FILTER CIRCUITS

Introduction

Circuits with a response that depends upon the frequency of the input voltage are known as **filters**. Filter circuits can be used to perform a number of important functions in a system. Although filters can be made from inductors, resistors and capacitors most filter circuits are based upon op-amps, resistors and capacitors.

Filter types and characteristics

A filter is a circuit whose transfer function, that is the ratio of its output to its input, depends upon frequency.

There are three broad categories of filter which are widely used:

> **<u>Low-pass filters</u> allow any input at a frequency below a characteristic frequency to pass to its output unattenuated or even amplified.**
>
> **<u>High-pass filters</u> allow signals above a characteristic frequency to pass unattenuated or even amplified.**
>
> **<u>Band-pass filters</u> allow frequencies in a particular range to pass unattenuated or even amplified.**

The transfer function of a circuit is usually expressed on a logarithmic scale in decibels, and since the fundamental quantity of interest is power, a filter is characterised by

$$G(j\omega) \;=\; 20\ \log_{10}\ \frac{V_{out}(j\ \omega)}{V_{in}(j\omega)}$$

The symbols and Bode diagrams for the ideal transfer functions for these filters are shown in Figure (18)

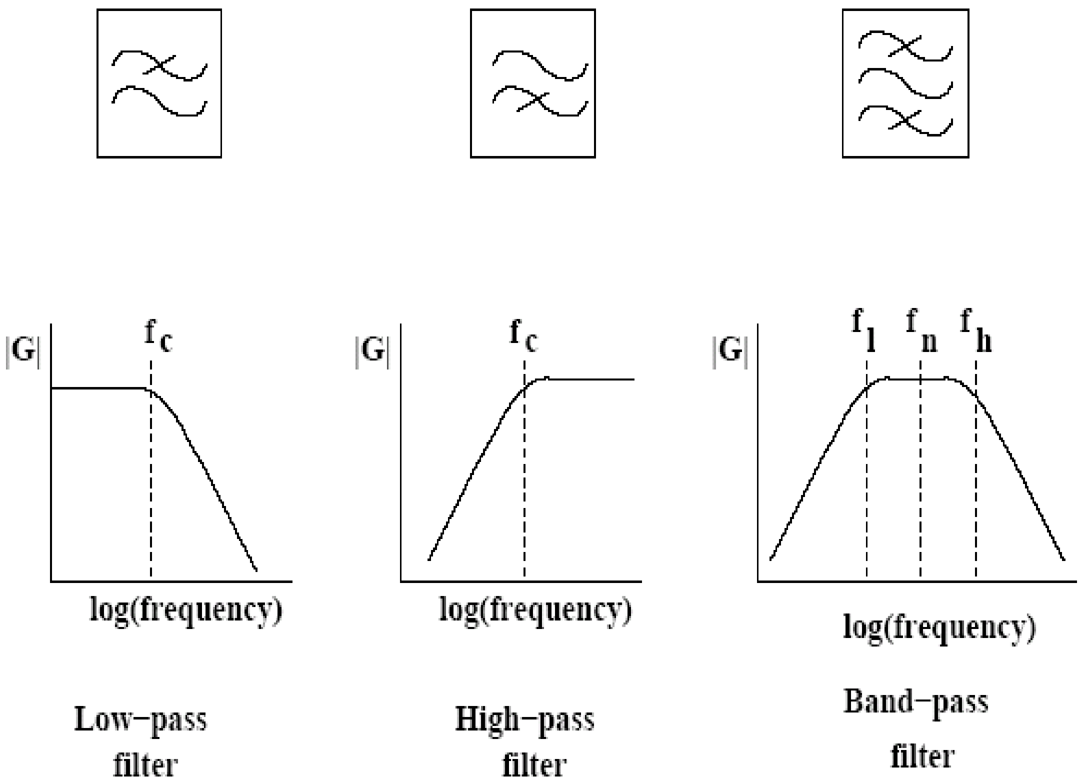

Figure 18 The symbols and characteristics of three types of filters

Applications of Filters

Circuits that attenuate undesirable frequency components of an input signal can fulfil three important functions:

Anti-Alias Filtering

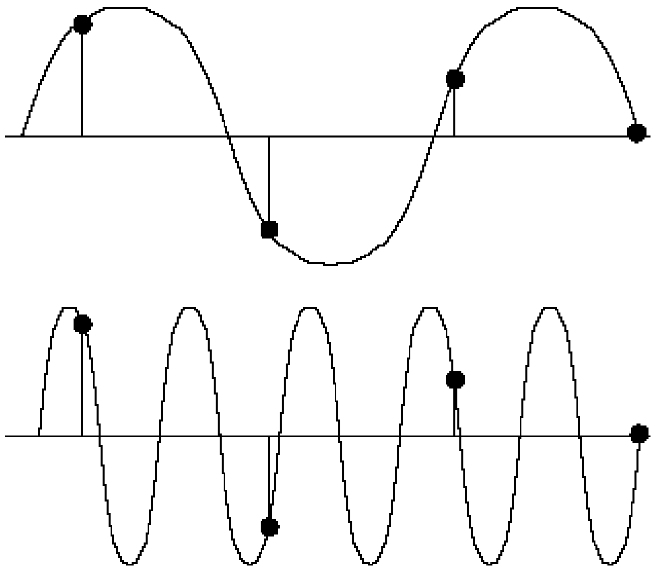

Figure 19: A schematic diagram showing signals of two different frequencies which are indistinguishable when they are sampled at a finite number of points.

Aliasing is a phenomenon that arises because all digital systems can only represent a signal at particular instances in time. An important consequence of this sampling is seen in Figure (19). This shows two different signals sampled at the same rate, i.e. with the same time interval between each sample. Although it is clear that the input signals are at different frequencies, it is equally clear that the four data points acquired by the digital system are the same. To the digital system these two signals are therefore indistinguishable. The conditions necessary to avoid this confusion are stated formally in the **sampling theorem**:

If the continuous input signal contains no frequency components higher than f_c, then it can be completely recovered without distortion if it is sampled at the rate of at least $2f_c$ **samples per second**

However, input signals can potentially contain frequency components at any frequency. Thus before a signal is sampled into a digital system the designer must determine the maximum frequency, f_{max}, in the input signal that will contain useful information. **A filter must then be designed which attenuates all frequencies above f_{max}, prior to sampling by the ADC at a frequency of at least 2*f_{max}. This type of <u>anti-aliasing filter</u> <u>must</u> be present in any well designed system that includes an analogue to digital converter.**

Noise Rejection

Ohms law is based upon the average behaviour of the electrons flowing through a resistor and it predicts the average current that will flow when a particular voltage difference is applied to a resistor. However, the behaviour of electrons is similar to atoms in a gas and the random thermal agitation of the electrons leads to fluctuations in the instantaneous current flow. **These random fluctuations in the current, and hence the voltage, are known as noise.** Noise is generated within the components of a circuit and contains components at all frequencies. This means that a circuit that responds to a smaller frequency range will have less noise in its output. Filters that limit the range of frequencies in their output are therefore useful for limiting the amount of noise.

The quality of a signal in the presence of noise is most often specified by the signal-to-noise ratio (SNR) which is

$$SNR = 10\ log_{10}(V_s^2/V_n^2)$$

where V_s is the rms value of the signal and V_n is the rms value of the noise.

