

Invariant Generation for Complexity Analysis of Python Programs

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Project Description

The high level goal of this project is to implement several functions. Given a Python program P ,

- `ub_time_complexity(P)` returns a symbolic upper bound on the *run time* of P .
- `ub_output_complexity(P)` returns a symbolic upper bound on the *size* of the output of P .
- `lb_time_complexity(P)` returns a symbolic lower bound on the *run time* of P .
- `lb_output_complexity(P)` returns a symbolic lower bound on the *size* of the output of P .

For example, consider the following code:

```
1 def f(n):  
2     out = 1  
3     for i in range(n):  
4         out = out * n  
5     return out
```

We can see that `ub_time_complexity(f)` should return $n \in \mathcal{O}(n)$, because the loop runs n iterations. However, f returns out which is 1 initially and is multiplied by n , n times. So, `ub_output_complexity(f)` $= n^n \in \mathcal{O}(n^n)$.

0.1 Restrictions

In order to keep the scope of this assignment reasonable, we will restrict input programs in the following ways:

- All functions will be of type `int, int, ..., int \rightarrow int`.
- We will only simplify recursive programs of several particular forms (discussed below).
- We will ignore certain dynamic features of Python.
- If termination/invariant detection is too difficult to infer, we will output `?`, meaning “we do not know”.

The difficulty arises in handling loops and recursion.

0.2 Loops

To effectively bound the runtime of a loop, we must bound the number of iterations. We propose doing this by using a variety of AI techniques and ideas from [8, 6, 5] to generate invariants on the values of variables after multiple iterations. In particular, we aim to be able to handle *binary search*-like functions, and *early terminations* of loops. We intend to use search techniques on the AST of the loop, planning

to prove the invariants we find correct, and possibly machine learning to classify programs as terminating or not.

0.3 Recursion

To effectively bound the runtime of a recursive function, we need to be able to bound the number of recursive calls. We only consider the class of programs where one of the following is true, for $f(\mathbf{x})$. We say $f(a_1, a_2, \dots, a_n) \rightsquigarrow f(b_1, b_2, \dots, b_n)$, when $f(\mathbf{b})$ is called as part of executing $f(\mathbf{a})$:

- $\exists i, m. \forall \mathbf{y}. f(\mathbf{x}) \rightsquigarrow f(\mathbf{y}) \implies m \leq y_i < x_i$
- $\exists i, M. \forall \mathbf{y}. f(\mathbf{x}) \rightsquigarrow f(\mathbf{y}) \implies x_i < y_i \leq M$

As with loops, we will use planning and search to determine and prove whether programs are of this form or not.

References

- [1] Baudouin L Charlier, Kaninda Musumbu, and Pascal Van Hentenryck. *A Generic Abstract Interpretation Algorithm and Its Complexity Analysis (Extended Abstract)*. Tech. rep. Providence, RI, USA, 1990.
- [2] Aziem Chawdhary. “Proving Termination using Abstract Interpretation”. PhD thesis. Queen Mary University of London, 2010.
- [3] P. Cousot. “Semantic Foundations of Program Analysis”. In: *Program Flow Analysis: Theory and Applications*. Ed. by S.S. Muchnick and N.D. Jones. Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1981. Chap. 10, pp. 303–342.
- [4] Michael Gorbovitski et al. “Alias Analysis for Optimization of Dynamic Languages”. In: *Proceedings of the 6th Symposium on Dynamic Languages*. DLS '10. Reno/Tahoe, Nevada, USA: ACM, 2010, pp. 27–42. ISBN: 978-1-4503-0405-4. DOI: 10.1145/1869631.1869635. URL: <http://doi.acm.org/10.1145/1869631.1869635>.
- [5] Sumit Gulwani. “SPEED: Symbolic Complexity Bound Analysis”. In: *Proceedings of the 21st International Conference on Computer Aided Verification*. CAV '09. Grenoble, France: Springer-Verlag, 2009, pp. 51–62. ISBN: 978-3-642-02657-7. DOI: 10.1007/978-3-642-02658-4_7. URL: http://dx.doi.org/10.1007/978-3-642-02658-4_7.
- [6] Sumit Gulwani and Florian Zuleger. “The Reachability-bound Problem”. In: *SIGPLAN Not.* 45.6 (June 2010), pp. 292–304. ISSN: 0362-1340. DOI: 10.1145/1809028.1806630. URL: <http://doi.acm.org/10.1145/1809028.1806630>.
- [7] Eva Maia, Nelma Moreira, and Rog  rio Reis. “A Static Type Inference for Python”. In: *Proceedings of 5th Workshop on Dynamic Languages and Applications*. 2011.
- [8] Florian Zuleger et al. “Bound Analysis of Imperative Programs with the Size-change Abstraction”. In: *Proceedings of the 18th International Conference on Static Analysis*. SAS'11. Venice, Italy: Springer-Verlag, 2011, pp. 280–297. ISBN: 978-3-642-23701-0. URL: <http://dl.acm.org/citation.cfm?id=2041552.2041574>.