Philosophy Honours Thesis

Adam Chalmers

2016

1 Exploiting expected utility maximisers with infinite-value lotteries

1.1 What is decision theory and why does it matter?

Decision theory is the study of how to make the right choice in a particular situation. Economists, politicians, scientists, financial planners and doctors all use decision theory to choose which possible action will best let them achieve their goals.

Making a plan is simple when one knows all the relevant information. Imagine a doctor choosing between two medical treatments. If we knew which of them would help her patient more, how much each would cost, and exactly what the side effects would be, her decision would be easy. There's no need to resort to decision theory. However, if the doctor is unsure exactly what disease her patient has, or if each treatment has a wide range of possible prices and side-effects, then her decision becomes much more complicated. In situations like this, decision theory offers precise mathematical bounds on each treatment's relative advantages and risks. It allows people to replace their intuitions - which are often flawed and deceptive - with mathematical guides that quantify and limit uncertainty. Decision theory has proved incredibly effective at helping people in uncertain situations make important choices.

Decision theory is one of the greatest successes of philosophy and has proved incredibly effective at solving real world problems, as evidenced by its popularity amongst mathematicians, computer scientists, economists and scientists. However, philosophers have devised some unusual decision problems which decision theory seems to provide misguided solutions to. Thought experiments like the St. Petersburg lottery present two hypothetical choices, one of which philosophers agree is a clearly superior option to the other. However, standard decision theory endorses the apparently inferior option. This is alarming, because while decision theory still works exceptionally well in real-life problems, its failure at these 'paradox' problems hints that we may not have completely finished formulating decision theory. Perhaps, like Newtonian astrophysics, standard decision theory effective at real-world problems on Earth, but will fail to model more exotic problems we could face in future decades.

I believe decision theory can be modified to better advise agents facing these paradoxes without losing its effectiveness at solving real-world problems. In this paper, I will examine show that some paradoxes can be used to exploit people who strictly follow decision theory. They can, for example, be forced to give free money to exploiters who offer them clearly fraudulent bets and gambles. Even if the decision-theoretic agent is overwhelmingly sure that they're being exploited, decision theory still tells them to hand their money over. Exploiting an agent like this is easy: you simply have to offer them an infinite amount of money if they give you a few dollars. No matter how little they believe you, their distrust will never be high enough to cancel out the desirability of an infinite reward. The cost-benefit analysis will always advise them to try for infinite money, regardless of how unlikely it is.

This is a grave problem for decision theory because this exploitation is simple and effective. This section will outline decision theory and its problems. Section 2 will examine the problem in more technical detail. However, in sections 3 and 4 I will review Dr. Nicholas Smith's modified decision theory [11] and show that agents who subscribe to it can escape this sort of exploitation. Section 5 will address possible objections to my analysis and look at my work's broader implications for decision theory.

1.2 Expected utility

Normative decision theory is the study of the mathematical processes of making the ideal decision in a range of different scenarios. Decision theory is applied to decision problems, which comprise of:

- An agent, the decision maker

- A set of actions the decision maker can take

- A set of states: mutually-exclusive propositions which describe ways the world could be

- A set of outcomes that may result from a given action being taken while a given state obtains

- A mapping from outcomes to utilities (numerical measures of desirability)

In decisions under risk, the agent has a probabilistic model which tells the agent the probability that a particular outcome will occur, given the agent takes a certain action while a certain state obtains.

If an agent knows the world's possible states, understands the actions available to them, the probabilities with which each action produces each outcome, and has assigned utilities to each outcome, then the agent is able to apply decision theory to their current situation. A decision algorithm takes these facts as inputs and ranks each action in order of how useful they are to achieving the agent's goals.