The effectiveness of using filters to reduce noise is demonstrated in Figure (20). In this case the original signal that is the input to the filter looks like noise, with a slowly varying mean value that hints at the presence of a low frequency component in the signal. When this signal is filtered using a low-pass filter with a characteristic frequency of 100Hz, it becomes clear that there is a small 10Hz signal buried in the noise. **Thus despite an original signal-to-noise ratio of 0.2 it is possible to recover a signal by filtering out the noise.**

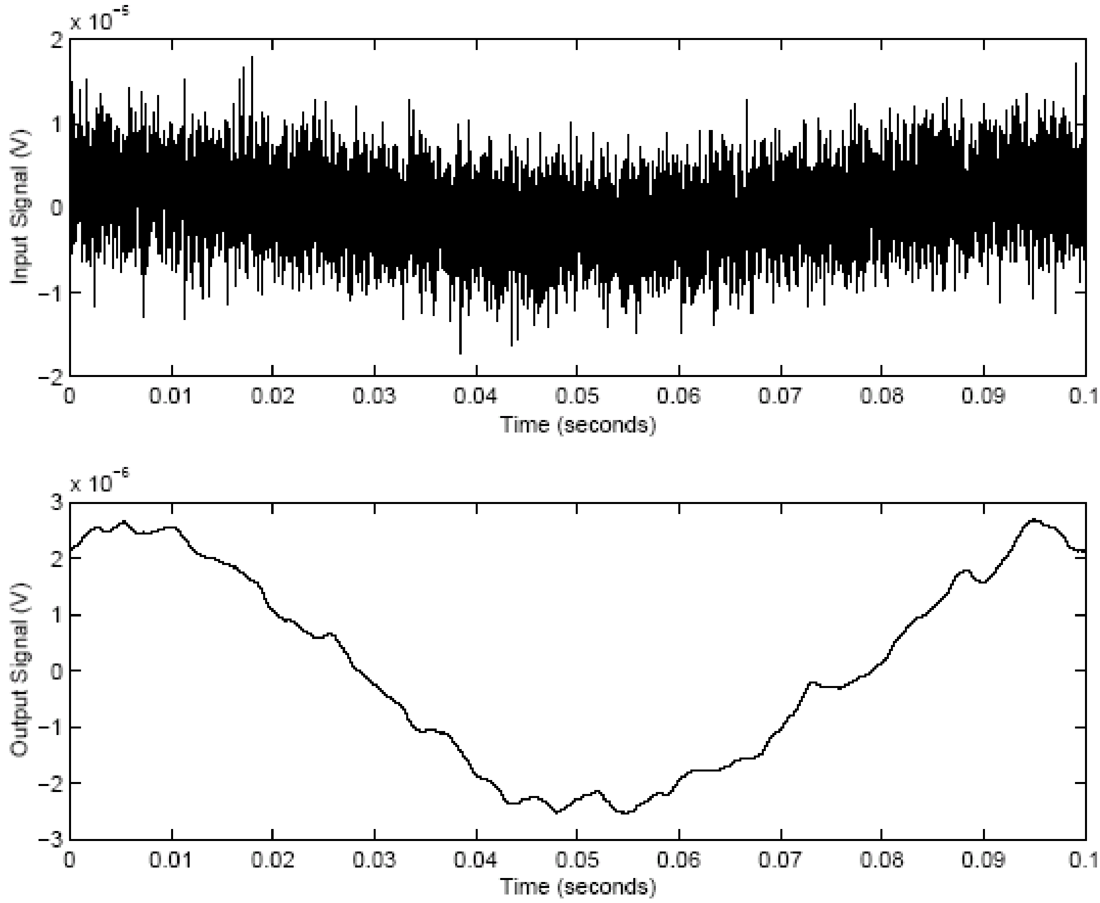

Figure 20: The input signal at the top is difficult to distinguish from noise. This is in sharp contrast to the filter output at the bottom that clearly shows a 10Hz signal.

Interference Rejection

Although interference can be minimised by careful design it is sometimes necessary to also employ filters. Often the dominant source of interference is the mains power supply and so it can be necessary to use a filter to attenuate the frequency of the local mains power supply, 50 Hz in the UK, and/or some of its harmonics.

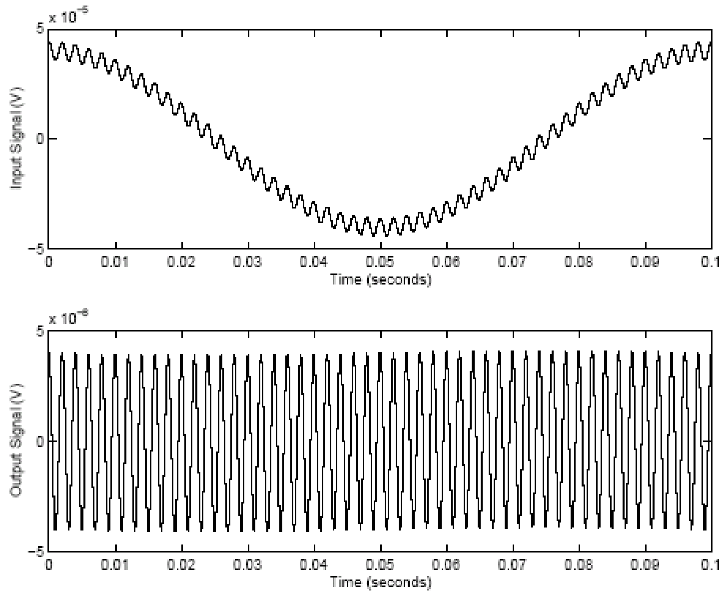

Figure 21 At the top of the figure is a 500Hz input signal with a 10Hz source of interference which is an order of magnitude larger. After filtering with a band-pass filter the interference is dramatically reduced.

A simple example of the use of a band-pass filter to remove interference is shown in Figure (21). In this example the voltage signal at a particular point in the system consists of a small signal at 500Hz and a strong source of interference at 10Hz. **Using a band-pass filter with a characteristic frequency of 500Hz it is possible to reduce the amount of interference by a factor of 400 without attenuating the signal at 500Hz**.

Passive filters

In principle, filters can be made from passive components, that is resistors, capacitors and inductors. However, **at low frequencies, typically below 100MHz, the inductors required to generate a reasonable impedance are bulky. Furthermore, they will include significant resistance that will limit the performance of any filter. Most filters therefore contain resistors and capacitors.**

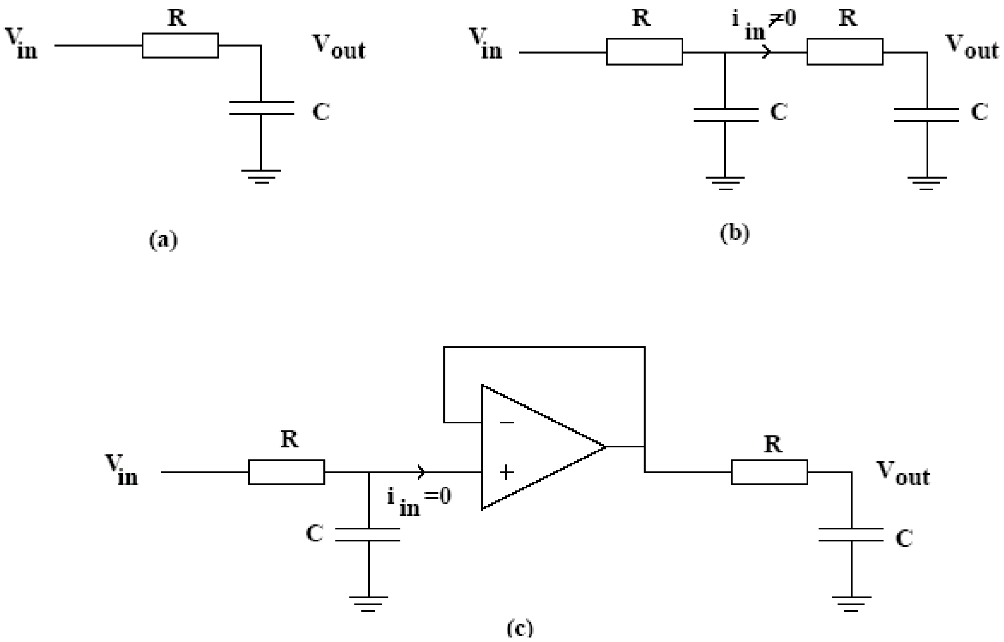

Figure 22: Various passive low-pass filters.

