ADC part 1 impedance model
16.10.2025

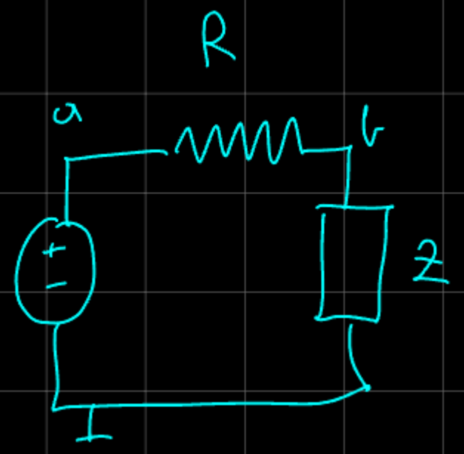
Objectives; Measure the impedance of different resistors

Background; The impedance model allows to extend the notion of resistance to all elements assuming they are operating in sinusoidal steady state. For different elements it has different forms

$V_{in} = e^{st}$ $\quad s \in \mathbb{C}$ $\quad i = C \cdot \frac{dV}{dt}$ $\quad i = C \cdot s \cdot e^{st}$ $\quad \frac{e^{st}}{i} = \frac{1}{Cs} = \frac{1}{j\omega C}$

$R = \frac{U}{I} = \frac{U}{I}$ $\quad V = L \cdot \frac{di}{dt}$ $\quad i = \frac{1}{L}\int e^{st}\,dt = \frac{1}{Ls} e^{st}$ $\quad \frac{e^{st}}{i} = Ls = j\omega L$

Since the oscilloscope measures the voltage relative to ground it's impossible to directly measure voltage across R since it would be a-b and the oscilloscope can only measure voltage x relative to ground. For this reason I measure node a with reference to ground and then node b with reference to ground and them subtract them using the MATH function on the



$R = \frac{U}{I}$ $\quad I = \frac{U_{ab}}{R}$

$Z = \frac{U_b}{I} = \frac{U_b}{U_{ab}} \cdot R$

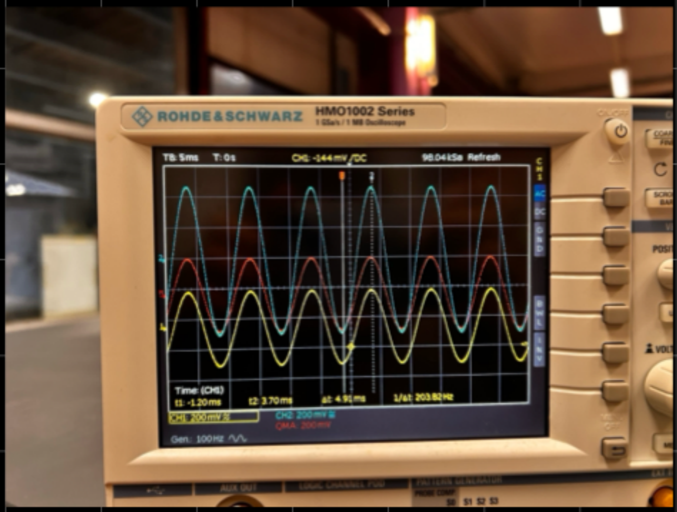
Connecting two probes at a and b would short circuit Z

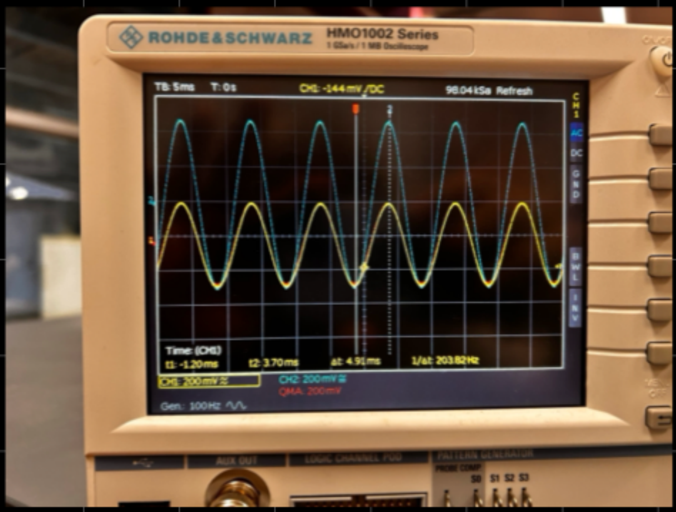
Additionally the oscilloscope can't measure current so the way to find a current going through a element is to put a resistor (with a known resistance) in series with it and using ohms law find the current through it.

Materials;

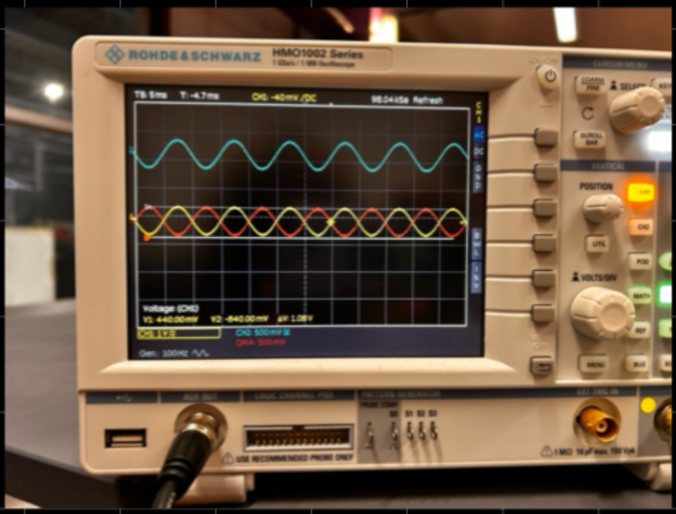
2x 1kOhm resistor, breadboard, oscilloscope probes, oscilloscope function generator

Results;





On the oscilloscope there are three waveforms. Blue is the voltage across both resistors, yellow is the voltage across the impedance and red is their difference which should be equal to the voltage on the other resistor. By analysing the waveforms it is visible that they are in fact the same: they have the same frequency and amplitude and phase.



On two photos above I measure the voltage of the input signal using the oscilloscope and the multimeter to challenge its accuracy. The amplitude recorded by the oscilloscope is 1.0V and the one recorded by the multimeter is 1.043V which is within the desired level of accuracy and is satisfactory to confirm the reliability of the measurements provided by the oscilloscope.

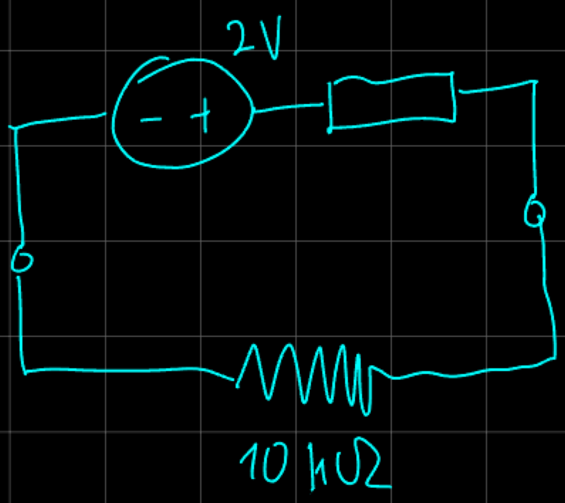
Analysis; Small discrepancies of 1.0V (amplitude measured using the oscilloscope) and 1.043V (amplitude measurement using multimeter) arise from imprecise measurements with the oscilloscope, since the signal is fuzzy it is hard to exactly see where it begins and ends.

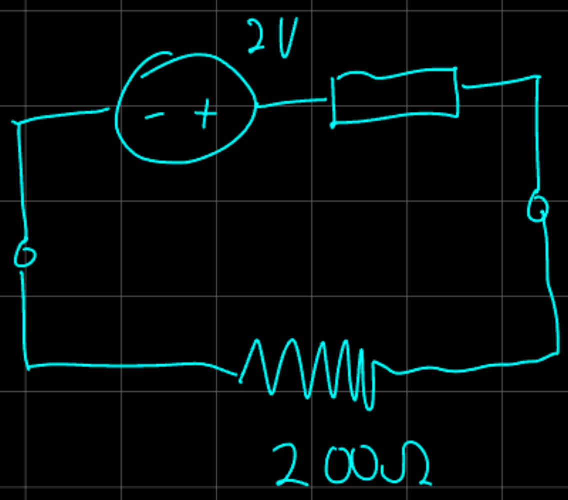
Challenge question; source impedance

In text it is suggested that the function generator provided within the oscilloscope also has some impedance. It can be measured by using a series connection with a resistor of a known resistance and a voltage source with a known value. Than it is sufficient to apply a simple voltage divider pattern to estimate the sources impedance .

pattern to estimate the sources impedance .

