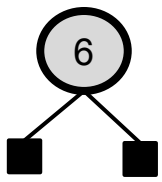
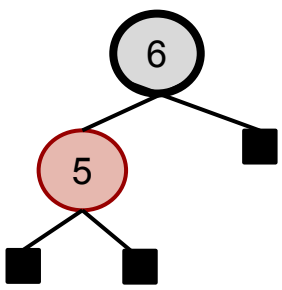
Solutions: Class 03/02, Balanced Trees



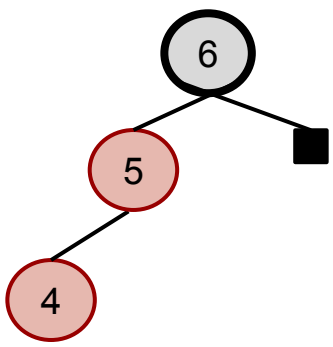
Insert 6:
← (a) RBT after insert
(b) **no** property is violated
(c) **no** remedy needed
(d) RBT after fixup is the same.
(e) (2,4): →

6

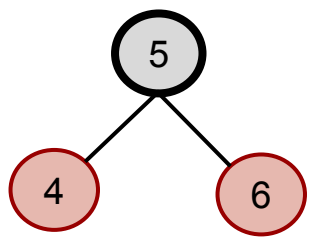


Insert 5:
← (a) RBT after insert
(b) **no** property violated
(c) **no** remedy needed
(d) RBT after fixup is the same.
(e) (2,4): →

6, 5

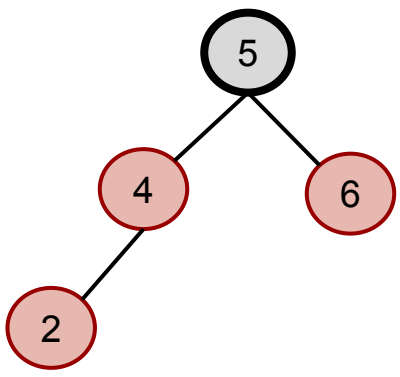


Insert 4:
← (a) RBT after insert
(b) **double red = incorrect representation**
(c) **black uncle** (sibling of red parent): **restructure**
(d,e) RBT and (2,4): →



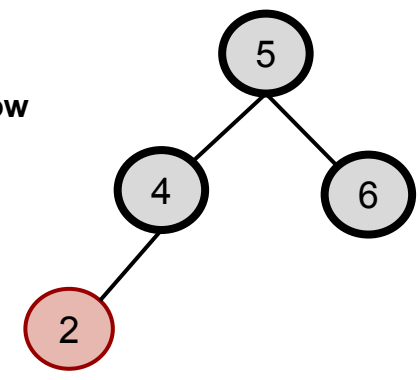
4, 5, 6

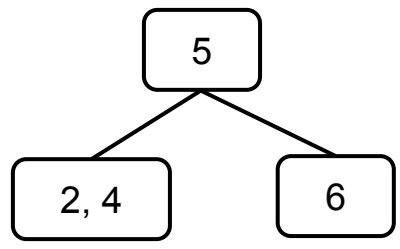
(You may leave out the black leaf nodes in the remainder. Show them only when they are the “uncle”)

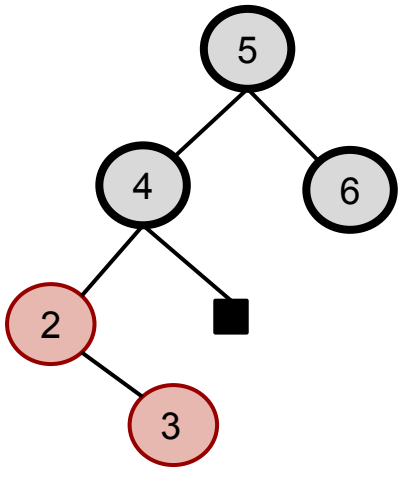


Insert 2:
← (a) RBT after insert
(b) **double red = overflow**
(c) **red uncle: recolor**
(d,e) RBT and (2,4): →

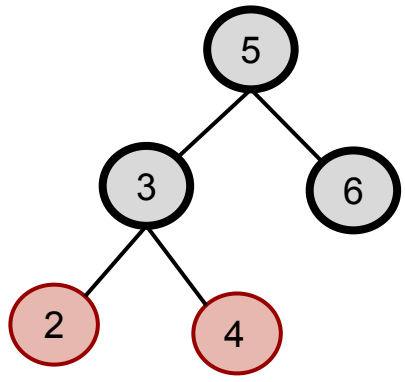
Note: 5 stays black because it is the root!

