Analog Filters

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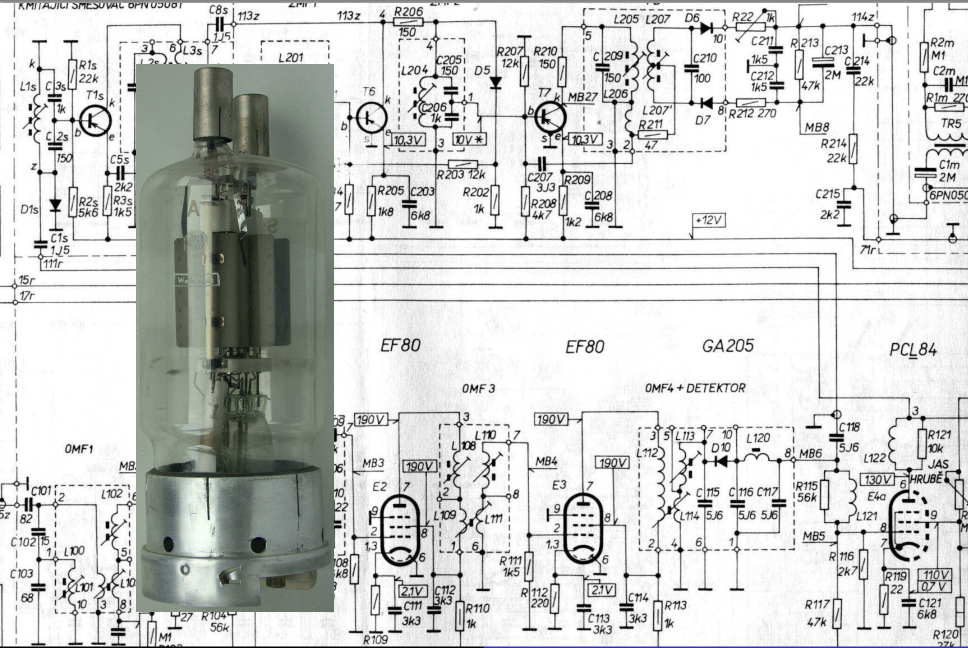
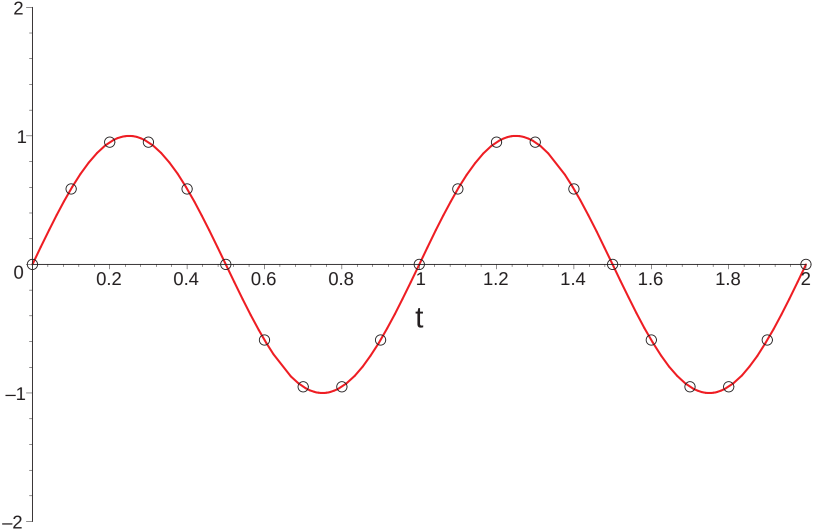
KATEDRA TEORIE OBVODŮ
Fakulta elektrotechnická

Old Electronic circuits

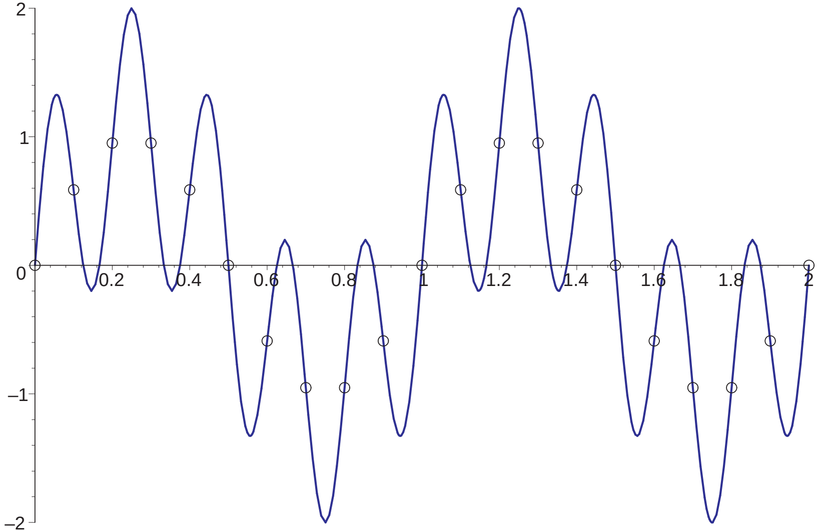


Where the analog filters are used?

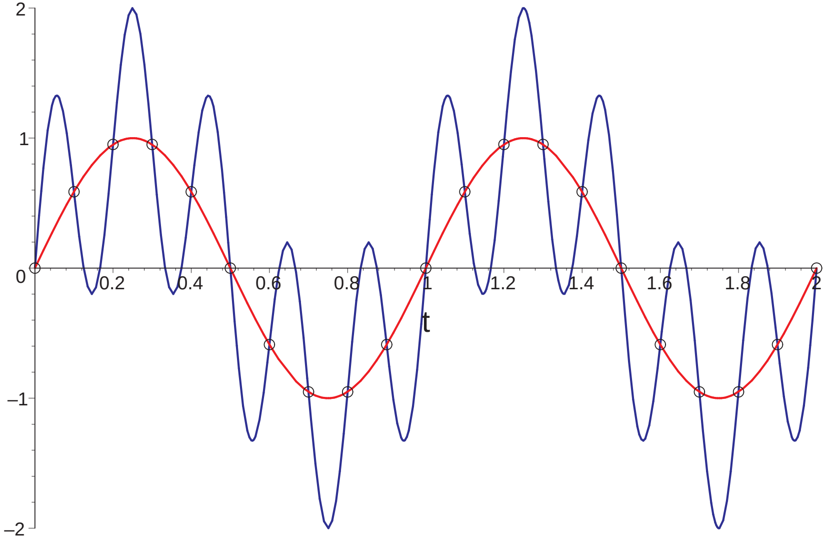
- Anti-aliasing filters – before sampling (and conversion to digital) of analog signal,
- In communication – frequency multiplex $\equiv$ transmitting multiple signals (with different frequency) over a single line – for special applications or high frequency signals, where digital filters cannot be used,
- Interference (DC component, 50 Hz, noise, ...) suppression,
- Reconstruction filters, conversion of digital signals back to analog signal.



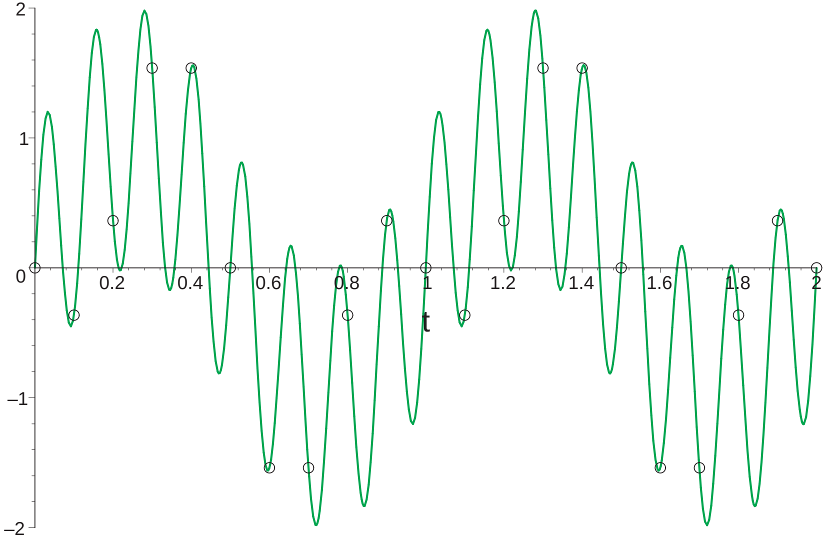
$f_c = 10$, resp. $f_c/2 = 5$ $\sin(2\pi 1 t)$



$f_c = 10,$ resp. $f_c/2 = 5$ $\sin(2\pi 1 t) + \sin(2\pi 5 t)$

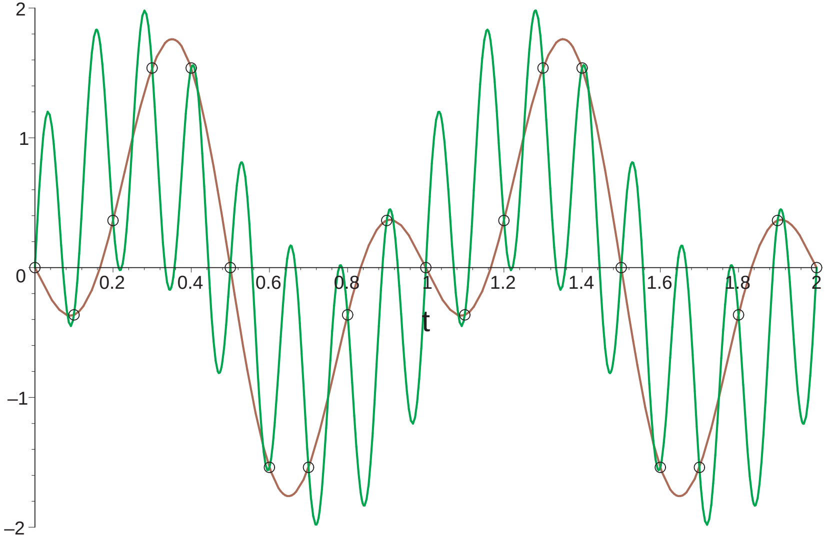


$f_c = 10$, resp. $f_c/2 = 5$ $\sin(2\pi 1 t) + \sin(2\pi 5 t)$, resp. $\sin(2\pi 1 t)$

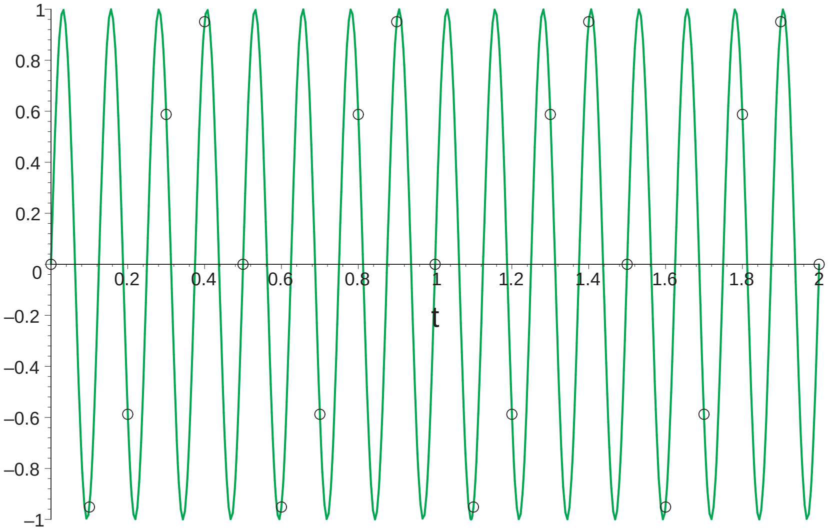


$f_c = 10,$ resp. $f_c/2 = 5$

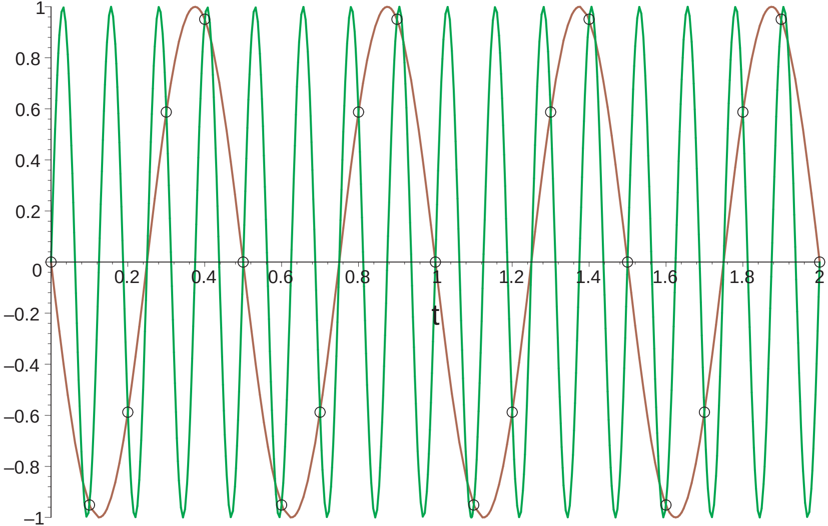
$\sin(2\pi 1 t) + \sin(2\pi 8 t)$



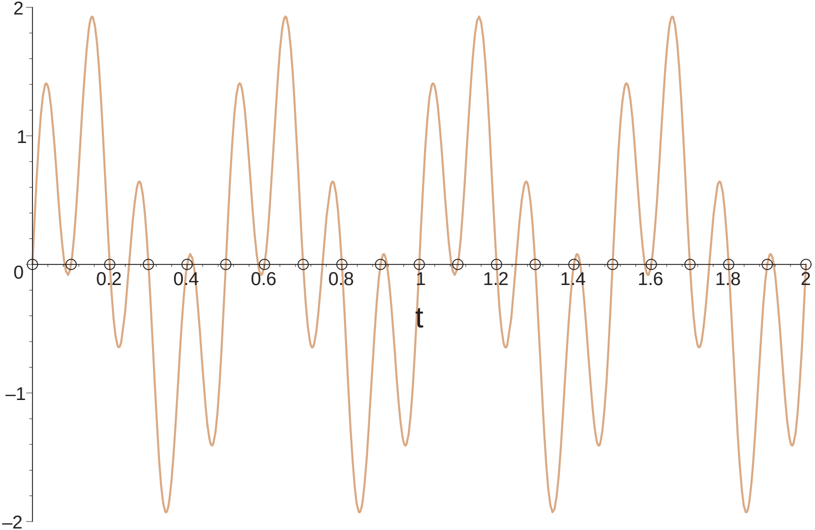
$f_c/2 = 5$ $\sin(2\pi t) + \sin(2\pi 8t)$, resp. $\sin(2\pi 1t) - \sin(2\pi 2t)$



$f_c = 10,$ resp. $f_c/2 = 5$ $\sin(2\pi 8t)$

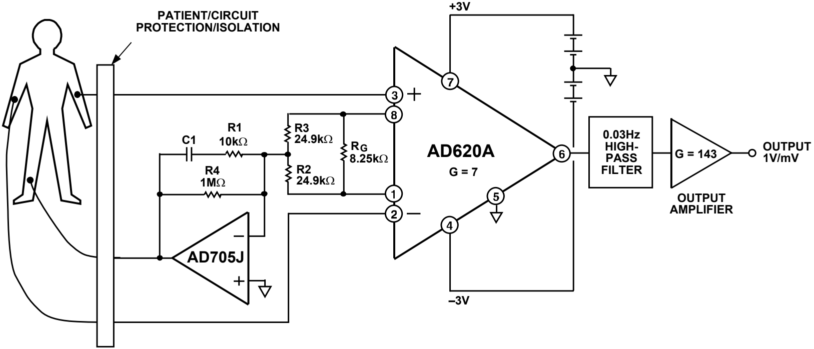


$f_c = 10$, resp. $f_c/2 = 5$ $\sin(2\pi 8t)$, resp. $-\sin(2\pi 2t)$

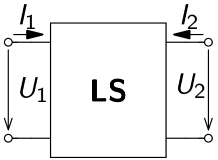


$f_c = 10,$ resp. $f_c/2 = 5$ $\sin(2\pi 2t) + \sin(2\pi 8t)$

Amplifier for Electrocardiograph (ECG) sensing



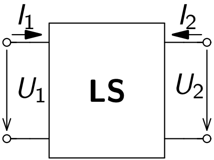
Transfer characteristics of the linear systems



$$U_1(s) = \mathcal{L}\{u_1(t)\}, \quad U_2(s) = \mathcal{L}\{u_2(t)\},$$
$$I_1(s) = \mathcal{L}\{i_1(t)\}, \quad I_2(s) = \mathcal{L}\{i_2(t)\}.$$

$$\begin{bmatrix} I_1 \\ I_2 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} & y_{13} & \cdots & y_{1n} \\ y_{21} & y_{22} & y_{23} & \cdots & y_{2n} \\ y_{31} & y_{32} & y_{33} & \cdots & y_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ y_{n1} & y_{n2} & y_{n3} & \cdots & y_{nn} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ \vdots \\ U_n \end{bmatrix} \equiv \mathbf{I} = \mathbf{Y}\ \mathbf{U}$$

Transfer characteristics of the linear systems



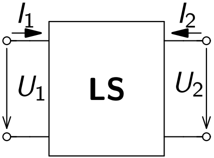
$$U_1(s) = \mathcal{L}\{u_1(t)\}, \quad U_2(s) = \mathcal{L}\{u_2(t)\},$$
$$I_1(s) = \mathcal{L}\{i_1(t)\}, \quad I_2(s) = \mathcal{L}\{i_2(t)\}.$$

$$\mathbf{I} = \mathbf{Y}\ \mathbf{U} \Rightarrow U_k = \frac{1}{|\mathbf{Y}|} \sum_{i=1}^{n} |\mathbf{Y}|_{ik}\ I_i\,,$$

two-port description we get for reduction $k = 1{,}2$, and $i = 1{,}2$:

$$U_1 = \frac{|\mathbf{Y}|_{11}}{|\mathbf{Y}|} I_1 + \frac{|\mathbf{Y}|_{21}}{|\mathbf{Y}|} I_2 = z_{11} I_1 + z_{12} I_2,$$
$$U_2 = \frac{|\mathbf{Y}|_{12}}{|\mathbf{Y}|} I_1 + \frac{|\mathbf{Y}|_{22}}{|\mathbf{Y}|} I_2 = z_{21} I_1 + z_{22} I_2\,.$$

Transfer characteristics of the linear systems



$$U_1(s) = \mathcal{L}\{u_1(t)\}, \quad U_2(s) = \mathcal{L}\{u_2(t)\},$$
$$I_1(s) = \mathcal{L}\{i_1(t)\}, \quad I_2(s) = \mathcal{L}\{i_2(t)\}.$$

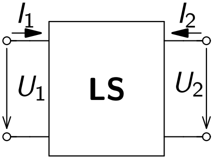
Impedance parameters (z-parameters)

$$\begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$z_{11} = \left.\frac{U_1}{I_1}\right|_{I_2=0}, \quad z_{12} = \left.\frac{U_1}{I_2}\right|_{I_1=0},$$
$$z_{21} = \left.\frac{U_2}{I_1}\right|_{I_2=0}, \quad z_{22} = \left.\frac{U_2}{I_2}\right|_{I_1=0},$$

For reciprocal networks $z_{12} = z_{21}$. For symmetrical networks $z_{11} = z_{22}$. For lossless networks all the z_{mn} are purely imaginary.

