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PROLOG PROGRAMMING FOR ARTIFICIAL INTELLIGENCE, 4TH EDN.
ADDITIONAL EXERCISES IN BAYESIAN NETWORKS

Note: These exercises are supplementary to those in the book: I. Bratko, Prolog Programming for Artificial Intelligence, 4th edn., Pearson Education 2011.

Problem

Let there be variables X and Y in a dynamic system. The behaviour of X in time is:

$$X = \sin(t)$$

(a) What is the corresponding *qualitative* behaviour in time of X? Give the first 7 qualitative states, starting with t = 0.

(b) Let there be the qualitative constraint $Y = M^{+}(X)$, and the initial value $Y(0) \geq 0$. Give all the possible qualitative behaviours of both X and Y in time, starting with t=0 (the first 7 qualitative states).

Answers

(a) zero/inc, zero..inf/inc, zero..inf/std, zero..inf/dec, zero/dec, minf..zero/dec, minf..zero/std

(b)

	X	Y
1.	zero/inc	zero..inf/inc
2.	zero..inf/inc	zero..inf/inc
3.	zero..inf/std	zero..inf/std
4.	zero..inf/dec	zero..inf/dec
5.	zero/dec	zero..inf/dec
6.	minf..zero/dec	zero..inf/dec
7.	minf..zero/std	zero..inf/std or zero/std

Also possible:

7.	minf..zero/dec	zero/dec

Problem

Consider a QDE-type (Qualitative Differential Equations) qualitative model with three variables x, y and z, and the following constraints:

$M_0^-(x, y)$	(monotonically decreasing functions; if x=0 then y=0)
plus(y, z, x)	(y + z = x)
deriv(y, z)	(dy/dt is “qualitatively equal” z)

The landmarks for these variables, ordered from smallest to largest, are:

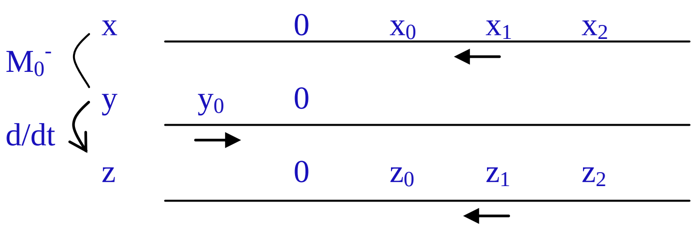
x: minf, 0, x_0, x_1, x_2, inf
y: minf, y_0, 0, inf
z: minf, 0, z_0, z_1, z_2, inf

At time t_0 the qualitative values of x, y and z are equal x_1, y_0 and z_1, respectively.

(a) What are all the possible qualitative states (i.e. values and directions of change) of these three variables at the initial time $t=t_0$.
(b) What are all the possible qualitative states of the three variables in the time interval $t_0..t_1$?
(c) What are all the possible qualitative states of the three variables at time t_1?
(d) Can the system reach, from the given initial state at time t0, a steady state (that is a state where all the variables are constant – not changing in time – and from that point stay the same for ever. If yes, what is this steady state?

Answers

In answering the questions, the representation below helps. The horizontal arrows indicate the directions of change at time t_0.



In addition to the graphically indicated constraints, it also holds: x = y + z.

(a) The qualitative state at time t0 is as indicated in the diagram.
(b) It is easy to see from the diagram that all the qualitative values are corresponding intervals (e.g. $x = x_0..x_1$) and the directions of change are the same as at time t_0, as indicated by the horizontal arrows.
(c) There are three alternatives possible: (1) x reaches x_0, or (2) z reaches z_0, or (3) both x and z reach these landmarks simultaneously. The directions of change remain the same. y cannot become std because z > 0. In all the three cases for the values of x and z, $y = y_0..0$. y cannot be 0 because x > 0.

(d) The possible stationary state is x = y = z = 0, all std.

Problem

Suppose we have a qualitative model with variables X, Y and Z. The landmarks for all three variables are:

minf, 0, inf

where ‘inf’ stands for infinity, ‘minf’ stands for minus infinity. There are the following constraints in the model:

$Y = M_0^+(X)$
$Z = M_0^-(Y)$
deriv(X, Z)   (dX/dt is “qualitatively equal” Z)

At the initial time t0, the qualitative value of X is 0..inf.