The simplest passive filter can be created using a resistor and capacitor. An example of this type of circuit designed to be a low-pass filter is shown in Figure (22 (a)). The response of this type of filter, is

$$G(\omega) = \frac{V_{out}}{V_{in}} = \frac{1/j\omega C}{R + 1/j\omega C}$$

which can be re-written as

$$G(\omega) = \frac{V_{out}}{V_{in}} = \frac{1}{1 + j\omega RC}$$

to show that the circuit attenuates high frequency signals. Unfortunately, the frequency dependence of these simple filters is relatively weak. Such filters may be adequate when an unwanted interfering signal is far removed in frequency from the desired signal.

If a sharper response is required, the simple solution would appear to be to connect several simple RC filters in series, see for example the two circuits in Figure (22 (b)). **In this situation the input signal to each stage is the output signal from the previous stage and it might be expected that the response of n identical RC circuits connected in series would be**

$$G_n(\omega) = G^n(\omega)$$

However, this simple approach to calculating the response of this system will give an incorrect answer. This failure arises because this simple calculation ignores the current that each circuit draws from the previous filter.

This problem can be solved by inserting an op-amp voltage follower buffer between the various filters as shown in Figure (22 (c)). **Now the infinite input impedance of the ideal op-amp means that the current drawn from each RC circuit is zero, as assumed in the analysis of the single RC circuit, and**

$$G_n(\omega) = G^n(\omega)$$

can be used to design filters with sharper roll-offs. The sharper roll-off of the high frequency response resulting from connecting two and three filters in series is clearly shown in Figure (23).

The problem with **these passive filters is that frequency selectivity is achieved by dissipated unwanted frequencies and it is impossible for any frequency components in the input signal to be amplified by this type of filter.**

To amplify some frequencies whilst attenuating others alternative op-amp based filters are therefore preferred. In these filters the capacitors and resistors are connected to form circuits around op-amps. With these <u>active</u> filters it is then possible to provide gain over a selected frequency range.

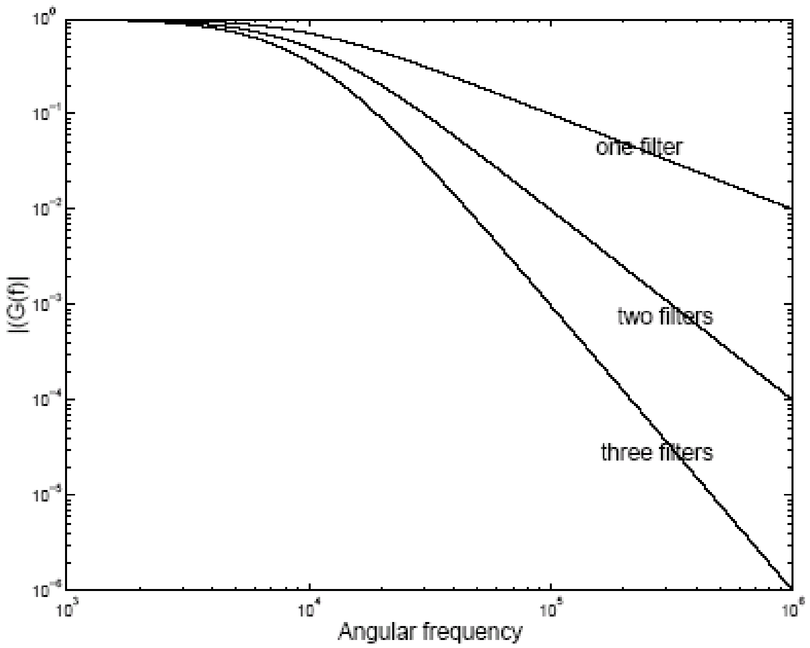

Figure 23: The response of one low-pass filter compared to a cascade of two and three identical filters**.**

Simple op-amp filter circuits

In order to create a filter that is capable of both amplifying the required frequency range and attenuating the undesirable frequencies it is necessary to include frequency dependant feedback within op-amp circuits.

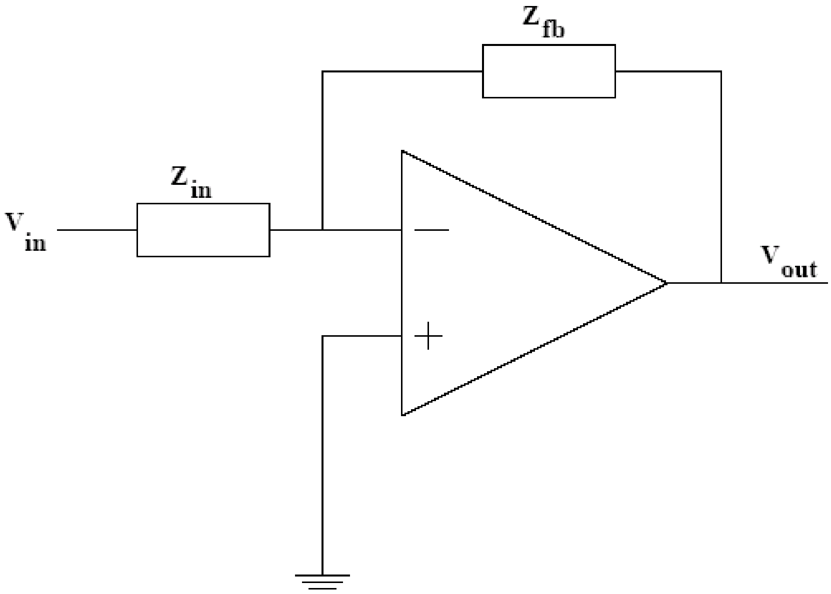

Figure 24: A simple active filter circuit.

Consider the circuit in Figure (24), which is a generalised version of the inverting amplifier, in particular it becomes the inverting amplifier if we take the special case $Z_{in} = R_1$ and $Z_{fb} = R_2$.

Assume that the op-amp is ideal. The inverting input is therefore a virtual earth that means that

$$\frac{V_{in}}{Z_{in}} = -\frac{V_{out}}{Z_{fb}}$$

and the transfer function is therefore

$$\frac{V_{out}}{V_{in}} = -\frac{Z_{fb}}{Z_{in}}$$

This general result can be used to create filters with various properties.

A low-pass filter with a constant gain below a controlled characteristic frequency can be created by a placing a resistor and capacitor in parallel in the feedback loop and using another resistor as the input component, see Figure (25).

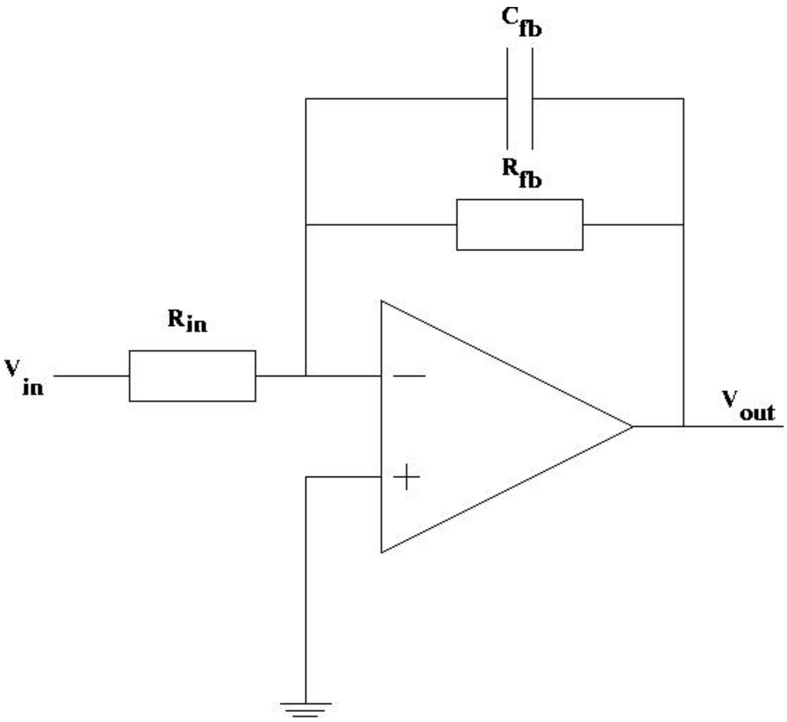

Figure 25: A simple active low pass filter circuit.