Transfer characteristics of the linear systems



$$U_1(s) = \mathcal{L}\{u_1(t)\}, \quad U_2(s) = \mathcal{L}\{u_2(t)\},$$
$$I_1(s) = \mathcal{L}\{i_1(t)\}, \quad I_2(s) = \mathcal{L}\{i_2(t)\}.$$

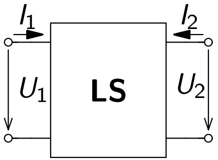
Admittance parameters (y-parameters)

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$$

$$y_{11} = \left.\frac{I_1}{U_1}\right|_{U_2=0}, \; y_{12} = \left.\frac{I_1}{U_2}\right|_{U_1=0},$$
$$y_{21} = \left.\frac{I_2}{U_1}\right|_{U_2=0}, \; y_{22} = \left.\frac{I_2}{U_2}\right|_{U_1=0},$$

For reciprocal networks $y_{12} = y_{21}$. For symmetrical networks $y_{11} = y_{22}$. For lossless networks all the z_{mn} are purely imaginary ($\mathbf{Z} = \mathbf{Y}^{-1}$).

Transfer characteristics of the linear systems



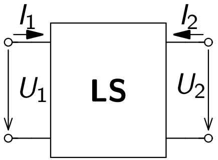
$$U_1(s) = \mathcal{L}\{u_1(t)\}, \quad U_2(s) = \mathcal{L}\{u_2(t)\},$$
$$I_1(s) = \mathcal{L}\{i_1(t)\}, \quad I_2(s) = \mathcal{L}\{i_2(t)\}.$$

Hybrid parameters (h-parameters)

$$\begin{bmatrix} U_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ U_2 \end{bmatrix}$$

$$h_{11} = \left.\frac{U_1}{I_1}\right|_{U_2=0}, \quad h_{12} = \left.\frac{U_1}{U_2}\right|_{I_1=0},$$
$$h_{21} = \left.\frac{I_2}{I_1}\right|_{U_2=0}, \quad h_{22} = \left.\frac{I_2}{U_2}\right|_{I_1=0},$$

Transfer characteristics of the linear systems



$$U_1(s) = \mathcal{L}\{u_1(t)\}, \quad U_2(s) = \mathcal{L}\{u_2(t)\},$$
$$I_1(s) = \mathcal{L}\{i_1(t)\}, \quad I_2(s) = \mathcal{L}\{i_2(t)\}.$$

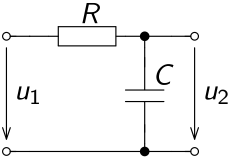
Cascade (chain, or transmission line) parameters (a-parameters)

$$\begin{bmatrix} U_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} U_2 \\ -I_2 \end{bmatrix}$$

$$a_{11} = \left.\frac{U_1}{U_2}\right|_{I_2=0}, \quad a_{12} = \left.\frac{U_1}{-I_2}\right|_{U_2=0},$$
$$a_{21} = \left.\frac{I_1}{U_2}\right|_{I_2=0}, \quad a_{22} = \left.\frac{I_1}{-I_2}\right|_{U_2=0},$$

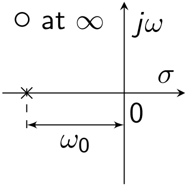
For reciprocal networks $a_{11}a_{22} - a_{12}a_{21} = 1$. For symmetrical networks $a_{11} = a_{22}$. For networks which are reciprocal and lossless, a_{11} and a_{22} are purely real while a_{12} and a_{21} are purely imaginary.

Description and transfer characteristics of the RC network

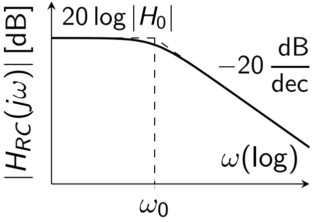


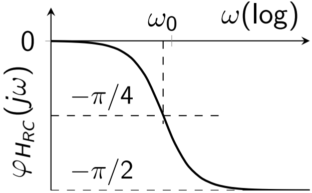
$$H_{RC}(s) = \frac{U_2(s)}{U_1(s)} = \frac{H_0}{1 + sRC}$$

$$H_0 = 1, \quad \omega_0 = 2\pi f_0 = \frac{1}{RC}$$

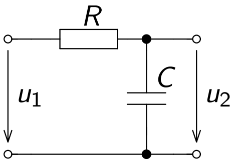


$s = j\omega \;\Rightarrow$



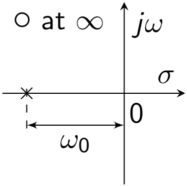


Description and transfer characteristics of the RC network



$$H_{RC}(s) = \frac{U_2(s)}{U_1(s)} = \frac{H_0\omega_0}{s+\omega_0}$$

$$H_0 = 1, \quad \omega_0 = 2\pi f_0 = \frac{1}{RC}$$

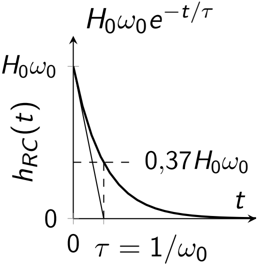


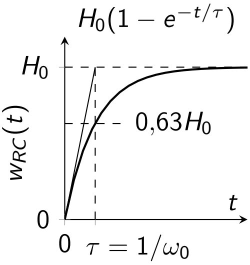
$$h(t) = \mathcal{L}^{-1}\left\{H(s)\cdot 1\right\}$$

$$w(t) = \mathcal{L}^{-1}\left\{H(s)/s\right\}$$

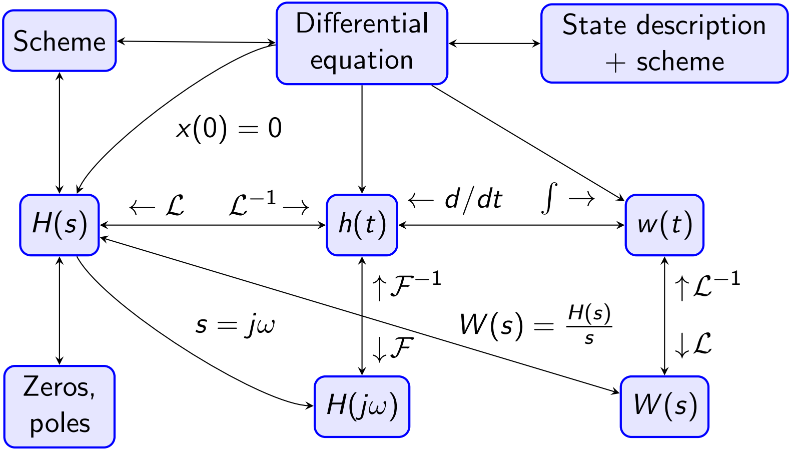
$$h(t) = \frac{dw(t)}{dt}$$

$$w(t) = \int_0^t h(\tau)d\tau.$$





Transfer characteristics of the linear systems



Laplace transform Definition

$$F(s) = \mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}\,dt$$

Properties

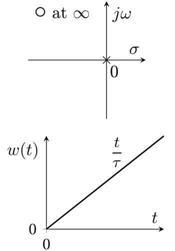
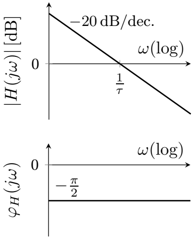
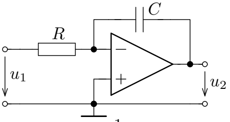
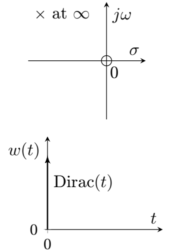
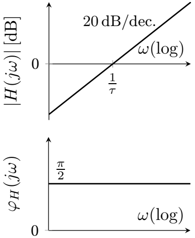
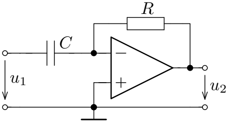
Linearity	$af(t) + bg(t)$	$F(s) + bG(s)$
Differentiation	$\dfrac{df(t)}{dt}$	$sF(s) - f(0^-)$
Integration	$\int_0^t f(\tau)\,d\tau = \mathbf{1}(t) * f(t)$	$\dfrac{1}{s}F(s)$
Convolution	$(f * g)(t) = \int_0^t f(\tau)g(t-\tau)\,d\tau$	$F(s) \cdot G(s)$
Damping	$e^{at}f(t)$	$F(s-a)$
Time scaling	$f(at)$	$1/a\,F(s/a)$
Time shifting	$f(t-a)$	$e^{-as}F(s)$

Initial value	$f(0^+) = \lim\limits_{s\to\infty} sF(s)$
Final value	$f(\infty) = \lim\limits_{s\to 0} sF(s);\quad \lim\limits_{s\to 0} s\dfrac{1}{s}H(s) = H(0) = \text{DCgain}$

Table of selected Laplace transforms

Function	Time domain	Laplace s-domain
Unit impulse	$\delta(t)$	1
Unit step	$\mathbf{1}(t)$	$\frac{1}{s}$
Delayed unit step	$\mathbf{1}(t-\tau)$	$e^{-\tau s}\dfrac{1}{s}$
ramp	$t \cdot \mathbf{1}(t)$	$\dfrac{1}{s^2}$
exponential decay	$e^{-\alpha t} \cdot \mathbf{1}(t)$	$\dfrac{1}{s+\alpha}$
exponential approach	$(1-e^{-\alpha t}) \cdot \mathbf{1}(t)$	$\dfrac{\alpha}{s(s+\alpha)} = \dfrac{1}{s} - \dfrac{\alpha}{s+\alpha}$
sine	$\sin(\omega t) \cdot \mathbf{1}(t)$	$\dfrac{\omega}{s^2+\omega^2}$
cosine	$\cos(\omega t) \cdot \mathbf{1}(t)$	$\dfrac{s}{s^2+\omega^2}$

see more, e.g. http://en.wikipedia.org/wiki/Laplace_transform

Transfer function	Poles, Zeros, Unit step response	Module and phase frequency response	Passive and active circuit realization
Ideal integrator $H(s) = \frac{1}{s\tau}$			Realization by passive circuit does not exist. Inverting integrator  $H(s) = -\frac{1}{s\tau}$, $\tau = \frac{1}{RC}$
Ideal derivator $H(s) = s\tau$			Realization by passive circuit does not exist. Inverting derivator  $H(s) = -s\tau$, $\tau = \frac{1}{RC}$

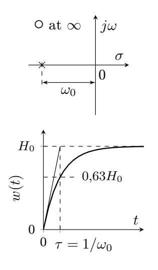
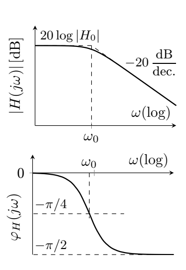
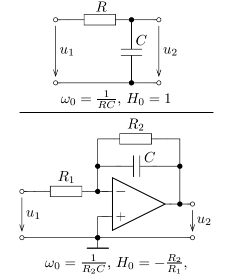
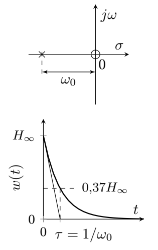
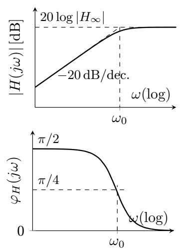
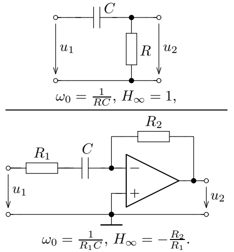
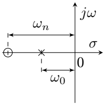
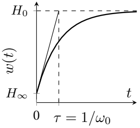
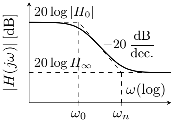
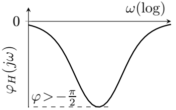
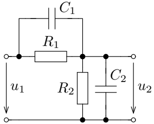
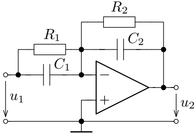
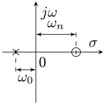
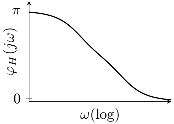
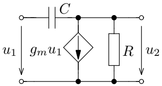
Transfer function	Poles, Zeros, Unit step response	Module and phase frequency response	Passive and active circuit realization
Low pass (LP_1) $H(s) = \dfrac{H_0 \omega_0}{s + \omega_0}$ $\omega_0 > 0,\ H_\infty = 0$			
High pass (HP_1) $H(s) = \dfrac{H_\infty s}{s + \omega_0}$ $\omega_0 > 0,\ H_0 = 0$			

Table: First order filter sections

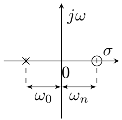
Dynamic System Description

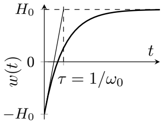
Transfer function	Poles, Zeros, Unit step response	Module and phase frequency response	Passive and active circuit realization
$H(s) = H_\infty \frac{s+\omega_n}{s+\omega_0}$ $\omega_0 > 0,\ \omega_n > 0$	  $H_0 = H_\infty \frac{\omega_n}{\omega_0}$	 	 $\omega_0 = \frac{1}{(R_1\|\|R_2)(C_1+C_2)}$, $\omega_n = \frac{1}{R_1C_1}$, $H_0 = \frac{R_2}{R_1+R_2}$, $H_\infty = \frac{C_1}{C_1+C_2}$,  $\omega_0 = \frac{1}{R_2C_2}$, $\omega_n = \frac{1}{R_1C_1}$, $H_0 = -\frac{R_2}{R_1}$ $H_\infty = -\frac{C_1}{C_2}$.
$H(s) = H_\infty \frac{s-\omega_n}{s+\omega_0}$ $\omega_0 > 0,\ \omega_n > 0$	 step resp. is the same $H_0 = -H_\infty \frac{\omega_n}{\omega_0}$	module resp. is the same 	 $\omega_0 = \frac{1}{RC}$, $\omega_n = \frac{g_m}{C}$, $H_0 = -\frac{g_m}{R}$, $H_\infty = 1$.

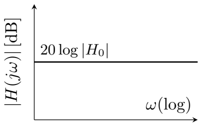
Dynamic System Description

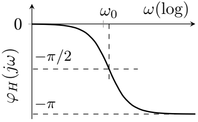
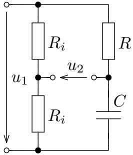
<table>
<tr><th>Transfer function</th><th>Poles, Zeros,
Unit step response</th><th>Module and phase
frequency response</th><th>Passive and active
circuit realization</th></tr>
<tr>
<td>All pass (AP_1)

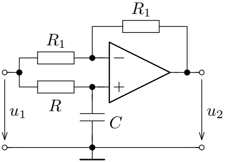
$H(s) = H_\infty \frac{s-\omega_0}{s+\omega_0}$

$\omega_0 > 0$, $\omega_n > 0$</td>
<td>



$H_0 = -H_\infty$</td>
<td>

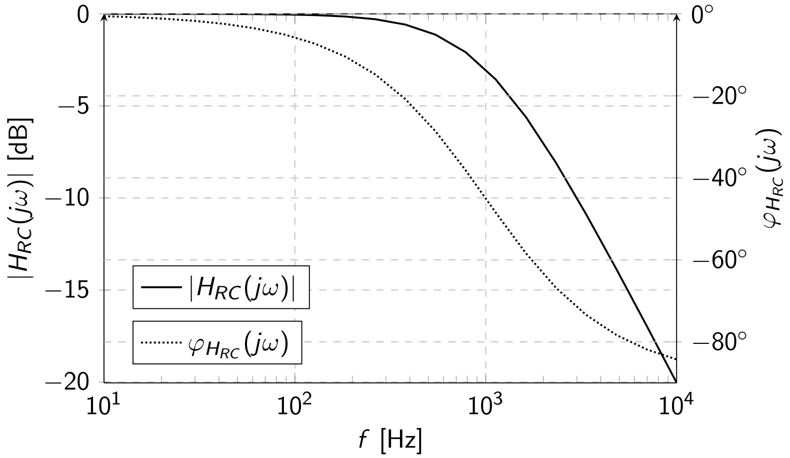

</td>
<td>

$\omega_0 = \frac{1}{RC}$, $H_0 = 1/2$,


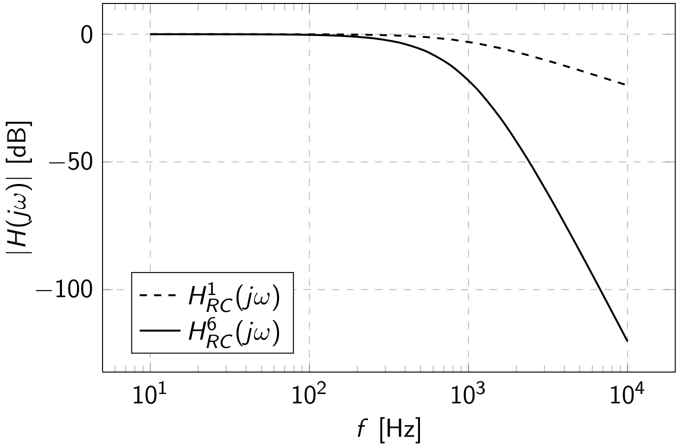
$\omega_0 = \frac{1}{RC}$, $H_0 = 1$.</td>
</tr>
</table>

Table: First-order all-pass section.

