實驗2

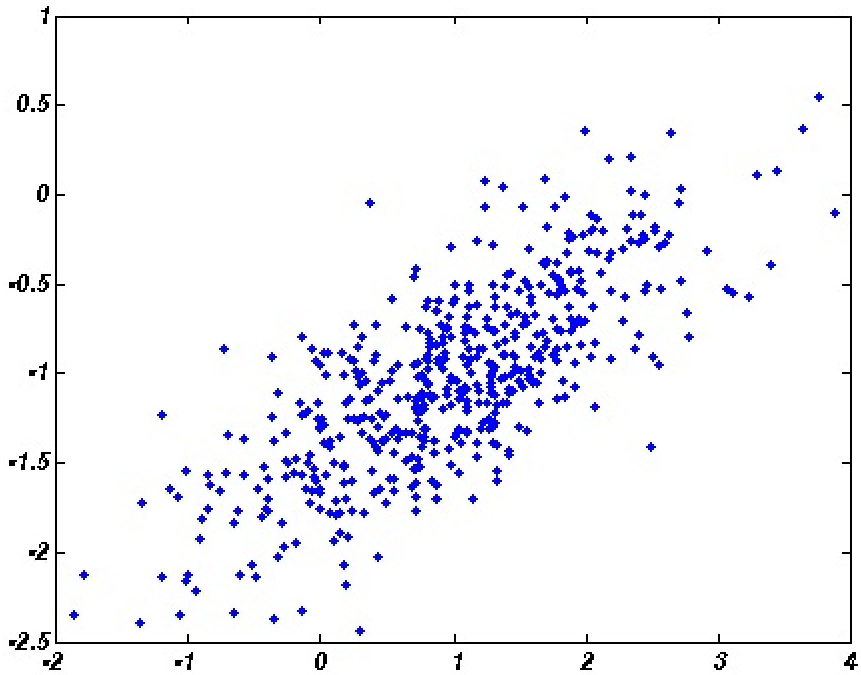
習題No. 2.1

實驗代碼：

以下爲幫助文檔中的示例代碼，mu爲mean vector(均值向量), Sigma爲covariance matrix協方差矩陣

```
mu = [1 -1];
Sigma = [.9 .4; .4 .3];
r = mvnrnd(mu, Sigma, 500);
plot(r(:,1),r(:,2),'.');
```

實驗結果截圖：



習題No. 2.2

實驗代碼：

數學理論：

多元正态分布

- *d* 维的多元正态分布

$$p(\mathbf{x}) = \frac{1}{(2\pi)^{d/2}|\boldsymbol{\Sigma}|^{1/2}} \exp\left[-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^{\mathsf{T}}\boldsymbol{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})\right]$$

- 简写为

$$p(\mathbf{x}) \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$$

$$\Sigma = \begin{bmatrix} \sigma_{11}^2 & \sigma_{12}^2 & \cdots & \cdots & \sigma_{1d}^2 \\ \sigma_{12}^2 & \sigma_{22}^2 & \cdots & \cdots & \sigma_{2d}^2 \\ \vdots & \vdots & & & \vdots \\ \vdots & \vdots & & & \vdots \\ \sigma_{1d}^2 & \sigma_{2d}^2 & \cdots & \cdots & \sigma_{dd}^2 \end{bmatrix}$$

- 充分统计量

$$\boldsymbol{\mu} \equiv \mathcal{E}[\mathbf{x}] = \int \mathbf{x}p(\mathbf{x})d\mathbf{x}$$

$$\boldsymbol{\Sigma} \equiv \mathcal{E}[(\mathbf{x}-\boldsymbol{\mu})(\mathbf{x}-\boldsymbol{\mu})^{\mathsf{T}}] = \int (\mathbf{x}-\boldsymbol{\mu})(\mathbf{x}-\boldsymbol{\mu})^{\mathsf{T}}p(\mathbf{x})d\mathbf{x}$$