Many specific decision algorithms exist, including causal decision theory, evidential decision theory, the dominance principle and expected utility maximisation. Expected utility maximisation states that agents should always choose the action with the highest expected utility. An action's expected utility is the average of each possible outcome's utility, weighted by how likely that outcome is. Mathematically, it is defined as

$$EU(a) = \sum_{\substack{s \in S \\ o \in O(a)}} P(o|a \wedge s) \times U(o)$$

where a is an action and o is any outcome which may arise as a result of that action. In economics, mathematics and computer science, expected utility maximisation is often suggested to be a *norm* of decision theory: a correct and rational mode of decision making that agents should strive to emulate. This is based on two arguments.

Firstly, agents who maximise their expected utility will obtain maximal utility in the long run. This is demonstrated in [12] which shows that maximising expected utility maximises utility. Given that an agent assigns utilities to outcomes in a way that accurately reflects their desires, expected utility maximisation therefore effectively guides agents towards achieving their desires.

Secondly, expected utility theory is a logical implication of basic axioms of rational choice [12]. This means any agent who doesn't take an action of maximal expected utility is violating one of the axioms of rational choice, which are all intuitively reasonable.

In many real-world situations expected utility theory seems to correctly value probabilistic actions such as gambles, medical interventions and business decisions. However, philosophers have devised many decision problems where expected utility seems to dramatically overvalue particular actions and endorse irrational choices. Even worse, this paper shows that expected utility maximisers can be systematically and repeatedly exploited by malicious agents who put the maximisers into particular decision problems.

1.3   Overvalued paradoxes

Expected utility theory systematically overvalues a class of decision theory problems I call High Utility, Low Probability (HULP) problems. I will briefly explain three exemplars of HULP problems and then discuss the features they share which are constitutive of HULP problems.

1.3.1   St. Petersburg Paradox

In the St. Petersburg paradox (first described in [1]) an agent is offered a gamble where a fair coin is repeatedly flipped until it lands tails-up. The agent is then paid $\$2^N$, where N is the number of heads that were flipped.

The expected utility of this gamble is $\$(\frac{1}{2} \times 2 + \frac{1}{4} \times 4 + ...) = \infty$ [10], and therefore an expected utility maximiser should be willing to pay any finite

amount to purchase it. However, this seems like a gross overvaluation because the incredibly high-value outcomes of this lottery have incredibly low probability of occurring. 97% of the time, the agent will make $32 or less from the gamble. As Ian Hacking wrote, 'few of us would pay even $25 to enter such a game' [5].

1.3.2 Pascal's Wager

Blaise Pascal proposed treating theism as a decision problem: an agent's decision to participate in religious observance is motivated by their desire to enter heaven, conditional on God existing [9]. If the cost of performing religious duty is C, then Pascal's Wager has the following decision table:

	God exists	God doesn't exist
Worship	$\infty - C$	$-C$
Don't worship	0	0

By this, worshipping God dominates non-worship. Heaven is presumed to have infinite utility, therefore the expected utility of worship will always be infinite (because an agent's disbelief in God could only ever be finite). This remains true no matter how tiny the agent's estimate of God's existence is. It also remains even if the religious observance is costly or demanding—even if the agent has to pay a large (but finite) amount of utility to perform religious observance, worshipping still maximises expected utility.

1.3.3 Pascal's Mugging

Many objections to Pascal's Wager dispute specific properties of God ([6]) or the possibility of infinite utility ([7])). Pascal's Mugging was proposed by [2] in order to focus criticism towards the decision-theoretic aspects of Pascal's Wager and away from metaphysical or mathematical analysis.

In Pascal's Mugging, the agent is confronted by a Mugger who claims to be a wizard whose powers can magically multiplying money. She asks the agent for a loan of $10, promising to use her magical powers to give the agent a fantastic sum of money in return. The agent has no reason to believe in magic and the Mugger offers no evidence of her wizardry, and so it appears rational for the agent to reject her offer.