The transfer function for the circuit can be written down once an expression has been obtained for the impedance of the feedback loop. Since the two elements of the feedback loop are in parallel

$$1/Z_{fb} = 1/R_{fb} + 1/(1/j\omega\ C_{fb})$$

which means that

$$Z_{fb} = \frac{R_{fb}}{1 + j\omega R_{fb} C_{fb}}$$

This can then be used to calculate the resulting transfer function

$$\frac{V_{out}}{V_{in}} = \frac{-R_{fb}}{R_{in}(1 + j\omega R_{fb} C_{fb})}$$

The characteristic frequency of this filter is the frequency at which $\omega R_{fb} C_{fb} = 1$. Below this frequency the transfer function can be approximated by $-R_{fb}/R_{in}$ whilst at frequencies higher than this critical frequency the transfer function becomes $-1/j\omega R_{in}\ C_{fb}$. This circuit therefore has a fixed gain at low frequencies whilst it attenuates higher frequencies.

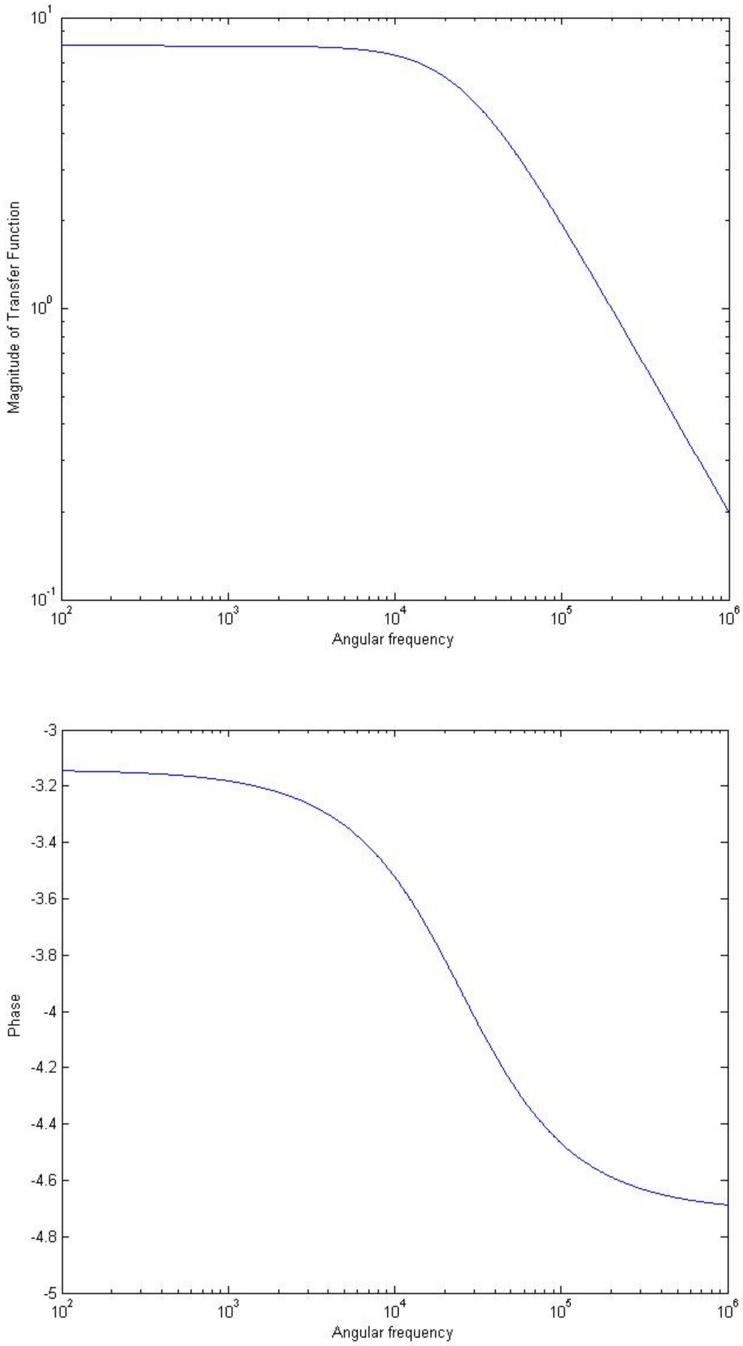

Figure 26: The amplitude and phase response of a low-pass filter

The operation of the circuit in Figure (25) can be understood by considering the characteristic frequency $\omega R_{fb} C_{fb} = 1$. This is the frequency at which the impedances of the resistor and the capacitor in the feedback network are the same. Below this frequency the impedance of the capacitor is higher than that of the resistor and most of the current flows through the resistor. In effect at these frequencies the presence of the capacitor has little effect and the circuit is equivalent to the inverting amplifier. At frequencies above the characteristic frequency the impedance of the capacitor is lower than that of the resistor and most of the current flows through the capacitor. In addition because the impedance is lower than the resistor it requires a smaller voltage to draw a current through the capacitor and so the output voltage can be lower. Hence once the impedance of the capacitor is less than that of the resistor the gain of the circuit starts to decrease.

By modifying the elements in the input and feedback networks of a single op-amp it is also possible to create high-pass and band-pass filters with fixed gains in the pass-bands.

Sallen-Key Filters

In order to limit noise or avoid interference **it may be necessary to employ a filter which rolls-off faster than 20dB/decade. A faster roll-off can be achieved by cascading several identical circuits. However, a more economical approach is to us more sophisticated RC circuits with a single op-amp.** Placing different components at the various positions in the feedback network then leads to circuits that act as low-pass, high-pass or band-pass filters.

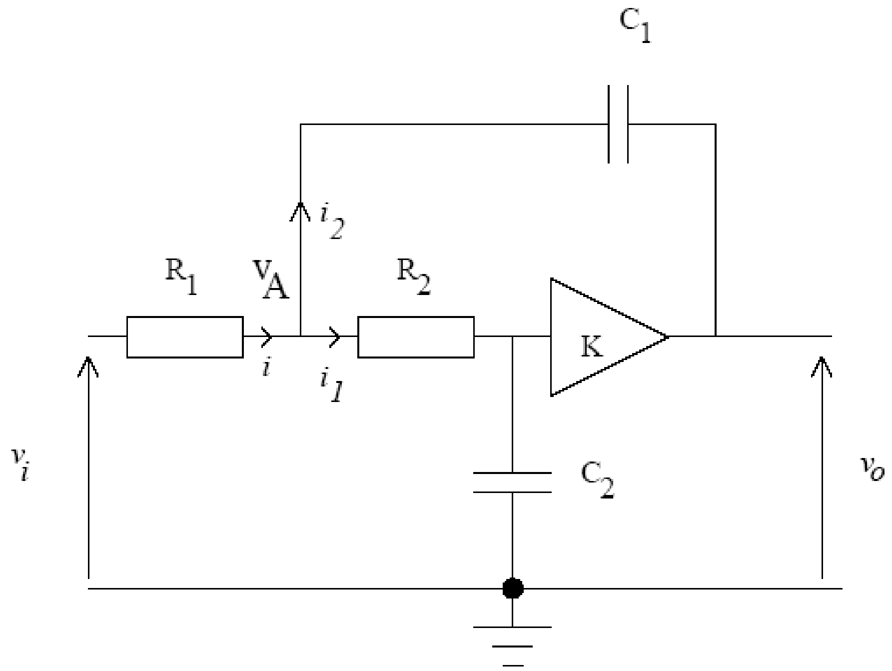

Figure 27: Second-order low-pass filter circuit including an amplifier circuit with a gain of K (shown as a triangular symbol).