代碼中解釋如下：

```
function val = multi_var_gaussian_probability_density(mu, Sigma, x)
% multi_var_gaussian_probability_density
%   Computes multi-variable Gaussian Probability Density Value.
%   Formular:
%       p(x) ~ N(mu, Sigma) where d is dimension
%       p(x) = 1./((2 * pi) ^(d/2) * det(Sigma) ^ (1./2) ...
%           * exp(-1./2 * (x - mu)' * Sigma^(-1) * (x - mu))
% Input:
%   mu - mean vector of the Gauss distribution
%   Sigma - covariance matrix of the Gauss distribution
%   x - the independent variable of p(x), multi-var Gaussian probability density value
[dimension, ~] = size(mu);

%val = (1/((2*pi) ^ (dimension/2) * det(Sigma) ^ 0.5)) * exp(-0.5 * (x-mu)' * inv(Sigr
val = (1/((2 * pi) ^ (dimension/2) * det(Sigma) ^ 0.5)) ...
    * exp(-0.5 * (Sigma \ (x - mu))' * (x - mu));

end
```

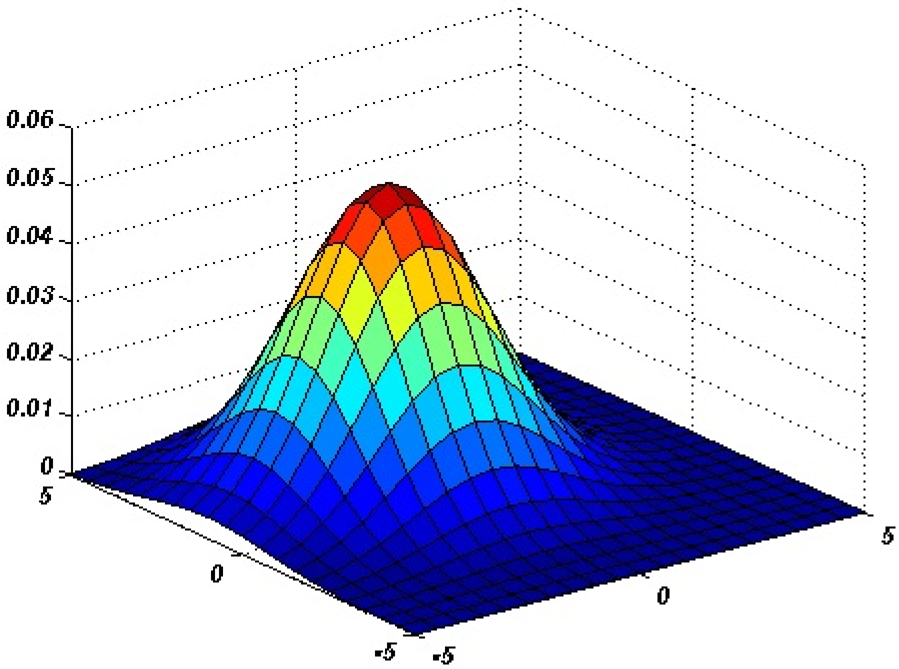
實驗結果：

1 *一個簡單的腳本來繪出多維高斯分佈的圖像：*

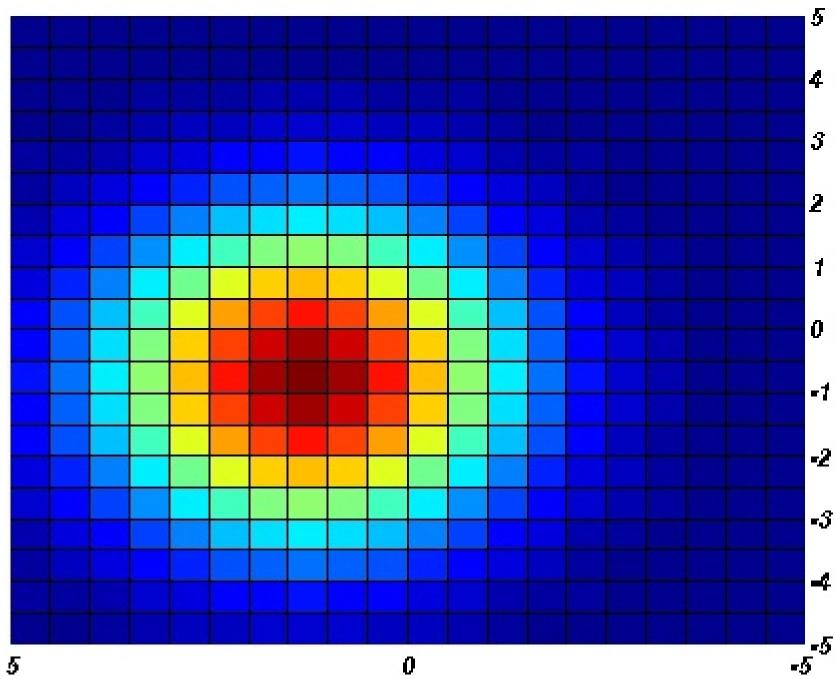
```
maxn = 5;
minn = -5;
step = 0.5;

x1 = minn:step:maxn;
x2 = minn:step:maxn;
[~, sz] = size(x1);
z = zeros(sz, sz);

icnt = 1;
jcnt = 1;
for i = minn:step:maxn
    jcnt = 1;
    for j = minn:step:maxn
        z(icnt, jcnt) = multi_var_gaussian_probability_density([1;-1], [3 0; 0 3], [i;

        jcnt = jcnt + 1;
    end
    icnt = icnt + 1;
end

surf(x1, x2, z)
```

