Unit 4 Pre Class Warm-up.

The 'Pyramid' Distribution

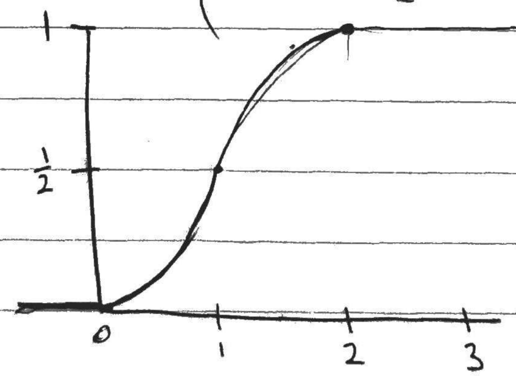
X is a continuous random variable with PDF:

$$f_X(x) = \begin{cases} x, & 0 \le x < 1 \\ 2-x, & 1 \le x < 2 \\ 0, & \text{otherwise.} \end{cases}$$

a) $F_X(x) = \int_{-\infty}^{x} f_X(x)\,dx$

$$= \int_0^1 x\,dx + \int_1^2 2-x\,dx = \left[\tfrac{1}{2}x^2\right]_0^1 + \left[2x - \tfrac{1}{2}x^2\right]_1^2$$

$$F_X(x) = \begin{cases} \tfrac{1}{2}x^2, & 0 \le x < 1 \\ \tfrac{1}{2} + 2x - \tfrac{1}{2}x^2 - 2 + \tfrac{1}{2} \end{cases} \Rightarrow F_X(x) = \begin{cases} 0, & x < 0 \\ \tfrac{1}{2}x^2, & 0 \le x < 1 \\ -\tfrac{1}{2}x^2 + 2x - 1, & 1 \le x < 2 \\ 1, & x \ge 2 \end{cases}$$



b) $E(x) = \int_{-\infty}^{\infty} x f(x)\,dx = \int_0^1 x^2\,dx + \int_1^2 2x - x^2\,dx = \left[\tfrac{1}{3}x^3\right]_0^1 + \left[x^2 - \tfrac{1}{3}x^3\right]_1^2$

$$= \tfrac{1}{3} - 0 + (2)^2 - \tfrac{1}{3}(2)^3 - (1)^2 + \tfrac{1}{3}(1)^3 = \tfrac{1}{3} + 4 - \tfrac{8}{3} - 1 + \tfrac{1}{3}$$

$$= 3 - \tfrac{6}{3} = 3 - 2 = 1$$

$E(x) = 1$

c. $Var(x) = E[(x-\mu)^2] = E(x^2) - [\overset{1}{\cancel{E(x)}}]^2$

$$E(h(x)) = \int_{-\infty}^{\infty} h(x)\, f(x)\, dx$$

$$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\, dx = \int_0^1 x^3\, dx + \int_1^2 2x^2 - x^3\, dx$$

$$= \left[\tfrac{1}{4}x^4\right]_0^1 + \left[\tfrac{2}{3}x^3 - \tfrac{1}{4}x^4\right]_1^2 = \tfrac{1}{4} + \tfrac{2}{3}(8) - \tfrac{1}{4}(16) - \tfrac{2}{3} + \tfrac{1}{4}$$

$$= \tfrac{1}{4} + \tfrac{16}{3} - 4 - \tfrac{2}{3} + \tfrac{1}{4} = \tfrac{1}{2} + \tfrac{14}{3} - 4 = \tfrac{3}{6} + \tfrac{28}{6} - \tfrac{24}{6}$$

$E(x^2) = \frac{7}{6}$, $Var(x) = \frac{7}{6} - 1 = \frac{1}{6}$ $\boxed{Var(x) = \frac{1}{6}}$

d) $Y(x) = x^2$, Y is also a random variable because a random variable is the function from a probability space to the real numbers space. $Y(x) = x^2$ is also a function that maps a real numbers space to the real numbers space.

e) $E(Y) = E(x^2)$, as shown in c), $\boxed{E(x^2) = \frac{7}{6}}$