Source model





$$1.949 = 2 \cdot \frac{10^4}{Z + 10^4}$$

$Z \approx 261{,}673\,\Omega$

$$1.552 = 2 \cdot \frac{200}{Z + 200} \Rightarrow Z = 57{,}73\,\Omega$$

ADC PART 2 REACTIVE COMPONENTS

HEADER)

20.10.2025 Imperial College London main campus

OBJECTIVE)

Find how accurate the impedance model is for capacitors and inductors. Perform this through measuring the impedance of both elements at different frequencies.

BACKGROUND)

When a device operates in SSS we can leverage the impedance model to avoid having to solve differential equations to find state variables.

Mathematically this can be described as ;

$V_i = e^{st} \quad s \in \mathbb{C} \qquad i = C\cdot\frac{dV}{dt} \qquad i = C\cdot s\cdot e^{st} = CsV_i \Rightarrow \frac{1}{Cs} = \frac{V_i}{i} = Z$

$V_i = e^{st} \quad s \in \mathbb{C} \qquad V = L\cdot\frac{di}{dt} \qquad i = \frac{1}{L}\int e^{st}dt = i = \frac{1}{Ls}V_i \Rightarrow \frac{V_i}{i} = Ls = Z$

Sadly often real life components don't behave idealy and have resistances as well, for this reason they can be modelled with a impedance with a resistor in series.

$Z_L = j\omega L + R \qquad Z_C = \frac{1}{j\omega C} + R$

MATERIALS AND METHODS)

Materials;

10 ohm Resistor, 1kOhm,

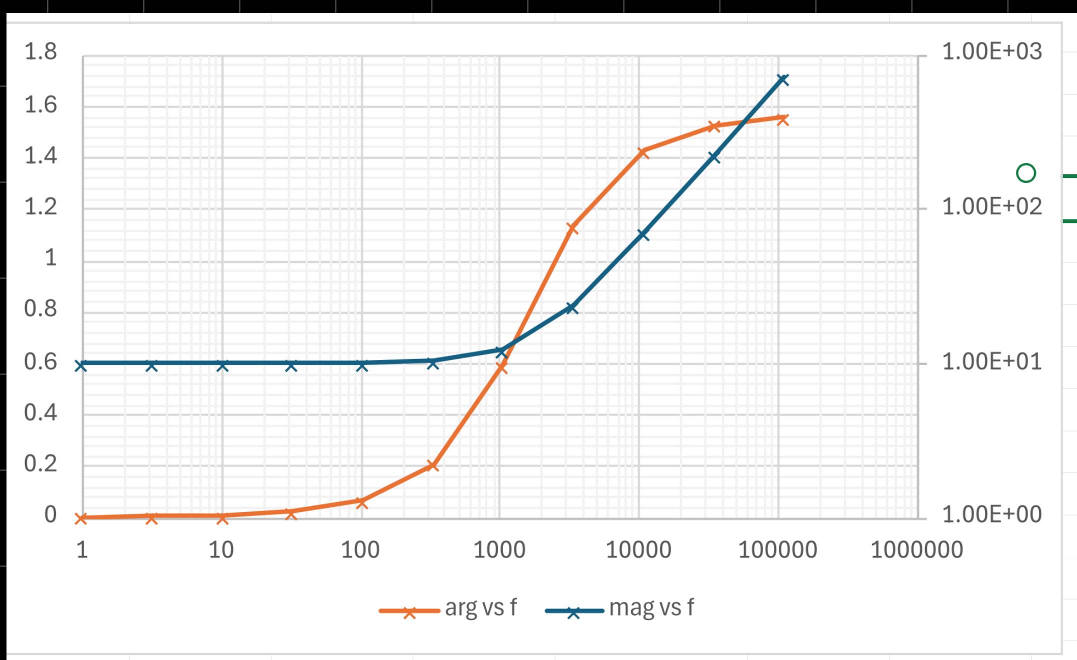
Methods;

Firstly I'm going to make a theoretical spreadsheet which follows the equation

To see how an ideal inductor in series with a resistor behaves.

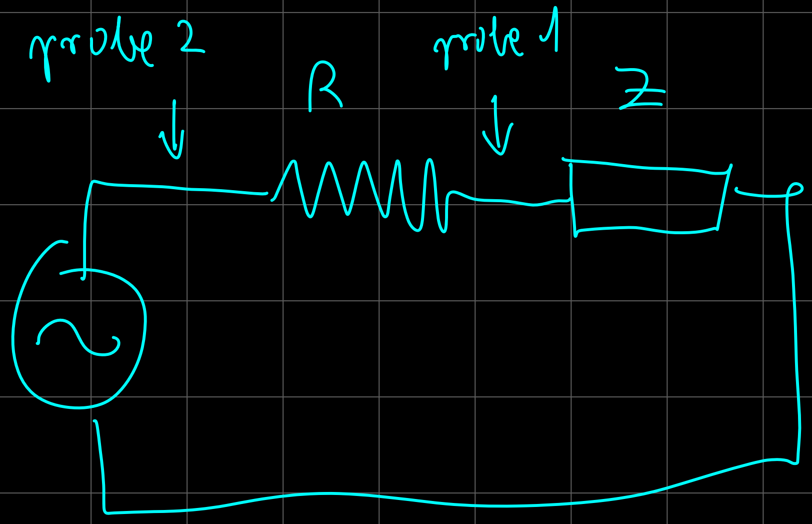
$|Z_L|(f) = \sqrt{4\pi^2\cdot10^{-6}f^2 + 100} \qquad arg(Z) = \arctan\left(\frac{\omega L}{R}\right)$

f	$\lvert Z_L \rvert$	$arg(Z_L)$ deg
1	10,000002	0.036
10	10.000197	0.3599
100	10.01792	3,595
1000	11.81	32.141
10000	63,623	80.957



arg(Z) and mag as a function of f

To check the impedance model I will employ the following method. I will make a series RC and RL circuit and use a voltage divider to establish an easy relationship between measured voltages and the impedance.