The circuit in Figure (27) is a low-pass filter that includes an amplifier circuit with a gain of K, represented in the circuit diagram by the triangle, which gives the filter a low-frequency (or dc) gain of K.

To analyse the circuit, start from the right and work towards the left; the definition of the gain of the amplifier means that the voltage at the input of the amplifier circuit is

$$\frac{v_o}{K}$$

and if no current flows into the amplifier (this limits the type of amplifier circuit that can be used):

$$i_1 = sC_2 \frac{v_o}{K}$$

which means that

$$v_A = (1 + sC_2R_2)\frac{v_o}{K}$$

This can then be used to calculate the current flowing round the feedback loop

$$i_2 = sC_1[v_A - v_o]$$

which becomes

$$i_2 = sC_1[(1 + sC_2R_2)\frac{v_o}{K} - v_o]$$

Finally

$$i = i_1 + i_2$$

which means that the input voltage is

$$v_i = v_A + iR_1$$

The above equations combine to give the following transfer function:

$$\frac{v_o}{v_i}(s) = \frac{K}{1 + s[C_2(R_1 + R_2) + (1 - K)C_1R_1] + s^2C_1C_2R_1R_2}$$

This can be compared with the standard form of the transfer function of a system whose response can be represented by a second order differential equation. The standard form for the denominator is always the same for the different types of filters. It is the form of the numerator which determines the type of filter. In the case of a low-pass filter the standard form for a low-pass filter with a dc gain of *K*:

$$\frac{v_o}{v_i}(s) = \frac{K}{1 + 2\zeta(s/\omega_c) + (s/\omega_c)^2}$$

At low frequencies $s=j\omega \approx 0$ and so the denominator is dominated by the first term and the gain of the circuit is *K.* At frequencies above ω_c the last term in the denominator dominates and the gain of the circuit falls off at a rate of 40dB/decade. The gain at the characteristic frequency, $\omega = \omega_c$, $1/2\zeta$ times the dc gain. Since **the characteristic frequency of an active filter is usually defined to be the frequency at which the power gain has been reduced by 3dB the low pass filter is usually designed with $\zeta = 0.707$ so that ω_c is the characteristic frequency of the filter.**

Comparing the standard form for a low-pass filter to the transfer function for the specific filter circuit leads to the conclusion that:

$$\omega_c = \frac{1}{\sqrt{C_1 C_2 R_1 R_2}} \qquad \zeta = \sqrt{\frac{C_2}{C_1}} \cdot \frac{R_1 + R_2}{2\sqrt{R_1 R_2}} - \frac{(K-1)}{2}\sqrt{\frac{C_1 R_1}{C_2 R_2}}$$

which means that different parameters of the filter can be controlled using the values of various components.

There are two different variations of the circuit in Figure (27), depending on whether *K* > 1 or *K* = 1:

Voltage follower (the original Sallen-Key filter)

If the triangle represents a simple voltage follower then K=1, and

$$\zeta = \frac{1}{2}\sqrt{\frac{C_2}{C_1}} \cdot \frac{R_1 + R_2}{\sqrt{R_1 R_2}}$$

which is greater than

$$\sqrt{\frac{C_2}{C_1}}$$

since

$$\frac{R_1 + R_2}{2} > \sqrt{R_1 R_2}$$

So for $\zeta < 1$, we need $C_1 > C_2$. Since there are more component values to select than filter parameters the design can be simplified by reducing the number of component values that need to be selected by setting $R_1 = R_2 = R$ and this resistor value can be chosen to be as large as possible so that the capacitors will be small (in general, the capacitors will have to be made from a combination of individual components).

Non-inverting amplifier (K > 1)

If the triangle in the filter represents a non-inverting amplifier then we can now set $C_1 = C_2$ and $R_1 = R_2$. The disadvantage is that although the dc gain, K, can be greater than one it is constrained to be some odd values which are difficult to obtain precisely unless close tolerance resistors are used in the inverting amplifier circuit.

High-pass filters

The general transfer function for a second-order <u>high-pass</u> filter is:

$$G(s) = \frac{K(s/\omega_c)^2}{1 + 2\zeta(s/\omega_c) + (s/\omega_c)^2}$$

At high signal frequencies the denominator is dominated by the term $(s/\omega_c)^2$ and the transfer function becomes

$$G(s) = K$$

K is therefore the high-frequency gain.

At low-frequencies the denominator is approximately 1 and in this frequency range

$$G(s) \ = \ K(s/\omega_c)^2$$

and since at low frequencies $(s/\omega_c)^2$ is less than one, if this wasn't true then the denominator would not be dominated by the 1, the response at low frequencies is less than the response at high-frequencies. However, since $s = j\omega$ the filter response is increasing at 40dB/decade at low frequencies.

A filter with this type of response can be realised simply by interchanging the resistors and capacitors in the low-pass circuit in Figure (27).

Band-pass filters

The general transfer function for a second-order band-pass filter is:

$$G(s) \ = \ \frac{K \ . \ 2\zeta(s/\omega_n)}{1 + 2\zeta(s/\omega_n) + (s/\omega_n)^2}$$

In this case, *K* is the gain in the middle of the pass-band and ω_n is the centre frequency of the pass-band.

For a band-pass filter the transfer function is usually re-written in terms of the quality factor $Q = 1/2\zeta = \omega_n/\Delta\omega$**, where** $\Delta\omega$ **is the 3 dB bandwidth of the bandpass filter, i.e.** $\omega_u - \omega_l$**where** ω_u **is the upper limit of the passband and** ω_l **the lower limit. (Note that since the frequency**

characteristics of a band-pass filter are symmetrical on a logarithmic scale ($\omega_n = \sqrt{\omega_l . \omega_u}$) The transfer function is then:

$$G(s) \;=\; \frac{(K/Q)\,.\,(s/\omega_n)}{1+\frac{1}{Q}(s/\omega_n)+(s/\omega_n)^2} \;=\; \frac{K}{1+Q(s/\omega_n+\omega_n/s)}$$

Band-pass filters could be constructed from cascaded connections of low-pass and high-pass filters, however, high *Q* values, that is relatively narrow pass-bands, are not then possible (because of the sensitivity to component values of the individual filter sections). Instead, we could use a circuit such as the one shown in Figure (28).

