1. Add missing annotations.

1. Γ	$\vdash \neg P \vee \neg Q$	. . . . premise
2. $P \wedge Q$	$\vdash P \wedge Q$	. . . . A
3. $\neg P$	$\vdash \neg P$	. . . . A
4. $P \wedge Q$	$\vdash P$	. . . . 2,$\wedge$E
5. $\neg P, P \wedge Q$	$\vdash \neg P$	 3
6. $\neg P$	$\vdash \neg(P \wedge Q)$	. . . . 4,5,$\neg$I
7. $\neg Q$	$\vdash \neg Q$	. . . . A
8. $P \wedge Q$	$\vdash Q$	. . . . 2,$\wedge$E
9. $\neg Q, P \wedge Q$	$\vdash \neg Q$	 7
10. $\neg Q$	$\vdash \neg(P \wedge Q)$	. . . . 8,9,$\neg$I
11. Γ	$\vdash \neg(P \wedge Q)$	. . . . 1,6,10,$\vee$E

2. Add annotations.

1. Γ	$\vdash \neg(P \wedge Q)$	. . . . premise
2. $\neg(\neg P \vee \neg Q)$	$\vdash \neg(\neg P \vee \neg Q)$	. . . . A
3. $\neg P$	$\vdash \neg P$	. . . . A
4. $\neg P$	$\vdash \neg P \vee \neg Q$	. . . . 3,$\vee$I
5. $\neg(\neg P \vee \neg Q), \neg P$	$\vdash \neg(\neg P \vee \neg Q)$	 2
6. $\neg(\neg P \vee \neg Q)$	$\vdash \neg\neg P$	. . . . 4,5,$\neg$I
7. $\neg(\neg P \vee \neg Q)$	$\vdash P$	. . . . 6,$\neg$E
8. $\neg Q$	$\vdash \neg Q$	. . . . A
9. $\neg Q$	$\vdash \neg P \vee \neg Q$	. . . . 8,$\vee$I
10. $\neg(\neg P \vee \neg Q), \neg Q$	$\vdash \neg(\neg P \vee \neg Q)$	 2
11. $\neg(\neg P \vee \neg Q)$	$\vdash \neg\neg Q$	. . . . 9,10,$\neg$I
12. $\neg(\neg P \vee \neg Q)$	$\vdash Q$	. . . . 11,$\neg$E
13. $\neg(\neg P \vee \neg Q)$	$\vdash P \wedge Q$	. . . . 7,12,$\wedge$I
14. $\Gamma, \neg(\neg P \vee \neg Q)$	$\vdash \neg(P \wedge Q)$	 1
15. Γ	$\vdash \neg\neg(\neg P \vee \neg Q)$	. . . . 13,14,$\neg$I
16. Γ	$\vdash \neg P \vee \neg Q$	. . . . 15,$\neg$E

3. Add missing datums.

1. Γ $\vdash \neg(P \wedge Q)$ .... premise
2. $\underline{\neg(\neg P \vee \neg Q)}$ $\vdash \neg(\neg P \vee \neg Q)$ .... A
3. $\underline{\neg P}$ $\vdash \neg P$ .... A
4. $\underline{\neg P}$ $\vdash \neg P \vee \neg Q$ .... 3,$\vee$I
5. $\underline{\neg(\neg P \vee \neg Q), \neg P}$ $\vdash \neg(\neg P \vee \neg Q)$ 2
6. $\underline{\neg(\neg P \vee \neg Q)}$ $\vdash \neg\neg P$ .... 4,5,$\neg$I
7. $\underline{\neg(\neg P \vee \neg Q)}$ $\vdash P$ .... 6,$\neg$E
8. $\underline{\neg Q}$ $\vdash \neg Q$ .... A
9. $\underline{\neg Q}$ $\vdash \neg P \vee \neg Q$ .... 8,$\vee$I
10. $\underline{\neg(\neg P \vee \neg Q), \neg Q}$ $\vdash \neg(\neg P \vee \neg Q)$ 2
11. $\underline{\neg(\neg P \vee \neg Q)}$ $\vdash \neg\neg Q$ .... 9,10,$\neg$I
12. $\underline{\neg(\neg P \vee \neg Q)}$ $\vdash Q$ .... 11,$\neg$E
13. $\underline{\neg(\neg P \vee \neg Q)}$ $\vdash P \wedge Q$ .... 7,12,$\wedge$I
14. $\underline{\Gamma, \neg(\neg P \vee \neg Q)}$ $\vdash \neg(P \wedge Q)$ 1
15. $\underline{\Gamma}$ $\vdash \neg\neg(\neg P \vee \neg Q)$ .... 13,14,$\neg$I
16. $\underline{\Gamma}$ $\vdash \neg P \vee \neg Q$ .... 15,$\neg$E

