

Note: A derivation in predicate logic need not use any of the quantifier-specific rules. The first four problems below are examples of this point.

1. Add missing annotations:

1. $\Gamma$	$\vdash Fc \vee Gd$	.....premise
2. $\Delta$	$\vdash Fc \supset Fk$	.....premise
3. $\Theta$	$\vdash Gd \supset He$	.....premise
4. $Fc$	$\vdash Fc$	.....__
5. $\Delta, Fc$	$\vdash Fk$	.....__
6. $\Delta, Fc$	$\vdash Fk \vee He$	.....__
7. $Gd$	$\vdash Gd$	.....__
8. $\Theta, Gd$	$\vdash He$	.....__
9. $\Theta, Gd$	$\vdash Fk \vee He$	.....__
10. $\Gamma, \Delta, \Theta$	$\vdash Fk \vee He$	.....__

2. Add missing annotations:

1. $\Gamma$	$\vdash \forall xFx \vee \exists xGx$	.....premise
2. $\Delta$	$\vdash \forall xFx \supset \neg \forall xFx$	.....premise
3. $\Theta$	$\vdash \exists xGx \supset \forall xHx$	.....premise
4. $\forall xFx$	$\vdash \forall xFx$	.....__
5. $\Delta, \forall xFx$	$\vdash \neg \forall xFx$	.....__
6. $\Delta, \forall xFx$	$\vdash \neg \forall xFx \vee \forall xHx$	.....__
7. $\exists xGx$	$\vdash \exists xGx$	.....__
8. $\Theta, \exists xGx$	$\vdash \forall xHx$	.....__
9. $\Theta, \exists xGx$	$\vdash \neg \forall xFx \vee \forall xHx$	.....__
10. $\Gamma, \Delta, \Theta$	$\vdash \neg \forall xFx \vee \forall xHx$	.....__

3. Fill in the missing items.

1. $\Gamma$	$\vdash Fa \vee (Lb \vee Kc)$	..... premise
2. $Fa$	$\vdash Fa$	..... A
3. $\underline{\quad}$	$\vdash \underline{\quad}$	..... A
4. $Kc$	$\vdash (Fa \vee Lb) \vee Kc$	..... 3, $\vee$
5. $\underline{\quad}$	$\vdash \underline{\quad}$	..... 2, $\vee$
6. $\underline{\quad}$	$\vdash \underline{\quad}$	..... A
7. $Lb$	$\vdash \underline{\quad}$	..... 6, $\vee$
8. $\underline{\quad}$	$\vdash \underline{\quad}$	..... 5, $\vee$
9. $Lb \vee Kc$	$\vdash Lb \vee Kc$	..... A
10. $\underline{\quad}$	$\vdash (Fa \vee Lb) \vee Kc$	..... 7, $\vee$
11. $\underline{\quad}$	$\vdash \underline{\quad}$	..... $\underline{\quad}$
12. $\Gamma$	$\vdash (Fa \vee Lb) \vee Kc$	..... 1, 8, 11, $\vee$

4. Fill in the missing items.

1.	$\Gamma$	$\vdash \exists xFx \vee (\forall xLx \vee \exists xKx)$	.....premise
2.	$\exists xFx$	$\vdash \exists xFx$	.....A
3.	—	$\vdash$ —	.....A
4.	$\exists xKx$	$\vdash (\exists xFx \vee \forall xLx) \vee \exists xKx$	.....3, —
5.	—	$\vdash$ —	.....2, $\vee I$
6.	—	$\vdash$ —	.....A
7.	$\forall xLx$	$\vdash$ —	.....6, —
8.	—	$\vdash$ —	.....5, $\vee I$
9.	$\forall xLx \vee \exists xKx$	$\vdash \forall xLx \vee \exists xKx$	.....A
10.	—	$\vdash (\exists xFx \vee \forall xLx) \vee \exists xKx$	.....7, $\vee I$
11.	—	$\vdash$ —	.....—
12.	$\Gamma$	$\vdash (\exists xFx \vee \forall xLx) \vee \exists xKx$	.....1,8,11, $\vee E$