$$V_{probe\,2} = V_R + V_Z \qquad V_{Probe\,1} = V_Z$$

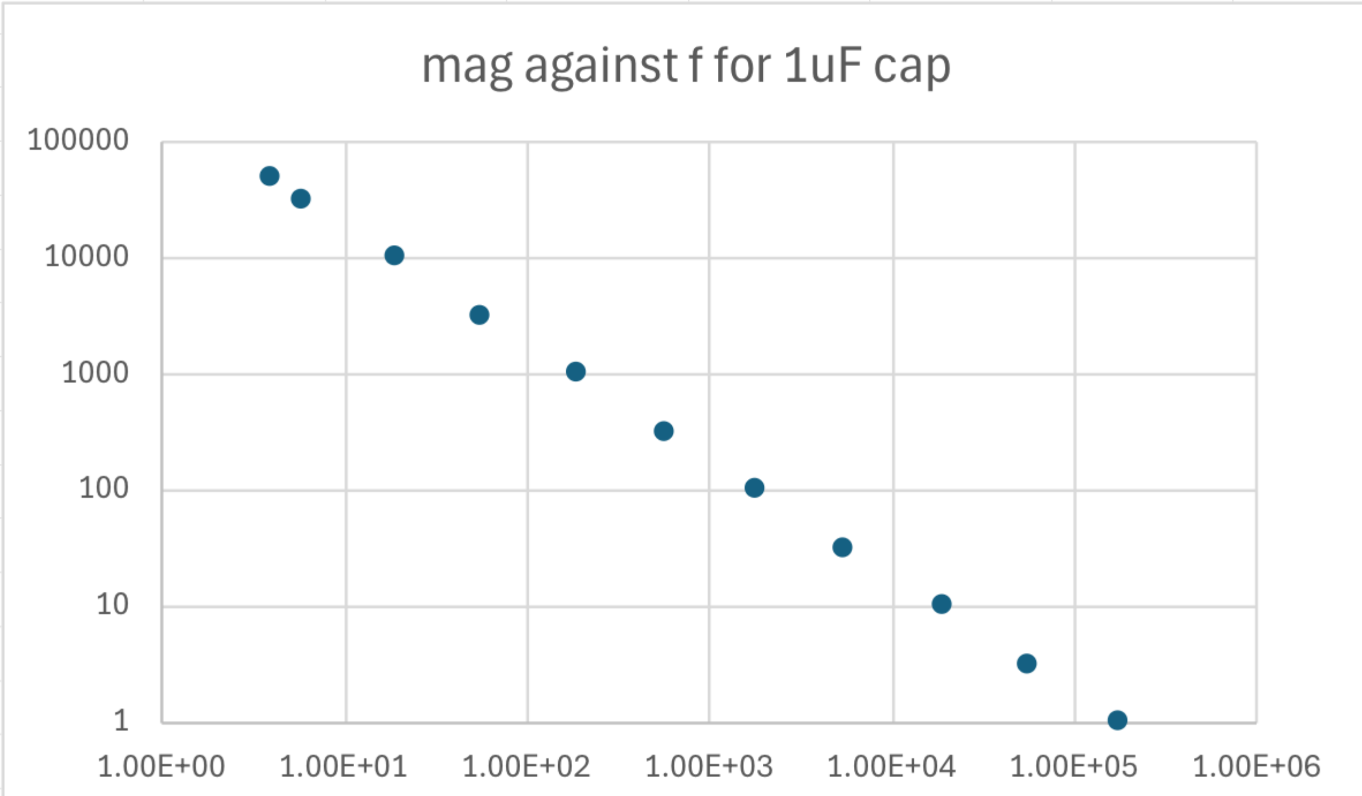
$$V_R = V_{Probe\,2} - V_{probe\,1}$$

$$I = \frac{V_R}{R} = \frac{V_Z}{Z} \Rightarrow Z = R \cdot \frac{V_Z}{V_R}$$

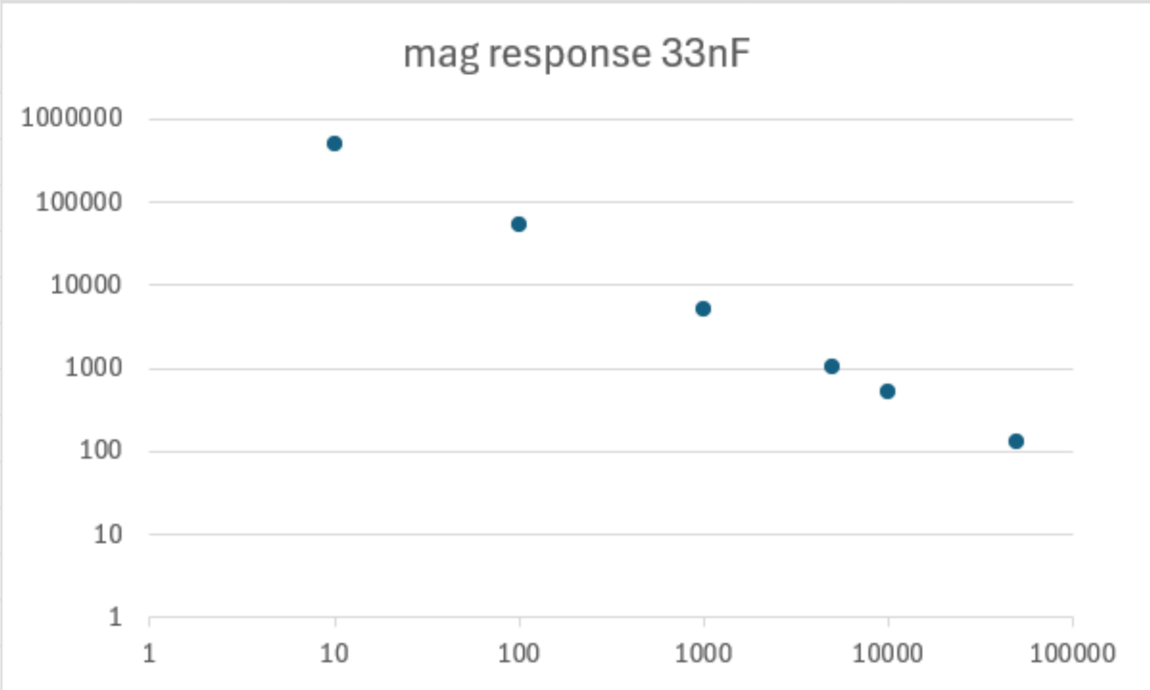
Then I will use the function generator to input sinusoid of different frequencies to the circuit to see how well the impedance model describes the actuall behaviour of the circuit.

Results;

	R	vr/mv	vz/mv	vr	vz	mag z	1/mag z	const
1	1.00E+05	1.70E+01	2.92E+01	1.70E-02	2.92E-02	1.72E+05	5.82E-06	1.72E+05
3.2	1.00E+05	3.09E+01	1.70E+01	3.09E-02	1.70E-02	5.49E+04	1.82E-05	1.76E+05
10	1.00E+05	35.14	6.44E+00	3.51E-02	6.44E-03	1.83E+04	5.46E-05	1.83E+05
32	1.00E+03	6.37	3.44E+01	6.37E-03	3.44E-02	5.40E+03	1.85E-04	1.73E+05
100	1.00E+03	1.66E+01	29.12	1.66E-02	2.91E-02	1.75E+03	5.71E-04	1.75E+05
320	1.00E+03	28.41	1.60E+01	2.84E-02	1.60E-02	5.64E+02	1.77E-03	1.80E+05
1000	1.00E+03	31.77	5.93E+00	3.18E-02	5.93E-03	1.87E+02	5.36E-03	1.87E+05
3200	1.00E+01	4.07	2.24E+01	4.07E-03	2.24E-02	5.50E+01	1.82E-02	1.76E+05
10000	1.00E+01	5.08	9.39E+00	5.08E-03	9.39E-03	1.85E+01	5.41E-02	1.85E+05
32000	1.00E+01	5.12	2.98E+00	5.12E-03	2.98E-03	5.82E+00	1.72E-01	1.86E+05
50000	1.00E+01	4.86	1.85E+00	4.86E-03	1.85E-03	3.81E+00	2.63E-01	1.90E+05

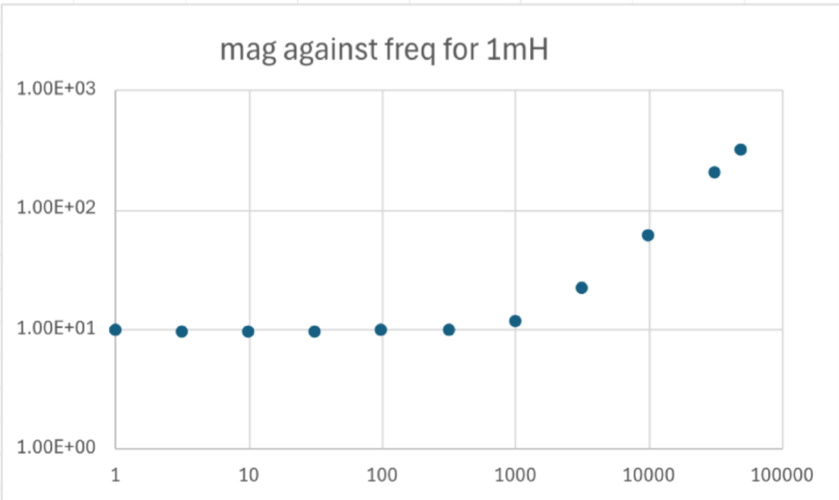


f	R	Vz (mV)	Vr (mV)	|Z|
10	100000	68,3	13,79	496945
100	100000	31,73	61,35	51719,64
1000	1000	67,88	13,5	5028,148
5000	1000	48	46,9	1023
10000	1000	30,55	58,6	521,33
50000	1000	7,6	59,8	130,43



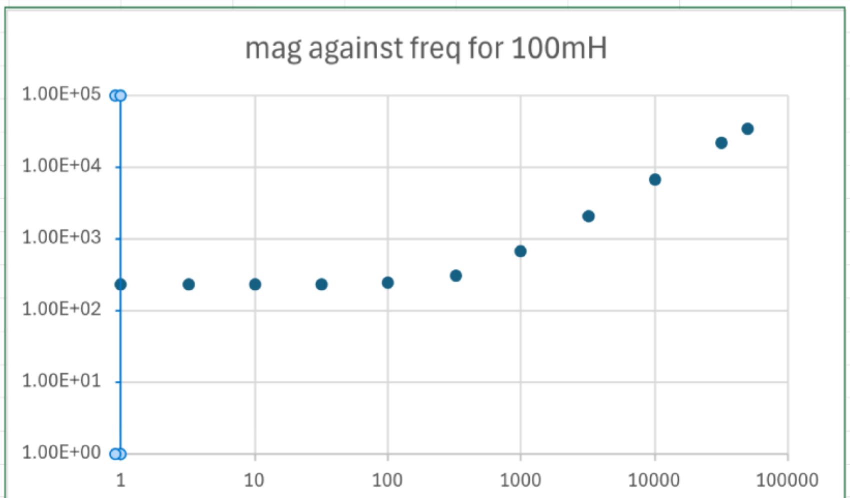
1mH:

f	R	vr/mv	vz/mv	vr	vz	mag z
1	1.00E+01	4.72E+01	4.55E+01	4.72E-02	4.55E-02	9.65E+00
3.2	1.00E+01	47.52	4.55E+01	4.75E-02	4.55E-02	9.57E+00
10	1.00E+01	47.3	45.37	4.73E-02	4.54E-02	9.59E+00
32	1.00E+01	47.44	4.53E+01	4.74E-02	4.53E-02	9.55E+00
100	1.00E+01	4.68E+01	4.52E+01	4.68E-02	4.52E-02	9.66E+00
320	1.00E+01	46.57	4.63E+01	4.66E-02	4.63E-02	9.93E+00
1000	1.00E+01	46.78	5.39E+01	4.68E-02	5.39E-02	1.15E+01
3200	1.00E+01	45.42	1.01E+02	4.54E-02	1.01E-01	2.22E+01
10000	1.00E+01	36.56	2.23E+02	3.66E-02	2.23E-01	6.09E+01
32000	1.00E+03	298.84	6.11E+01	2.99E-01	6.11E-02	2.04E+02
50000	1.00E+03	280	8.74E+01	2.80E-01	8.74E-02	3.12E+02



