1 Electroweak Unification

1.1 The electroweak interaction

Summary of fields and interactions:

Field	Current	Coupling	Charge
$W^{+,\mu}$	$j_\mu^+ = \bar{u}\gamma_\mu \frac{1}{2}(1-\gamma^5)u$	$g/\sqrt{2}$	$2I_3$
$W^{-,\mu}$	$j_\mu^- = \bar{u}\gamma_\mu \frac{1}{2}(1-\gamma^5)u$	$g/\sqrt{2}$	$2I_3$
$W^{0,\mu}$	$j_\mu^0 = \bar{u}\gamma_\mu \frac{1}{2}(1-\gamma^5)u$	g	I_3
B^μ	$j_\mu^Y = \bar{u}\gamma_\mu u$	$g^{'}/2$	Y
A^μ	$j_\mu^{EM} = \bar{u}\gamma_\mu u$	e	Q
Z^μ	$\{j_\mu^{NC}\}$	$\{g_z\}$	$\{\}$

Quantities in {curly braces} are to be found by the theory.

$$\boxed{Q = I_3 + \frac{1}{2}Y} \tag{1}$$

The full current is given by $J_\mu = \text{charge} \times j_\mu$ and the field is then

$$F^\mu = \frac{1}{q^2 - M^2}\tilde{g}J^\mu \tag{2}$$

where $\tilde{g}$ is the appropriate coupling constant.

In the GSW basis, the full electroweak interaction is

$$\frac{g}{\sqrt{2}}J_\mu^+ W^{+,\mu} + \frac{g}{\sqrt{2}}J_\mu^- W^{-,\mu} + gJ_\mu^0 W^{0,\mu} + \frac{g^{'}}{2}J_\mu^Y B^\mu \tag{3}$$

with neutral current

$$gJ_\mu^0 W^{0,\mu} + \frac{g^{'}}{2}J_\mu^Y B^\mu \tag{4}$$

1.2 Change of basis

The observed and GSW fields are related through the Weinberg angle, θ_W,

$$\begin{pmatrix} A^\mu \\ Z^\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} B^\mu \\ W^{0,\mu} \end{pmatrix} \tag{5}$$

In this basis, the electroweak neutral current becomes

$$gJ_\mu^0 \left(\sin\theta_W A^\mu + \cos\theta_W Z^\mu\right) + \frac{g^{'}}{2}J_\mu^Y \left(\cos\theta_W A^\mu - \sin\theta_W Z^\mu\right) \tag{6}$$

$$= \left(g\sin\theta_W J_\mu^0 + \frac{g^{'}}{2}\cos\theta_W J_\mu^Y\right) A^\mu + \left(g\cos\theta_W J_\mu^0 - \frac{g^{'}}{2}\sin\theta_W J_\mu^Y\right) Z^\mu \tag{7}$$

1.3 The electromagnetic interaction

The electromagnetic part of the electroweak interaction is associated with the photon (A^μ) field:

$$eJ_\mu^{EM} \equiv g\sin\theta_W J_\mu^0 + \frac{g'}{2}\cos\theta_W J_\mu^Y \tag{8}$$

Using $J_\mu^{EM} = J_\mu^0 + \frac{1}{2}J_\mu^Y$ from (1),

$$eJ_\mu^0 + \frac{e}{2}J_\mu^Y \equiv g\sin\theta_W J_\mu^0 + \frac{g'}{2}\cos\theta_W J_\mu^Y \tag{9}$$

Giving the unification condition:

$$\boxed{e = g\sin\theta_W = g'\cos\theta_W} \tag{10}$$

1.4 The weak neutral current

Now consider the part of the electroweak interaction associated with the Z^μ field:

$$g_Z J_\mu^Z = g\cos\theta_W J_\mu^0 - \frac{g'}{2}\sin\theta_W J_\mu^Y \tag{11}$$