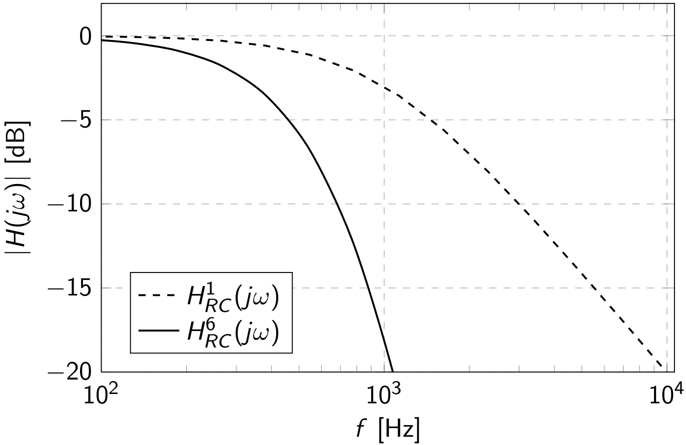
Frequency response of the one RC network, $f_0 = 10\,\mathrm{kHz}$



Module frequency response of the one and six RC networks



Module frequency response of the one and six RC networks

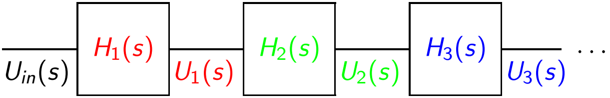


Cascade structure of the filter – cascading transfer function

$$H(s) = \frac{s^m + a_{m-1}s^{m-1} + \cdots + a_0}{b_n s^n + b_{n-1}s^{n-1} + \cdots + b_0} = H_\infty \frac{(s - s_{n_1})(s - s_{n_2})\cdots(s - s_{n_m})}{(s - s_{p_1})(s - s_{p_2})\cdots(s - s_{p_m})},$$

where $\quad s_{p_i} = \sigma_{p_i} \pm j\omega_{p_i}$, and $\omega_{0_i} = |s_{p_i}| = \sqrt{\omega_{p_i}^2 + \sigma_{p_i}^2}, \quad Q_i = \frac{1}{2}\frac{\omega_{0_i}}{|\sigma_{p_i}|} = \frac{1}{2\xi},$

$$\text{general } 2^{\text{nd}} \text{ order TF} \quad H_b(s) = H_\infty \frac{(s - \omega_n)^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}.$$



transfer function of the block k: $H_k(s) = \dfrac{U_k(s)}{U_{k-1}(s)}$,

tranfer function after the block k: $H_{1\to k}(s) = \dfrac{U_k(s)}{U_{in}(s)} = \prod_{n=1}^{k} H_n(s)$.

Time response of the 2nd order (biquadratic) transfer function – biquad

$$H(s) = \frac{C(s - s_{n_1})}{(s - s_{p_1})(s - s_{p_2})} \overset{\mathcal{L}^{-1}}{\Rightarrow} h(t) = C_1 \mathrm{e}^{s_{p_1} t} + C_2 \mathrm{e}^{s_{p_2} t},$$

$$H(s) = \frac{C}{(s - s_{p_1})(s - s_{p_2})} \overset{\mathcal{L}^{-1}}{\Rightarrow} h(t) = \Big|_{s_{p12} = \sigma_p \pm j\omega_p} \frac{C}{\omega_p} \mathrm{e}^{\sigma_p t} \sin(t\omega_p),$$

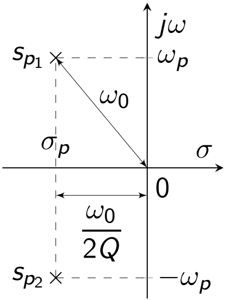
$$H_{LP}(s) = \frac{H_0 \omega_0^2}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2} \quad \forall\ \sigma_p < 0$$

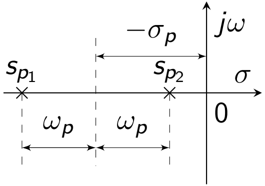
$$\text{where poles are} \quad s_{p12} = \frac{\left(-1 \pm \sqrt{1 - 4Q^2}\right)\omega_0}{2Q},$$

where $Q = \frac{1}{2\xi}$ – quality factor of the biquad and
$\xi = \frac{1}{2Q}$ – damping factor.

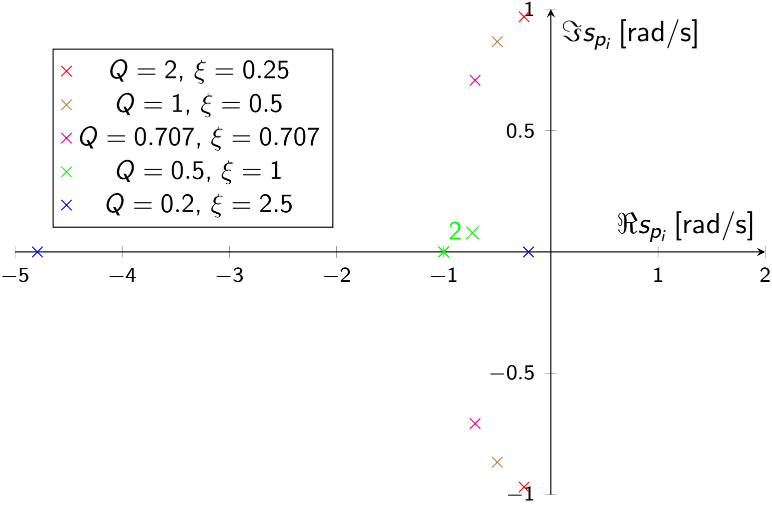
$$H_{LP}(s) = \frac{H_0 \omega_0^2}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2} \; \forall \, \sigma_p < 0, \quad \text{where} \quad s_{p_{12}} = \frac{\left(-1 \pm \sqrt{1 - 4Q^2}\right) \omega_0}{2Q},$$

$Q < 1/2$, $\xi > 1$ two real poles,
$Q = 1/2$, $\xi = 1$ double real pole,
$Q > 1/2$, $\xi < 1$ complex conjugate poles, where special case
$Q = \xi = 1/\sqrt{2}$ – complex conjugate poles with $\sigma_p = \omega_p$.

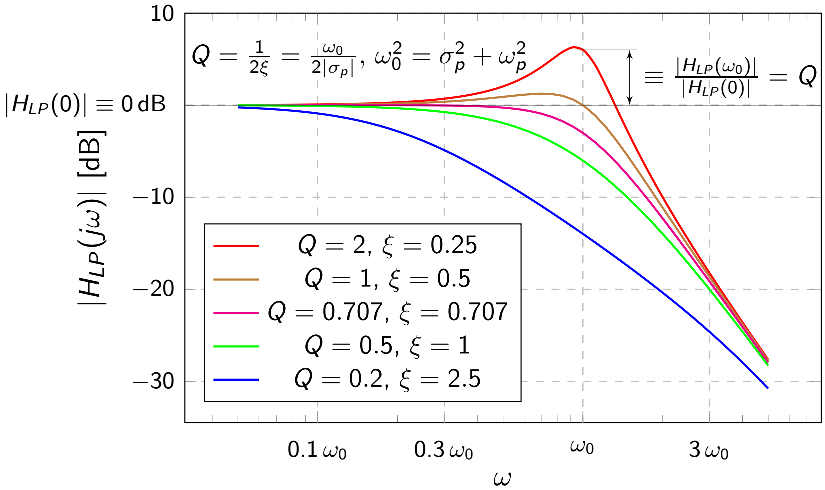




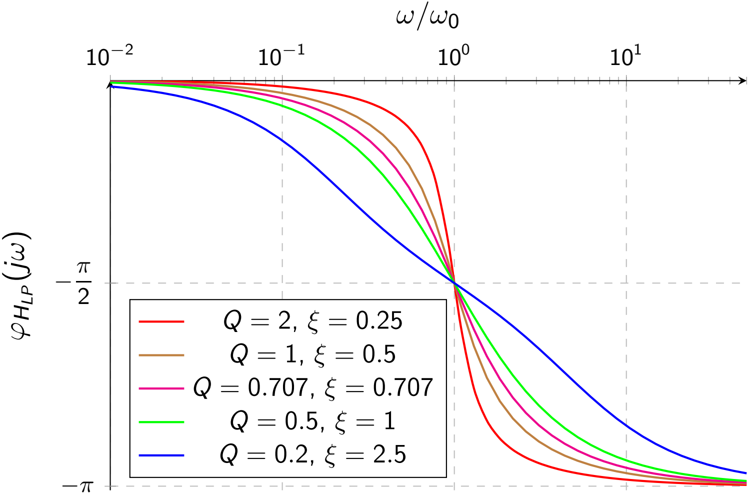
Poles location of the biquad for $\omega_0 = 1$



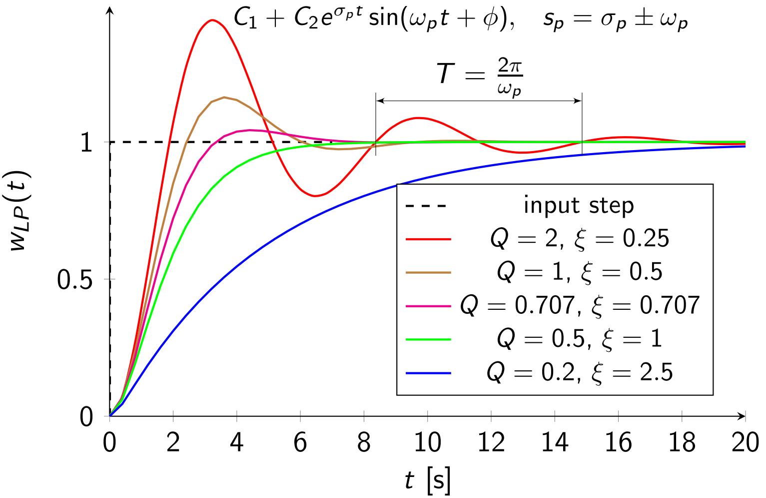
Magnitude frequency responses of the biquad for
$H_0 = H_{LP}(0) = 1$

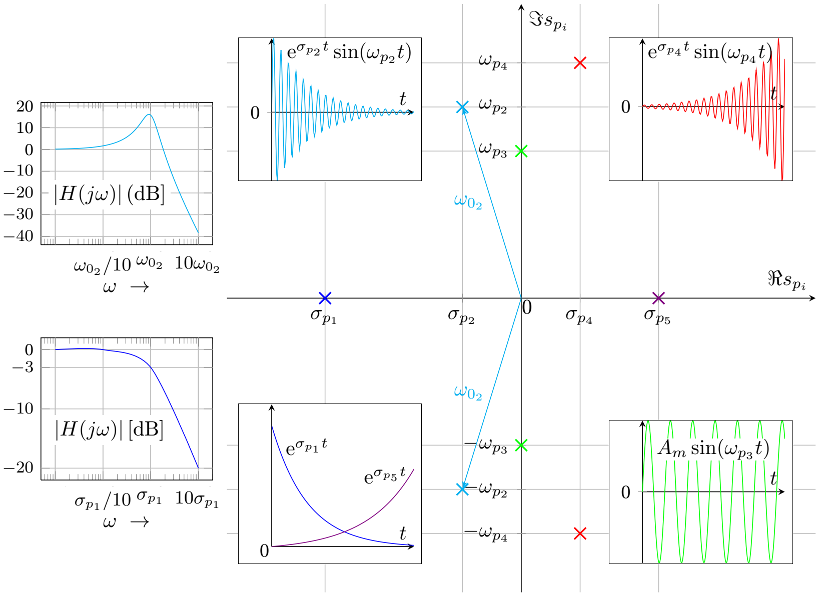


Phase frequency responses of the biquad



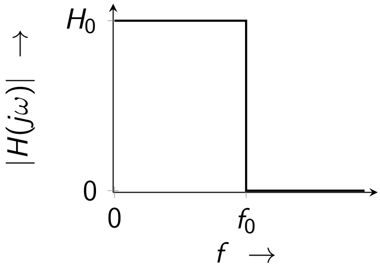
Time (unit step) responses of the biquad for $H_0 = 1$ and $\omega_0 = 1$

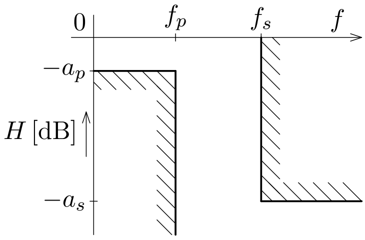




$\Im s_{p_i}$
$\Re s_{p_i}$
ω_{p_4}
ω_{p_2}
ω_{p_3}
$-\omega_{p_3}$
$-\omega_{p_2}$
$-\omega_{p_4}$
ω_{0_2}
ω_{0_2}
σ_{p_1}
σ_{p_2}
0
σ_{p_4}
σ_{p_5}
$e^{\sigma_{p_2} t} \sin(\omega_{p_2} t)$
$e^{\sigma_{p_4} t} \sin(\omega_{p_4} t)$
$e^{\sigma_{p_1} t}$
$e^{\sigma_{p_5} t}$
$A_m \sin(\omega_{p_3} t)$
t
0
20
10
0
-10
-20
-30
-40
$|H(j\omega)|$ (dB]
$\omega_{0_2}/10$ ω_{0_2} $10\omega_{0_2}$
ω $\rightarrow$
0
-3
-10
-20
$|H(j\omega)|$ [dB]
$\sigma_{p_1}/10$ σ_{p_1} $10\sigma_{p_1}$
ω $\rightarrow$

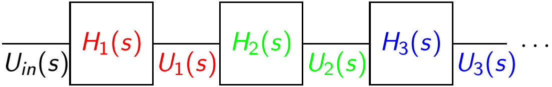
Magnitude Specification of the Ideal and Real Lowpass (LP) Filter





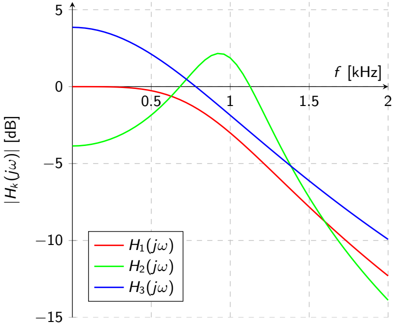
Ideal LP filter $(f_p = f_s = f_0)$
passband $\equiv \langle 0, f_0 \rangle$, $|H| = H_0$,
transition band $\equiv 0$,
stopband $\equiv \langle f_0, \infty)$, $|H| = 0$

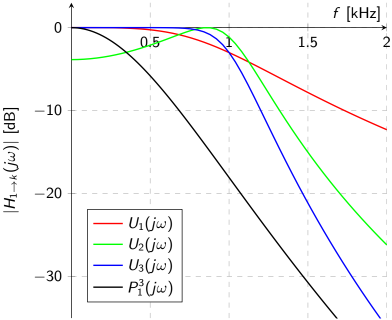
Real LP filter
passband $\equiv \langle 0, f_p \rangle$, $|H| = \langle H_0, H_0 - a_p \rangle$,
transition band $\equiv (f_p, f_s)$,
stopband $\equiv \langle f_s, \infty)$, $|H| < H_0 - a_s$



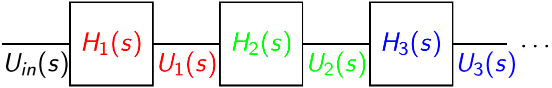
Butterworth filter

$$f_{01} = 1\,\text{kHz}, \quad f_{02} = 1\,\text{kHz}, \quad f_{03} = 1\,\text{kHz},$$
$$Q_1 = 0.71, \quad Q_2 = 1.9, \quad Q_3 = 0.52.$$



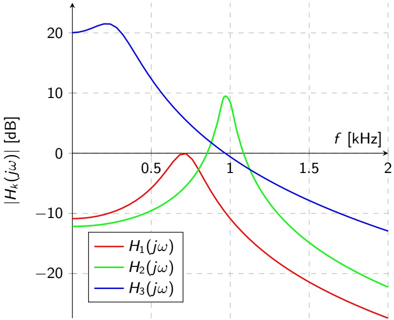


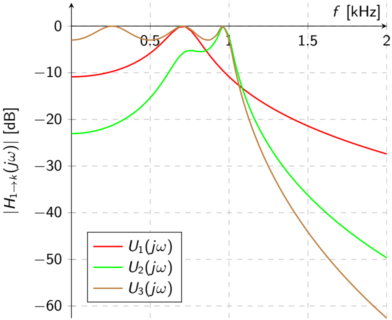
Frequency Filters

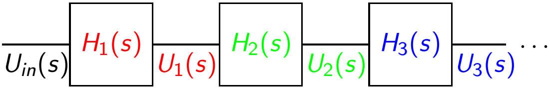


Chebyshev filter

$f_{01} = 0.7\,\text{kHz}$, $Q_1 = 3.5$, $f_{02} = 0.96\,\text{kHz}$, $Q_2 = 13$, $f_{03} = 0.3\,\text{kHz}$, $Q_3 = 1$.