100mH

f	R	vr/mv	vz/mv	vr	vz	mag z
1	1.00E+03	2.70E+02	6.10E+01	2.70E-01	6.10E-02	2.26E+02
3.2	1.00E+03	272	6.10E+01	2.72E-01	6.10E-02	2.24E+02
10	1.00E+03	271.9	6.10E+01	2.72E-01	6.10E-02	2.24E+02
32	1.00E+03	267.9	6.06E+01	2.68E-01	6.06E-02	2.26E+02
100	1.00E+03	2.67E+02	6.28E+01	2.67E-01	6.28E-02	2.35E+02
320	1.00E+03	263.39	8.07E+01	2.63E-01	8.07E-02	3.06E+02
1000	1.00E+03	240.5	1.62E+02	2.41E-01	1.62E-01	6.73E+02
3200	1.00E+03	143.28	2.91E+02	1.43E-01	2.91E-01	2.03E+03
10000	1.00E+03	53	3.56E+02	5.30E-02	3.56E-01	6.71E+03
32000	1.00E+03	15.6	3.29E+02	1.56E-02	3.29E-01	2.11E+04
50000	1.00E+03	9.1	3.14E+02	9.10E-03	3.14E-01	3.45E+04



ANALYSIS)

For both capacitors achieved results are very close (up to 3 decimal places) to the predicted by the impedance model value. This shows impedance can be used to analyse circuits with capacitors across a wide ranges of frequencies. For the inductor the behaviour at low frequencies (up to 1kHz) becomes unpredictable and chaotic due to the parasitic resistance of wires taking over.

Challenge:

I picked a capacitor of unknown capacitance. I again implemented the same setup and used a single frequency of a sinusoid. Leveraging an earlier established equation involving the voltage across the capacitor and the resistor and its resistance I found the impedance of the element. Than from that using the thereotical model i found its capacitance with a very small error of 3% which is an acceptable result.

$$C_M = 32\ nF \qquad C_T = 33\ nF \qquad E_{rr} = \frac{C_T - C_M}{C_T} \approx 3\%$$

ADC section 3

Before the lab;

Design a RLC circuit with a desired break frequency (where the power delivered by the signal drops to 1/2) and a desired resonant frequency

1 degree of freedom so pick $R = 10^3\,\Omega$

$$R = 2\sqrt{\frac{L}{C}} \qquad \omega_0 = \frac{1}{\sqrt{LC}} \qquad \omega_0 = 2\pi \cdot 3400\,Hz$$

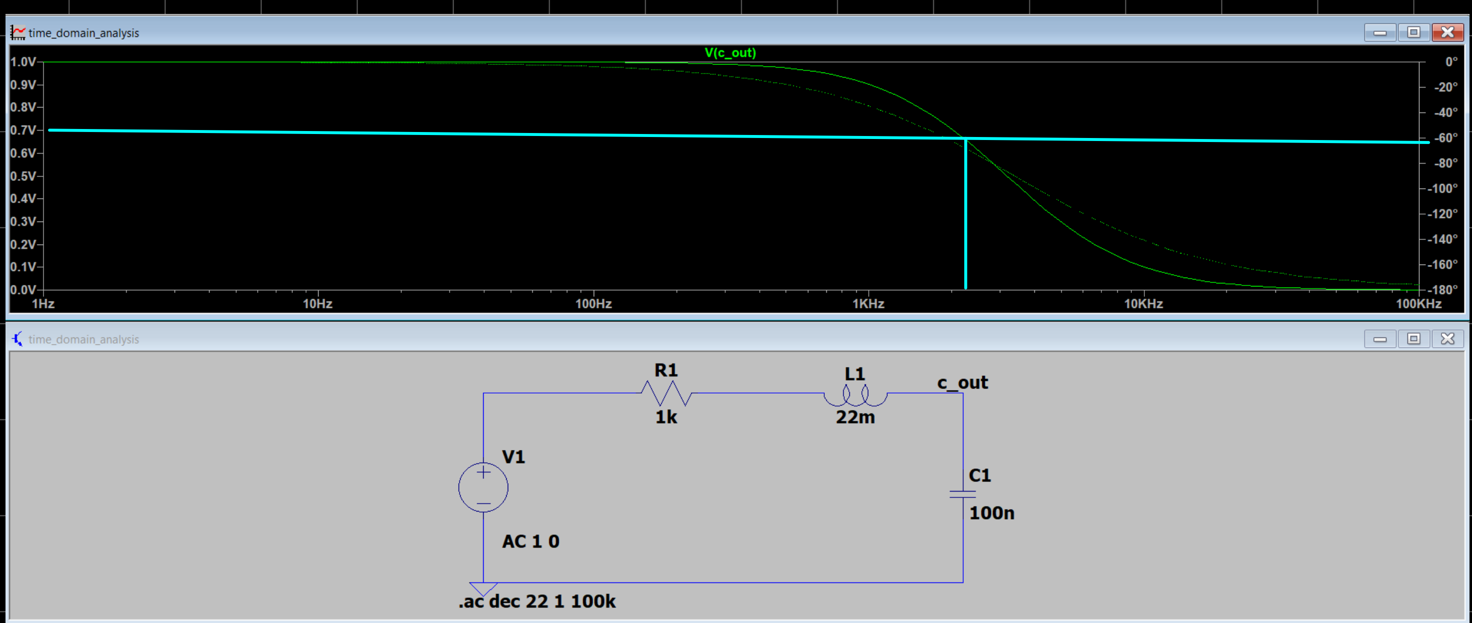
$R = 10^3\,\Omega$

$L = 23\ mH$

$$C\,\frac{10^6}{4} = L \qquad \frac{1}{\omega_0^2 C} = L \qquad \frac{10^6}{4} C^2 = \frac{1}{\omega_0^2} \qquad C = \frac{2}{\omega_0 10^3} = 93.6\ nF$$

Choosen Values ; $1k\Omega$ $\quad 22mH$ $\quad 100nF$

I built this circuit in LS spice and simulated it to check accuracy



HEADER)

Imperial College London, main campus, 23.10.2025

OBJECTIVES)

Asses and explore the limitations of the impedance model and analyse resonant circuits of second degree. This experiments will be conducted for a series RLC circuit as well as a series combination of a resistor and parallel capacitor and inductor. Additionally I will analyse the behaviour of a first order RC circuit which will serve as a high pass filter.

BACKGROUND)

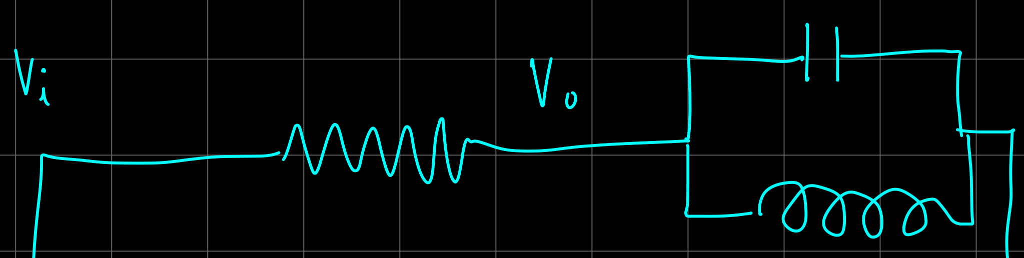
In a series RLC circuit the voltage measured across both the capacitor and inductor can be forecasted using the impedance model.

$$\frac{V_o}{V_i} = \frac{\frac{1}{j\omega C} + j\omega L}{R + \frac{1}{j\omega C} + j\omega L}$$

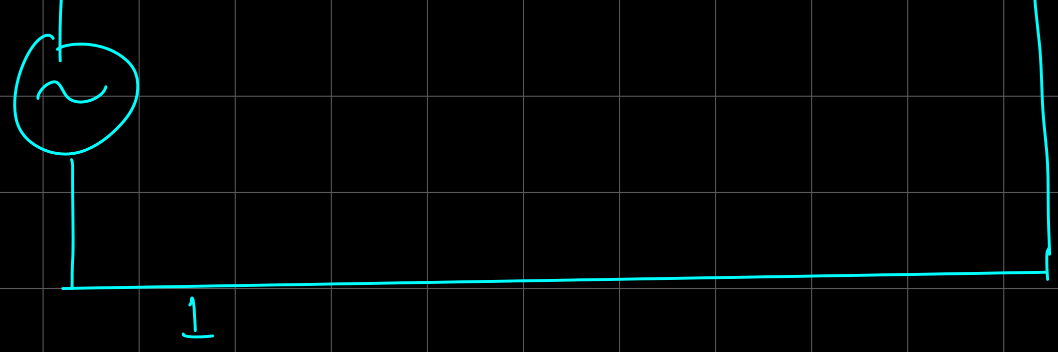
$$\lim_{\omega \to \omega_0} \frac{V_o}{V_i} = 0$$

For this reason as the input frequency approaches the resonant frequency the impedance of the capacitor and inductor cancel out leaving only the impedance coming from the resistor. However in real life idealised objects have parasitic impedances which don't allow for a exact zero resultant impedance of a series LC connection.

