad}$	$\vdash P \vee R$	.....5, $\vee$ I
13. $\underline{\quad}$	$\vdash P \vee R$	.....2,11,12, $\vee$ E
14. $\underline{\quad}$	$\vdash P \vee R$	.....3, $\vee$ I
15. $\Gamma, \Delta$	$\vdash P \vee R$	.....1,13,14, $\vee$ E

6. Add missing items.

1. $\Gamma$	$\vdash P \supset Q$	..... premise
2. $\Delta$	$\vdash R \vee \neg Q$	..... premise
3. $\underline{\quad}$	$\vdash \underline{\quad}$	..... A
4. $R, P$	$\vdash R$	..... 3
5. $R$	$\vdash \underline{\quad}$	..... 4, $\supset$ I
6. $\underline{\quad}$	$\vdash \underline{\quad}$	..... A
7. $\underline{\quad}$	$\vdash \underline{\quad}$	..... $\underline{\quad}$
8. $\Gamma, P$	$\vdash Q$	..... 1, 7, $\supset$ E
9. $\underline{\quad}, P$	$\vdash \neg Q$	..... 6
10. $\Gamma, \neg Q$	$\vdash \neg P$	..... 8, 9, $\neg$ I
11. $\Gamma, \neg Q$	$\vdash \neg P \vee R$	..... 10, $\vee$ I
12. $\underline{\quad}$	$\vdash \underline{\quad}$	..... $\underline{\quad}$
13. $\neg P, \underline{\quad}$	$\vdash \underline{\quad}$	..... 12
14. $P, \underline{\quad}$	$\vdash P$	..... 7
15. $\neg P, P$	$\vdash \underline{\quad}$	..... 13, 14, $\neg$ I
16. $\underline{\quad}$	$\vdash \underline{\quad}$	..... 15, $\neg$ E
17. $\neg P$	$\vdash P \supset R$	..... 16, $\supset$ I
18. $\Gamma, \neg Q$	$\vdash P \supset R$	..... $\underline{\quad}$
19. $\Gamma, \Delta$	$\vdash P \supset R$	..... $\underline{\quad}$

7. Continue the following derivation until you reach  $\Gamma \vdash P \supset Q$ .

1. $\Gamma$	$\vdash \neg P$	..... premise
2. $P$	$\vdash P$	..... A
3. $P, \neg Q$	$\vdash P$	..... 2
4. $\Gamma, \neg Q$	$\vdash \neg P$	..... 1

8. Derive from  $\Gamma \vdash P \supset Q$  and  $\Delta \vdash P \vee Q$  to  $\Gamma, \Delta \vdash Q$

9. Construct derivation from  $\Gamma \vdash P \supset Q$  and  $\Gamma \vdash Q \supset R$  to  $\Gamma, \Delta \vdash P \supset Q$ .

10. Derive from  $\Gamma \vdash P \vee Q$  and  $\Delta \vdash P \supset R$  to  $\Gamma, \Delta \vdash R \vee Q$ .

11. Derive from  $\Gamma \vdash P \vee Q$  and  $\Delta \vdash Q \supset R$  to  $\Gamma, \Delta \vdash P \vee R$
12. Derive from  $\Gamma \vdash \neg(P \supset Q)$  to  $\Gamma \vdash \neg Q$
13. Derive from  $\Gamma \vdash R \supset P$ ,  $\Delta \vdash \neg R \supset S$  and  $\Theta \vdash \neg P$  to  $\Gamma, \Delta, \Theta \vdash S \wedge \neg R$ .
14. Derive from  $\Gamma \vdash (P \vee Q) \vee R$  to  $\Gamma \vdash P \vee (Q \vee R)$ .