

# Symbols

## Introduction

Mathematics provides a very rich language for the communication of concepts and ideas, and a set of powerful tools for the solution of problems. In order to use this language it is essential to appreciate how **symbols** are used to represent quantities, and to understand the conventions which have been developed to manipulate them.

## Symbols

The choice of which letters or other symbols to use is largely up to the user although it is helpful to choose letters which have some meaning in any particular context. For instance if we wish to choose a symbol to represent an interest rate we might choose the letters  $i$  or  $r$ . Usually the lower case letter  $t$  is used to represent time. Because both time and interest rate can vary we refer to  $t$ ,  $i$  and  $r$  as **variables**. In a particular calculation some symbols represent fixed and unchanging quantities and we call these **constants**. Often we reserve the letters  $x$ ,  $y$  and  $z$  to stand for variables and use the earlier letters of the alphabet, such as  $a$ ,  $b$  and  $c$ , to represent constants. The Greek letter pi, written  $\pi$ , is used to represent the constant 3.14159.... which appears in the formula for the area of a circle. Other Greek letters are frequently used and for reference the Greek alphabet is given in Table 1.

Table 1: The Greek alphabet

$A$	$\alpha$	alpha	$I$	$\iota$	iota	$P$	$\rho$	rho
$B$	$\beta$	beta	$K$	$\kappa$	kappa	$\Sigma$	$\sigma$	sigma
$\Gamma$	$\gamma$	gamma	$\Lambda$	$\lambda$	lambda	$T$	$\tau$	tau
$\Delta$	$\delta$	delta	$M$	$\mu$	mu	$\Upsilon$	$\upsilon$	upsilon
$E$	$\epsilon$	epsilon	$N$	$\nu$	nu	$\Phi$	$\phi$	phi
$Z$	$\zeta$	zeta	$\Xi$	$\xi$	xi	$X$	$\chi$	chi
$H$	$\eta$	eta	$O$	$\omicron$	omicron	$\Psi$	$\psi$	psi
$\Theta$	$\theta$	theta	$\Pi$	$\pi$	pi	$\Omega$	$\omega$	omega

## Taking care over the position of symbols

Mathematics is a very precise language and care must be taken to note the exact position of any symbol in relation to any other. If  $x$  and  $y$  are two symbols, then the quantities  $xy$ ,  $x^y$ ,  $x_y$  can all mean different things. In the expression  $x^y$  you will note that the symbol  $y$  is placed to the right of and slightly higher than the symbol  $x$ . In this context  $y$  is called a **superscript**. In the expression  $x_y$ ,  $y$  is placed lower than and to the right of  $x$ , and is called a **subscript**. You must take the same care when writing mathematics to be precise and be clear about what you mean.

## The product of two quantities

The instruction to multiply two quantities  $x$  and  $y$  together is written as  $x \times y$ . Usually the multiplication sign is missed out altogether and we write simply  $xy$ . An alternative and acceptable notation is to use a dot to represent multiplication and so we could write  $x \cdot y$ . The quantity  $xy$  is called the **product** of  $x$  and  $y$ . Note that  $xy$  is the same as  $yx$  just as  $4 \times 3$  is the same as  $3 \times 4$ .

## The quotient of two quantities

The quantity  $x \div y$  means  $x$  divided by  $y$ . This is also written as  $x/y$  or  $\frac{x}{y}$  and is known as the **quotient** of  $x$  and  $y$ . In the expression  $\frac{x}{y}$  the top line is called the **numerator** and the bottom line is called the **denominator**. Note that  $x/y$  is not the same as  $y/x$ . Division by 1 leaves a quantity unchanged so that  $\frac{x}{1}$  is simply  $x$ .

The **reciprocal** of a fraction is found by inverting it so that the old numerator becomes the new denominator, and the old denominator becomes the new numerator. Thus the reciprocal of  $\frac{2}{x}$  is  $\frac{x}{2}$ . The reciprocal of  $\frac{1}{1+i}$  is  $\frac{1+i}{1}$  or simply  $1+i$ .

## The delta notation for the change in a variable

The **change** in the value of a quantity is found by subtracting its initial value from its final value. For example, if the value of an investment is initially £1300 and after some time is found to be £1700, the change in value is  $1700 - 1300 = £400$ . The Greek letter delta ( $\delta$ , or  $\Delta$ ) is often used to indicate such a change. If  $x$  is a variable we write  $\delta x$  to stand for a change in the value of  $x$ . We sometimes refer to  $\delta x$  as an **increment** in  $x$ . For example if the value of  $x$  changes from 3 to 3.01 we could write  $\delta x = 3.01 - 3 = 0.01$ . It is important to note that this is not the product of  $\delta$  and  $x$ , rather the whole quantity  $\delta x$  means the increment in  $x$ .

## Sigma notation

**Sigma notation** uses the Greek letter  $\Sigma$  to provide a concise and convenient way of writing long sums. Full details are given on the leaflet *Sigma Notation*.