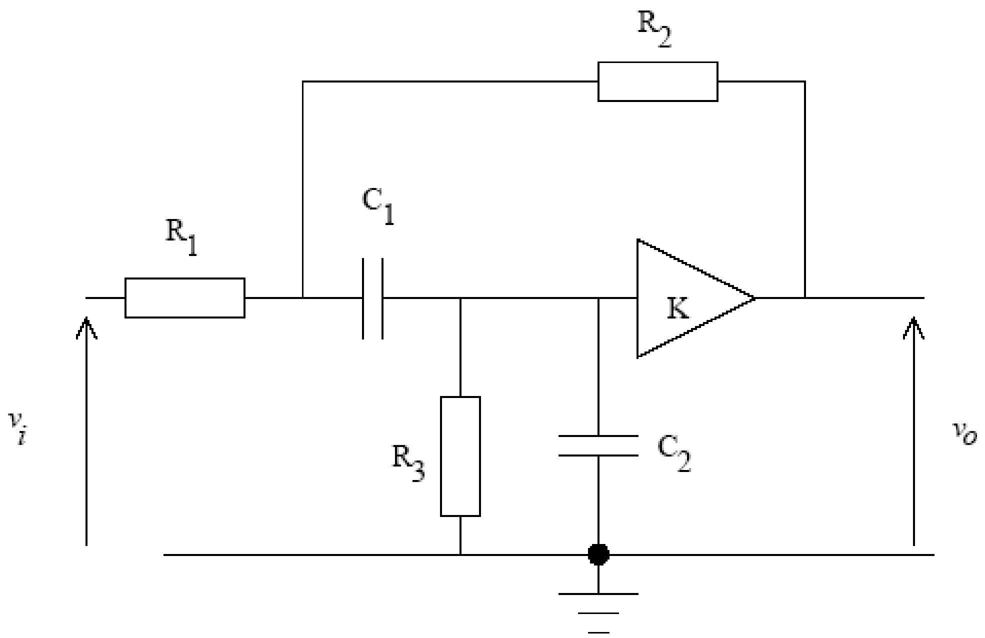

Figure 28: A band-pass filter circuit

The circuit can be analysed in the same way as the low-pass filter circuit, although one ends up with very cumbersome expressions for *K*, ω_n and *Q* and it is difficult in practice, to design and build such a band-pass filter.

Multiple Feedback Filter Circuits

An alternative to the Sallen-Key filter circuits is the multiple feedback design shown in Figure (29).

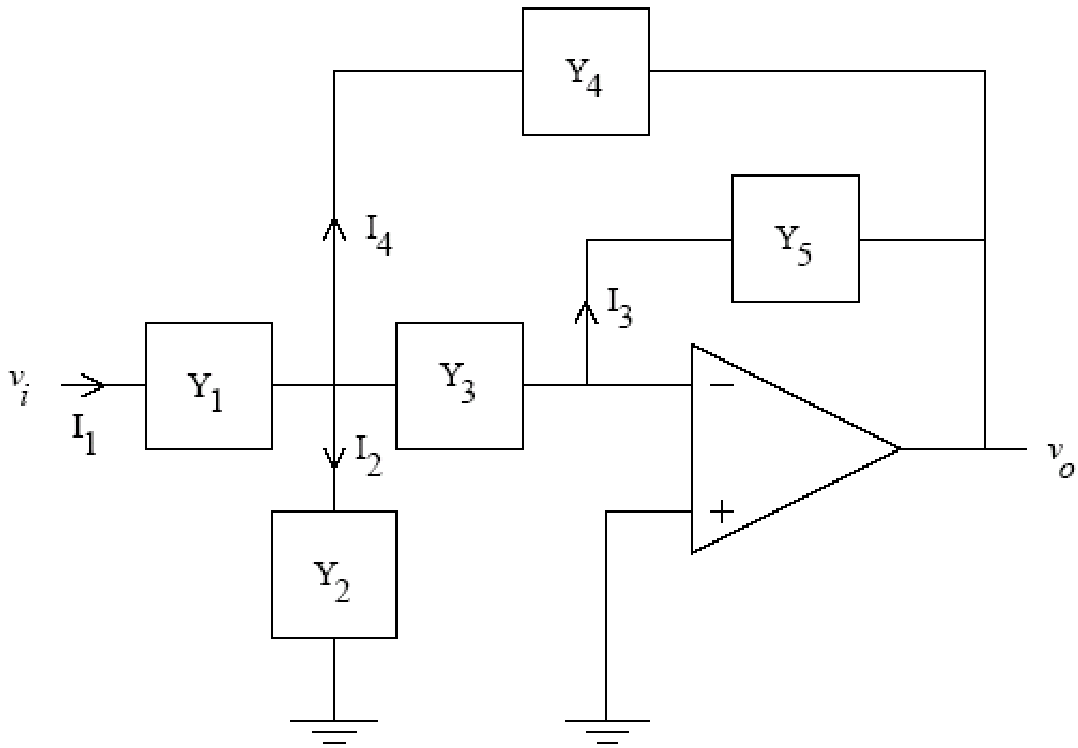

Figure 29: Generalised multiple feedback circuit.

To analyse this circuit **consider the admittance, Y_n, of the various components of the feedback network where the admittance is the inverse of the impedance**

$$Y_n = 1/Z_n$$

Summing the currents at the first node in the circuit, and remembering that the op-amp creates a virtual earth, gives:

$$I_1 \;=\; Y_1(v_i - v_N) \;=\; I_2 + I_3 + I_4 \;=\; Y_2 v_N \;+\; Y_3 v_N \;+\; Y_4(v_N - v_o)$$

Re-arranging the above leads to:

$$Y_1 v_i \;=\; v_N(Y_1 + Y_2 + Y_3 + Y_4) \;-\; Y_4 v_o$$

With a high input impedance at the inverting input we also have:

$$Y_3 v_N \;=\; -Y_5 v_o \quad \rightarrow \quad v_N \;=\; -\frac{Y_5}{Y_3} v_o$$

$$Y_1 Y_3 v_i \;=\; v_o[-Y_5(Y_1 + Y_2 + Y_3 + Y_4) \;-\; Y_3 Y_4]$$

and **so the transfer function is**

$$\frac{v_o}{v_i} \;=\; \frac{-Y_1 Y_3}{Y_5(Y_1 + Y_2 + Y_3 + Y_4) \;+\; Y_3 Y_4}$$

By comparing this expression with the general expressions for low-pass, high-pass and band-pass filters it is possible to determine which components are required to create each type of filter. For example, the general expression for the response of a high-pass filter is

$$G(s) \;=\; \frac{K(s/\omega_c)^2}{1 + 2\zeta(s/\omega_c) + (s/\omega_c)^2}$$

Comparing this equation and the transfer function for the multiple feedback filter indicates that the correct numerator for a high-pass filter can only be created if circuit elements 1 and 3 are both capacitors. There are then several different configurations that will generate the correct denominator. However, one simple approach is to use a capacitor as element 4 and make the other two elements resistors. Hence one possible component list for a high-pass filter is $Y_1 = sC_1;\ Y_3 = sC_3;\ Y_4 = sC_4$ and $Y_2 = R_2;\ Y_5 = R_5$

The equivalent procedure to that above can be followed to obtain a circuit configuration that corresponds to a low-pass filter. Alternatively, to create a low-pass filter, swap the R's and C's, in the high-pass filter.

The transfer function of a band-pass filter is

$$G(s) \ = \ \frac{(K/Q) \, . \, (s/\omega_n)}{1 + \frac{1}{Q}(s/\omega_n) + (s/\omega_n)^2}$$

Again there are several circuit configurations that will achieve the correct response. One approach is to create the quadratic part of the denominator by making elements 3 and 4 capacitors. Then to obtain the correct numerator element 1 must be a resistor. In this case the constant part of the denominator is obtained by making element 5 a resistor. Element 2 can then either be a capacitor or a resistor. Thus one possible circuit configuration for a band-pass filter is $Y_1 = 1/R_1; \ Y_2 = 1/R_2; \ Y_5 = 1/R_5$ and $Y_3 = sC_3; \ Y_4 = sC_4$.