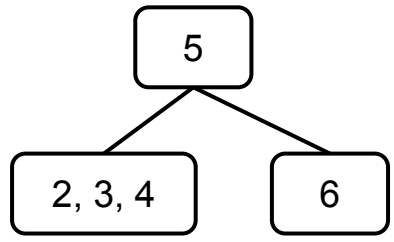






Insert 3:
← (a) RBT after insert
(b) **double red = incorrect representation**
(c) **black uncle: restructure**
(d,e) RBT and (2,4): →





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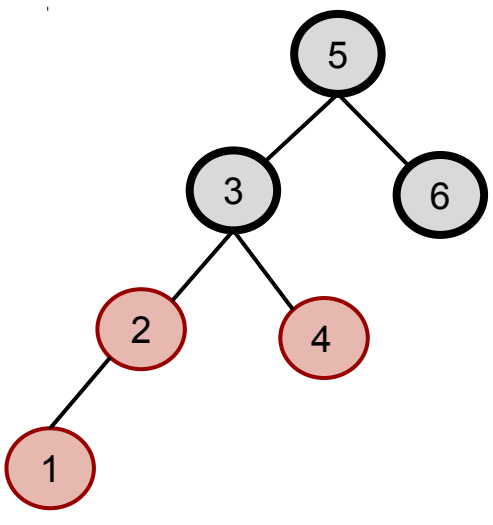
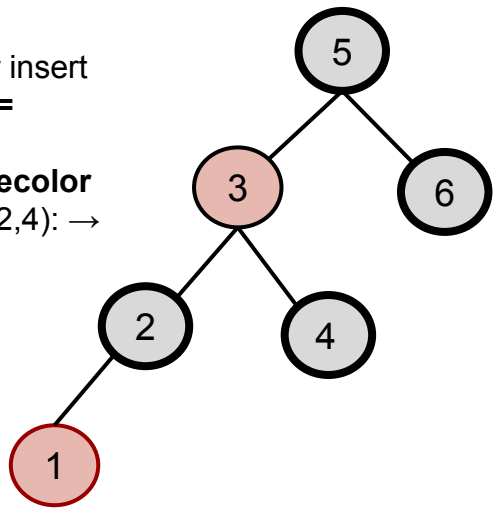
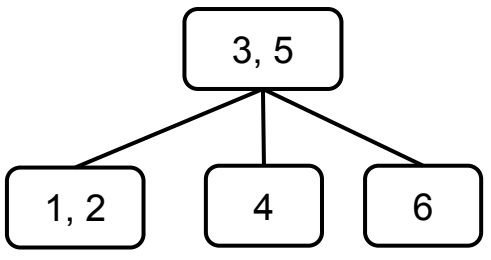
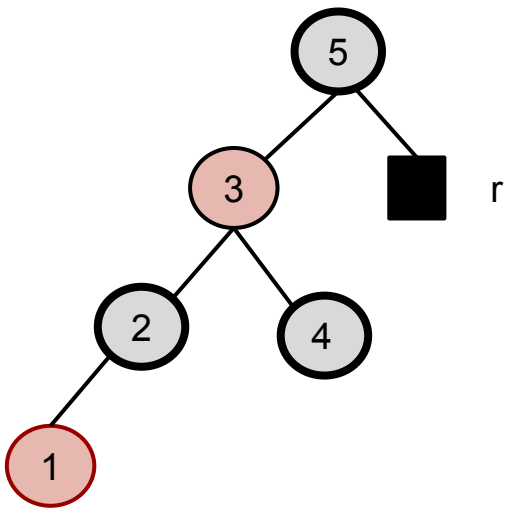


Insert 1:
← (a) RBT after insert
(b) **double red = overflow**
(c) **red uncle: recolor**
(d,e) RBT and (2,4): →