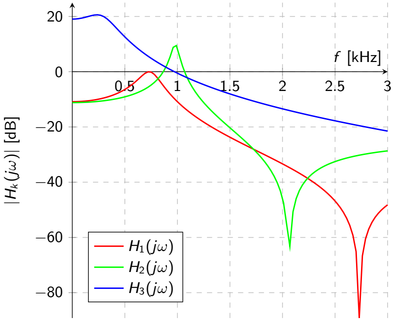


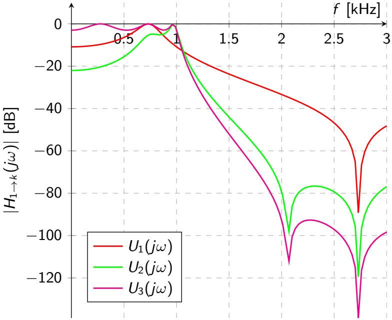




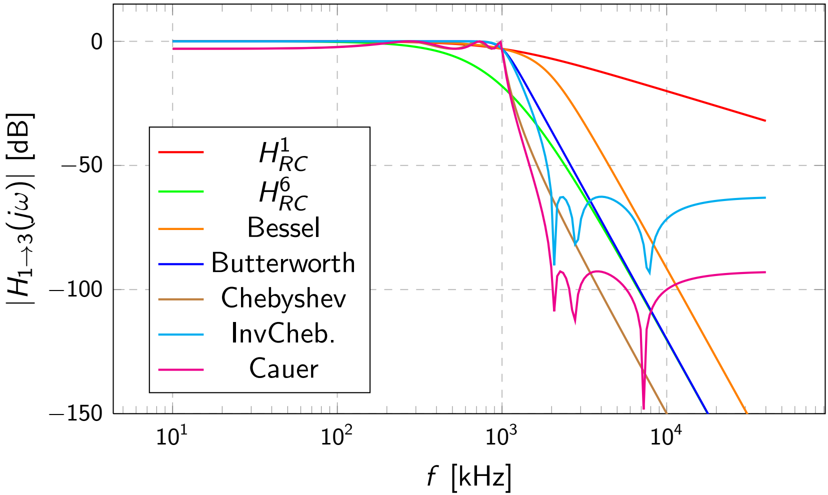
Cauer filter

$$f_{01} = 0.77\,\text{kHz}, \quad f_{02} = 0.99\,\text{kHz}, \quad f_{03} = 0.32\,\text{kHz},$$
$$f_{n1} = 2.7\,\text{kHz}, \quad f_{n2} = 2.1\,\text{kHz}, \quad f_{03} = 7\,\text{kHz},$$
$$Q_1 = 3.7, \quad Q_2 = 15, \quad Q_3 = 1.1.$$

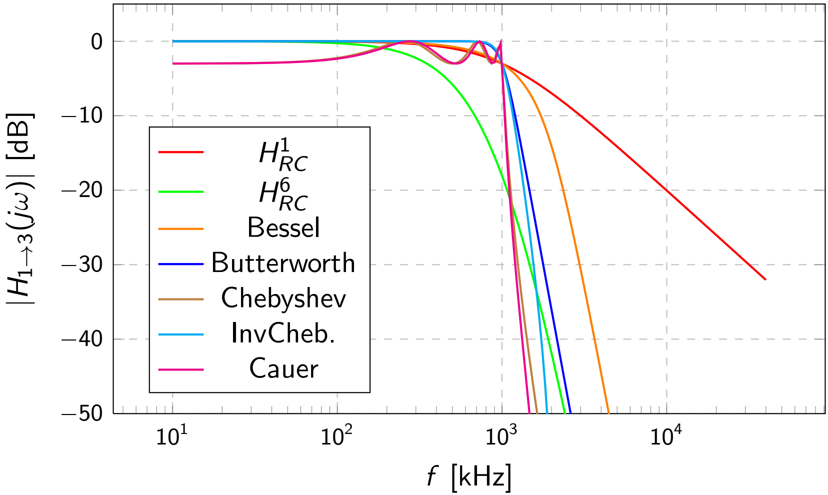




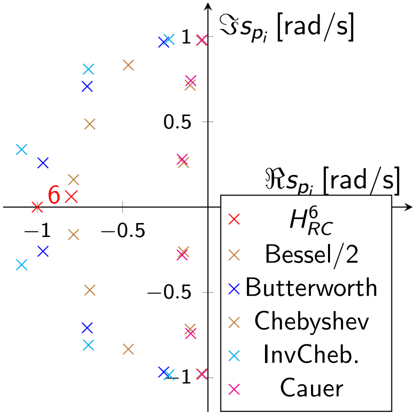
Magnitude frequency responses of the filters

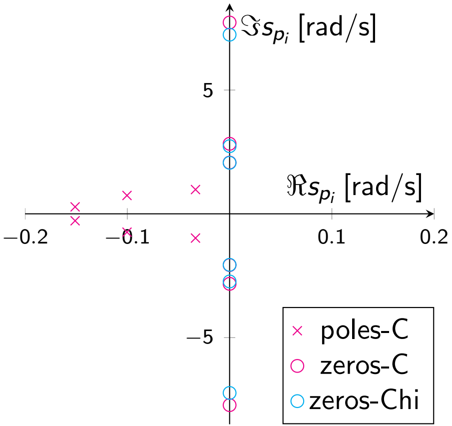


Magnitude frequency responses of the filters

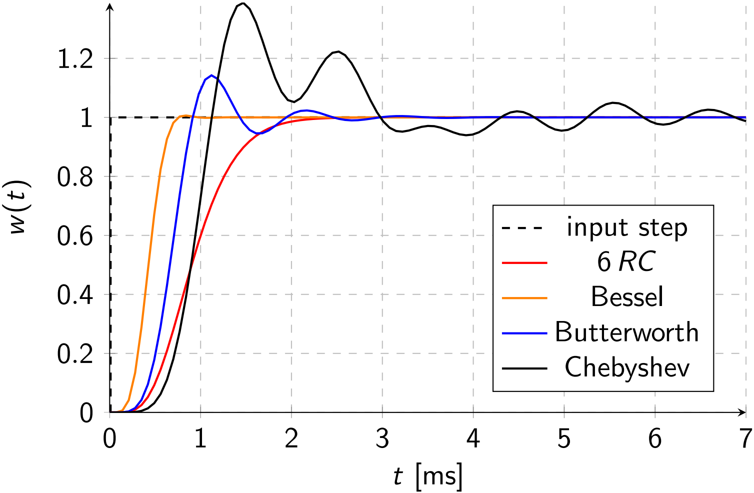


Poles of the filters, poles and zeros of the Cauer and Inverse Chebyshev filter in p-plane for normalized frequency $\omega_0 = 1$.

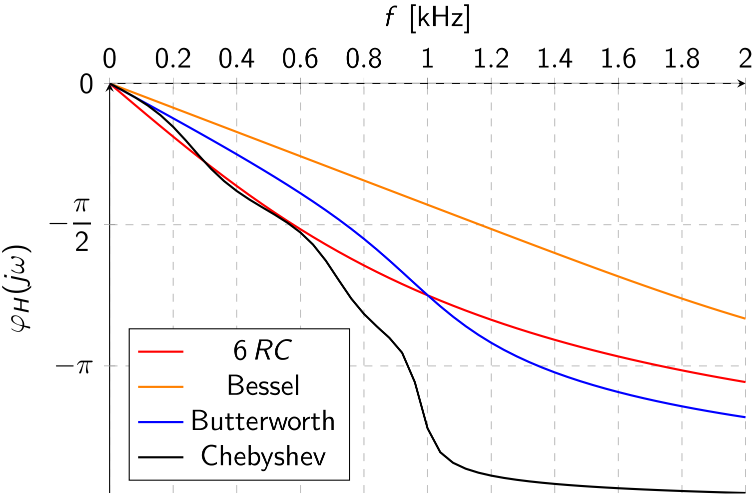




Time (unit step) responses of the filters (Phase- and Amplitude errors cause linear distortions)



Phase frequency responses of the filters



Why do we need a good time response?

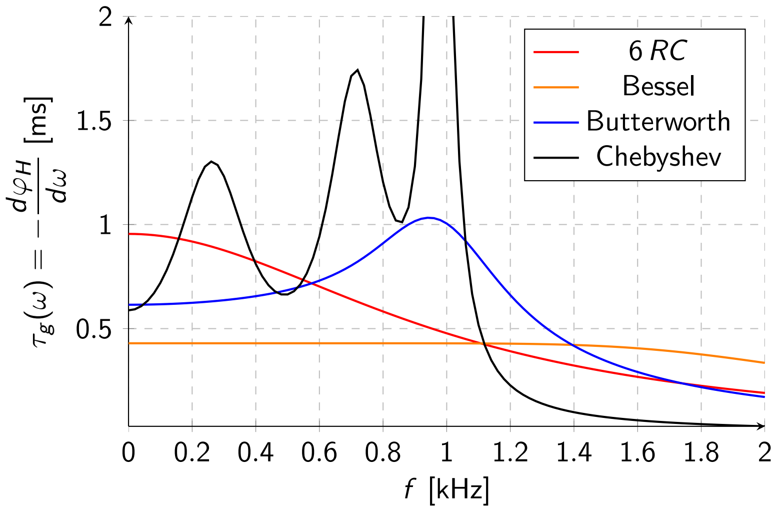
Filter for DC value of rectifier signal, . . .

Why the phase frequency responses should be linear (not constant)?

Phase values of sinusoidal waveform for delay e.g. $\tau = 1/2\,\mathrm{ms}$.

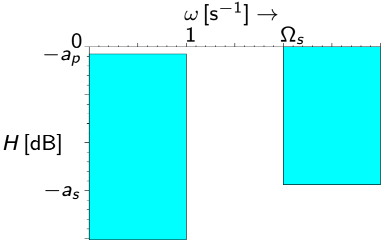
f [kHz]	T [ms]	$\tau = 1/2\,\mathrm{ms}$
1	1	$T/2 \equiv 1\pi$
2	$1/2$	$1T \equiv 2\pi$
6	$1/6$	$3T \equiv 6\pi$

Group delays of the filters



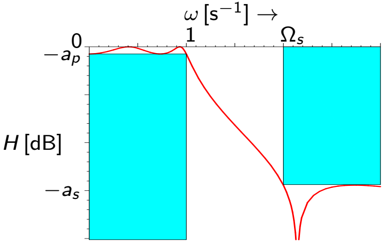
Filter Design Steps

Frequency transformations, normalization, solving of approximation task, electric filter synthesis, analysis resulted structures.



Filter Design Steps

Frequency transformations, normalization, solving of approximation task, electric filter synthesis, analysis resulted structures.

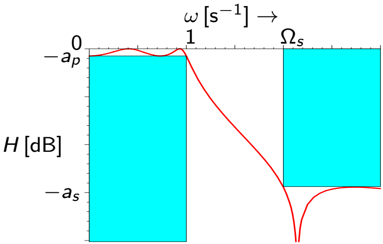


$\Rightarrow$
$\Leftarrow$

$$H(s) = \frac{s^m + a_{m-1}s^{m-1} + \cdots + a_0}{b_n s^n + b_{n-1}s^{n-1} + \cdots + b_0}$$

Filter Design Steps

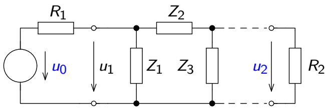
Frequency transformations, normalization, solving of approximation task, electric filter synthesis, analysis resulted structures.



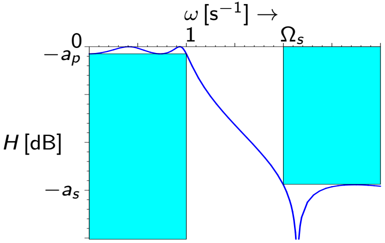
$\Rightarrow$

$\Leftarrow$

$$H(s) = \frac{s^m + a_{m-1}s^{m-1} + \cdots + a_0}{b_n s^n + b_{n-1}s^{n-1} + \cdots + b_0}$$

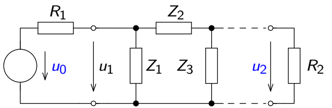


Frequency transformations, normalization, solving of approximation task, electric filter synthesis, analysis resulted structures.



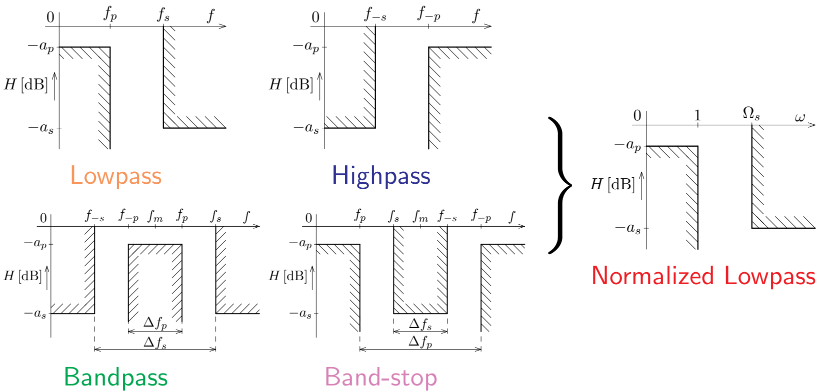
$\Rightarrow$ $\Leftarrow$

$$H(s) = \frac{s^m + a_{m-1}s^{m-1} + \cdots + a_0}{b_n s^n + b_{n-1}s^{n-1} + \cdots + b_0}$$



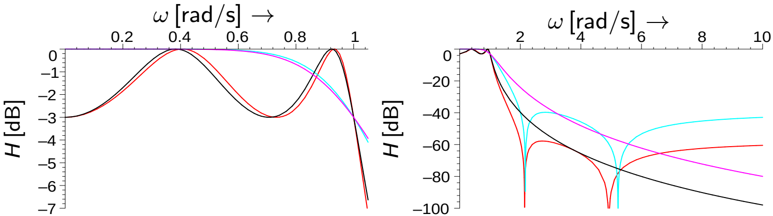
$\Leftarrow$

$$H(s) = \frac{U_2(s)}{U_0(s)} \left(\sqrt{\frac{4R_1}{R_2}}\right) = \sqrt{\frac{P_2}{P_{max}}}$$

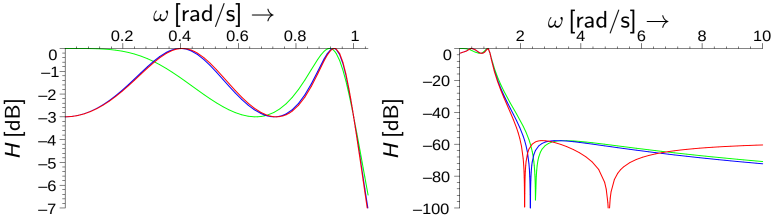


Filter Characteristic for Lowpass (LP), Highpass (HP), Bandpass (BP), Band-rejection (or Band-stop – BS) and Normalized Lowpass (NLP).