2 *圖像結果*：

• 圖像1：



• 圖像2 – 圖像1的俯視圖：



習題No. 2.3

實驗代碼：

- 由於書中的代碼是有問題的，它的fix函數會使得`X=[X t]`中的`t`矩陣成爲緯度不固定的矩陣，導致錯誤。
- (注：後來我發現這個`generate_gauss_classes`函數，作者只用在二維中， 所以不會發生`t`的緯度會改變的問題，我寫的以下函數是可以在任意緯度下工作的)
- 根據理解了意圖之後，我使用rand函數來完成對Gauss Distribution的選擇，以使得生成的DataSet符合Probability向量中的生成概率要求。
- 以下爲函數代碼：

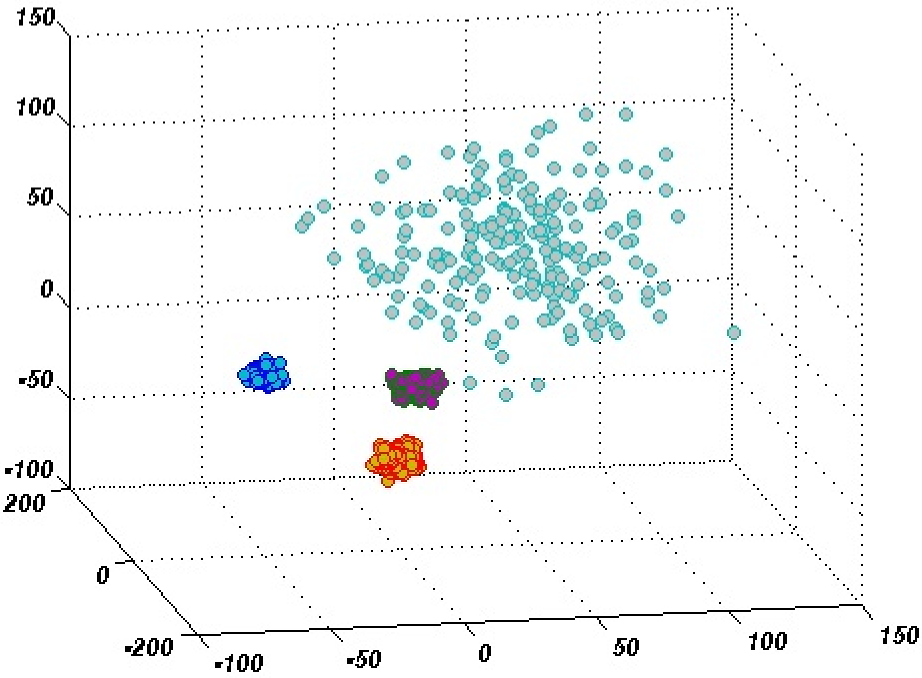
```
function [ DataSet, BasedGaussDistIndex ] = generate_gauss_classes(mu, Sigma, Probabil
% BRIEF: generate gauss data set
% `a` is the dimension of your data set, and we are going to make `N` of them
% `c` is the number of `gauss distribution`(s) that we should use
%
% INPUTS:
%    `mu` is  a `c*a` matrix, each row is the mean vector of the ith
% distribution.
%    `Sigma` is a `a*a*c` matrix, each S(:,:,i) is the covariance of the ith
% distribution.
%    `Probability` is a `c*1` vector, representing the corresponding probability
% of ith distribution.
%    `count` is the number of data sets that we are going to generate
%
% OUTPUTS:
%    `DataSet` is a `a*N` matrix, the corresponding data set (every row is
% a data set)
%    `BasedGaussDistIndex` is the index matrix, representing the ith data set
% is generated based on the jth gauss distribution.

if sum(Probability) ~= 1
    error(['The `Probability` vector indicating the probability should'...
    'sum up to 1.0']);
end

[row, column] = size(mu);
DataSet = zeros(Count, column);
BasedGaussDistIndex = zeros(Count, 1);

% Generate the Acucumulated Probability Vector
prob_sz = row;
ProbGenVec = zeros(prob_sz, 1);
accumulated_probability = 0;
for i = 1 : prob_sz
    accumulated_probability = accumulated_probability + Probability(i, 1);
    ProbGenVec(i, 1) = accumulated_probability;
end

% `ith` is a variable storing using of ith Gauss Distribution.
% (Bad design though because of matlab)
ith = 0;

for i = 1 : Count
    % rand to use ith Gauss Distribution
    rnd = rand();
    for j = 1 : prob_sz
        if rnd < ProbGenVec(j, 1)
            ith = j;
            break;
        end
    end

    temp = mvnrnd(mu(ith,:), Sigma(:,:,ith), 1);
    DataSet(i,:) = temp;
    BasedGaussDistIndex(i, 1) = ith;
end
```

```
end
```