However, the Mugger can promise an arbitrarily large amount of money, and our agent's skepticism, while strong, is fixed and non-zero in accordance with norms of rationality. So if the Mugger offers [equation here] such that [equation here], then the agent's expected utility is maximised by giving $10 to the Mugger, despite having no reason to believe her claims.

1.4 HULP Problems

These three problems all share some similar features. In all of them, a gamble's expected utility and its actual intuitively reasonable value appear to differ. All involve a choice between two options:

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- A walk away option with certain chance of zero payoff (and therefore expected utility of zero). Examples of this include not buying the St. Petersburg lottery, not worshipping God, and not paying the Mugger.

- A HULP option (High Utility payoff with Low Probability) with high expected utility. This includes buying the St. Petersburg lottery, worshipping God and paying the Mugger.

Expected utility maximisation instructs agents to choose the HULP action over the walk-away action because the HULP action has higher expected utility. However, agents who consistently choose the HULP action in HULP problems leave themselves open to alarming consequences. Here are some of them.

The St. Petersburg gamble can easily be modified to award payouts in utiles instead of dollars[1]. If an expected utility maximiser is offered the chance to play a Utility St. Petersburg gamble, they should pay any finite utility cost to do so. Such an agent should be willing to bear any utility cost—trading all their wealth, or murdering large numbers of innocent people—in order to play the gamble. As long as the price is finite, the agent must pay it or violate the expected utility maxim.

Pascal's Wager can similarly be used to compel an expected utility maximiser's options. An agent should easily give up their riches and material goods if such costs are necessary for the agent's actions to qualify as religious observance. Indeed, religious observance can involve any finitely-large utility cost and still dominate non-observance, due to the presumption that heaven is valued at infinite utility. Suppose an agent was considering the worship of Quetzalcoatl the Aztec serpent god. Quetzalcoatl's worship involves human sacrifice, which our agent thinks is abhorrent and values at, say, -9000 utiles. An expected utility maximiser should still prefer to perform human sacrifice and be rewarded with heaven rather than not worship, because a large finite utility cost still does not lessen the infinite utility of heaven.

Of course many responses to Pascal's Wager point out that the agent doesn't face a binary choice. There are many possible gods—Quetzalcoatl, Jesus, Poseidon—and not all of them demand human sacrifices. Many gods offer a chance at infinite utility. Alternative gods who do not require human sacrifice exist, and the decision maker is free to choose a god whose worship is more pleasing, since differences in each god's likelihood and worship-cost are cancelled out by the infinite utility reward in the expected utility calculations [4].

Pascal's Mugging cannot be resolved by the many gods objection. If the wizard offers to grant arbitrarily high utility instead of money, then an expected utility maximiser should be willing to pay any amount of utility to appease the mugger. By similar reasoning to the previous examples, an expected utility maximiser would sacrifice their family or perform any other arbitrarily undesir-

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[1]Imagine a sociopathic scientist is offering St. Petersburg gambles that pays out not dollars, but saved human lives (perhaps he knows the secret to manufacturing a much-needed vaccination, or perhaps he has some doomsday device that can kill an arbitrarily large number of humans).

able deed, because the Mugger can offer them utility high enough to perfectly balance out the expected utility equation.

None of these individual considerations is new. That an infinite utility gain can outweigh a finite utility cost is obvious. One could simply bite the bullet and claim expected utility maximisation is a correct norm of decision theory. However, the fact that this same problem keeps rearing its head in different decision theory paradoxes hints that there is a deeper, more systematic problem. These are not three seperate edge cases each designed to be overvalued by expected utility theory. Rather, they provide insight into a deeper problem with expected utility. To illustrate this, I will show how expected utility maximisers can be exploited by agents who can present them with HULP problems.

1.5 Systematically exploiting expected utility maximisers

Suppose there are two agents, an expected utility maximiser called Max, and an exploitative agent called Eliza. If Eliza knows Max is an expected utility maximiser, she can force him to undertake arbitrarily (but finitely) unpleasant actions by appealing to the norms of his decision theory.