In a circuit which contains a parallel inductor and capacitor in series with a resistor we can again employ the impedance model to make predictions about its behaviour.



$$\frac{V_o}{V_i} = \frac{\frac{L\omega j / j\omega C}{L\omega j + 1/j\omega C}}{R \quad L\omega j / j\omega C}$$



$$R + \frac{\quad}{L\omega j + 1/j\omega C}$$

$$\lim_{\omega \to \omega_0^+} \frac{L\omega j / j\omega C}{L\omega j + 1/j\omega C} = \infty$$

So this circuit is a duel of the earlier one. In the case of connecting LC in parallel their impedance tends to infinity as the angular frequencies approaches the resonant frequency. However this theoretical model has limitations. In the real world the impedance obviously can't be infinite and will be limited by parasitic impedances of elements.

For a first order RC circuit the transfer function is as follows;

$$\frac{V_o}{V_i} = \frac{1/j\omega C}{R + 1/j\omega C} = \frac{1}{1 + Rj\omega C}$$

According to the impedance model this circuit with a voltage measured across the capacitor will act as a low pass filter meaning that for inputs of low frequencies it will not significantly influence the amplitude of the output signal but for higher frequencies it will atennuate the output.

MATERIALS AND METHODS)

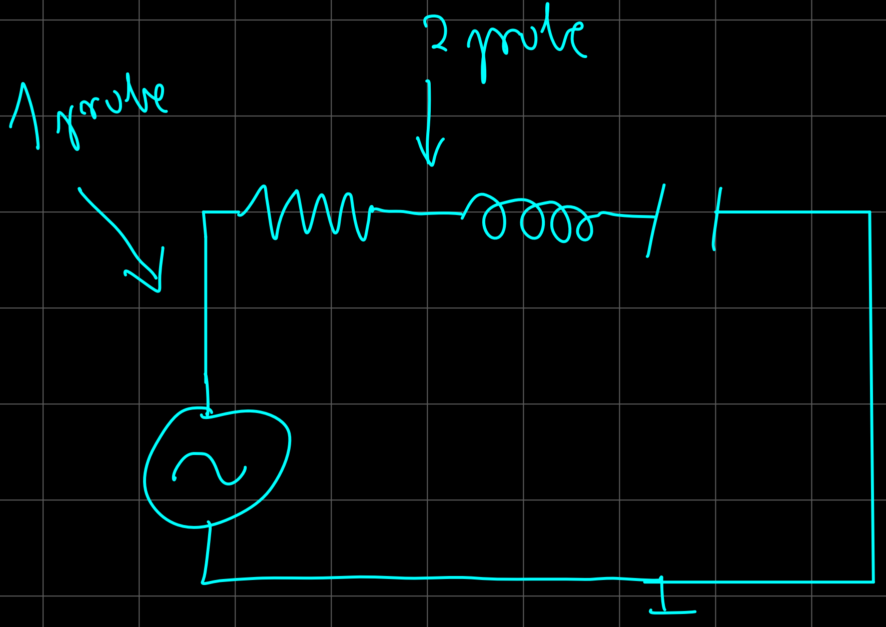
R; $10\,\Omega$, $1\,k\Omega$, $10\,k\Omega$, $100\,k\Omega$, $1\,M\Omega$

L; $100\,mH$

C; $100\,nF$

$$\omega_0 = 2\pi f = \frac{1}{\sqrt{LC}} \qquad f = 1{,}591\ kHz$$

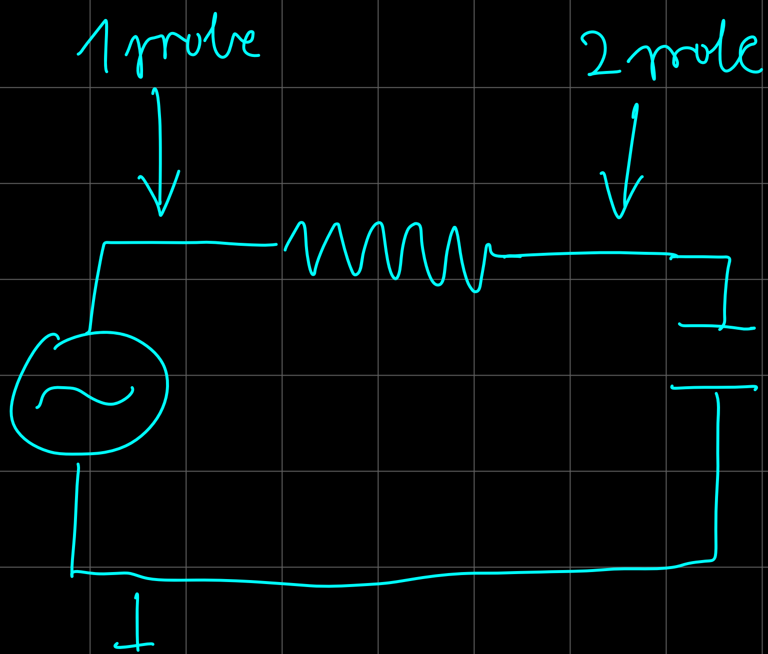
For the purpose of this laboratory we will employ a similar method of measuring impedance as in the previous labs. That is voltage input will be the function generator within the oscilloscope. Then two probes will be used to measure the voltage across the impedance and the other one will measure the sum of the voltages across the impedance and the resistance. Then by using the maths channel one will be subtracted from another to obtain the voltage across the resistor. Then since element are in series



$$I = \frac{U}{R} = \frac{V_R}{R} = \frac{V_Z}{Z} \qquad Z = \frac{V_Z}{V_R} \cdot R$$

Since impedance varies with frequency I will input sinusoids of different frequencies and observe for which the impedance is the lowest and highest for the first and second circuit respectively. This will allow me to find the resonant frequency.

For the filter circuit I will not need to measure the impedance. Measuring the voltage across a given element will be enough. For this purpose I will use two probes of the oscilloscope



$$T(\omega) = \frac{V_2}{V_1}$$

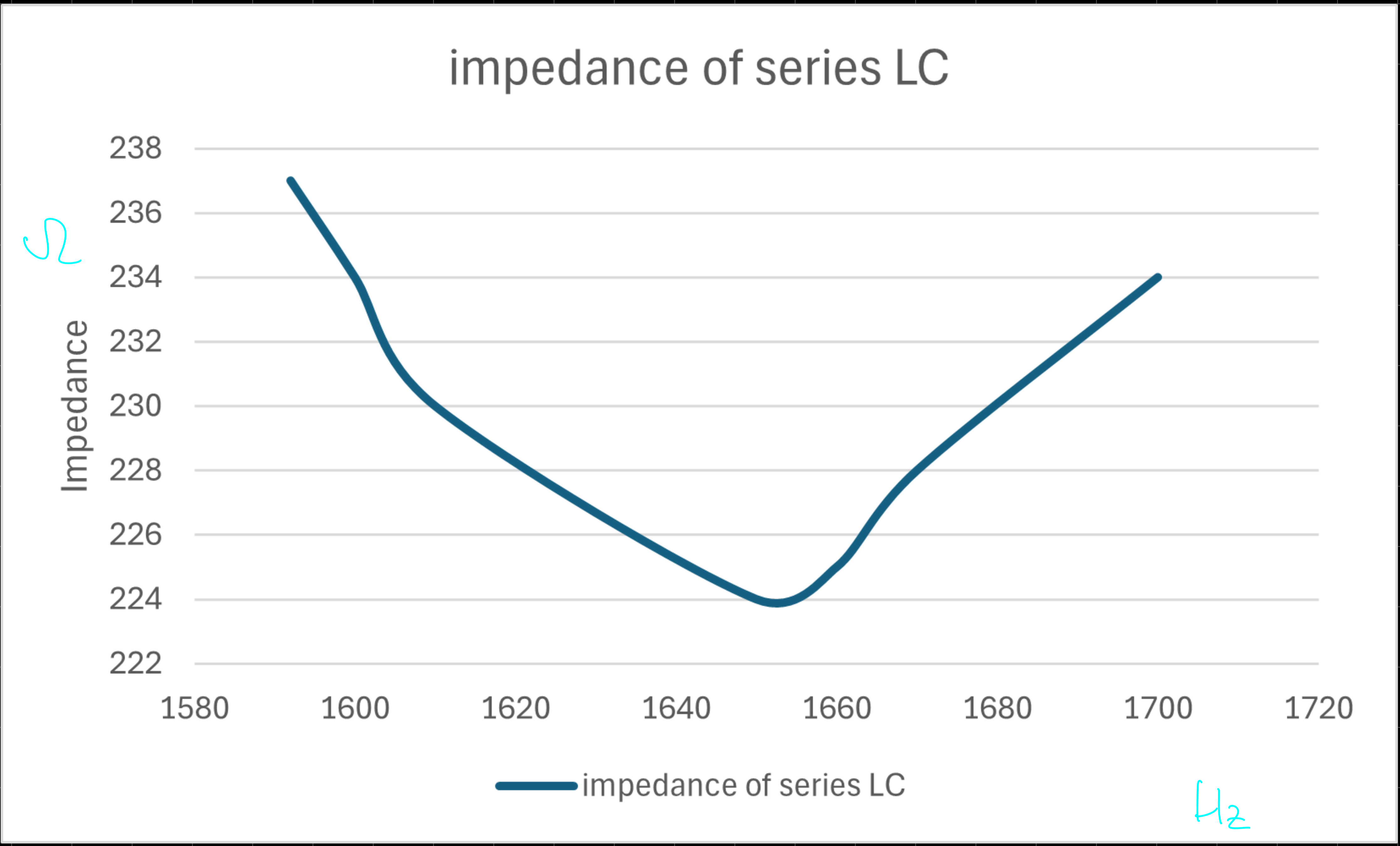
Then I will record the magnitude of the transfer function at different frequencies and plot its phase as well.