5. Fill in missing items.

1.  $\underline{\hspace{1cm}}$   $\vdash \forall xFx$  ..... A
2.  $\forall xGx$   $\vdash \underline{\hspace{1cm}}$  ..... A
3.  $\underline{\hspace{1cm}}$   $\vdash \underline{\hspace{1cm}}$  ..... 1,  $\forall E$
4.  $\underline{\hspace{1cm}}$   $\vdash \underline{\hspace{1cm}}$  ..... 2,  $\forall E$
5.  $\underline{\hspace{1cm}}$   $\vdash Fa \wedge Ga$  ..... 3, 4,  $\wedge I$
6.  $\forall xFx, \forall xGx$   $\vdash \forall x(Fx \wedge Gx)$  .....  $\underline{\hspace{1cm}}$

6. Fill in missing items.

1.  $\underline{\hspace{1cm}}$   $\vdash \underline{\hspace{1cm}}$  ..... A
2.  $\underline{\hspace{1cm}}$   $\vdash \forall xFx$  ..... A
3.  $\underline{\hspace{1cm}}$   $\vdash \underline{\hspace{1cm}}$  ..... 2,  $\forall E$
4.  $\underline{\hspace{1cm}}$   $\vdash \underline{\hspace{1cm}} \vee Gk$  ..... 3,  $\vee I$
5.  $\forall xGx$   $\vdash \forall xGx$  .....  $\underline{\hspace{1cm}}$
6.  $\underline{\hspace{1cm}}$   $\vdash \underline{\hspace{1cm}}$  ..... 5,  $\forall E$
7.  $\underline{\hspace{1cm}}$   $\vdash Fk \vee \underline{\hspace{1cm}}$  ..... 6,  $\vee I$
8.  $\forall xFx \vee \forall xGx$   $\vdash Fk \vee Gk$  ..... 1, 4, 7,  $\vee E$
9.  $\underline{\hspace{1cm}}$   $\vdash \forall x(Fx \vee Gx)$  ..... 8,  $\forall I$

7. Fill in missing items.

1.  $\forall xFx$   $\vdash \forall xFx$  .....  $\underline{\hspace{1cm}}$
2.  $\forall xFx$   $\vdash Fa$  .....  $\underline{\hspace{1cm}}$
3.  $\forall xFx$   $\vdash \exists xFx$  .....  $\underline{\hspace{1cm}}$

8. Fill in missing items.

- |     |   |                                      |                      |
|-----|---|--------------------------------------|----------------------|
| 1.  | $\neg \forall x Fx$                         | $\vdash \neg \forall x Fx$           | ..... A              |
| 2.  | $\neg \exists x \neg Fx$                    | $\vdash \neg \exists x \neg Fx$      | ..... A              |
| 3.  | ___   | $\vdash$ ___                         | ..... A              |
| 4.  | ___   | $\vdash$ ___                         | ..... 3, $\exists I$ |
| 5.  | ___   | $\vdash$ ___                         | ..... 2              |
| 6.  | $\neg \exists x \neg Fx$                    | $\vdash$ ___                         | ..... 4, 5, $\neg I$ |
| 7.  | $\neg \exists x \neg Fx$                    | $\vdash Fa$                          | ..... 6, $\neg E$    |
| 8.  | $\neg \exists x \neg Fx$                    | $\vdash \forall x Fx$                | ..... ___            |
| 9.  | $\neg \forall x Fx, \neg \exists x \neg Fx$ | $\vdash \neg \forall x Fx$           | ..... ___            |
| 10. | $\neg \forall x Fx$                         | $\vdash \neg \neg \exists x \neg Fx$ | ..... ___            |
| 11. | $\neg \forall x Fx$                         | $\vdash \exists x \neg Fx$           | ..... 10, $\neg E$   |

9. Fill in missing items.

- |    |                     |                            |                      |
|----|---------------------|----------------------------|----------------------|
| 1. | $\neg \exists x Fx$ | $\vdash \neg \exists x Fx$ | ..... A              |
| 2. | ___                 | $\vdash$ ___               | ..... A              |
| 3. | ___                 | $\vdash$ ___               | ..... 2, $\exists I$ |
| 4. | ___                 | $\vdash$ ___               | ..... 1              |
| 5. | $\neg \exists x Fx$ | $\vdash \neg Fa$           | ..... 3, 4, $\neg I$ |
| 6. | $\neg \exists x Fx$ | $\vdash \exists x \neg Fx$ | ..... 5, $\exists I$ |

10. Fill in missing items. Hint: Line 1 is a theorem of sentential logic. Lines 2 and 3 are simply citing the results of Q3 and Q4; leave them untouched.

