

Here are some more simple practice derivations. Do not use any theorems:

1. From $\Gamma \vdash p$ to $\Gamma \vdash q \supset p$ (Hint: you can add anything you want to the datum of a sequent).

1. Γ	$\vdash p$ premise
2. Γ, q	$\vdash p$ 1
3. Γ	$\vdash q \supset p$ 2, \supset I

2. From $\Gamma \vdash r$ and $\Delta \vdash \neg r$ to $\Gamma, \Delta \vdash s$ (Hint: you can add anything you want to the datum of a sequent).

1. Γ	$\vdash r$ premise
2. Δ	$\vdash \neg r$ premise
3. $\Gamma, \neg s$	$\vdash r$ 1
4. $\Gamma, \neg s$	$\vdash \neg r$ 2
5. Γ	$\vdash \neg \neg s$ 3,4, \neg I
6. Γ	$\vdash s$ 5, \neg E

3. From $\Gamma \vdash s$ to $\Gamma \vdash \neg\neg s$ (Hint: assume $\neg s$).

1.	Γ	$\vdash s$	premise
2.	$\neg s$	$\vdash \neg s$	A
3.	$\Gamma, \neg s$	$\vdash s$	1
4.	Γ	$\vdash \neg\neg s$	2,3, \neg I

4. From $\Gamma \vdash \neg\neg p \supset q$ to $\Gamma \vdash p \supset q$ (Hint: part of the derivation adapts the previous one).

1.	Γ	$\vdash \neg\neg p \supset q$	premise
2.	p	$\vdash p$	A
3.	$\neg p$	$\vdash \neg p$	A
4.	$p, \neg p$	$\vdash p$	2
5.	p	$\vdash \neg\neg p$	3,4, \neg I
6.	Γ, p	$\vdash q$	1,5, \supset E
7.	Γ	$\vdash p \supset q$	6, \supset I

5. From $\Gamma \vdash \neg\neg p \supset \neg\neg q$ to $\Gamma \vdash p \supset q$ (Hint: adapt the previous two derivations).

1. Γ	$\vdash \neg\neg p \supset \neg\neg q$ premise
2. p	$\vdash p$ A
3. $\neg p$	$\vdash \neg p$ A
4. $p, \neg p$	$\vdash p$ 2
5. p	$\vdash \neg\neg p$ 3,4, \neg I
6. Γ, p	$\vdash \neg\neg q$ 1,5, \supset E
7. Γ, p	$\vdash q$ 6, \neg E
8. Γ	$\vdash p \supset q$ 7, \supset I

6. From $\Gamma \vdash \neg\neg s \vee w$ to $\Gamma \vdash s \vee w$ (Hint: \vee E is your friend and don't forget about \vee I).

1. Γ	$\vdash \neg\neg s \vee w$ premise
2. $\neg\neg s$	$\vdash \neg\neg s$ A
3. $\neg\neg s$	$\vdash s$ 2, \neg E
4. $\neg\neg s$	$\vdash s \vee w$ 3, \vee I
5. w	$\vdash w$ A
6. w	$\vdash s \vee w$ 5, \vee I
7. Γ	$\vdash s \vee w$ 1,4,6, \vee E

7. From $\Gamma \vdash p \wedge (q \wedge r)$ to $\Gamma \vdash (p \wedge r) \wedge q$ (Hint: you only need $\wedge E$ and $\wedge I$).

1. Γ	$\vdash p \wedge (q \wedge r)$ premise
2. Γ	$\vdash p$ 1, $\wedge E$
3. Γ	$\vdash q \wedge r$ 1, $\wedge E$
4. Γ	$\vdash r$ 3, $\wedge E$
5. Γ	$\vdash p \wedge r$ 2, 4, $\wedge I$
6. Γ	$\vdash q$ 3, $\wedge E$
7. Γ	$\vdash (p \wedge r) \wedge q$ 5, 6, $\wedge I$

8. From $\Gamma \vdash (p \vee q) \vee r$ to $\Gamma \vdash q \vee (p \vee r)$ (Hint: apart from assumptions, you only need $\vee I$ and $\vee E$).

1. Γ	$\vdash (p \vee q) \vee r$ premise
2. $p \vee q$	$\vdash p \vee q$ A
3. p	$\vdash p$ A
4. p	$\vdash p \vee r$ 3, $\vee I$
5. p	$\vdash q \vee (p \vee r)$ 4, $\vee I$
6. q	$\vdash q$ A
7. q	$\vdash q \vee (p \vee r)$ 6, $\vee I$
8. $p \vee q$	$\vdash q \vee (p \vee r)$ 2, 5, 7, $\vee E$
9. r	$\vdash r$ A
10. r	$\vdash p \vee r$ 9, $\vee I$
11. r	$\vdash q \vee (p \vee r)$ 10, $\vee I$
12. Γ	$\vdash q \vee (p \vee r)$ 1, 8, 11, $\vee E$