Magnitude Approximation

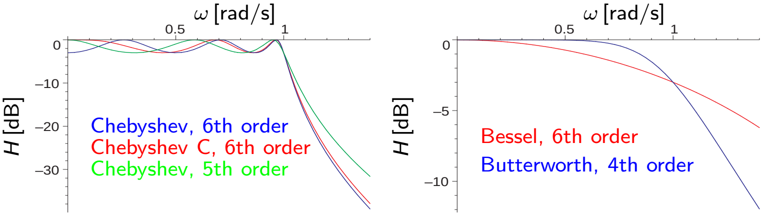


Butterworth, Chebyshev, Inverse Chebyshev A type and Cauer (A),

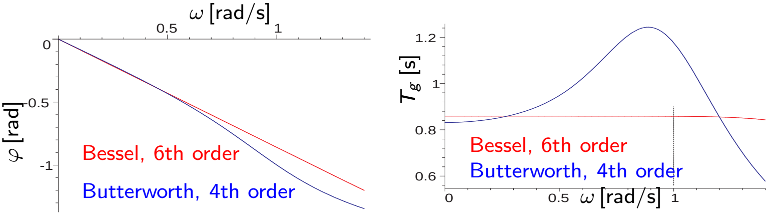


Cauer A, B and C type approximation of NLP filter magnitude

Magnitude Approximation



Comparison of Chebyshev A vs C type and Butterworth vs Bessel approximation



Comparison of phase and group delay characteristic of Butterworth and Bessel approximation

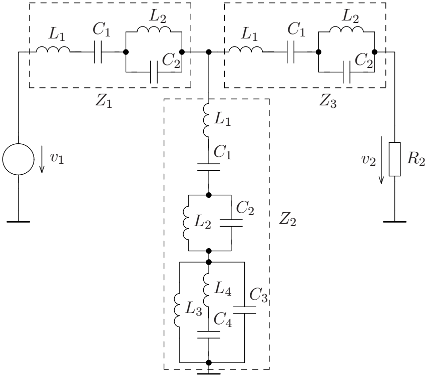
Results of Approximation Task

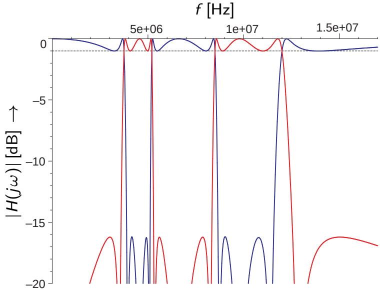
- secondary parameters of the filter
  (order, new stopband value, ...)
- zeros and poles of gain and characteristic function
- gain and characteristic function
- partial gain functions (parameters of biquads – ω_0, Q, $[\omega_n]$)
  appropriate for cascade filter synthesis

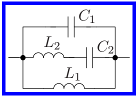
Analog filters realization

1. Passive filters
   - Electromechanical filters, $10^1\,\mathrm{kHz} < f < 10^2\,\mathrm{kHz}$,
   - LC filters, $10^0\,\mathrm{kHz} < f < 1\,\mathrm{GHz}$,
   - Crystal (piezoelectric) filters, $10^1\,\mathrm{kHz} < f < 1\,\mathrm{GHz}$,
   - Surface Acoustic Wave (SAW) filters, $10\,\mathrm{MHz} < f < 2\,\mathrm{GHz}$,
   - Microwave filter (cavity filters), $f > 1\,\mathrm{GHz}$.
2. Active building blocks using
   - Op Amps – ARC filters,
   - OTA – transkonductance apmlifiers filters,
   - Current conveyors (current-mode filters),
   - Log-domain and Square-root domain filters,
   - Disrete time filters using SC or SI technique.

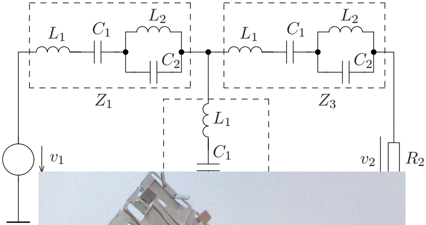
LC Filters

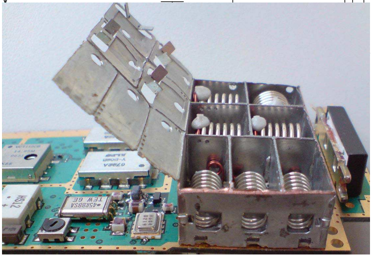

L_1
C_1
L_2
C_2
Z_1
Z_3
Z_2
L_3
L_4
C_3
C_4
v_1
v_2
R_2

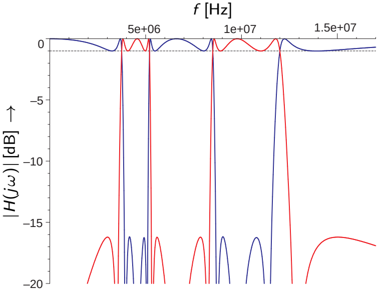

f [Hz]
5e+06
1e+07
1.5e+07
0
−5
−10
−15
−20
$|H(j\omega)|$ [dB] →

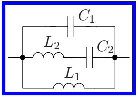

C_1
L_2
C_2
L_1

LC Filters

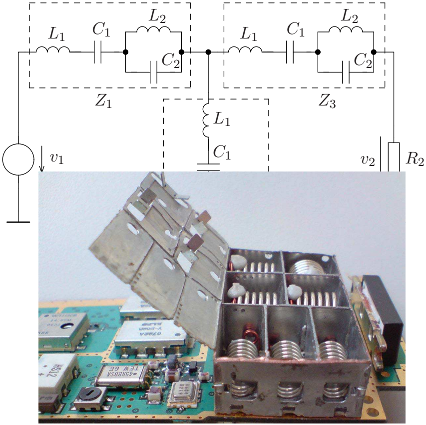

L_1
C_1
L_2
C_2
Z_1
Z_3
v_1
v_2
R_2

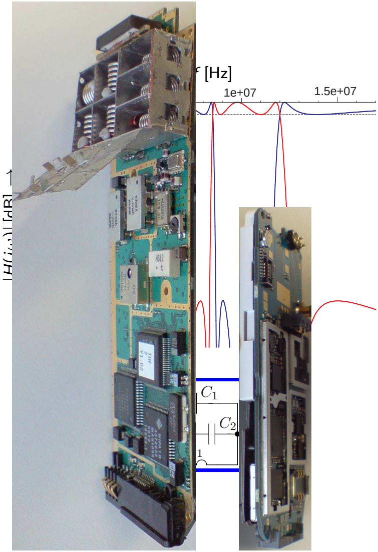



f [Hz]
5e+06
1e+07
1.5e+07
0
−5
−10
−15
−20
$|H(j\omega)|$ [dB] →


C_1
L_2
C_2
L_1

LC Filters

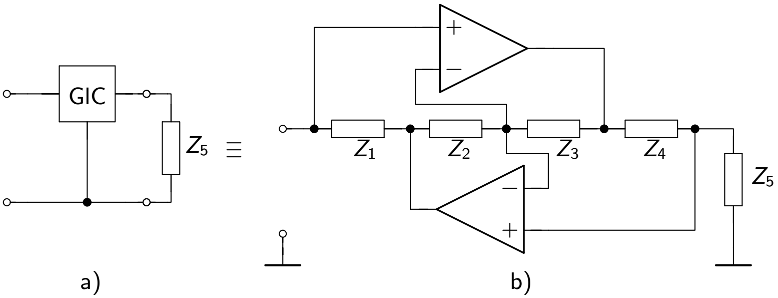




ARC filters: very good dynamic range at low frequencies, tuning is problem.

- LC ladder simulation by signal-flow graphs or by element substitution (synthetic inductors, Bruton's transformation),
- Cascade synthesis (forming biquadratic sections – biquads),
- Direct synthesis.

Impedance converter and its realization



$$Z_{vst}(s) = k(s)Z_5 = \frac{Z_1 Z_3 Z_5}{Z_2 Z_4}$$

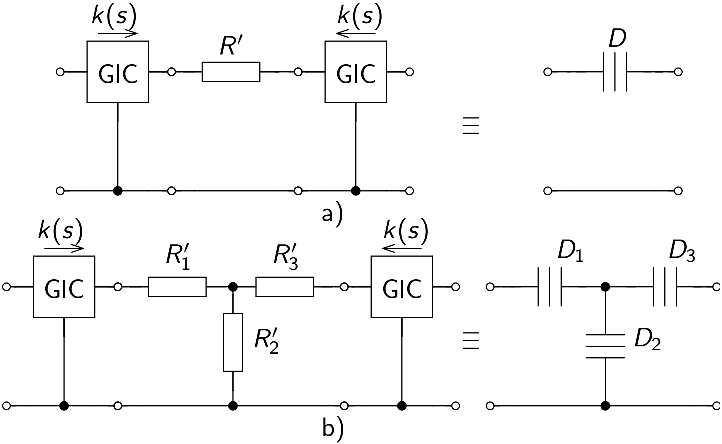
Inductor realization $Z_4 = 1/(sC)$ and $Z_1 = Z_2 = Z_3 = Z_5 = R$, then

$$Z_{vst}(p) = s\,CR^2$$

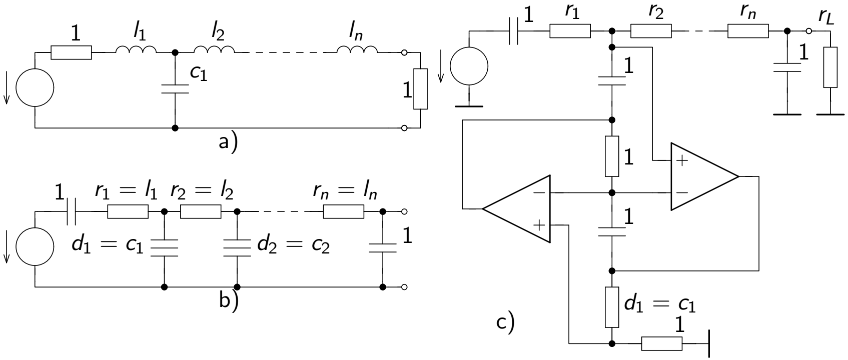
Double capacitor realization $Z_1 = Z_3 = 1/(sC)$ and $Z_1 = Z_2 = Z_4 = Z_5 = R$, then

$$Z_{vst}(s) = \frac{1}{s^2 C^2 R}$$

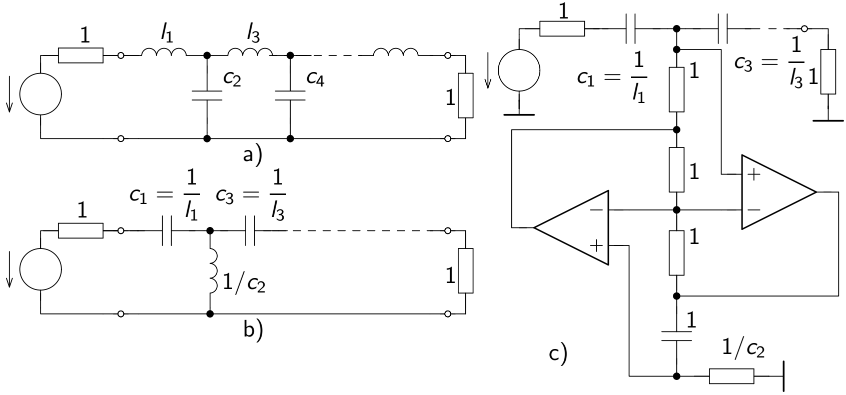
Circuit realization of no-grounded double capacitor

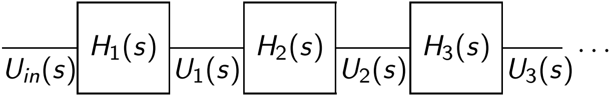


Circuit realization of LP filter



Circuit realization of HP filter



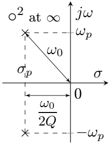
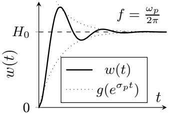
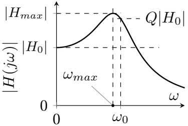
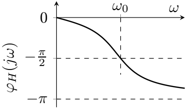
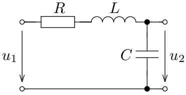
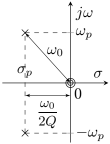
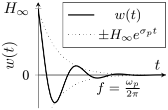
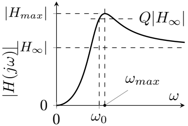
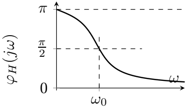
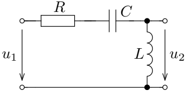


transfer function of the block k: $H_k(s) = \dfrac{U_k(s)}{U_{k-1}(s)}$,

tranfer function after the block k: $H_{1\to k}(s) = \dfrac{U_k(s)}{U_{in}(s)} = \displaystyle\prod_{n=1}^{k} H_n(s)$.

Figure: Cascade structure of a filter.

Cascade Filters

Transfer function	Poles, Zeros, Unit step response	Module and phase frequency response	Circuit realization
Low pass (LP_2) $H(s) =$ $= \dfrac{H_0\omega_0^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$ $Q > 0,\ \omega_0 > 0,$ $H_\infty = 0$	 	 	 $H_0 = 1,$ $\omega_0 = \frac{1}{\sqrt{LC}},\ Q = \omega_0\frac{L}{R}$ active realization ⇒ see below
High pass (HP_2) $H(s) =$ $= \dfrac{H_\infty s^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$ $Q > 0,\ \omega_0 > 0,$ $H_0 = 0$	 	 	 $H_\infty = 1,$ $\omega_0 = \frac{1}{\sqrt{LC}},\ Q = \omega_0\frac{L}{R}$ active realization ⇒ see below

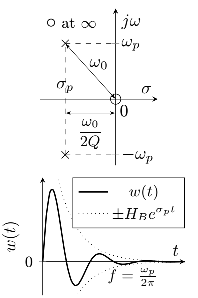
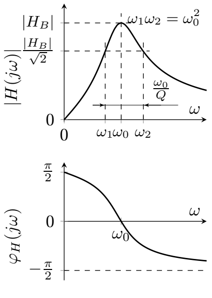
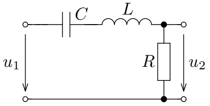
Transfer function	Poles, Zeros, Unit step response	Module and phase frequency response	Circuit realization
Band pass (BP_2) $H(s) =$ $= \frac{H_B \frac{\omega_0}{Q} s}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}$ $Q > 0,\ \omega_0 > 0,$ $H_0 = H_\infty = 0$			$H_B = 1,$ $\omega_0 = \frac{1}{\sqrt{LC}},\ Q = \omega_0 \frac{L}{R}$ active realization $\Rightarrow$ see below

Table: Second-order filter section.

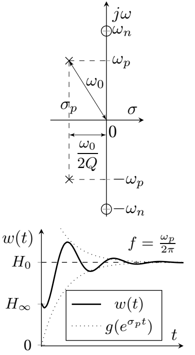
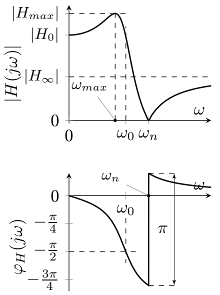
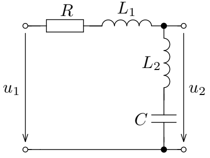
Transfer function	Poles, Zeros, Unit step response	Module and phase frequency response	Circuit realization
Low pass notch (LPN) $H(s) =$ $= \dfrac{H_0 \frac{\omega_0^2}{\omega_n^2}(s^2+\omega_n^2)}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$ $\omega_n > \omega_0 > 0,$ $Q > 0$			 $H_0 = 1,\ H_\infty = \frac{L_2}{L_1+L_2},$ $\omega_0 = \frac{1}{\sqrt{(L_1+L_2)C}},$ $\omega_n = \frac{1}{\sqrt{L_2C}},$ $Q = \omega_0 \frac{(L_1+L_2)}{R}$ active realization $\Rightarrow$ see below

Table: Second-order eliptic filter section.

Cascade Filters

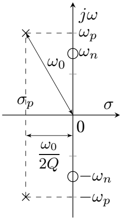
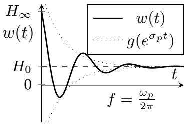
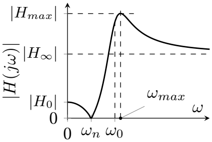
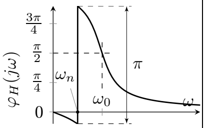
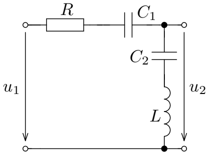
Transfer function	Poles, Zeros, Unit step response	Module and phase frequency response	Circuit realization
High pass notch (HPN) $H(s) = $ $= \dfrac{H_\infty (s^2 + \omega_n^2)}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}$ $\omega_0 > \omega_n > 0$, $Q > 0$	 	 	 $H_\infty = 1,\ H_0 = \frac{C_1}{C_1 + C_2}$, $\omega_0 = \sqrt{\frac{C_1 + C_2}{C_1 C_2 L}}$, $\omega_n = \frac{1}{\sqrt{L C_2}},\ Q = \omega_0 \frac{L}{R}$ active realization $\Rightarrow$ see below

Table: Second-order eliptic filter section.

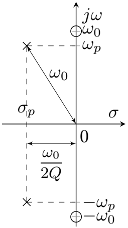
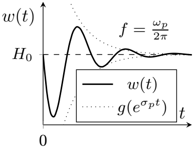
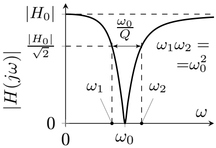
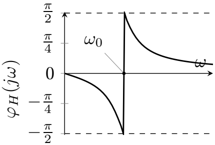
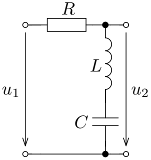
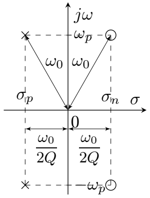
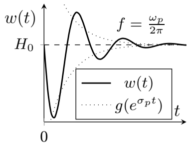
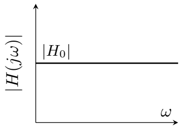
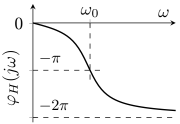
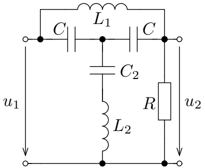
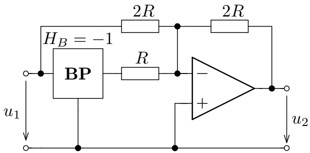
Transfer function	Poles, Zeros, Unit step response	Module and phase frequency response	Circuit realization
Notch (N) $H(s) =$ $= \dfrac{H_0(s^2+\omega_0^2)}{s^2+\frac{\omega_0}{Q}s+\omega_0^2}$ $\omega_0 > 0,\ Q > 0$ $H_0 = H_\infty$			$H_0 = H_\infty = 1$, $\omega_0 = \frac{1}{\sqrt{LC}}$, $Q = \omega_0 \frac{L}{R}$ active realization $\Rightarrow$ see below

Table: Second-order eliptic filter section.