- |    |  |   |
|----|--|---|
| 1. | $\vdash$ ___                                 | ..... ___                                 |
| 2. | $\forall x Fx$                               | $\vdash \exists x Fx$ ..... Q3 above      |
| 3. | $\neg \forall x Fx$                          | $\vdash \exists x \neg Fx$ ..... Q4 above |
| 4. | $\forall x Fx$                               | $\vdash$ ___                              |
| 5. | $\neg \forall x Fx$                          | $\vdash$ ___                              |
| 6. | $\vdash \exists x Fx \vee \exists x \neg Fx$ | ..... 1, 4, 5, $\vee E$                   |

11. Fill in missing items.

- |    |  |                                   |       |                  |
|----|--|-----------------------------------|-------|------------------|
| 1. | $\forall x(Fx \supset Gx)$                   | $\vdash \forall x(Fx \supset Gx)$ | ..... | __               |
| 2. | __   | $\vdash$ __                       | ..... | A                |
| 3. | __   | $\vdash$ __                       | ..... | 1, $\forall E$   |
| 4. | __   | $\vdash$ __                       | ..... | __               |
| 5. | $\forall x(Fx \supset Gx), Fa$               | $\vdash Ga$                       | ..... | 3,4, $\supset E$ |
| 6. | __   | $\vdash \neg Ga$                  | ..... | 2, $\forall E$   |
| 7. | __   | $\vdash \neg Ga$                  | ..... | __               |
| 8. | $\forall x(Fx \supset Gx), \forall x\neg Gx$ | $\vdash \neg Fa$                  | ..... | 5,7, $\neg I$    |
| 9. | __   | $\vdash \forall x\neg Fx$         | ..... | 8, $\forall I$   |

12. Fill in missing items.

- |     |                                |                                       |       |                   |
|-----|--------------------------------|---------------------------------------|-------|-------------------|
| 1.  | $\exists xFx \vee \exists xGx$ | $\vdash \exists xFx \vee \exists xGx$ | ..... | A                 |
| 2.  | __                             | $\vdash$ __                           | ..... | __                |
| 3.  | __                             | $\vdash$ __                           | ..... | A                 |
| 4.  | __                             | $\vdash$ __                           | ..... | 3, $\forall I$    |
| 5.  | $Fa$                           | $\vdash$ __                           | ..... | 4, $\exists I$    |
| 6.  | $\exists xFx$                  | $\vdash \exists x(Fx \vee Gx)$        | ..... | 2,5, $\exists E$  |
| 7.  | $\exists xGx$                  | $\vdash \exists xGx$                  | ..... | A                 |
| 8.  | __                             | $\vdash$ __                           | ..... | A                 |
| 9.  | __                             | $\vdash$ __                           | ..... | 8, $\forall I$    |
| 10. | __                             | $\vdash$ __                           | ..... | 9, $\exists I$    |
| 11. | $\exists xGx$                  | $\vdash \exists x(Fx \vee Gx)$        | ..... | 7,10, $\exists E$ |
| 12. | $\exists xFx \vee \exists xGx$ | $\vdash \exists x(Fx \vee Gx)$        | ..... | __                |

13.	1.	$\exists xFx \vee \exists xGx$	$\vdash \exists xFx \vee \exists xGx$	.....__
	2.	$\forall x(Fx \supset Gx)$	$\vdash \forall x(Fx \supset Gx)$	.....__
	3.	$\exists xFx$	$\vdash \exists xFx$	.....__
	4.	$Fa$	$\vdash Fa$	.....__
	5.	$\forall x(Fx \supset Gx)$	$\vdash Fa \supset Ga$	.....__
	6.	$\forall x(Fx \supset Gx), Fa$	$\vdash Ga$	.....__
	7.	$\forall x(Fx \supset Gx), Fa$	$\vdash \exists xGx$	.....__
	8.	$\forall x(Fx \supset Gx), \exists xFx$	$\vdash \exists xGx$	.....__
	9.	$\exists xGx$	$\vdash \exists xGx$	.....__
	10.	$\exists xFx \vee \exists xGx, \forall x(Fx \supset Gx)$	$\vdash \exists xGx$	.....__