4. Add missing items.

1. Γ $\vdash \neg(P \supset Q)$ .... premise
2. $\neg P$ $\vdash \neg P$ .... A
3. $\underline{\neg P, \neg Q}$ $\vdash \underline{\neg P}$ 2
4. $\underline{P}$ $\vdash \underline{P}$ .... A
5. $\underline{P, \neg Q}$ $\vdash \underline{P}$ 4
6. $\neg P, P$ $\vdash \neg\neg Q$ .... 3,5,$\neg$I
7. $\neg P, P$ $\vdash Q$ .... 6,$\neg$E
8. $\underline{\neg P}$ $\vdash \underline{P \supset Q}$ .... 7,$\supset$I
9. $\Gamma, \underline{\neg P}$ $\vdash \neg(P \supset Q)$ 1
10. Γ $\vdash \neg\neg P$ .... 8,9,$\neg$I

11. Γ $\qquad$ $\vdash P$ $\quad$ . . . . $\underline{10,\neg E}$

5. Add missing datums.

1. Γ	$\vdash P \vee Q$	. . . . premise
2. Δ	$\vdash \neg Q \vee R$	. . . . premise
3. $\underline{P}$	$\vdash P$	. . . . A
4. $\underline{Q}$	$\vdash Q$	. . . . A
5. $\underline{R}$	$\vdash R$	. . . . A
6. $\underline{\neg Q}$	$\vdash \neg Q$	. . . . A
7. $\underline{Q, \neg P}$	$\vdash Q$	 4
8. $\underline{\neg Q, \neg P}$	$\vdash \neg Q$	 6
9. $\underline{Q, \neg Q}$	$\vdash \neg\neg P$	. . . . 7,8,$\neg$I
10. $\underline{Q, \neg Q}$	$\vdash P$	. . . . 9,$\neg$E
11. $\underline{Q, \neg Q}$	$\vdash P \vee R$	. . . . 10,$\vee$I
12. $\underline{R}$	$\vdash P \vee R$	. . . . 5,$\vee$I
13. $\underline{\Delta, Q}$	$\vdash P \vee R$	. . . . 2,11,12,$\vee$E
14. $\underline{P}$	$\vdash P \vee R$	. . . . 3,$\vee$I
15. Γ, Δ	$\vdash P \vee R$	. . . . 1,13,14,$\vee$E