Transfer function	Poles, Zeros, Unit step response	Module and phase frequency response	Circuit realization
All pass (AP_2) $H(s) =$ $= H_0 \dfrac{s^2 - \frac{\omega_0}{Q}s + \omega_0^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$ $\omega_0 > 0,\ Q > 0$ $H_0 = H_\infty$	 	 	 $H_0 = H_\infty = 1,$ $C = \frac{Q}{\omega_0 R},\ C_2 = \frac{2Q}{\omega_0 R(Q^2-1)},$ $L_1 = \frac{2R}{\omega_0 Q},\ L_2 = \frac{QR}{2\omega_0}$ active realization $\Rightarrow$ see below

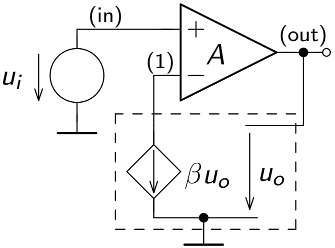


$$H(s) = \frac{U_2(s)}{U_1(s)} - \left(\frac{-2\frac{\omega_0}{Q}s}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2} + 1 \right) =$$

$$= -\frac{s^2 - \frac{\omega_0}{Q}s + \omega_0^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

- Assignment particular zeros to corresponding poles, so-called pole-zero pairing to biquadratic functions are formed,
- determination of order in which the biquadratic functions (biquads) should be cascaded,
- choosing optimum gain distribution to particular biquads and
- electric circuit synthesis of particular biquad (choosing of appropriate circuit and element values calculation).

Frequency Response of the Feedback Structure

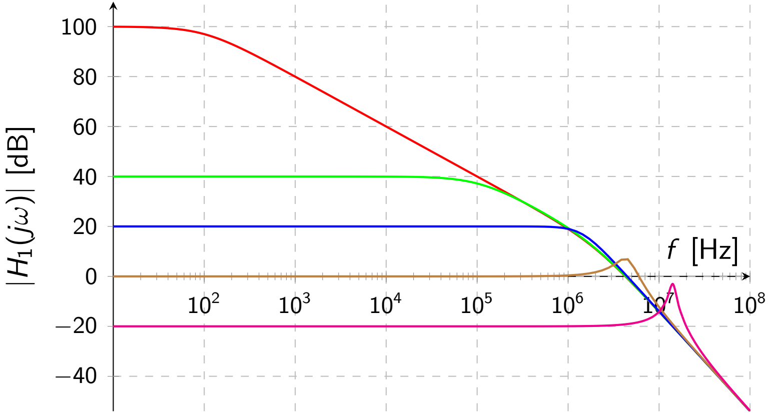


$$H(s) = \frac{U_o(s)}{U_i(s)} = \frac{A(s)}{A(s)\beta + 1}$$

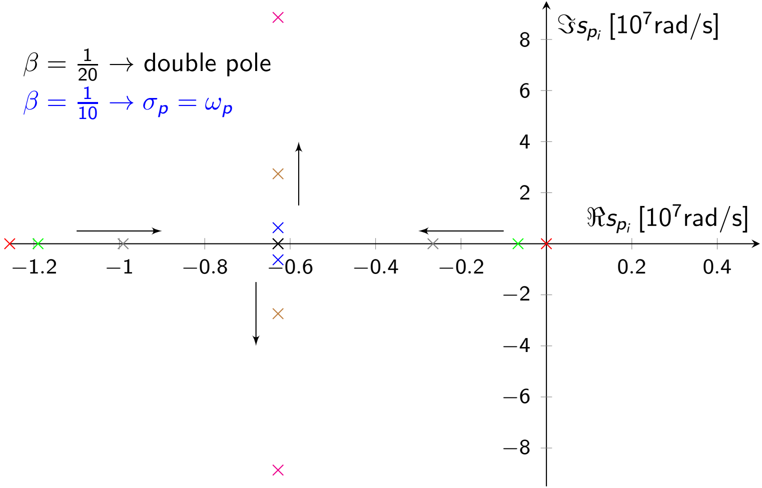
$$H(0) \doteq \frac{1}{\beta} \quad \forall \quad A(0)\beta \gg 1$$

1. $H_1(s) = \dfrac{A_0}{(1 + s/\omega_1)(1 + s/\omega_2)}$,
   where $A_0 = 10^5$, $\omega_1 = 2\pi 100$, $\omega_2 = 2\pi 2 \cdot 10^6$
2. $H_2(s) = \dfrac{A_0}{(1 + s/\omega_1)(1 + s/\omega_2)(1 + s/\omega_3)}$,
   where $A_0 = 10^5$, $\omega_1 = 2\pi 100$, $\omega_2 = 2\pi 2 \cdot 10^6$, $\omega_2 = 2\pi 4 \cdot 10^6$

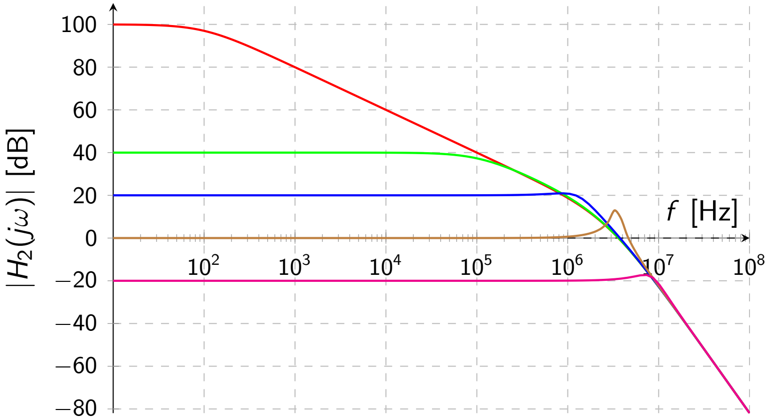
Magnitude frequency responses of the feedback system $H_1(s)$ for $\beta = 0,\ 1/100,\ 1/10,\ 1,\ 10$.



Poles location of the feedback system $H_1(s)$ for
$\beta = 0,\ 1/100,\ 1/30,\ 1/20,\ 1/10,\ 1,\ 10.$



Magnitude frequency responses of the feedback system $H_2(s)$ for $\beta = 0,\ 1/100,\ 1/10,\ 1,\ 10$.

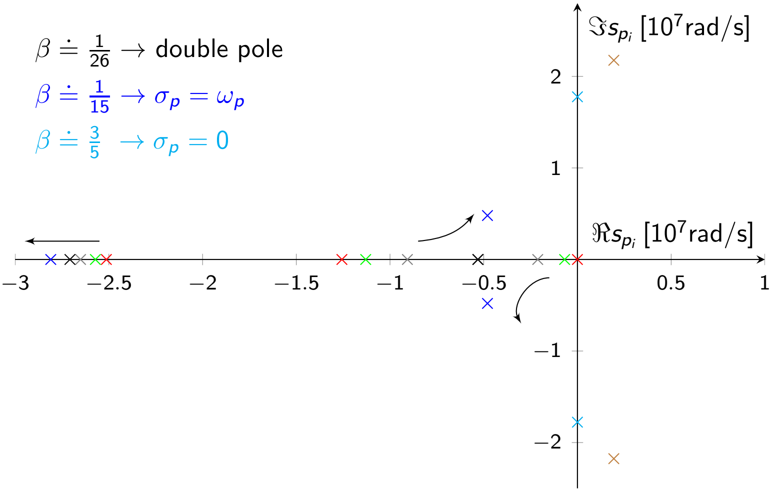


Poles location of the feedback system $H_2(s)$ for
$\beta = 0, 1/100, 1/40, 1/26, 1/15, 3/5, 1$.

$\beta \doteq \frac{1}{26} \rightarrow$ double pole

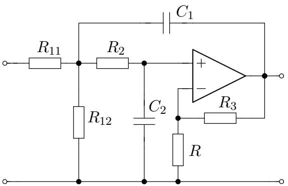
$\beta \doteq \frac{1}{15} \rightarrow \sigma_p = \omega_p$

$\beta \doteq \frac{3}{5} \rightarrow \sigma_p = 0$



Biquad Realization using Circuits Working in Continuous Time

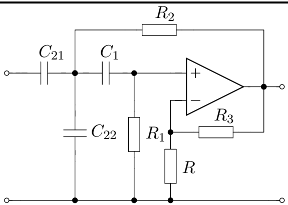
- Single amplifier Sallen-Key circuit for LP, HP and BP filters with a small Q-factor
- Generalized Impedance Converter (GIC) biquad for BP filters with $Q < 60$.
- Åckerberg-Mossberg (AM) or Tow-Thomas (TT) circuit for LP and BP filters with higher Q-factors.
- General biquad based on the TT circuit for eliptic and HP filters.
- Alternatively a single amplifier circuit using double T-network for elliptic filters with small Q-factors.



LP1

metod "K=1": C_1, $[R, A_0, f_t]$, $(Q<5)$

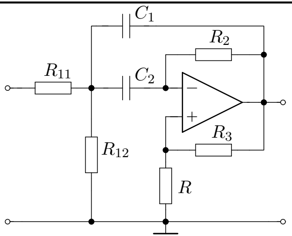
metod "min. Γ_K^Q": C_1, C_2, $[R, A_0, f_t]$, $(Q<10)$



HP1

metod "K=1": C_1, $[R, A_0, f_t]$, $(Q<5)$

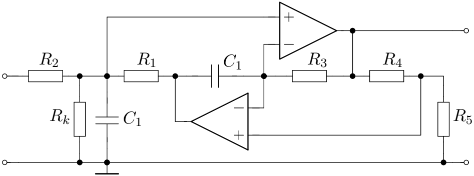
metod "min. Γ_K^Q": C_1, C_2, $[R, A_0, f_t]$, $(Q<10)$



BP1

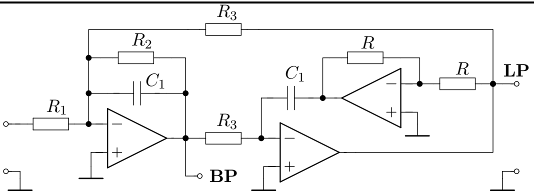
metod "K=1": C_1, $[R, A_0, f_t]$, $(Q<5)$

metod "min. Γ_K^Q": C_1, C_2, $[R, A_0, f_t]$, $(Q<15)$

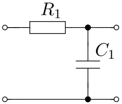


BP2

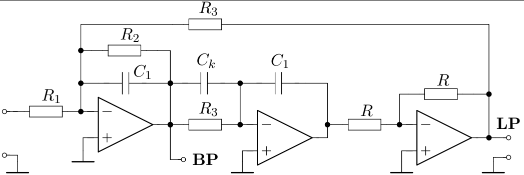
C_1, $[\underline{A_0}, \underline{f_t}]$, $(Q<60)$



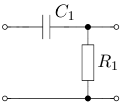
LP2, BP3
C_1, $[R,\ A_0,\ f_t]$



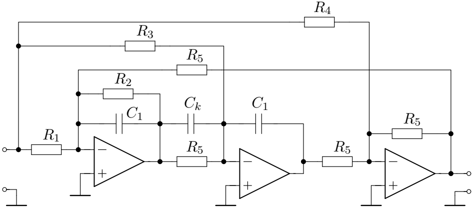
LP0
C_1



LP3, BP4
C_1, $[R,\ A_0,\ f_t]$

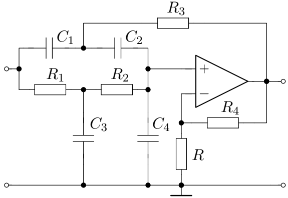


HP0
C_1



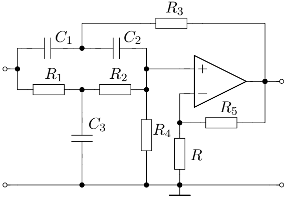
ES1

C_1, [R, A_0, f_t]



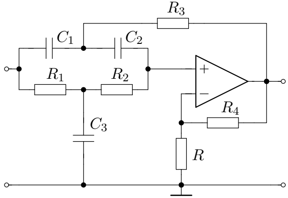
ESLP

C_1, C_2, R, [A_0, f_t]



ESHP

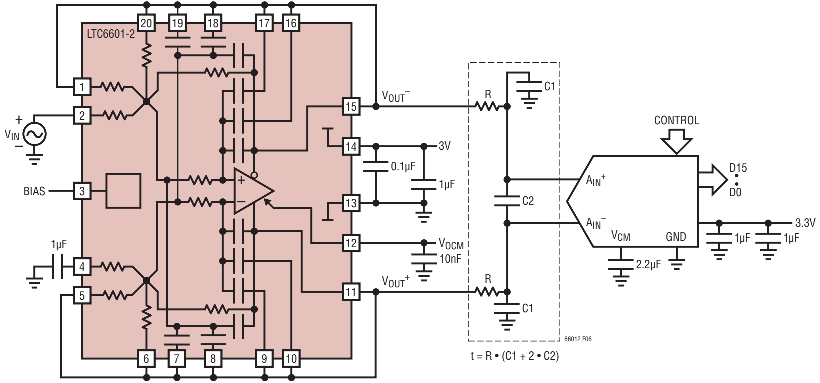
C_1, C_2, R, [A_0, f_t]



ESNotch

C_1, C_2, R, [A_0, f_t]

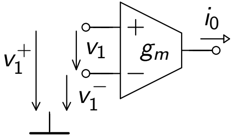
Example of biquad realization, *LTC6601 – pin configurable gain and filter response up to 27 MHz*: anti-aliasing filter for ADC.

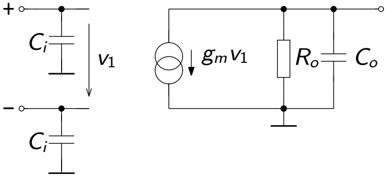


ARC filters: very good dynamic range at low frequencies, tuning is problem.

New blocks for analog filter design

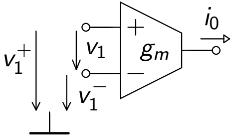
OTA – Operational Transkonductance Amplifier

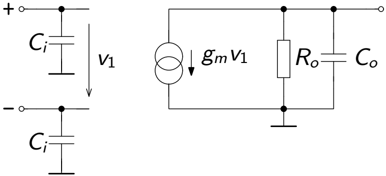


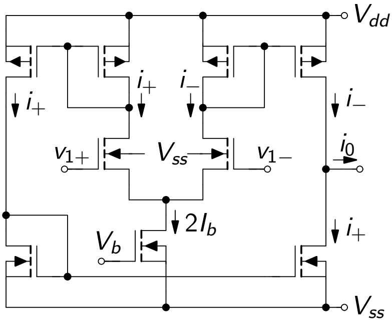


New blocks for analog filter design

OTA – Operational Transkonductance Amplifier

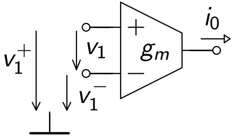


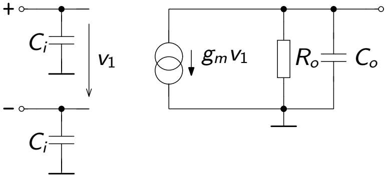


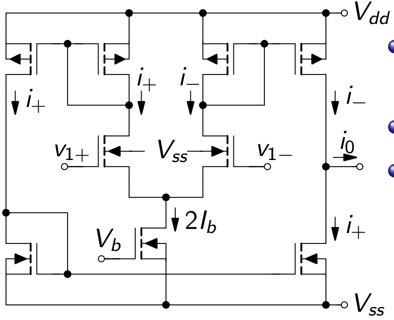


New blocks for analog filter design

OTA – Operational Transkonductance Amplifier





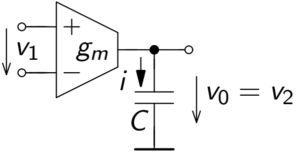


- $C_i = 100^1$ fF, $C_o = 100^1$ fF to 10^0 pF,
- $g_m = 10^1\ \mu$S, $g_o = 10^1\ nS$,
- OPA860: B 80 MHz, SR 900 V/μs, g_m 95 mS, bipolar OT for wide-bandwith systems (video and RF, IF circuitry).

New blocks for analog filter design

OTA – Operational Transkonductance Amplifier

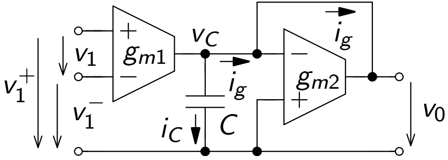
OTA integrator



$$v_0(t) = \frac{1}{C}\int i(t)\,dt = \\ = \frac{1}{C}\int g_m v_1(t)\,dt$$

$$H(s) = \frac{v_2}{v_1} = \frac{g_m}{sC}$$

OTA damped integrator

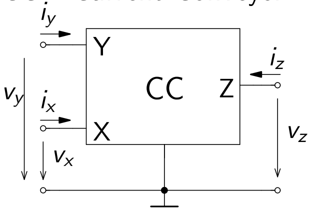


$$i_g = g_{m2} v_C \Rightarrow \frac{v_C}{i_g} = \frac{1}{g_{m2}}$$

$$v_0 = \frac{g_{m1}}{sC + g_{m2}}(v_1^+ - v_1^-)$$

New blocks for analog filter design

CC – Current Conveyor

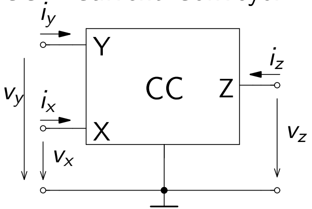


$v_x = v_y,\ i_z = \pm i_x,\ i_y = \alpha i_x,$

- $\alpha \leq 1$, CCI,
- $\alpha = 0$, CCII,
- $\alpha = -1$, CCIII.