(a) What are the qualitative values of Y and Z at t0?
(b) What is the direction of change (inc, std, or dec) of X at t0? Explain why.
(c) What are the qualitative states (that is qualitative values and directions of change) of the three variables in time interval t0..t1?
(d) What are all possible qualitative states of the three variables at time point t1?

Problem

Let there be three variables x, y and z in a QDE-type qualitative model. Landmark values for these variables are:

x: minf, 0, x0, inf
y: minf, 0, y0, inf
z: minf, 0, inf

“minf” means minus infinity, “inf” means infinity.

There are the following constraints in the model:

$y = M_0^-(x)$ (y is monotonically decreasing function of x, with y = 0 when x = 0)
plus(x,z,y)   (x + z = y)
deriv(x,z)   (dx/dt is qualitatively equal z)

At time t0, the qualitative value of x is: $x(t_0) = x_0$/dec

(a) What are all possible qualitative values $y(t_0)$ and $z(t_0)$ that satisfy the above qualitative constraints?
(b) What are all possible qualitative values of x, y and z in the interval $t_0..t_1$ that immediately follows time t_0.
(c) Is it possible that this system gets into a steady state at time point t_1 (all variables are steady)? If yes, what are the qualitative values of x, y and z in this steady state?

Problem

Let there be three variables x, y and z in a QDE-type qualitative model. Landmark values for these variables are:

x: minf, 0, x0, inf
y: minf, y0, 0, inf
z: minf, 0, inf

"minf" means minus infinity, "inf" means infinity.

There are the following constraints in the model:

$y = M_0^-(x)$	% y is monotonically decreasing function of x, with y=0 when x=0
plus(x,z,y)	% x + z = y
deriv(x,z)	% dx/dt = z

At time t_0 the qualitative value of x is: $x(t_0) = x_0$..inf/dec.
It is also given that the value of $y(t_0)$ is less than y_0, but the direction of change (inc, dec, std) of y at t_0 is not given.

(a) What are all possible qualitative values $y(t_0)$ and $z(t_0)$ that satisfy the above qualitative constraints?
(b) What are all possible qualitative values of x, y and z in the interval $t_0..t_1$ that immediately follows time t_0?
(c) What are all possible qualitative values of x, y and z at time point t_1?

Problem

Let there be four variables, X, Y, Z and D, in a QDE-type qualitative model. The landmarks for all these variables are: minf, 0, inf (minus infinity, zero and infinity). The landmarks for D are: minf, 0, d0, inf. The model consists of the following constraints:

deriv(Z, D)	(D is qualitatively equal dZ/dt)
add(X, Y, Z)	(X+Y = Z)
Y = M^-(X)	(Y is monotonically decreasing function of X)
D = const.	(D does not change)

At the initial time t0, the values of the variables are X = Y = Z = zero, D = d_0.

(a) What are all possible qualitative states (that is qualitative values and directions of change) of the four variables at time t_0 so that they satisfy the given constraints?

(b) What are all possible qualitative states of the four variables in the time interval between t_0 and t_1, so that they satisfy the given constraints?

Problem

There are three variables x, y and z in a qualitative model which includes the following constraints:

M^-(x, y)
plus(y, z, x)
deriv(y, z)

The landmark values for the variables are:

x: minf, 0, x_0, inf
y: minf, 0, y_0, inf
z: minf, 0, inf

In the initial state of the system at time t_0, the qualitative values of x and y are x_0 and y_0.

(a) Determine all possible qualitative states (that is qualitative values and directions of change) of the three variables at time t = t_0.

(b) What are all possible qualitative states of the three variables in the time interval $t_0..t_1$?

(c) What is the smallest qualitative value that the variable y can reach from the initial state?

Answers

The following answers were obtained with the Prolog qualitative simulation program in the book:

(a)

S0 = [x:x0/dec, y:y0/inc, z:zero..inf/dec] ;
S0 = [x:x0/std, y:y0/std, z:zero/std] ;
S0 = [x:x0/inc, y:y0/dec, z:minf..zero/inc] ;

no

(b)
S1 = [x:zero..x0/dec, y:y0..inf/inc, z:zero..inf/dec] ? ;
S1 = [x:x0..inf/inc, y:zero..y0/dec, z:minf..zero/inc] ? ;
no

Another possibility is for the variables to stay unchanged for ever:

S1 = [x:x0/std, y:y0/std, z:zero/std]

The qualitative simulator in the book does not produce this state because there is no change from the previous steady state (at t_0).

(c) y = zero..y0 (y can never be less or equal 0)