Exercises for 5.1–5.4

1. Let x, y be variables and let the following keys explain the meanings of some predicates and constants:

Nx:	$x$is a narcissist
Lxy:	x loves y
Px:	is popular
d:	Donald
f:	Francis
m:	Magdalena

   Translate the following sentences into $\mathcal{L}_Q$ (it is often helpful to translate back into English to see if it comes out meaning the same as the original):

   (a) Donald is a narcissist.
   (b) If Donald is a narcissist, then Donald loves Donald.
   (c) Narcissists love themselves.
   (d) If someone is a narcissist, they are not popular.
   (e) Narcissists are not popular.
   (f) Everyone loves Magdalena.
   (g) Magdalena loves everyone.
   (h) There is someone Francis loves.
   (i) There is someone who loves Francis.
   (j) Everyone loves someone.
   (k) Someone is loved by everyone.
   (l) If someone is loved by everyone, then everyone loves someone.

Answer Key

(a) Nd
(b) $Nd \supset Ldd$
(c) $\forall x(Nx \supset Lxx)$
(d) $\forall x(Nx \supset \neg Px)$
(e) $\forall x(Nx \supset \neg Px)$
(f) $\forall x Lxm$

(g) $\forall x Lmx$

(h) $\exists x Lfx$

(i) $\exists x Lxf$

(j) $\forall x \exists y Lxy$

(k) $\exists x \forall y Lyx$

(l) $\exists x \forall y Lyx \supset \forall x \exists y Lxy$

2. Consider the following formula:

$$\exists x\Big[Fx \wedge \exists y\Big(Fy \wedge \{\neg Gxy \wedge \neg \exists z[\neg Gxk \wedge (\neg Gyz \wedge Fz)]\}\Big)\Big]$$

(a) What is the scope of the left-most quantifier?

(b) What is the scope of the right-most quantifier?

(c) True of false: the formula contains free variables.

Answer Key

(a) The entire rest of the sentence.

(b) $\neg Gxk \wedge (\neg Gyz \wedge Fz)$

(c) True. k is free (should have specified k is not a constant).

3. Consider the following formula (x, y, z are variables):

$$\exists x Fx \supset \forall x[\neg Fx \supset \exists y(\neg Gxz \wedge Fy)]$$

(a) Which variable in the formula is free?

(b) What is the scope of the left-most existential quantifier?

(c) What is the scope of the universal quantifier?

Answer Key

(a) z.

(b) Just the Fx left of the first $\supset$.

(c) $\neg Fx \supset \exists y(\neg Gxz \wedge Fy)$