New blocks for analog filter design

CC – Current Conveyor



$$v_x = v_y,\ i_z = \pm i_x,\ i_y = \alpha i_x,$$

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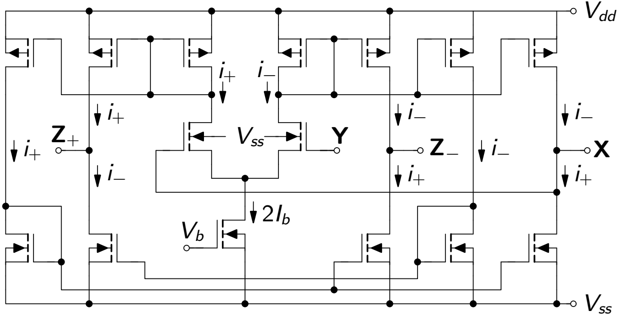
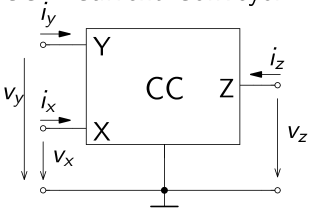


New blocks for analog filter design

CC – Current Conveyor



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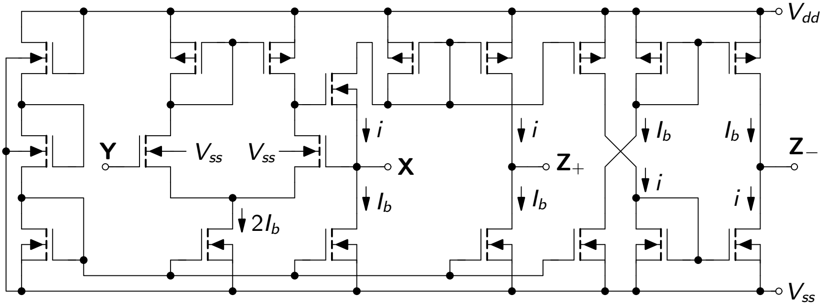
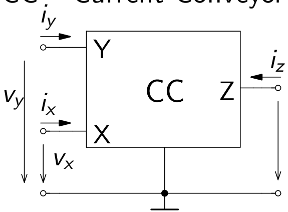


New blocks for analog filter design

CC – Current Conveyor



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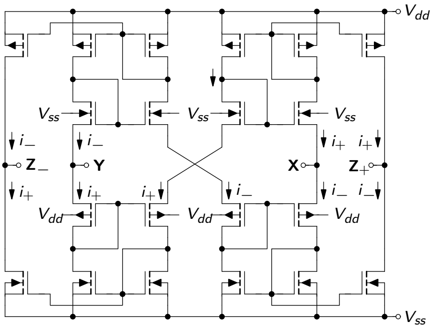
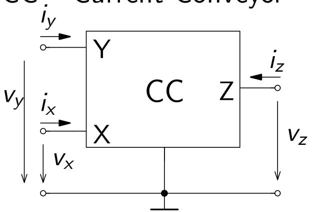


New blocks for analog filter design

CC – Current Conveyor



$$v_x = v_y,\ i_z = \pm i_x,\ i_y = \alpha i_x,$$

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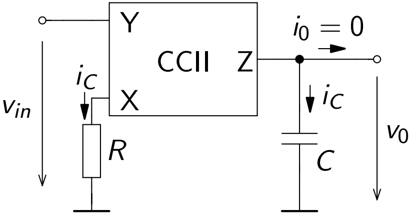
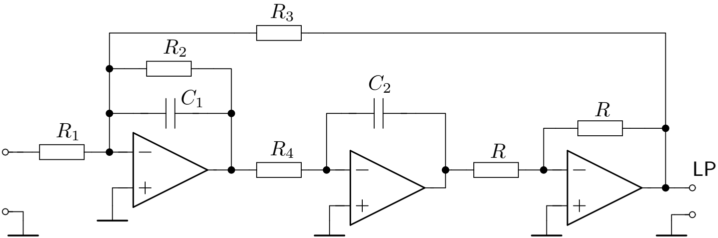
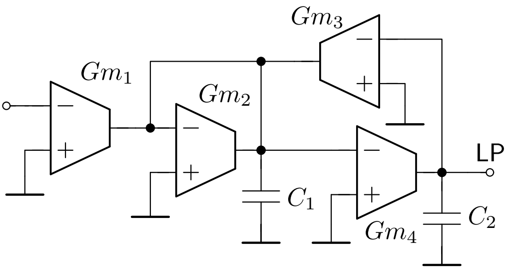


$$v_0(t) = \frac{1}{C}\int i_C(t)\,dt = \\ = \frac{1}{RC}\int v_{in}(t)\,dt$$

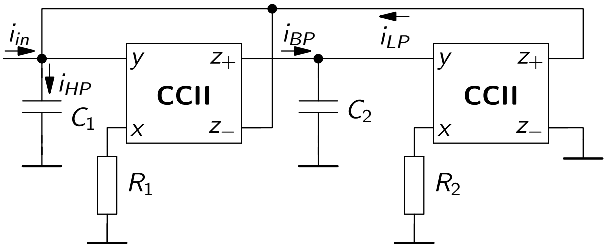
$$H(s) = \frac{V_2}{V_1} = \frac{1}{sC\,R}$$

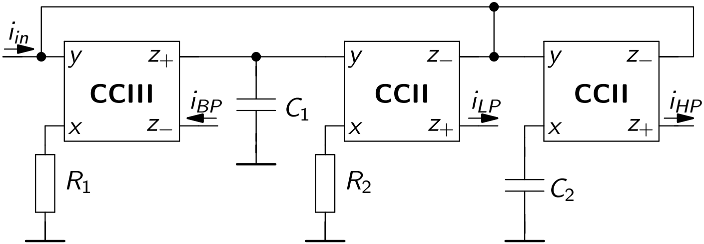
Biquad realization using ARC, OTA



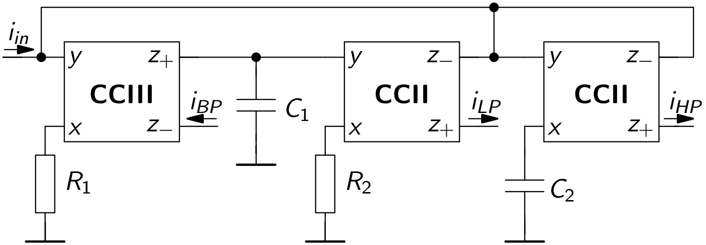


Biquad realization in "mixed" current mode using CC





Biquad realization in "mixed" current mode using CC

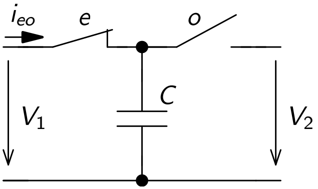


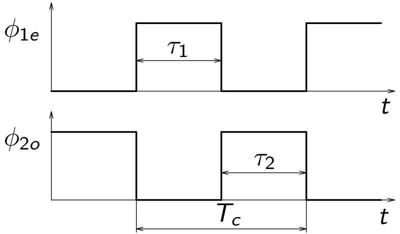
$$\frac{i_{LP}}{i_{in}} = \frac{1}{s^2 R_1 R_2 C_1 C_2 + sR_1 C_1 + 1}, \quad \frac{i_{BP}}{i_{in}} = \frac{sR_1 C_1}{s^2 R_1 R_2 C_1 C_2 + sR_1 C_1 + 1}$$

$$\frac{i_{HP}}{i_{in}} = \frac{s^2 R_1 R_2 C_1 C_2}{s^2 R_1 R_2 C_1 C_2 + sR_1 C_1 + 1}, \quad \frac{i_{BS}}{i_{in}} = \frac{i_{LP} + i_{HP}}{i_{in}} =$$

$$= \frac{s^2 R_1 R_2 C_1 C_2 + 1}{s^2 R_1 R_2 C_1 C_2 + sR_1 C_1 + 1}, \quad \frac{i_{AP}}{i_{in}} = \frac{i_{LP} + i_{HP} - i_{BP}}{i_{in}}$$

Principle of Switched Capacitor Circuits



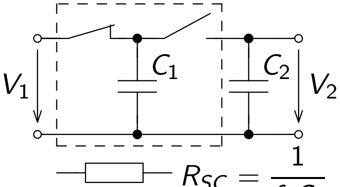


$$Q_1 = C\,V_1, \quad Q_2 = C\,V_2, \;\Rightarrow\; \Delta Q = Q_1 - Q_2 = C(V_1 - V_2)$$

Relation for equivalent average current i_{eo} in one switched period $T_c = 1/f_c$ can be derived

$$i_{eo} = \frac{\partial Q}{\partial t} \doteq \frac{\Delta Q}{\Delta t} = \frac{C(V_1 - V_2)}{T_c} \Rightarrow \frac{V_1 - V_2}{R_{SC}} \Rightarrow R_{SC} = \frac{1}{f_c C}\,.$$

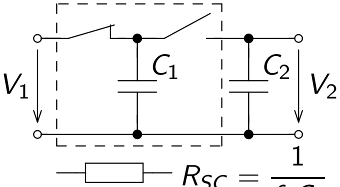
Principle of Switched Capacitor Circuits



$$R_{SC} = \frac{1}{f_c C_1}$$

$$K(\omega) = \frac{V_2}{V_1} = \frac{1}{1 + j\omega R_{SC} C_2} = \frac{1}{1 + j\frac{\omega}{f_c}\frac{C_2}{C_1}}$$

Principle of Switched Capacitor Circuits

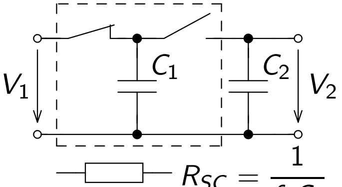


$$R_{SC} = \frac{1}{f_c C_1}$$

$$K(\omega) = \frac{V_2}{V_1} = \frac{1}{1 + j\omega R_{SC} C_2} = $$
$$= \frac{1}{1 + j\frac{\omega}{f_c}\frac{C_2}{C_1}}$$

$$\boxed{\omega_0 \sim \frac{f_c C_1}{C_2}}$$

Principle of Switched Capacitor Circuits



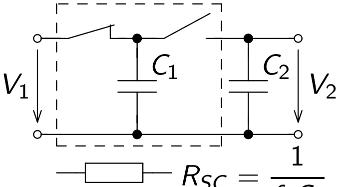
$$R_{SC} = \frac{1}{f_c C_1}$$

$$K(\omega) = \frac{V_2}{V_1} = \frac{1}{1 + j\omega R_{SC} C_2} = \frac{1}{1 + j\frac{\omega}{f_c}\frac{C_2}{C_1}}$$

$$\boxed{\omega_0 \sim \frac{f_c C_1}{C_2}}$$

- Easy tunable, easy to integrate!
- high accuracy only for ratio's of C_i, could be 0.2 %,
- possibility to realize high values of resistance ($f_c = 100\,\mathrm{kHz}$ and $C = 1\,\mathrm{pF} \rightarrow 10\,\mathrm{M\Omega}$),
- only capacitors to drive $\rightarrow$ low power,
- only for $f \ll f_c$.

Principle of Switched Capacitor Circuits



$$R_{SC} = \frac{1}{f_c C_1}$$

$$K(\omega) = \frac{V_2}{V_1} = \frac{1}{1 + j\omega R_{SC} C_2} = \frac{1}{1 + j\frac{\omega}{f_c}\frac{C_2}{C_1}}$$

$$\boxed{\omega_0 \sim \frac{f_c C_1}{C_2}}$$

- Easy tunable, easy to integrate!
- high accuracy only for ratio's of C_i, could be 0.2 %,
- possibility to realize high values of resistance ($f_c = 100\,\mathrm{kHz}$ and $C = 1\,\mathrm{pF} \rightarrow 10\,\mathrm{M\Omega}$),
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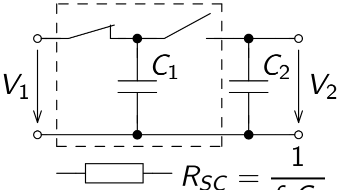
- only for $f \ll f_c$.

Principle of Switched Capacitor Circuits



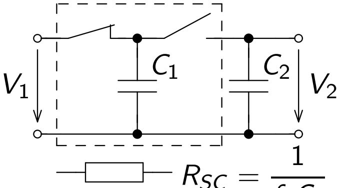
$$R_{SC} = \frac{1}{f_c C_1}$$

$$K(\omega) = \frac{V_2}{V_1} = \frac{1}{1 + j\omega R_{SC} C_2} = \frac{1}{1 + j\frac{\omega}{f_c}\frac{C_2}{C_1}}$$

$$\boxed{\omega_0 \sim \frac{f_c C_1}{C_2}}$$

- Easy tunable, easy to integrate!
- high accuracy only for ratio's of C_i, could be 0.2 %,
- possibility to realize high values of resistance ($f_c = 100\,\text{kHz}$ and $C = 1\,\text{pF} \rightarrow 10\,\text{M}\Omega$),
- only capacitors to drive $\rightarrow$ low power,
- only for $f \ll f_c$.

Principle of Switched Capacitor Circuits



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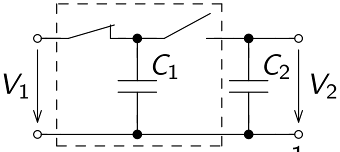
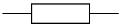
Principle of Switched Capacitor Circuits





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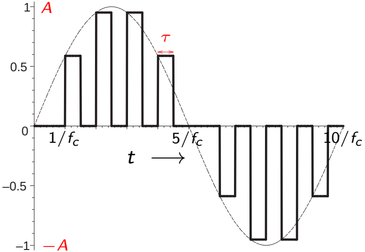
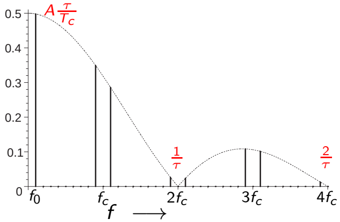
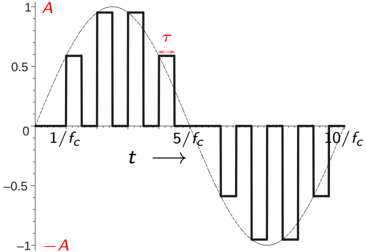
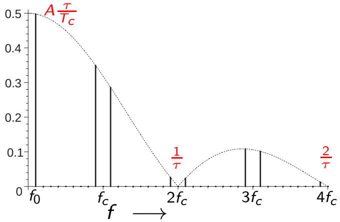
Spectrum of switched sinusoidal signal





Spectrum of switched sinusoidal signal





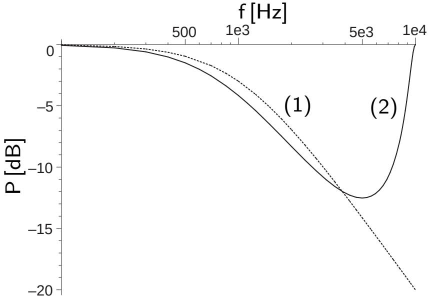
$$\approx A\frac{\tau}{T_c}\sum_{k=0}^{\infty}\frac{\sin(k\omega_c \pm \omega_0)\frac{\tau}{2}}{(k\omega_c \pm \omega_0)\frac{\tau}{2}}\sin\big((k\omega_c \pm \omega_0)t + \varphi(k)\big)$$

- Spectrum of sampled signal is periodical – period given by f_c.
- The spectrum is constrictive with factor $\frac{\tau}{T_c}\frac{\sin(\pi f \tau)}{\pi f \tau}$,
- i.e. $\frac{1}{2}\frac{\sin\left(\frac{\pi f}{2f_c}\right)}{\frac{\pi f}{2f_c}}$ for $\tau = T_c/2$.

Frequency response

$$\frac{V_3}{V_1} = \frac{1}{1 + j\omega C_2 R_{eq}} = \frac{1}{1 + j\frac{\omega}{f_c}\frac{C_2}{C_1}} \tag{1}$$

$$K_{13} = \frac{V_{3e}}{V_{1e}} = \frac{C_1}{z(C_1 + C_2) - C_2}, \qquad z = e^{j\omega T_c} \tag{2}$$



for $f_c = 10^4$ [Hz], $C_1 = 6{,}2$ [nF] and $C_2 = 10$ [nF].

Frequency response

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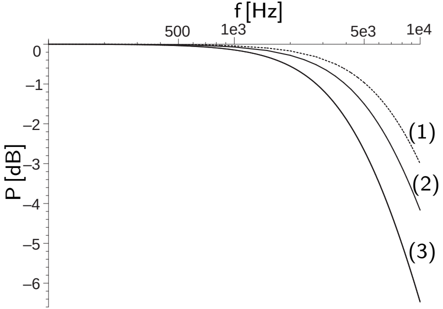
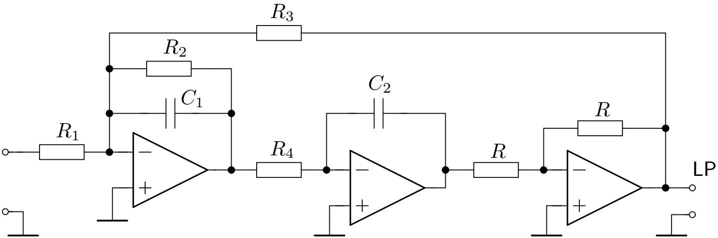
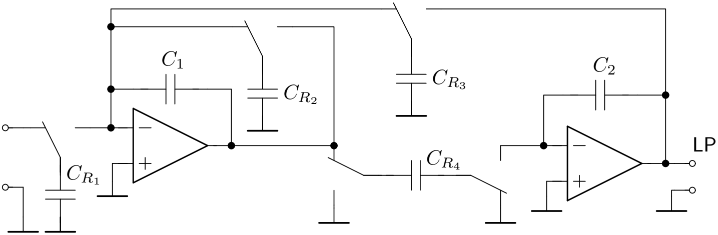


$$R_{on} = 5\,\mathrm{k}\Omega \tag{3}$$

for $f_c = 10^5$ [Hz], $C_1 = 6{,}2$ [nF] and $C_2 = 10$ [nF].