- 我嘗試用一個簡單的腳本將數據可視化，腳本如下：

```
a = 3;
c = 4;

m = zeros(c, a);
m(1,:) = [-50 0 0];
m(2,:) = [0 -50 0];
m(3,:) = [0 0 -50];
m(4,:) = [50 50 50];

S = zeros(a,a,c);
S(:,:,1) = [10 0 0; 0 10 0; 0 0 10];
S(:,:,2) = [10 0 0; 0 10 0; 0 0 10];
S(:,:,3) = [10 0 0; 0 10 0; 0 0 10];
S(:,:,4) = [1000 0 0; 0 1000 0; 0 0 1000];

P = zeros(c, 1);
P(1) = 0.1;
P(2) = 0.2;
P(3) = 0.5;
P(4) = 0.2;

[X, y] = generate_gauss_classes(m,S,P,1000);
disp('Base: ');
disp(y);
disp('DataSet: ');
disp(X);

g1index = find(y == 1);
g2index = find(y == 2);
g3index = find(y == 3);
g4index = find(y == 4);

g1 = X(g1index, :);
g2 = X(g2index, :);
g3 = X(g3index, :);
g4 = X(g4index, :);

figure
grid on
hold all
scatter3(g1(:,1), g1(:,2), g1(:,3), 'MarkerFaceColor', [0, 0.75, 0.75]);
scatter3(g2(:,1), g2(:,2), g2(:,3), 'MarkerFaceColor', [0.75, 0, 0.75]);
scatter3(g3(:,1), g3(:,2), g3(:,3), 'MarkerFaceColor', [0.75, 0.75, 0]);
scatter3(g4(:,1), g4(:,2), g4(:,3), 'MarkerFaceColor', [0.75, 0.75, 0.75]);
```

實驗最終可視化效果：



- 藍色的點爲g1型，即均值爲`[-50 0 0]`的點。
- 紫色的點爲g2型，即均值爲`[0 -50 0]`的點。
- 橙色的點爲g3型，即均值爲`[0 0 -50]`的點。
- 灰色的點爲g4型，即均值爲`[50 50 50]`的點。
- 另外，藍色、紫色，和橙色的點的協方差都是`[10 0 0; 0 10 0; 0 0 10]`而，灰色的點，我們將其協方差擴大到`[1000 0 0; 0 1000 0; 0 0 1000]`，這樣大家就可以非常顯著地看出他們的差別。

習題No 2.4

實驗代碼：

- 由於我的generate_gauss_classes是自己重寫的，接口有一點點不一樣。
- 這個和習題2.3中，我用來可視化的代碼差不多，所以沒什麼問題，也不用太多加說明。
- 另外，書上的代碼有小小的一點點性能問題，所以我稍微改動了一下