Use $g' = g\frac{\sin\theta_W}{\cos\theta_W}$ from the unification condition,

$$g_z J_\mu^Z = \frac{g}{\cos\theta_W}\left[\cos^2\theta_W J_\mu^0 - \sin^2\theta_W \frac{J_\mu^Y}{2}\right] \tag{12}$$

Then $J_\mu^Y = 2(J_\mu^{EM} - J_\mu^0)$ from (1),

$$\begin{aligned} g_z J_\mu^Z &= \frac{g}{\cos\theta_W}\left[\cos^2\theta_W J_\mu^0 - \sin^2\theta_W \left(J_\mu^{EM} - J_\mu^0\right)\right] && (13)\\ &= \frac{g}{\cos\theta_W}\left[J_\mu^0 - \sin^2\theta_W J_\mu^{EM}\right] && (14)\end{aligned}$$

Now expand out the currents as

$$\boxed{\begin{aligned} J_\mu^0 &= \bar{u}\gamma_\mu \frac{1}{2}(1-\gamma^5)I_3 u && (15)\\ J_\mu^{EM} &= \bar{u}\gamma_\mu Q u && (16)\end{aligned}}$$

$$\begin{aligned} g_z J_\mu^Z &= \frac{g}{\cos\theta_W}\bar{u}\gamma_\mu\left[\frac{1}{2}(1-\gamma^5)I_3 - \sin^2\theta_W Q\right]u && (17)\\ &= \frac{g}{\cos\theta_W}\bar{u}\gamma_\mu\left[\frac{1}{2}(1-\gamma^5)I_3 - \sin^2\theta_W\left\{\frac{1}{2}\left(1+\gamma^5\right) + \frac{1}{2}\left(1-\gamma^5\right)\right\}Q\right]u && (18)\\ &= \frac{g}{\cos\theta_W}\bar{u}\gamma_\mu\left[\frac{1}{2}(1-\gamma^5)(I_3 - Q\sin^2\theta_W) - \frac{1}{2}(1+\gamma^5)Q\sin^2\theta_W\right]u && (19)\end{aligned}$$

From this, the right- and left-handed couplings can be deduced:

$$\boxed{\begin{aligned} g_L &= I_3 - Q\sin^2\theta_W && (20) \\ g_R &= -Q\sin^2\theta_W && (21) \end{aligned}}$$

1.5 Z boson mass

The degeneracy on the mass of the Z boson is lifted by electroweak symmetry breaking. From the form of the weak neutral current (19), the vertex factor is $g/\cos\theta_W$.

Considering the return to Fermi theory in the low-energy limit, for the process $\nu_e e \to \nu_e e$, we get $G_F/\sqrt{2} = g^2/(8M_Z^2 \cos_W^\theta)$.

For a charged-current weak interaction, we get $G_F/\sqrt{@} = g^2/(8M_W^2)$.

Therefore, comparing these,

$$\boxed{M_W = M_Z\cos\theta_W} \tag{22}$$

1.6 Vector and axial couplings

Above, the couplings of the weak neutral field Z_μ to right- and left-handed fermions was deduced. Alternatively, we can find its coupling to the vector (γ_μ) and axial ($\gamma_u\gamma^5$) parts:

$$\begin{aligned} g_z J_\mu^Z &= \frac{g}{\cos\theta_W}\bar{u}\gamma_\mu\left[\frac{1}{2}(1-\gamma^5)I_3 - \sin^2\theta_W Q\right]u && (23) \\ &= \frac{g}{\cos\theta_W}\bar{u}\gamma_\mu\left[\left(\frac{1}{2}I_3 - Q\sin^2\theta_W\right) - \frac{1}{2}I_3\gamma^5\right]u && (24) \\ &\equiv \frac{g}{\cos\theta_W}\bar{u}\gamma_\mu\left[\frac{c_V}{2} - \frac{c_A}{2}\gamma^5\right]u && (25) \end{aligned}$$

Therefore, we can identify the vector and axial coupling constants:

$$\boxed{\begin{aligned} c_V &= I_3 - 2Q\sin\theta_W && (26) \\ c_A &= I_3 && (27) \end{aligned}}$$