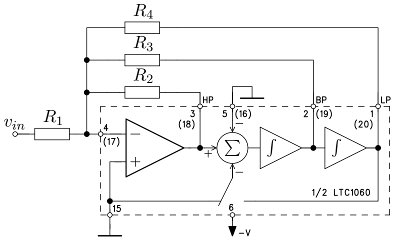
Biquad realization using ARC, SC

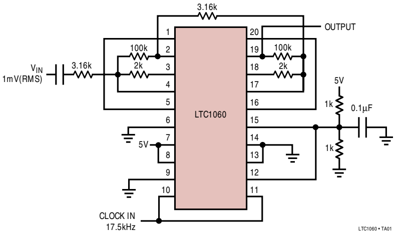




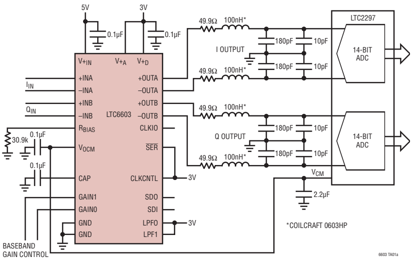
Realization using Switched Capacitor Circuits based on LTC 1060 Chip

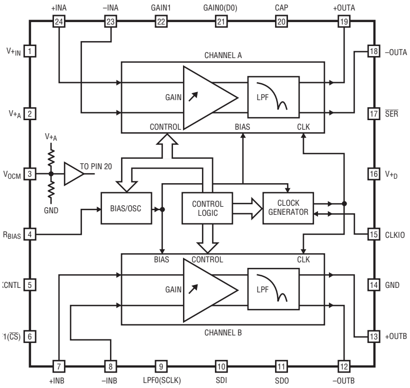
- LTC1060 consists of two SC biquads
- LP, HP, BP, BS and allpass
- operates up to 30 kHz ($f_{clk} = 1.5$ MHz)
- center frequency Q-product up to 1.6 MHz
- 88 dB dynamic range at ±2.5 V supply





Example of SC filter realizarion, LTC6603 – programmable lowpass filter for communications receivers and transmitters





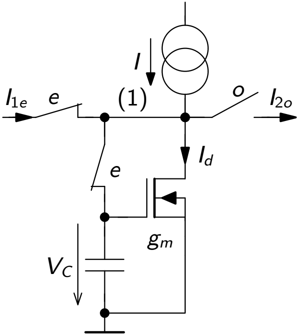
BW up to 2.5MHz

Gain (0/6/12/24 dB)

9th Order Linear Phase Response

APP.: WCDMA, UMTS, radio links, modems, 802.11x receivers

Principle of Switched Current Circuits

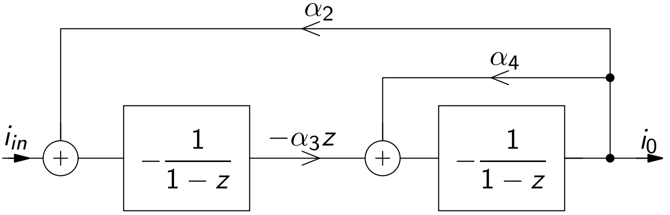


$$\Phi_1: \quad t = (n - 1/2)T_c \qquad I_d = I_{1e}\big((n - 1/2)T_c\big) + I$$

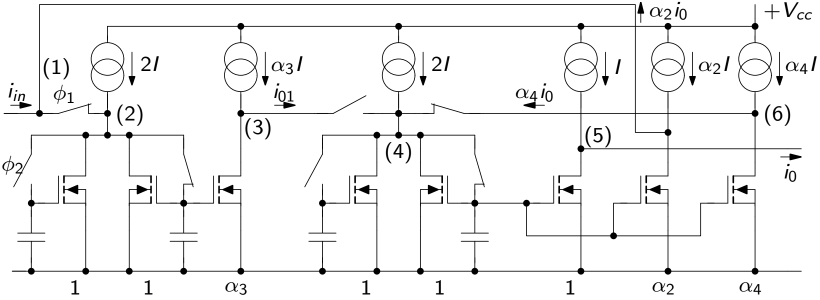
$$\Phi_2: \quad t = (n)T_c \qquad I_{2o} = I - I_d = -I_{1e}\big((n - 1/2)T_c\big)$$

$$I_{2o}(z) = -z^{-1/2} I_{1e}(z) \quad \Rightarrow \quad \frac{I_{2o}(z)}{I_{1e}(z)} = -z^{-1/2} = -\frac{1}{\sqrt{z}}$$

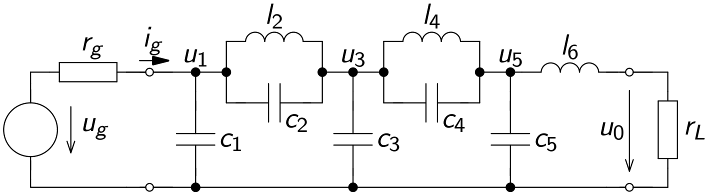
Biquad realization in "clear" current mode using SI



S



Active realization of the LC ladder by OTAs



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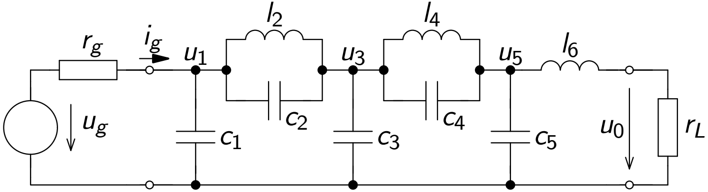

$$v_g = \frac{R}{r_g}(u_g - u_1); \quad u_1 = \frac{1}{sc_1 R}(v_g - v_2 - sc_2 R(u_1 - u_3));$$

$$v_2 = \frac{R}{sl_2}(u_1 - u_3); \quad u_3 = \frac{1}{sc_3 R}(v_2 - v_4 + sc_2 R(u_1 - u_3) - sc_4 R(u_3 - u_5));$$

$$v_4 = \frac{R}{sl_4}(u_3 - u_5); \quad u_5 = \frac{1}{sc_5 R}(v_4 - v_6 + sc_4 R(u_3 - u_5));$$

$$v_6 = \frac{R}{sl_6}(u_5 - u_7); \quad u_7 = \frac{r_2}{R} v_6.$$

Active realization of the LC ladder by OTAs

$$v_g = \frac{R}{r_g}(u_g - u_1); \quad u_1 = \frac{1}{sc_1 R}(v_g - v_2 - sc_2 R(u_1 - u_3));$$

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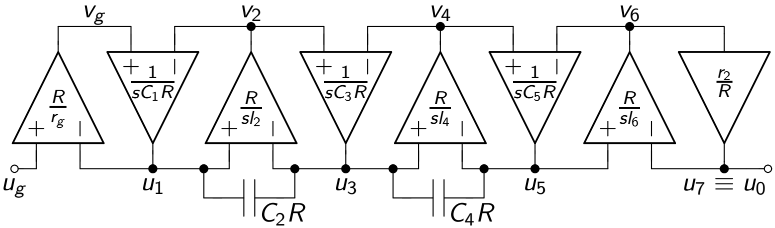
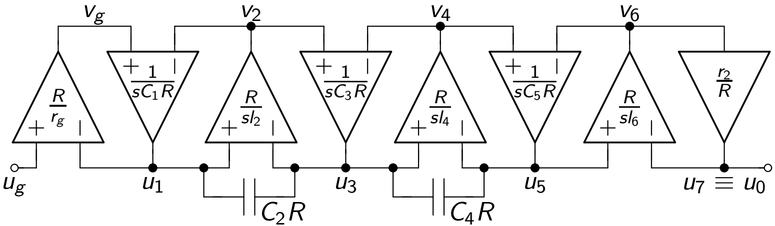
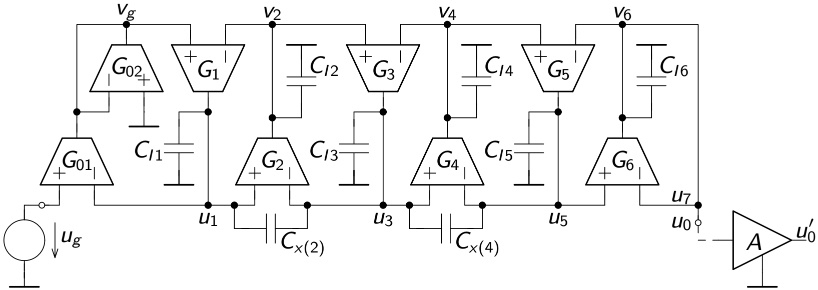
$$v_4 = \frac{R}{sl_4}(u_3 - u_5); \quad u_5 = \frac{1}{sc_5 R}(v_4 - v_6 + sc_4 R(u_3 - u_5));$$

$$v_6 = \frac{R}{sl_6}(u_5 - u_7); \quad u_7 = \frac{r_2}{R} v_6.$$



Active realization of the LC ladder by OTAs



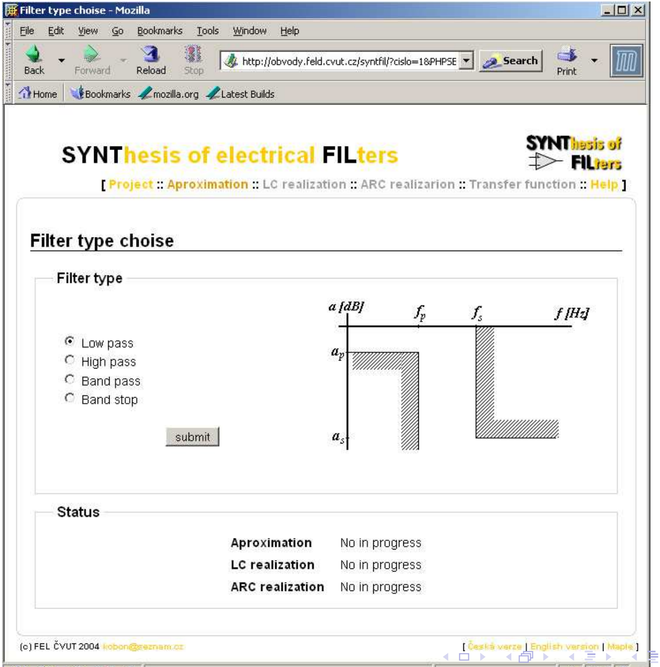


$$C_{lj} = c_j g_{mj} R,\ {}_{j=1,3,5}\ ;\quad C_{lk} = l_k g_{mk} / R,\ {}_{k=2,4,6}\ ;\quad C_{x(m)} = C_m\, g_m R,\ {}_{m=2,4}\ .$$

SYNThesis of FILters

http://syntfil.feld.cvut.cz

WWW Interface – Filter Type Selection



WWW Interface – Filter Specification

SYNThesis of electrical FILters

[Project :: Aproximation :: LC realization :: ARC realizarion :: Transfer function :: Help]

Setting of required values

Frequence

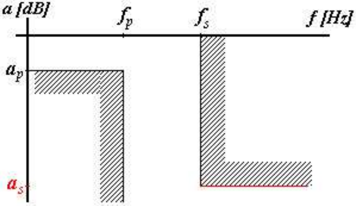
f_p = 5000 Hz

f_s = 7000 Hz

a_p = 2 dB

a_s = 20 dB

submit



Status

Aproximation	Low pass
LC realization	No in progress
ARC realization	No in progress

Choosing selection requiest approximate and order filter

Approximation

Approximation	Keeping of required parameters			Filter with lower order		
		Order	New value of a_s		Order	New value of a_s
Butterworth	○	8	21.085211	○	7	18.195003
Chebyshev	◉	4	21.810501	○	3	14.449014
Inverzní Chebyshev A	○	4	21.810501	○	3	14.449014
Inverzní Chebyshev B	○	4	21.810501 (Ω_s=1.457638)	○	2	7.772117 (Ω_s=1.708801)
Cauer A	○	3	26.194390	○	2	12.957894
Cauer B	○	4	39.698595 (Ω_s=1.480785)	○	2	12.957894 (Ω_s=1.825298)
Cauer C	○	4	39.698595 (Ω_s=1.566231)	○	2	12.957894 (Ω_s=2.379796)

Result from aproximate analyzis

NLP (Norm Low pass)

Transfer function

$$H(p)=\frac{0.163445}{1.000000p^4+0.716215p^3+1.256482p^2+0.516798p+0.205765}$$

graph

Zeros and poles

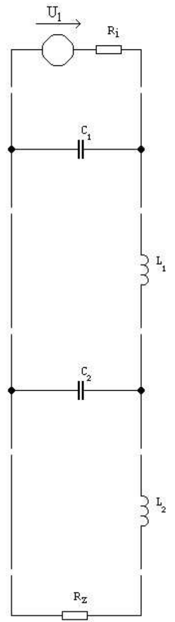
Zeros	Poles
	-.104887252238877+.957952960166620*I
	-.104887252238877-.957952960166620*I
	-.253220226875144+.396797108216470*I
	-.253220226875144-.396797108216470*I

graph

Coeficient Gc

6.11826481263365

Scheme of LC filter



R_i = 1k [Ω]

C_1 = 88.887n [F]

L_1 = 28.03m [H]

Q_1 = infinity [-]

R_1 = 0 [Ω]

C_2 = 114.793r [F]

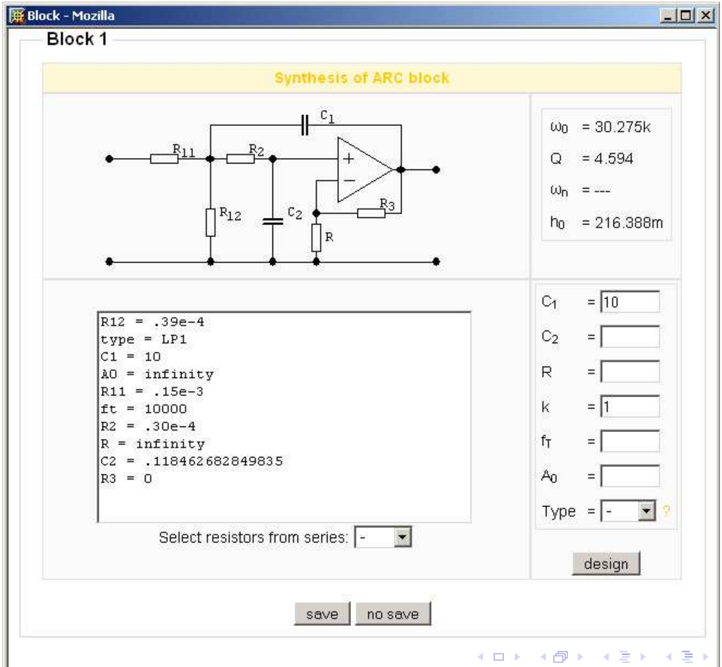
L_2 = 21.704m [H]

Q_2 = infinity [-]

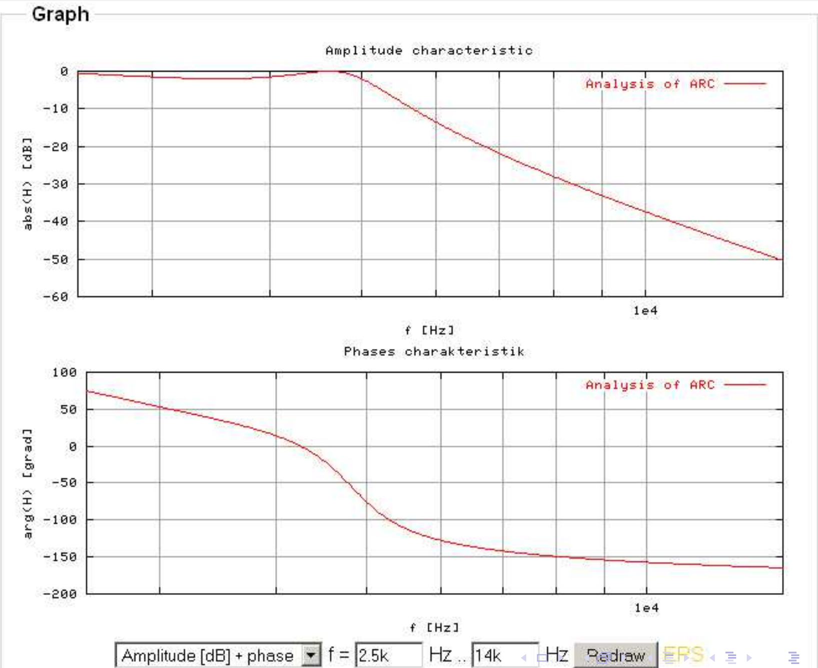
R_2 = 0 [Ω]

R_z = 244.177 [Ω]

WWW Interface – ARC Cascade Filter Synthesis



WWW Interface – Frequency Response of Designed Filter



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- Bičák J., Laipert M., Vlček M.: *Lineární obvody a systémy*, Monografie ČVUT, Praha 2006.
- Sedra, A. S., Smith, K. C.: *Microelectronic Circuits*, 5th edition, Oxford University Press, Inc., New York 2004. ISBN 0-19-514-251-9.

Thank you for your attention !

KATEDRA TEORIE OBVODŮ

Fakulta elektrotechnická