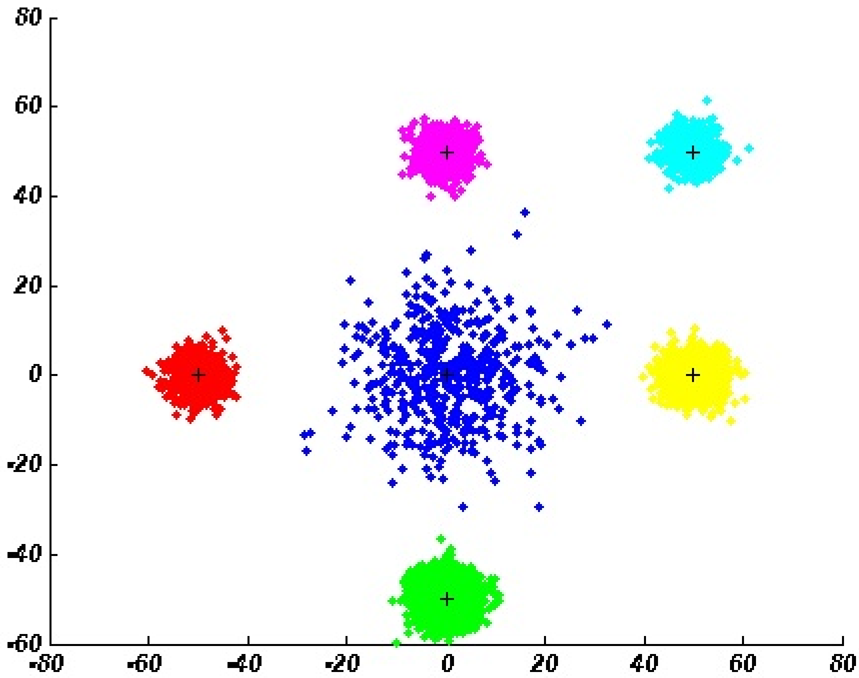
```
function hw24_plot_data(DataSet, BasedGaussDistIndex, mu)
    [N, col] = size(DataSet);
    [c, ~] = size(mu);

    if c > 6
        disp(['Warning: This function supports only a maximum'...
        'of 6 gauss dist classes']);
        disp('We will draw the first 6 classes');
    end

    pale = ['r.'; 'g.'; 'b.'; 'y.'; 'm.'; 'c.'];

    g1index = find(BasedGaussDistIndex == 1);
    g2index = find(BasedGaussDistIndex == 2);
    g3index = find(BasedGaussDistIndex == 3);
    g4index = find(BasedGaussDistIndex == 4);
    g5index = find(BasedGaussDistIndex == 5);
    g6index = find(BasedGaussDistIndex == 6);

    g1 = DataSet(g1index, :);
    g2 = DataSet(g2index, :);
    g3 = DataSet(g3index, :);
```

```
    g4 = DataSet(g4index, :);
    g5 = DataSet(g5index, :);
    g6 = DataSet(g6index, :);

    figure(1);
    % plot the DataSet
    hold on
    plot(g1(:, 1), g1(:, 2), 'r.');
    plot(g2(:, 1), g2(:, 2), 'g.');
    plot(g3(:, 1), g3(:, 2), 'b.');
    plot(g4(:, 1), g4(:, 2), 'y.');
    plot(g5(:, 1), g5(:, 2), 'm.');
    plot(g6(:, 1), g6(:, 2), 'c.');

    % plot the `mu`
    plot(mu(:, 1), mu(:, 2), 'k + ')
end
```

- 以下是driver script:

```
a = 2;
c = 6; % maximum is 6 because of colors

m = zeros(c, a);
m(1,:) = [-50 0];
m(2,:) = [0 -50];
m(3,:) = [0 0];
m(4,:) = [50 0];
m(5,:) = [0 50];
m(6,:) = [50 50];

S = zeros(a,a,c);
S(:,:,1) = [10 0; 0 10];
S(:,:,2) = [10 0; 0 10];
S(:,:,3) = [100 0; 0 100];
S(:,:,4) = [10 0; 0 10];
S(:,:,5) = [10 0; 0 10];
S(:,:,6) = [10 0; 0 10];

P = zeros(c, 1);
P(1) = 0.1;
P(2) = 0.4;
P(3) = 0.1;
P(4) = 0.2;
P(5) = 0.1;
P(6) = 0.1;

[X, y] = generate_gauss_classes(m,S,P,5000);
disp('Base: ');
disp(y);
disp('DataSet: ');
disp(X);

hw24_plot_data(X, y, m);
```