Example of a band-pass filter using multiple feedback design

The multiple feedback circuit is one of many circuits in which the designer has more variables, i.e. component values, than constraints from the specification. In these situations the designer will simply assume that two components have the same value. For the example circuit shown in Figure (30) it has been assumed that $C_3 = C_4$:

You should check for yourself that, if $R_{12} = R_1R_2/(R_1 + R_2)$

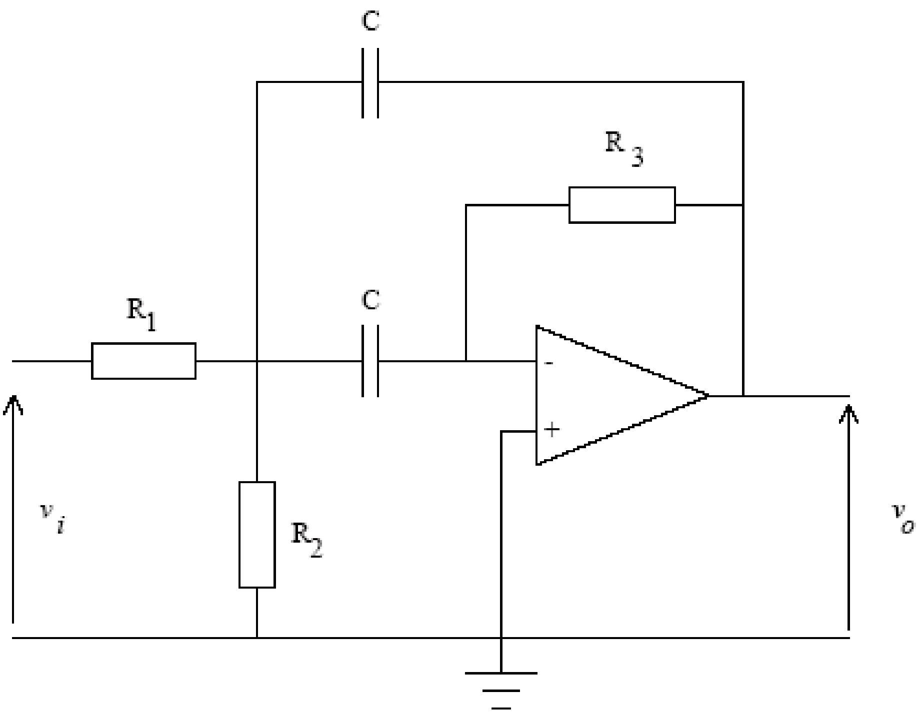

Figure 30: Multiple feedback 2nd-order band-pass filter circuit

then the gain at resonance is:

$$K = -\frac{1}{2}\frac{R_3}{R_1}$$

the resonant frequency is

$$\omega_n = \frac{1}{C\sqrt{R_{12}.R_3}}$$

and the filter has a *Q* value of

$$Q = 0.5\sqrt{R_3/R_{12}}$$

Since $Q^2 = R_3/4R_{12}$ and $\mid K/2 \mid = R_3/4R_1$, we must have $Q^2 > \mid \frac{K}{2} \mid$

Note that, since Q depends on the square root of component ratios, a high-Q filter (with a Q of around 100) requires very high component ratios.

KHN Filters

The **Sallen-Key and Multiple Feedback filters** have the advantage that they use a small number of components, especially op-amps. They **were particularly popular when op-amps were relatively expensive**. **However, it may be impossible to design a circuit with a particular transfer function or the resulting design may be sensitive to variations in component values**. In these situations the best approach may be to take advantage of reductions in component costs, which mean that op-amps are now commonly available with up to four op-amps in a single package and use a filter design which contains several op-amps.

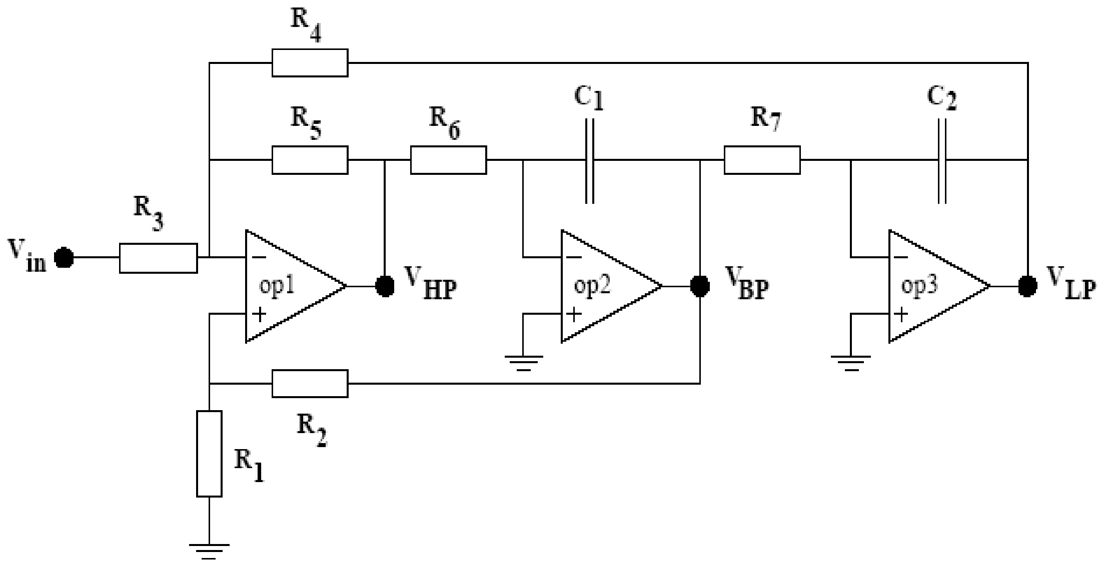

Figure 31: An inverting state-variable or KHN filter.

One type of filter that contains several op-amps in a single stage is the state-variable or KHN filter, first proposed by Kerwin, Huelsman and Newcomb. As shown in Figure (31) **this filter contains three op-amps from which high-pass, band-pass and low-pass responses are available simultaneously.**

The first step in analysing the circuit is to break it down into the different functional blocks. The op-amp labelled op1 has quite a complicated looking set of connections. However, the connections to the other two op-amps are much simpler, for example isolating the part of the circuit containing op3 leads to the simple circuit shown in Figure (32).

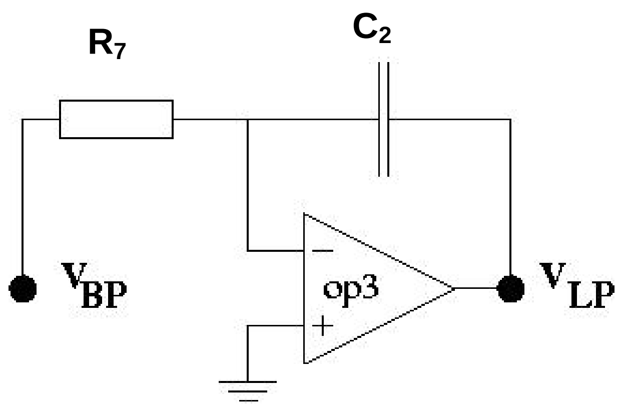

Figure (32) The connections in op-amp op3 in the KHN Filter in Figure (31)

This is simply the circuit shown in Figure (24) with a capacitor acting as the input impedance and a resistor in the feedback loop. The relationship between the input and the output of this circuit is therefore

$$V_{LP} = \frac{-1}{R_7\ C_2\ s}\ V_{BP}$$

In fact op2 has equivalent connections, and so the relationship between the output of this circuit block and its input is

$$V_{BP} = \frac{-1}{R_6\ C_1\ s}\ V_{HP}$$

Now consider the remaining op-amp in the circuit, op1. The components connected to op1 mean that the voltage at the output of this op-amp depends upon a linear combination of the input to the filter circuit and the outputs from the two other op-amps within the filter. This op-amp therefore has three inputs and <u>linear superposition</u> can be used to derive an expression for its output.

In linear superposition the total output is found by considering the response of the circuit to each voltage or current source separately. Any voltage source that is not being considered acts as a short-circuit and any current source that is not being considered acts as an open-circuit. For the three inputs to the circuit around op1, short-circuiting the output from op2 shows that for V_{in} and V_{LP}, op1 is configured as a summing amplifier that adds the two components.