6. Add missing items.

1. Γ	$\vdash P \supset Q$	$\dots\dots\dots\dots\dots\dots\dots$	premise
2. Δ	$\vdash R \vee \neg Q$	$\dots\dots\dots\dots\dots\dots$	premise
3. $\underline{R}$	$\vdash \underline{R}$	$\dots\dots\dots\dots\dots\dots\dots\dots\dots$	A
4. R, P	$\vdash R$	$\dots\dots\dots\dots\dots\dots\dots\dots\dots$	3
5. R	$\vdash \underline{P \supset R}$	$\dots\dots\dots\dots\dots\dots\dots\dots$	4,$\supset$I
6. $\underline{\neg Q}$	$\vdash \underline{\neg Q}$	$\dots\dots\dots\dots\dots\dots\dots\dots$	A
7. $\underline{P}$	$\vdash \underline{P}$	$\dots\dots\dots\dots\dots\dots\dots\dots\dots$	$\underline{A}$
8. Γ, P	$\vdash Q$	$\dots\dots\dots\dots\dots\dots\dots\dots$	1,7,$\supset$E
9. $\underline{\neg Q}, P$	$\vdash \neg Q$	$\dots\dots\dots\dots\dots\dots\dots\dots\dots$	6
10. $\Gamma, \neg Q$	$\vdash \neg P$	$\dots\dots\dots\dots\dots\dots\dots$	8,9,$\neg$I
11. $\Gamma, \neg Q$	$\vdash \neg P \vee R$	$\dots\dots\dots\dots\dots\dots\dots$	10,$\vee$I
12. $\underline{\neg P}$	$\vdash \underline{\neg P}$	$\dots\dots\dots\dots\dots\dots\dots\dots$	$\underline{A}$
13. $\neg P, \underline{\neg R}$	$\vdash \underline{\neg P}$	$\dots\dots\dots\dots\dots\dots\dots\dots$	12
14. $P, \underline{\neg R}$	$\vdash P$	$\dots\dots\dots\dots\dots\dots\dots\dots$	7
15. $\neg P, P$	$\vdash \underline{\neg\neg R}$	$\dots\dots\dots\dots\dots\dots$	13,14,$\neg$I
16. $\underline{\neg P, P}$	$\vdash \underline{R}$	$\dots\dots\dots\dots\dots\dots\dots$	15,$\neg$E
17. $\neg P$	$\vdash P \supset R$	$\dots\dots\dots\dots\dots\dots\dots$	16,$\supset$I
18. $\Gamma, \neg Q$	$\vdash P \supset R$	$\dots\dots\dots\dots\dots$	$\underline{11,17,5,\vee E}$
19. Γ, Δ	$\vdash P \supset R$	$\dots\dots\dots\dots\dots\dots$	$\underline{2,5,18,\vee E}$

7. Continue the following derivation until you reach $\Gamma \vdash P \supset Q$.

1. Γ	$\vdash \neg P$	$\dots\dots\dots\dots\dots\dots\dots$	premise
2. P	$\vdash P$	$\dots\dots\dots\dots\dots\dots\dots\dots$	A
3. $P, \neg Q$	$\vdash P$	$\dots\dots\dots\dots\dots\dots\dots\dots$	2
4. $\Gamma, \neg Q$	$\vdash \neg P$	$\dots\dots\dots\dots\dots\dots\dots\dots$	1

Answer Key			
5. Γ, P	$\vdash \neg\neg Q$	$\dots\dots\dots\dots\dots\dots$	3,4,$\neg$I
6. Γ, P	$\vdash Q$	$\dots\dots\dots\dots\dots\dots\dots$	5,$\neg$E
7. Γ	$\vdash P \supset Q$	$\dots\dots\dots\dots\dots\dots\dots$	6,$\supset$I

8. Derive from $\Gamma \vdash P \supset Q$ and $\Delta \vdash P \vee Q$ to $\Gamma, \Delta \vdash Q$

Answer Key

1. Γ	$\vdash P \supset Q$	.... premise
2. Δ	$\vdash P \vee Q$	.... premise
3. P	$\vdash P$	.... A
4. Γ, P	$\vdash Q$	.... 1,3,$\supset$E
5. Q	$\vdash Q$	.... A
6. Δ, Γ	$\vdash Q$	.... 2,4,5,$\vee$E

9. Construct derivation from $\Gamma \vdash P \supset Q$ and $\Gamma \vdash Q \supset R$ to $\Gamma, \Delta \vdash P \supset Q$.