實驗效果圖：



- 和前面習題2.3的可視化一樣，我將其中的一個的協方差調整的比較大，這樣，可以看出不同。

習題 2.5

實驗代碼：

```
function result = bayes_classifier(mu, Sigma, Probability, DataSet)
% BRIEF: Use Bayes Classifier to classify DataSet
%
% `a` is the dimension of your data set
% `c` is the number of `gauss distribution`(s) that we should use
%
% INPUTS:
%    `mu` is  a `c*a` matrix, each row is the mean vector of the ith
% distribution.
%    `Sigma` is a `a*a*c` matrix, each S(:,:,i) is the covariance of the ith
% distribution.
%    `Probability` is a `c*1` vector, representing the corresponding probability
% of ith distribution.
%    `DataSet` is a `a*N` matrix, the input DataSet,
% each row contains one sample data

    [N, ~] = size(DataSet);
    [c, ~] = size(mu);

    t = zeros(1, c);
    result = zeros(1, N);

    for i = 1 : N
        for j = 1 : c
            t(j) = Probability(j) * mvnpdf(DataSet(i, :), mu(j, :), Sigma(:,:,j));
        end
        [~, result(i)] = max(t);
    end
end
```

自動化腳本：

```
a = 3;
c = 4;

m = zeros(c, a);
m(1,:) = [-50 0 0];
m(2,:) = [0 -50 0];
m(3,:) = [0 0 -50];
m(4,:) = [50 50 50];

S = zeros(a,a,c);
S(:,:,1) = [10 0 0; 0 10 0; 0 0 10];
S(:,:,2) = [10 0 0; 0 10 0; 0 0 10];
S(:,:,3) = [10 0 0; 0 10 0; 0 0 10];
S(:,:,4) = [1000 0 0; 0 1000 0; 0 0 1000];

P = zeros(c, 1);
P(1) = 0.1;
P(2) = 0.2;
P(3) = 0.5;
P(4) = 0.2;

[X, y] = generate_gauss_classes(m,S,P,1000);

ret = bayes_classifier(m, S, P, X);

disp('sum(abs(bayes_classifer_result - answer)): ');
disp(sum(abs(ret' - y)));
```

實驗結果：

```
sum(abs(bayes_classifer_result - answer)):
     0
```

- 我相信畫一張和習題2.3一樣的圖片已經沒什麼太大意思了，大家也看不出什麼不同
- 我做的事情是這樣的，我拿Bayes分類器做出來的結果和原先用來生成的Based Gauss Distribution Index相減（爲了抵消正負我做了絕對值）
- 得到的結果爲0，就很能說明問題，這個驗證實驗是成功的，成功用Bayes算法找到了原本使用的Gauss Distribution。

習題 1.3.1

實驗代碼：

```
m = [0 1]';
S = eye(2);
X1 = [0.2 1.3]';
X2 = [2.2 -1.3]';

pg1 = multi_var_gaussian_probability_density(m, S, X1);
pg2 = multi_var_gaussian_probability_density(m, S, X2);
disp(pg1);
disp(pg2);
```