Here is a specific example. Eliza would like \$100 from Max, and knowing that he is an expected utility maximser, offers to sell him a St. Petersburg lottery for \$100. Max knows buying the gamble would maximise his utility in this situation, but he doubts she has the financial backing to guarantee he'd receive 2^n dollars after flipping n heads. Eliza retorts that, even if Max's belief in her is in 10^{-20}, the expected utility of paying her is still infinite regardless of her honesty.

$$EU(\text{Pay Eliza}) = (10^{-20} \times (\infty - C)) = \infty$$

$$EU(\text{Don't pay}) = 0$$

As an expected utility maximiser Max is forced to agree that giving Eliza \$100 is the highest-utility option available to him, and hands it over. Eliza, of course, reveals she was lying and walks away with Max's \$100.

Why was Eliza successful? If Max was not an expected utility maximser, or didn't value the St. Petersburg gamble at infinite expected utility, he would have grounds to deny Eliza. Unfortunately, he is neither, and Eliza can thus compel any action from him by offering him the chance at a St. Petersburg lottery in return.

Note that Eliza could have performed a Pascal's Mugging just as easily by promising to use her magical powers to grant Max a large reward, the size of which would be calculated to dominate Max's skepticism. In fact, constructing any HULP problem will allow Eliza to exploit Max and force his action. This is because in all HULP problems, the HULP action has higher expected utility than the walk-away action regardless of which particular large finite cost is attached to performing the HULP action. Eliza could ask Max to do anything and rest assured that, despite the incredibly large utility costs Max would pay

in carrying out these actions, Max would be compelled to obey her if he is a genuine expected utility maximiser.

These concerns demonstrate why expected utility maximisation is inadequate as a norm of decision theory. Any expected utility maximising agent can have their agency hijacked by an exploiter who is able to offer them HULP problems, and as we have seen, such problems are trivial and cheap to offer. In the next chapter I will formalise my notion of HULP problems and go over the mathematics behind HULP exploitation.

2 Formalising exploitation

In the previous section, I provided an intuitive explanation of HULP exploitation. To summarise:

- The victimised agent has some available action A which is undesirable because of its low (perhaps negative) expected utility

- The exploiter agent offers the victim an infinite or arbitrarily large reward just in case the victim performs A.

- The victim now evaluates that performing A has infinite (or arbitrarily large) utility, and performs A.

This section formalises the notion of HULP exploitation and explains why immunity to such exploitation is a desirable norm of rationality. I will begin by outlining the generic form of HULP exploitation, and showing how the Max-Eliza exploitation in the previous section is a specific instance of this general exploitation method. I will then show that HULP-exploitable agents can have their agency hijacked and their preferences reordered at no cost to the exploiter. Agents who wish to preserve their utility function should thus be careful to avoid HULP-exploitation, and we should enshrine immunity to HULP-exploitation as a norm of rationality and desirable property of any decision agents.

2.1 HULP-exploitation

2.2 How bad is HULP-exploitation?

Agents should avoid specific situations of HULP-exploitation, and if possible, avoid being HULP-exploitable at all. This is because HULP-exploitation can lead to the victim modifying and reordering their preferences in a way which leads to them avoiding opportunities to greatly advance their interests, or even taking actions which greatly set back their interests. This is informally demonstrated in the Max-Eliza story above, where Max initially prefers keeping his money to giving it away, but after Eliza offers him the gamble, he prefers giving his money away to keeping it.

We can put this formally like so:

FORMAL PROOF GOES HERE

This demonstrates that Eliza can reverse Max's preferences at no cost to herself. Goal-driven agents seek to maintain their preferences [8, 3] (EXPANDED EXPLANATION OF THIS GOES HERE - ASK MARK FOR READINGS ON THIS TOPIC), and therefore an ideal goal-driven agent should not allow themselves to be HULP-exploited.

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