W203 Unit 3 HW: Probability Theory

1. R = event where customers uses Regular gas
M = event where customers uses mid-grade gas
P = event where customer uses premium gas
F = event where customer fills tank.

$P(R) = 40\%$ $P(M) = 35\%$ $P(P) = 25\%$

$P(F|R) = 30\%$

$P(F|M) = 60\%$

$P(F|P) = 50\%$

a) Find $P(F \wedge R)$

$$P(F|R) = \frac{P(F \wedge R)}{P(R)} \rightarrow P(F \wedge R) = P(F|R)P(R)$$

$$P(F \wedge R) = (0.3)(0.4) = \underline{0.12}$$

b)
$$\begin{aligned} P(F) &= P(F \wedge R) + P(F \wedge M) + P(F \wedge P) \\ &= 0.12 + P(F|M)P(M) + P(F|P)P(P) \\ &= 0.12 + (0.6)(0.35) + (0.5)(0.25) \end{aligned}$$

$$P(F) = \underline{0.455}$$

c) Find $P(R|F)$

$$P(R|F) = \frac{P(R \wedge F)}{P(F)} = \frac{0.12}{0.455} = \underline{0.2637}$$

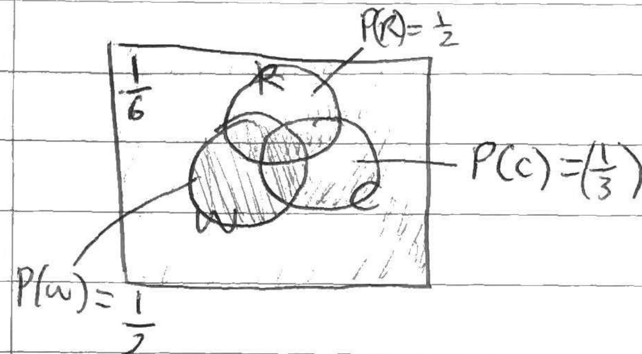
2. R= event where toy is red
W = event where toy is waterproof
C = event where toy is cool

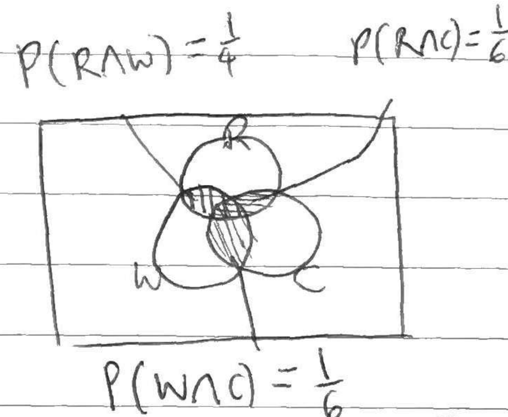
$P(R) = \frac{1}{2} \quad P(W) = \frac{1}{2} \quad P(C) = \frac{1}{3}$

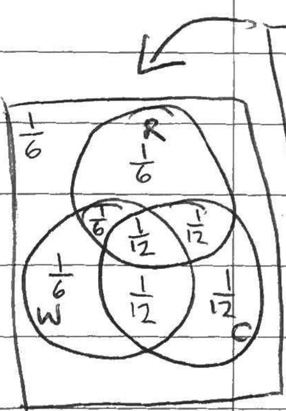
$P(R \cap W) = \frac{1}{4} \quad P(R \cap C) = \frac{1}{6} \quad P(W \cap C) = \frac{1}{6}$

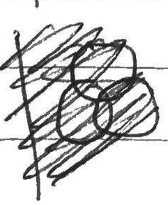
$P(R' \cap W' \cap C') = \frac{1}{6}$

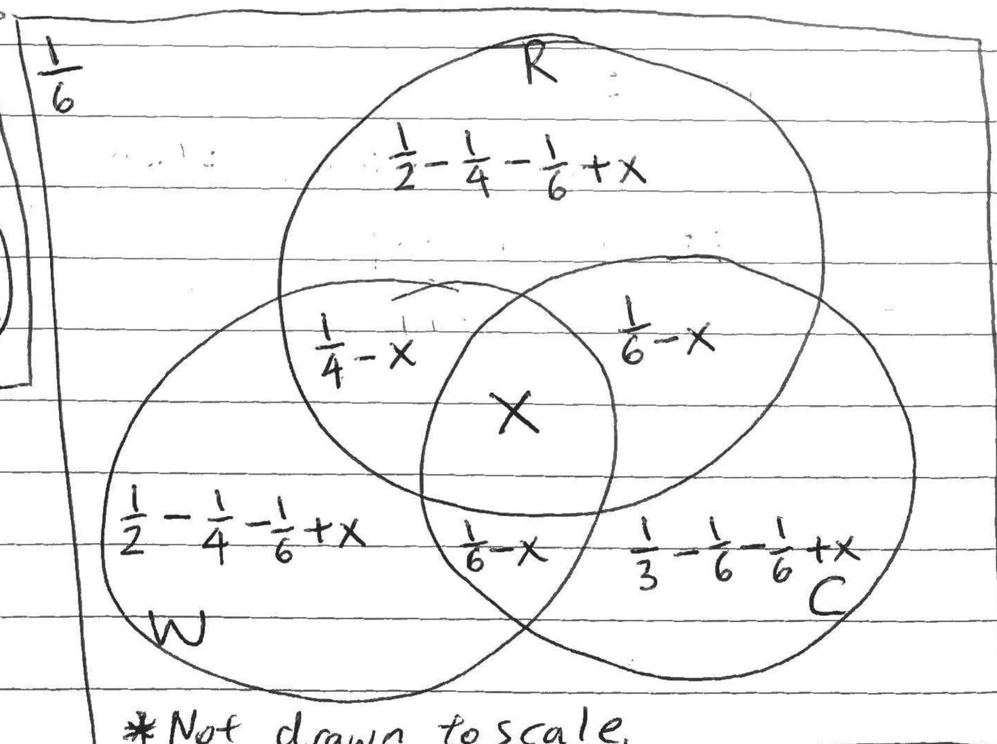
a)