實驗結果：

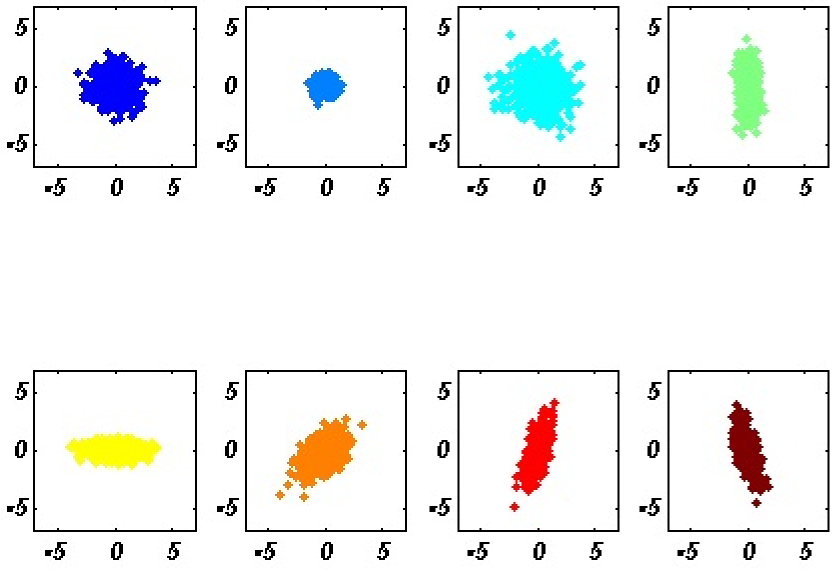
```
    0.1491

    0.0010
```

習題 1.3.3

實驗代碼：

```
N = 500;
m = [0 0];
S = zeros(2,2,8);
DataSet = zeros(N, 2, 8);

S(:,:,1) = [1.0  0.0;  0.0 1.0];
S(:,:,2) = [0.2  0.0;  0.0 0.2];
S(:,:,3) = [2.0  0.0;  0.0 2.0];
S(:,:,4) = [0.2  0.0;  0.0 2.0];
S(:,:,5) = [2.0  0.0;  0.0 0.2];
S(:,:,6) = [1.0  0.5;  0.5 1.0];
S(:,:,7) = [0.3  0.5;  0.5 2.0];
S(:,:,8) = [0.3 -0.5; -0.5 2.0];

cc = jet(8);
figure(1);
hold on;
for i = 1 : 8
    DataSet(:,:,i) = mvnrnd(m, S(:,:,i), N);
    subplot(2, 4, i);
    plot(DataSet(:,1,i), DataSet(:,2,i), '.', 'color', cc(i,:));
    axis equal;
    axis([-7 7 -7 7]);
end
```

實驗效果圖：



- 由於方便描述，我們將Sigma矩陣標誌爲`[a b; c d]`形式，a爲x1和自己的協方差（即x1的方差），b和c爲x1和x2的協方差，d爲x2和自己的協方差（即x2的方差）
- 從形象化的角度來看：
  - a和d的值越大，數據就越分散，反之就越密集
  - a和d分別操縱着x軸和y軸的離散度，即a越大的時候x方向上的點就越分散，反之則越密集，同理可得d。
  - b和c操控着以b(c)的斜率方向上的點的密集程度（b=c!=0）

- 通俗的來講：
    - 方差，就是自己和自己的相聯繫程度的大小，方差越大，自己和自己的聯繫程度越小。
    - 協方差，就是A和B之間的聯繫程度的大小，協方差越大，兩者的關係越密切，這將使得點集更多地聚集在以協方差爲斜率的直線上

習題 1.3.4

實驗代碼：

```
Prob = [0.5 0.5];
mu = [1 1; 3 3];
Sig = zeros(2,2,2);
Sig(:,:,1) = [1 0; 0 1];
Sig(:,:,2) = [1 0; 0 1];

x = [1.8 1.8];

disp('mu1:'); disp(mu(1,:));
disp('mu2:'); disp(mu(2,:));
disp('Sigma1:'); disp(Sig(:,:,1));
disp('Sigma2:'); disp(Sig(:,:,2));
disp('x: '); disp(x);

fprintf('----------\n');
ret = bayes_classifier(mu, Sig, Prob, x);
disp('Probability: '); disp(Prob);
fprintf('Class: %i\n', ret);
fprintf('----------\n');

Prob = [1/6 5/6];
ret = bayes_classifier(mu, Sig, Prob, x);
disp('Probability: '); disp(Prob);
fprintf('Class: %i\n', ret);
fprintf('----------\n');

Prob = [5/6 1/6];
ret = bayes_classifier(mu, Sig, Prob, x);
disp('Probability: '); disp(Prob);
fprintf('Class: %i\n', ret);
fprintf('----------\n');
```

實驗結果：

```
mu1:
     1     1

mu2:
     3     3

Sigma1:
     1     0
     0     1

Sigma2:
     1     0
     0     1

x:
    1.8000    1.8000

----------
Probability:
    0.5000    0.5000
```

```
Class: 1
----------
Probability:
    0.1667    0.8333

Class: 2
----------
Probability:
    0.8333    0.1667

Class: 1
----------
```

實驗結果的啓示：

- 先驗概率，正比地影響着，在同等結果的條件下，使用生成之條件的概率。
- 在我們使用的Bayes算法中，我們分別求出了使用第`i`種類型，且生成的數據是`x`的概率密度，即`P(A[i], x)`。
- 由於，`P(A[i]|x) = P(A[i], x)/p(x)`，所以，`P(A[i]|x)`正比於`P(A[i], x)`
- 同時，`P(A[i],x) = p(x|A[i]) * P(A[i])`，所以，`P(A[i], x)`正比於`P(A[i])`