Seven Sketches in Compositionality – Exercises

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1 Chapter 1 — Generative Effects: Orders and Galois Connections

(1.1)

(a) *order-preserving* $f : x \mapsto x + 1$
non-order-preserving $f : x \mapsto -x$

(b) *metric-preserving* $x \mapsto x + 2$
non-metric-preserving $x \mapsto 2x$

(c) *addition-preserving* $x \mapsto x$
non-addition-preserving $x \mapsto 2x$

(1.2)

Circle 21, Circle the rest, box around the whole thing. i.e.

$$\{\{21\}, \{11, 12, 13, 22, 23\}\}$$

(1.6)

1. True
2. False
3. True

(1.7)

1. $\{\}, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}$
2. $\{1\} \cup \{1, 3\} = \{1, 3\}$
3. $(h, 1), (h, 2), (h, 3), (1, 1), (1, 2), (1, 3)$
4. $(h, 1), (1, 1), (1, 2), (2, 2), (3, 2)$

5. $A \cup B = \{h, 1, 2, 3\}$

1.11

1. If there were more than one $p' \in P'$ such that $A_p = A'_{p'}$ (i.e. p'_1 and p'_2 such that $A_p = A'_{p'_1}$ and $A_p = A'_{p'_2}$), then $p'_1 \neq p'_2$, so necessarily $A'_{p'_1} \cap A'_{p'_2} = \emptyset$. But then since $A'_{p'_1} = A_p = A'_{p'_2}$, then $A'_{p'_1} = A'_{p'_2}$, thus $A'_{p'_1} \cap A'_{p'_2} = A_p \cap A_p = A_p \neq \emptyset$, by definition of partition, therefore there cannot be more than one $p' \in P'$ such that $A_p = A'_{p'}$. ■

2. Since there exists a $p' \in P'$ such that $A_p = A'_{p'}$, and by 1.11.1, there is at most one such p', it follows that there is a bijection between these $p \in P$ and $p' \in P'$, thus $\forall p' \in P'$ there exists a $p \in P$ such that $A_p = A'_{p'}$. ■

1.12

$(11, 11), (12, 12), (13, 13), (21, 21), (22, 22), (23, 23), (11, 12), (12, 11), (22, 23), (23, 22)$

1.15

1. Each A_p is $(\sim)$-connected, therefore they are nonempty. ■

2. If $A_p \cap A_q \neq \emptyset$, then for any $x \in A_p \cap A_q$, it follows that $x \in A_p$ and $x \in A_q$. Thus for any $x_p \in A_p$, we have $x_p \sim x$, and under $(\sim)$-closure and $x \in A_q$, it follows that $x_p \in A_q$, and the same for $x_q \in A_q$. Therefore, $A_p = A_q \vDash p = q$, which violates the contextual assertion that $p \neq q$, therefore $p \neq q \implies A_p \cap A_q = \emptyset$. ■

3. Each A_p is nonempty and $\sim$ is reflexive, i.e. we have at least for each x that $x \sim x$, therefore we have at least that A is the union of singleton sets that cover A, therefore $A = \bigcup_{p \in P} A_p$. ■

1.19

1. $f : \mathbb{Z} \to \mathbb{R} : x \mapsto x + 1$

2. $f : \mathbb{Q} \to \mathbb{Z} : x \mapsto x - \frac{x}{10}$

3. (1): yes (2): no (3): no (4): yes

4. (1): neither (2): top dot's targets are not unique, bottom dot has no target (3): top dot has no target (partial function?) (4) bijective

1.20

By function definition, each $a \in A$ has a unique $y \in \emptyset$ such that $(a, y) \in f$. But there are no $y \in \emptyset$, therefore there cannot exist any such $a \in A$, therefore A is empty. ■