RESULTS)

Series RLC circuit result (impedance measured across L and C)

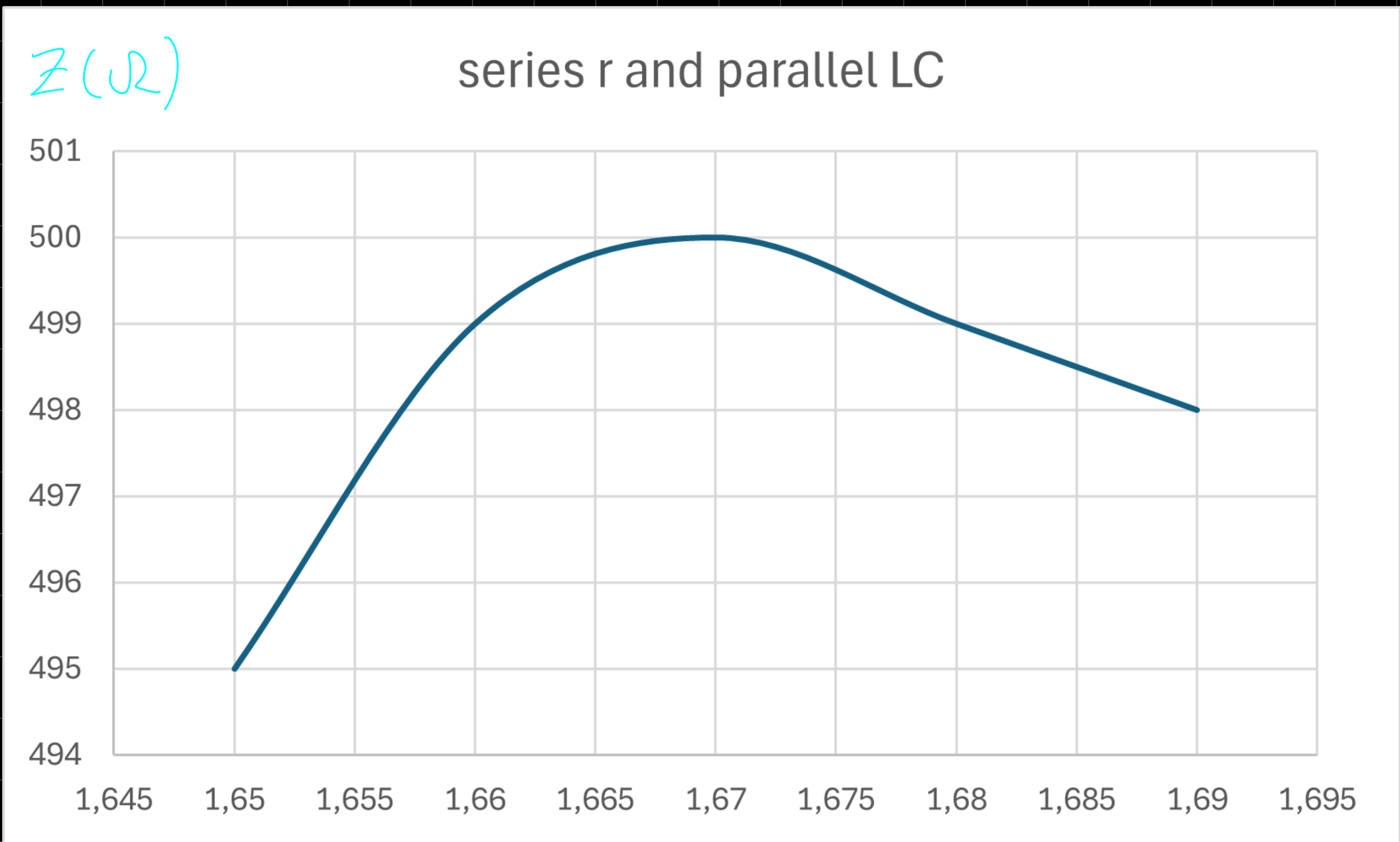
$R\ (\Omega)$	$V_Z\ (V)$	$V_R\ (V)$	$f\ (kHz)$	$Z\ (\Omega)$
10^3	31.9	134.1	1.592	237

10^3	31.5	134.5	1.6	234
10^3	31.1	134.9	1.61	230
10^3	30.4	135.6	1.65	224
10^3	30.5	135.5	1.66	225
10^3	30.5	133.5	1.67	228
10^3	31.5	134.5	1.7	234



Series R and parallel L and C results (impedance across parallel L and C)

R (Ω)	V_Z (V)	V_R (V)	f (MHz)	Z (Ω)
10^3	57	115	1.65	495
10^3	57.3	114.7	1.66	499
10^3	57.4	114.6	1.67	500
10^3	57.3	114.7	1.68	499
10^3	57.2	114.8	1.69	498