Answer Key

1. Γ	$\vdash P \supset R$	.... premise
2. Δ	$\vdash R \supset Q$	.... premise
3. P	$\vdash P$	.... A
4. Γ, P	$\vdash R$	.... 1,3,$\supset$E
5. Γ, Δ, P	$\vdash Q$	.... 2,4,$\supset$E
6. Γ, Δ	$\vdash P \supset Q$	.... 5,$\supset$I

10. Derive from $\Gamma \vdash P \vee Q$ and $\Delta \vdash P \supset R$ to $\Gamma, \Delta \vdash R \vee Q$.

Answer Key

1. Γ	$\vdash P \vee Q$	.... premise
2. Δ	$\vdash P \supset R$	.... premise
3. P	$\vdash P$	.... A
4. Δ, P	$\vdash R$	.... 2,3,$\supset$E
5. Δ, P	$\vdash R \vee Q$	.... 4,$\vee$I
6. Q	$\vdash Q$	.... A
7. Q	$\vdash R \vee Q$	.... 6,$\vee$I
8. Γ, Δ	$\vdash R \vee Q$	.... 1,5,7,$\vee$E

11. Derive from $\Gamma \vdash P \vee Q$ and $\Delta \vdash Q \supset R$ to $\Gamma, \Delta \vdash P \vee R$

5

Answer Key

1. Γ	⊢ $P \lor Q$	. . . . premise
2. Δ	⊢ $Q \supset R$	. . . . premise
3. P	⊢ P	. . . . A
4. P	⊢ $P \lor R$	. . . . 3,∨I
5. Q	⊢ Q	. . . . A
6. Δ, Q	⊢ R	. . . . 2,5,⊃E
7. Δ, Q	⊢ $P \lor R$	. . . . 6,∨I
8. Γ, Δ	⊢ $P \lor R$	. . . . 1,4,7,∨E

12. Derive from $\Gamma \vdash \neg(P \supset Q)$ to $\Gamma \vdash \neg Q$

Answer Key

1. Γ	⊢ $\neg(P \supset Q)$	. . . . premise
2. Q	⊢ Q	. . . . A
3. Q, P	⊢ Q	 2
4. Q	⊢ $P \supset Q$	. . . . 3,⊃I
5. Γ, Q	⊢ $\neg(P \supset Q)$	 1
6. Γ	⊢ $\neg Q$	. . . . 4,5,¬I

13. Derive from $\Gamma \vdash R \supset P$, $\Delta \vdash \neg R \supset S$ and $\Theta \vdash \neg P$ to $\Gamma, \Delta, \Theta \vdash S \land \neg R$
.

Answer Key

1. Γ	⊢ $R \supset P$	. . . . premise
2. Δ	⊢ $\neg R \supset S$	. . . . premise
3. Θ	⊢ $\neg P$	. . . . premise
4. R	⊢ R	. . . . premise
5. Γ, R	⊢ P	. . . . 1,4,⊃E
6. Θ, R	⊢ $\neg P$	 3
7. Γ, Θ	⊢ $\neg R$	. . . . 5,6,¬I
8. Γ, Δ, Θ	⊢ S	. . . . 2,7,⊃E

9. $\Gamma, \Delta, \Theta \qquad \vdash S \wedge \neg R \qquad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots 7,8,\wedge I$

14. Derive from $\Gamma \vdash (P \vee Q) \vee R$ to $\Gamma \vdash P \vee (Q \vee R)$.

Answer Key		
1. Γ	$\vdash P \vee (Q \vee R)$	$\dots\dots\dots\dots\dots\dots$ premise
2. P	$\vdash P$	$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$ A
3. P	$\vdash P \vee Q$	$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots 2,\vee I$
4. P	$\vdash (P \vee Q) \vee R$	$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots 3,\vee I$
5. $Q \vee R$	$\vdash Q \vee R$	$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$ A
6. Q	$\vdash Q$	$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$ A
7. Q	$\vdash P \vee Q$	$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots 6,\vee I$
8. Q	$\vdash (P \vee Q) \vee R$	$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots 7,\vee I$
9. R	$\vdash R$	$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$ A
10. R	$\vdash (P \vee Q) \vee R$	$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots 9,\vee I$
11. $Q \vee R$	$\vdash (P \vee Q) \vee R$	$\dots\dots\dots\dots\dots\dots\dots 5,8,10,\vee E$
12. Γ	$\vdash (P \vee Q) \vee R$	$\dots\dots\dots\dots\dots\dots\dots\dots 1,4,11,\vee E$