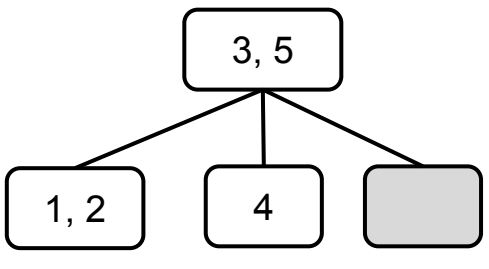


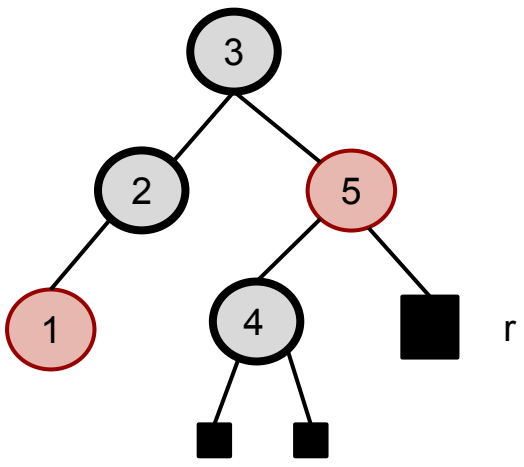


DELETION

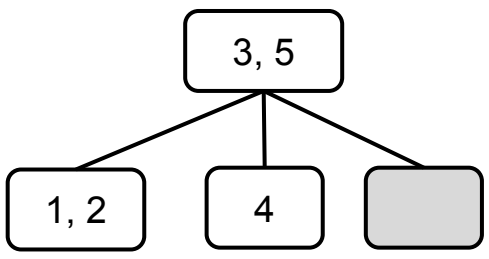


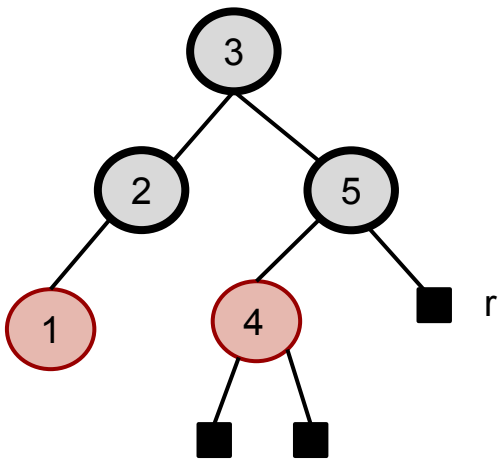
Delete 6:
(b): **double black at r**, corresponding to **underflow**
(c): **Case 3: sibling "3" is red: perform RBT adjustment.**
(Another case will then apply.)





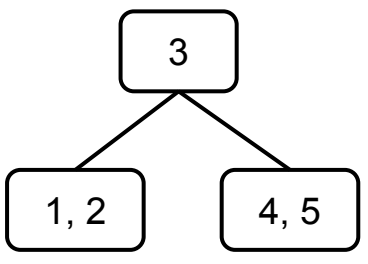
New Case:
(b): still **double black at r**, corresponding to **underflow,** but now ...
(c): **Case 2: sibling "4" is black with black children: perform RBT recoloring, equivalent to 2-4 fusion.**
(Essentially, push the blackness of both 4 and r up to parent 5.)

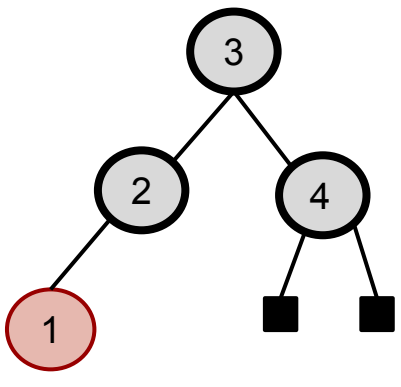




Now we are good. A red node absorbed the extra black by turning black.

(if 5 had been black the double black violation would propagate up the tree, potentially O (lg n) times.)





Delete 5:
BST deletion says when there is one child, replace it with its child.
RBT deletion says the child should take on its parent' s color.
All is good.