

## Exercises for 4.5

1. Suppose there is a derivation from  $\Gamma \vdash s$  to  $\Gamma \vdash t$ . Explain why it follows that  $\vdash s \supset t$ . (Hint: you can plug anything you want into  $\Gamma$ .)
2. Suppose  $s \supset t$  is a tautology. Explain why in that case the truth of  $s$  guarantees the truth of  $t$ .
3. Suppose we can show that if  $\vdash s \supset t$ , then  $s \supset t$  is a tautology. Explain why this would show that if there is a derivation from  $\Gamma \vdash s$  to  $\Gamma \vdash t$ , then the truth of  $s$  guarantees the truth of  $t$ .
4. Suppose we can show that if  $s \supset t$  is a tautology, then  $\vdash s \supset t$ . Explain why this would show that if  $s \supset t$  is a tautology, then there is a derivation from  $\Gamma \vdash s$  to  $\Gamma \vdash t$ .