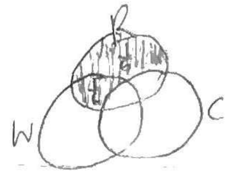


b) $1 - \frac{1}{6} = P(R) + P(W \cap R') + P(C \cap W' \cap R')$

$1 - \frac{1}{6} = \frac{1}{2} + \left[\frac{1}{2} - \frac{1}{4}\right] + \left[\frac{1}{3} - \frac{1}{6} - \frac{1}{6} + x\right]$

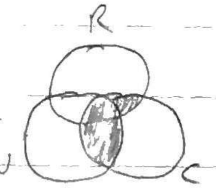
$x = \frac{5}{6} - \frac{1}{2} - \frac{1}{2} + \frac{1}{4} - \frac{1}{3} + \frac{1}{6} + \frac{1}{6} = \frac{1}{12}$

$\boxed{P(R \cap W \cap C) = x = \frac{1}{12}}$



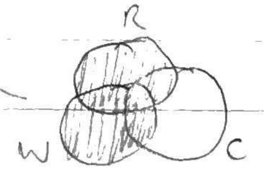
2c) solve for $P(C'|R)$

$$P(C'|R) = \frac{P(C' \wedge R)}{P(R)} = \frac{\frac{1}{6}+\frac{1}{6}}{\frac{1}{2}} = \frac{2}{3}$$



2d) solve for $P(C \mid R \cup W)$

$$P(C \mid R \cup W) = \frac{P(C \wedge (R \cup W))}{P(R \cup W)}$$



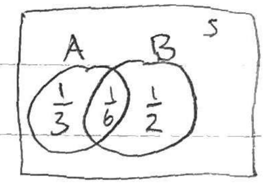
$$= \frac{\frac{1}{12}+\frac{1}{12}+\frac{1}{12}}{\frac{1}{2}+\frac{1}{6}+\frac{1}{12}} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$$

3. $P(A) = \frac{1}{2}$, $P(B) = \frac{2}{3}$, No more info about A and B.

a) Assuming events A and B are in the same state space, then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

$P(A \cup B) \leq 1$ so $\frac{1}{2} + \frac{2}{3} - P(A \cap B) \leq 1$

$\therefore P(A \cap B) \geq \frac{1}{6}$.

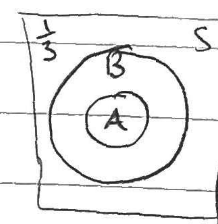


The min possible value for $P(A \cap B)$ is $\underline{\frac{1}{6}}$

* Note: we are not told that A and B are in the same state space so it is possible for $P(A \cap B) = 0$ if A and B are in seperate state spaces.

If A is a subset of B, then $P(A \cap B) = \frac{1}{2}$

$\therefore$ the max possible value for $P(A \cap B) = \frac{1}{2}$



b) $P(A|B) = \dfrac{P(A \cap B)}{P(B)} \Rightarrow \begin{cases} \text{min}: \dfrac{\frac{1}{6}}{\frac{2}{3}} = \frac{1}{4} \\ \text{max}: \dfrac{\frac{1}{2}}{\frac{2}{3}} = \frac{3}{4} \end{cases}$

* Note: if A and B are in separate state spaces, the min value for $P(A|B)$ would be 0.

4. L = event where student likes stats
C = event where student completed w203

$P(L|C) = \frac{3}{4}$ $\quad$ $P(C) = \frac{1}{100}$

$P(L|C') = \frac{1}{4}$

Find $P(C|L)$ $\quad$ $P(C|L) = \frac{P(C \cap L)}{P(L)}$

$$P(C \cap L) = P(L \cap C) = P(L|C)P(C) = \frac{3}{4} \cdot \frac{1}{100} = 0.0075$$

$$P(L) = P(\underbrace{L \cap C}_{0.0075}) + P(L \cap C')$$

$$P(L \cap C') = P(L|C')P(C')$$
$$= \frac{1}{4} \cdot \frac{99}{100}$$

$$P(L) = 0.0075 + \frac{1}{4} \cdot \frac{99}{100} = 0.255$$

$$\therefore P(C|L) = \frac{P(C \cap L)}{P(L)} = \frac{0.0075}{0.255} = 0.02941$$