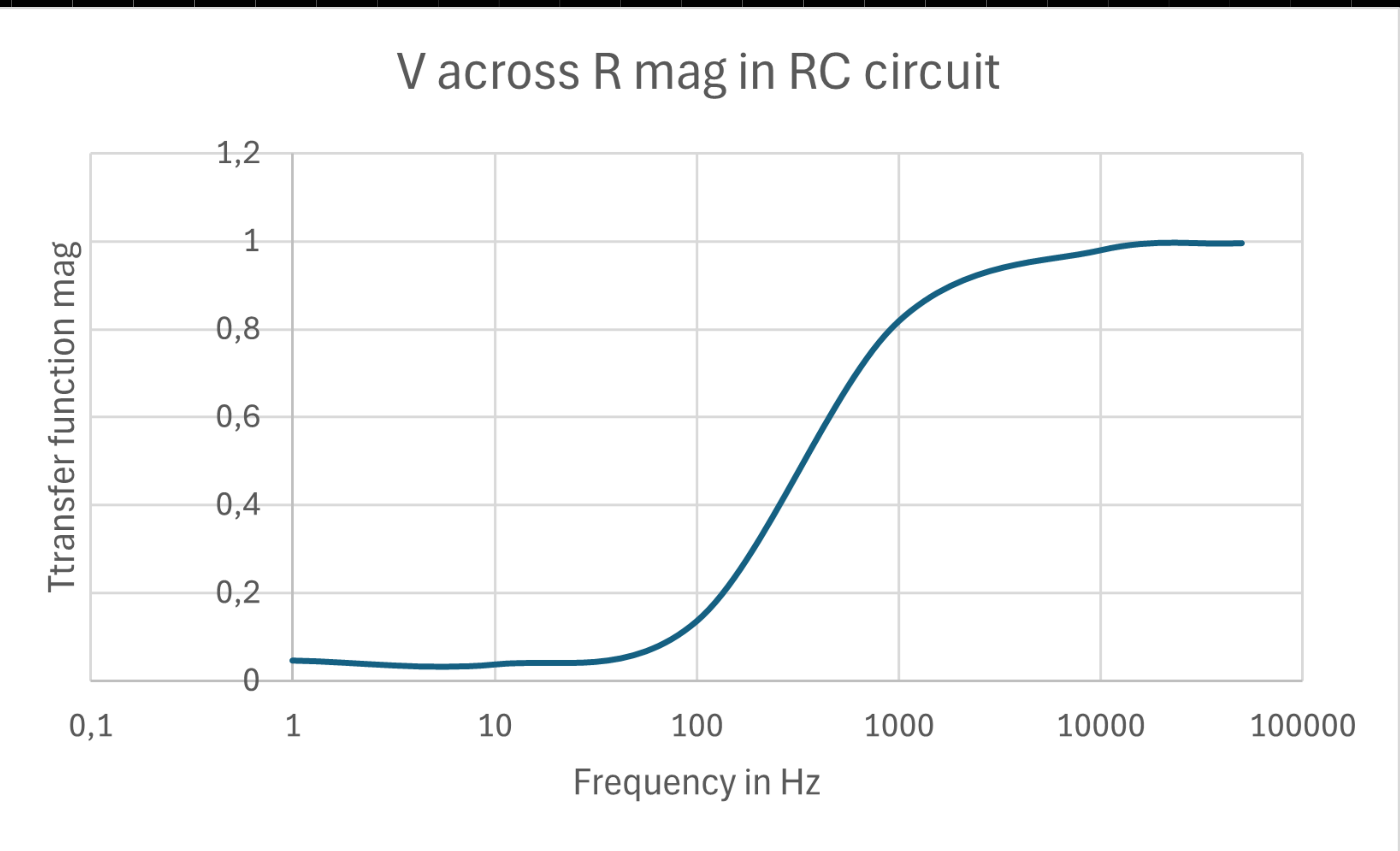


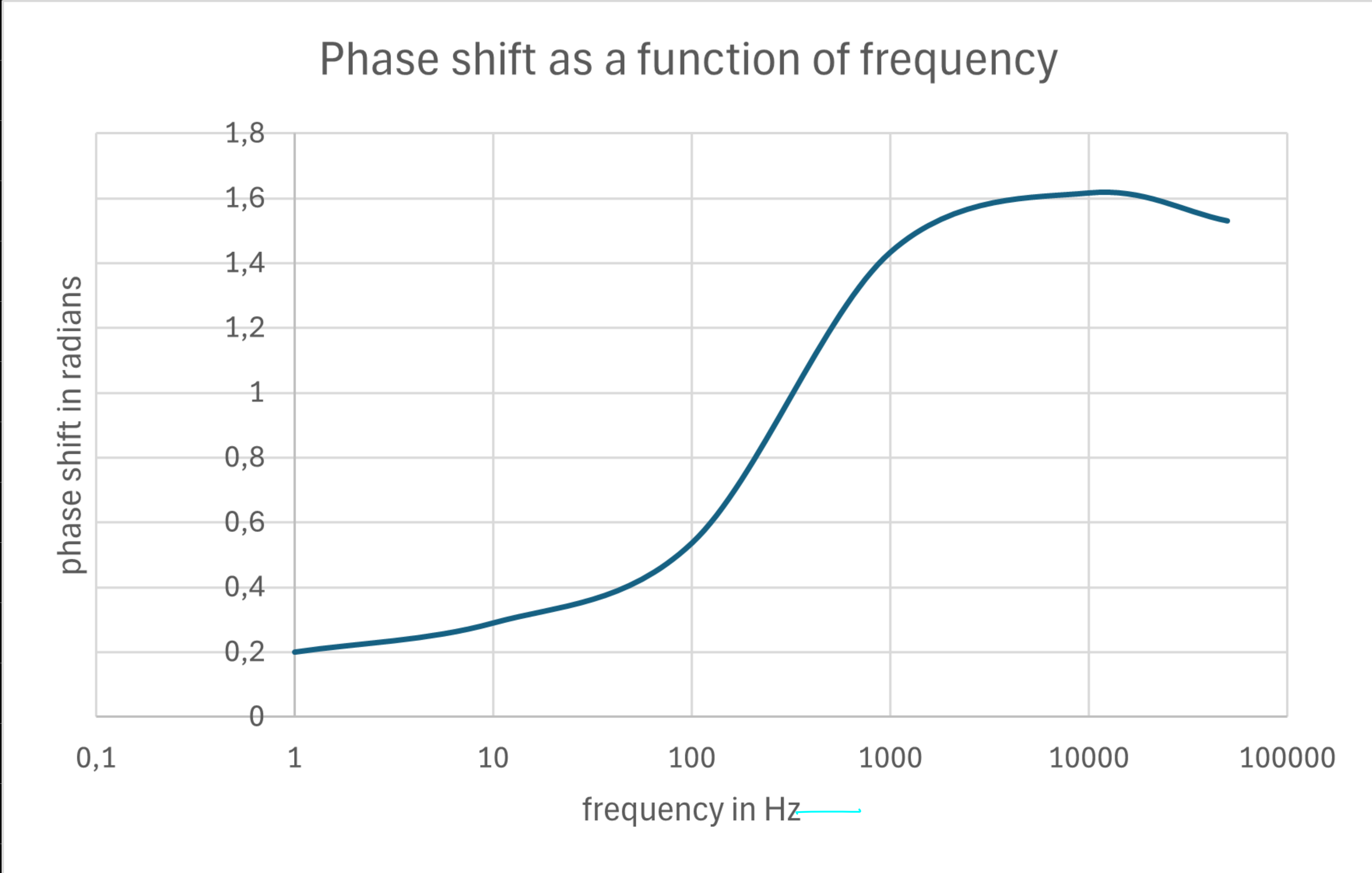
f (MHz)

Measurements across R in the first order RC filter

f (Hz)	V_i (mV)	V_o (mV)	$\lvert T(\omega) \rvert$	ΔT

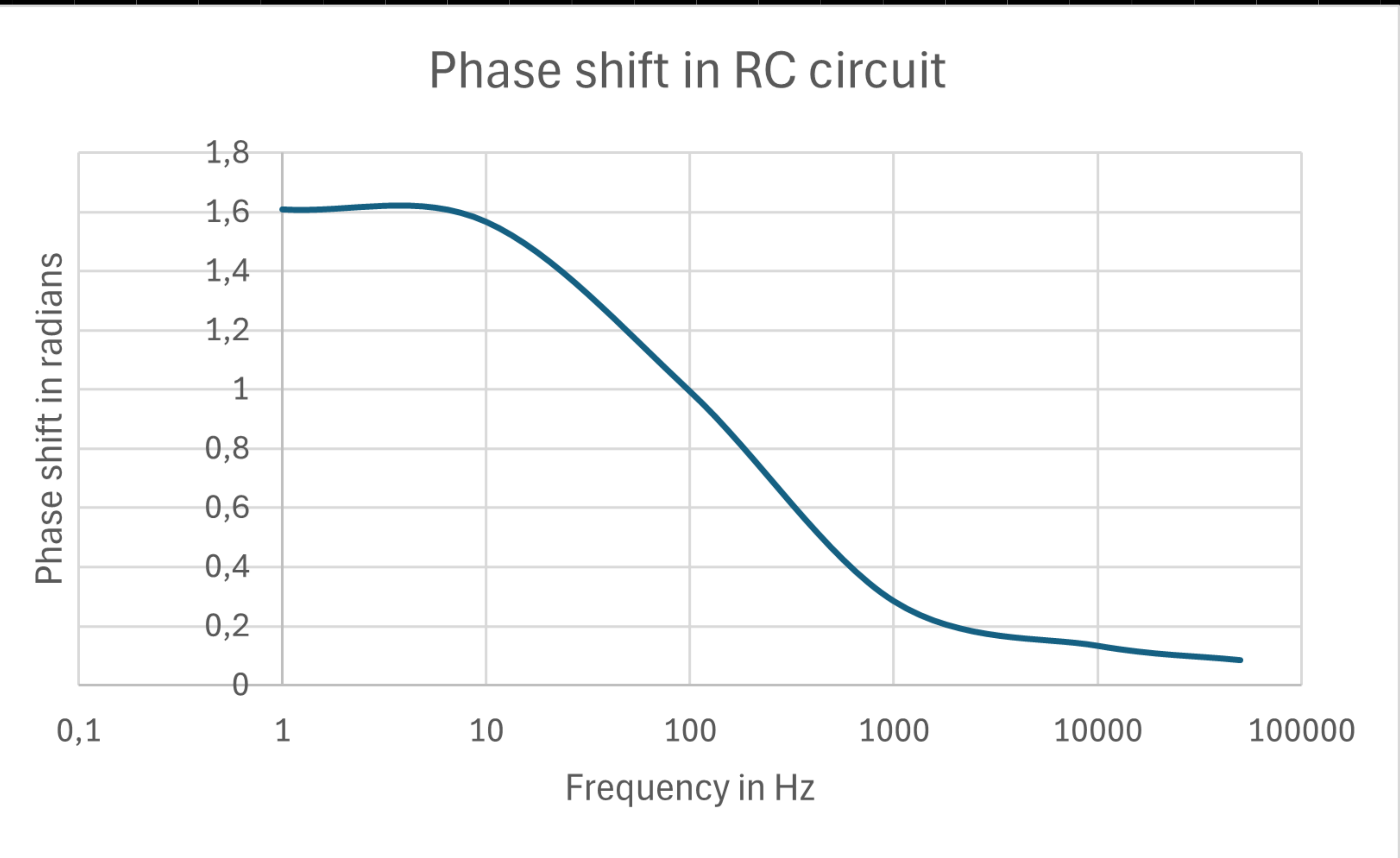
10^0	331	15	0.04531	256 ms
10^1	357	13	0.036414	24.93 ms
10^2	341	46	0.134897	1.58 ms
10^3	325	266,2	0.81846	45.2 µs
10^4	323	316.85	0.98095	2.12 µs
$5 \cdot 10^4$	297	295.85	0.9961	270 ns