To determine the contribution to the output of op1 from the output from op2 by superposition, short-circuit both V_{in} and V_{LP}. This means that both R_3 and R_4 connect the inverting op-amp input to ground, these two resistors therefore act in parallel so that the voltage at the inverting op-amp input is

$$V^- = \frac{R_3//R_4}{R_5 + R_3//R_4}\, V_{HP}$$

similarly, the voltage at the non-inverting input is

$$V^+ = \frac{R_1}{R_1 + R_2}\, V_{BP}$$

For the ideal op-amp $V^- = V^+$ so that

$$V_{HP} = (1 + \frac{R_5}{R_3//R_4})\ \frac{R_1}{R_1 + R_2}\ V_{BP}$$

This contribution can then be combined with the other two contributions to give

$$V_{HP} = -\frac{R_5}{R_3}\ V_{in} - \frac{R_5}{R_4}\ V_{LP} + (1 + \frac{R_5}{R_3//R_4})\ \frac{R_1}{R_1 + R_2}\ V_{BP}$$

Then remember that the analysis of the responses of the other two op-amps gave

$$V_{BP} = \frac{-1}{R_6\ C_1\ s}\ V_{HP}$$

and

$$V_{LP} = \frac{-1}{R_7\ C_2\ s}\ V_{BP}$$

These two expressions can then be included into

$$V_{HP} = -\frac{R_5}{R_3}\ V_{in} - \frac{R_5}{R_4}\ V_{LP} + (1 + \frac{R_5}{R_3//R_4})\ \frac{R_1}{R_1 + R_2}\ V_{BP}$$

to give

$$\frac{V_{HP}}{V_{in}} = -\frac{R_5}{R_3} \times$$

$$\frac{R_4 R_6 C_1 R_7 C_2 s^2 / R_5}{R_4 R_6 C_1 R_7 C_2 s^2 / R_5 + R_4 (1 + R_5/R_3 + R_5/R_4) s / (1 + R_2/R_1) R_5 + 1}$$

This is the transfer function of a second-order high-pass filter, with

$$gain = -\frac{R_5}{R_3}$$

$$\omega_c = \frac{(R_5/R_4)^{1/2}}{(R_6 C_1 R_7 C_2)^{1/2}}$$

$$Q = \frac{(1 + R_2/R_1(R_5R_6C_1/R_4R_7C_2)^{1/2}}{1 + R_5/R_3 + R_5/R_4}$$

The band-pass filter response can then be obtained from the high-pass response using

$$V_{BP} = \frac{-1}{R_6\ C_1\ s}\ V_{HP}$$

In turn the expression for the band-pass response of the system can be used to obtain an expression for the low-pass filter response using

$$V_{LP} = \frac{-1}{R_7\ C_2\ s}\ V_{BP}$$

Switch-Capacitor Filters (for information only)

The last topic that should be mentioned is a technique that has become popular for implementing filters in microelectronic integrated circuits. Using the processes available when manufacturing an integrated circuit it is difficult to create a small, and hence cheap, high value resistance. However, with these manufacturing processes it is easy to manufacture small switches. To understand how these can be used to mimic a resistor consider the circuit in Figure (33), which contains a capacitor and two switches.

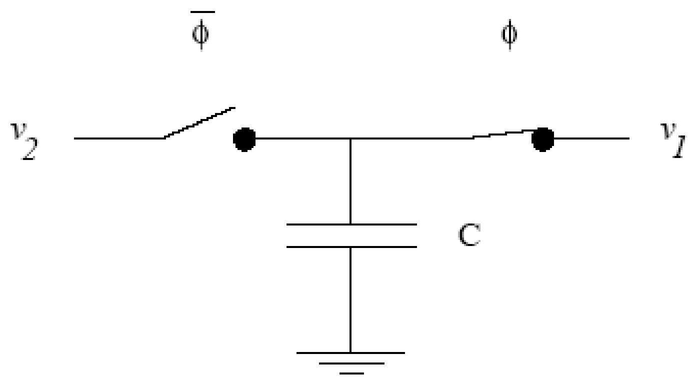

Figure 33: A schematic diagram of the combination of a capacitor and two resistors used to mimic a resistor in a switched capacitor circuit.

Both the switches are controlled by the same signal, usually referred to as the clock signal, however, they are connected so that one switch has a zero resistance when the clock is high and an infinite resistance when the clock is low, whilst the other switch has a zero resistance when the clock is low. **If the capacitor is connected to the right hand node it will be charged to voltage v_1, when the clock signal changes the capacitor will be disconnected from the right hand node and connected to the left hand node. The voltage on the capacitor will then change from v_1 to v_2 and**

a charge

$$\delta q = C \ (v_1 - v_2)$$

will have moved from right to left. Since this happens every clock period this process is equivalent to an <u>average</u> current flow of

$$i = \frac{C \ (v_2 - v_1)}{1/f_{clock}}$$

which means that on a timescale longer than the clock period the combination of the capacitor and the two switches is equivalent to a resistance of

$$R_{eq} = \frac{1}{C f_{clock}}$$

One advantage of this approach is that a small capacitance can be used to create a large effective resistance. The only disadvantage of this technique is that it is based upon the average current over a timescale longer than the clock period. The clock period of the switches must therefore be significantly higher than the highest signal frequency, for example if the highest frequency of interest in the input signal is 20kHz then the clock frequency should be more than 100kHz.

Summary

Filters fulfil three important functions:

(i) **They must limit the maximum frequency at the input of the ADC in order to avoid aliasing**. This means that if the ADC acquires N samples of a signal per second, so that the interval between samples is 1/N seconds, then the maximum frequency that can be represented in the sampled signal is (N/2) Hz. In order to avoid aliasing all frequencies above (N/2) Hz should be removed from the input signal before sampling.

(ii) **They can limit the noise bandwidth of the input signal in order to reduce the amount of noise in the signal.**

(iii) **They can attenuate frequencies at which interference is expected**. Usually, this means attenuating the frequency of the local mains power supply, 50 Hz in the UK, and/or some of its harmonics.

A filter is characterised by

$$G(j\omega) \;=\; 20\,\log_{10}\,\frac{V_{out}(j\omega)}{V_{in}(j\omega)} \qquad (\mathrm{dB})$$

There are three broad categories of filter which are widely used: **Low-pass** filters allow any signal at a frequency below a characteristic frequency to pass to its output unattenuated. These low-pass filters are therefore often used as the anti-alias filter immediately before the ADC, but, they can also be used to limit the bandwidth of a signal in order to reduce the amount of noise. **High-pass** filters allow signals above a characteristic frequency to pass unattenuated. They can be used to reject noise and/or interference at frequencies below the minimum signal frequency of interest. Finally, **band-**

pass filters allow frequencies in a particular range to pass unattenuated. This type of filter is often useful to limit noise bandwidth for applications when it is known that the input signal of interest only occurs in a restricted frequency range. However, when designing a filter to reduce noise it is important to remember that the noise bandwidth of the filter will be larger than the signal bandwidth.

Passive filters can be created by combining capacitors and resistors. However, there are two problems with these filters. Loading between stages means that it can be difficult to design a filter of this type with a rapid roll-off. More importantly these filters act by rejecting unwanted frequencies, it is therefore impossible to amplify the desirable frequency range whilst rejecting unwanted frequencies.

Active filters that contain op-amps can amplify signals and they can be cascaded to simply the design of a filter with a rapid roll-off.

Different types of active filters can be created using different feedback loops around an op-amp. These include first-order filters that have the same frequency response as passive filters, except they can provide gain when required, and second order filters. One problem that can arise with any filter based on a single op-amp is a limitation in the combination of filter parameters that can be implemented. These problems can be avoided using filters that incorporate several different op-amps. Another advantage of these filters is that they can simultaneously provide low-pass, band-pass and high-pass filtering of a signal.