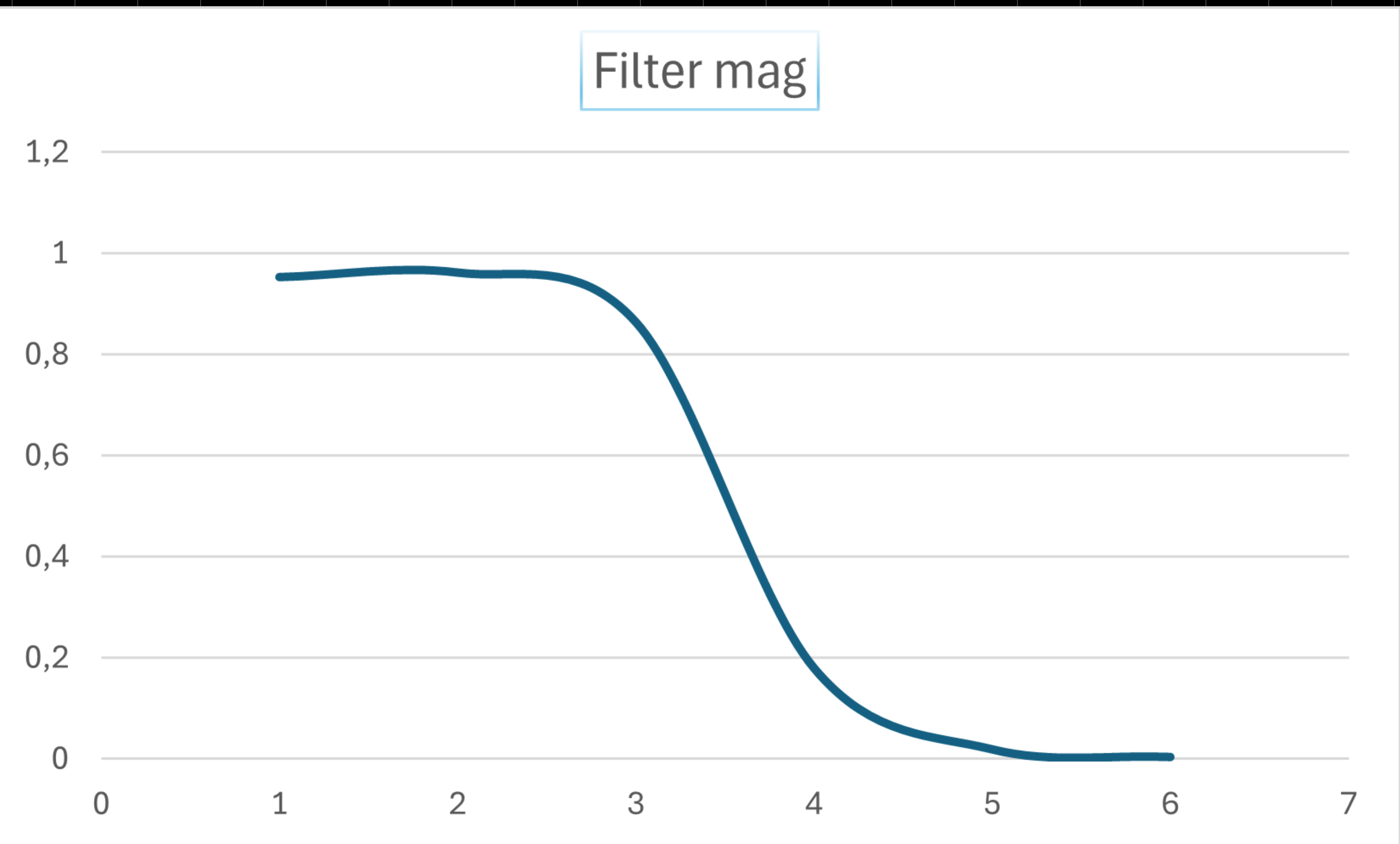




Measurement across the capacitor in a first order series RC circuit

f (Hz)	V_i $(V)_{RMS}$	V_o $(V)_{RMS}$	$\lvert H(\omega)\rvert$	Δt
10^0	331	316	0.95468	31.84 ms
10^1	357	344	0.96358	1.6 ms
10^2	341	295	0.8651026	852,7 μs
10^3	325	58.8	0.180923	228 μs
10^4	323	6.15	0.019040	25.72 μs
$5\cdot10^4$	297	1.15	0.0038720	4.87 μs


Phase shift in RC circuit
Phase shift in radians
1,8
1,6
1,4
1,2
1
0,8
0,6
0,4
0,2
0
0,1
1
10
100
1000
10000
100000
Frequency in Hz


Filter mag
1,2
1
0,8
0,6
0,4
0,2
0
0
1
2
3
4
5
6
7

ANALYSIS)

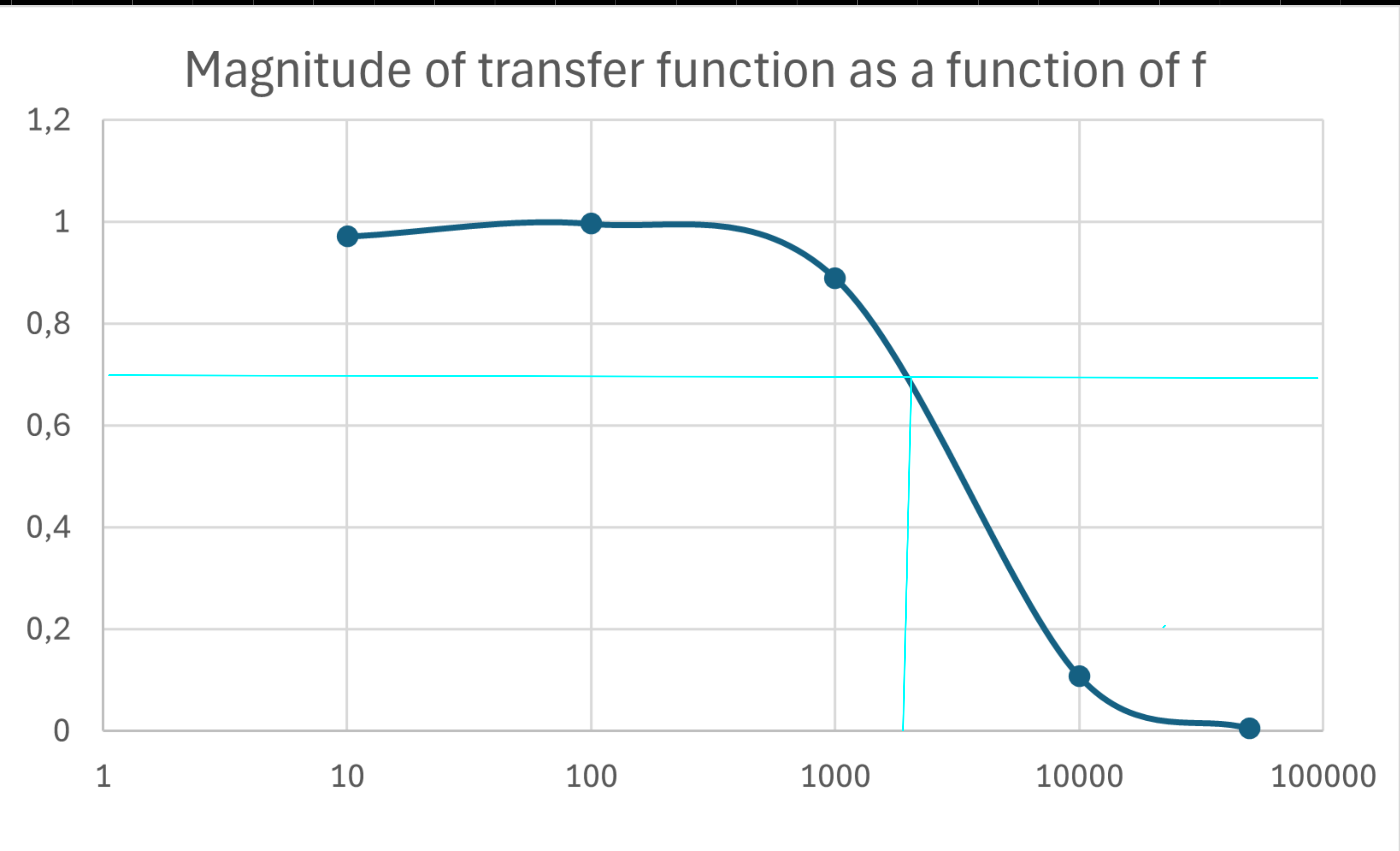
After measuring the impedances across a wide range of values I found the following; for the first circuit even when resonance occurs the impedance doesn't drop to zero. The lowest recorded value was 224Ohms which is the parasitic resistance of the inductor and the capacitor, at 1.65kHz which differs from the expected resonant frequency predicted by the impedance model at 1.591kHz. However since capacitors and inductors have 5% of uncertainty tied to their value the recorded frequency is acceptable.

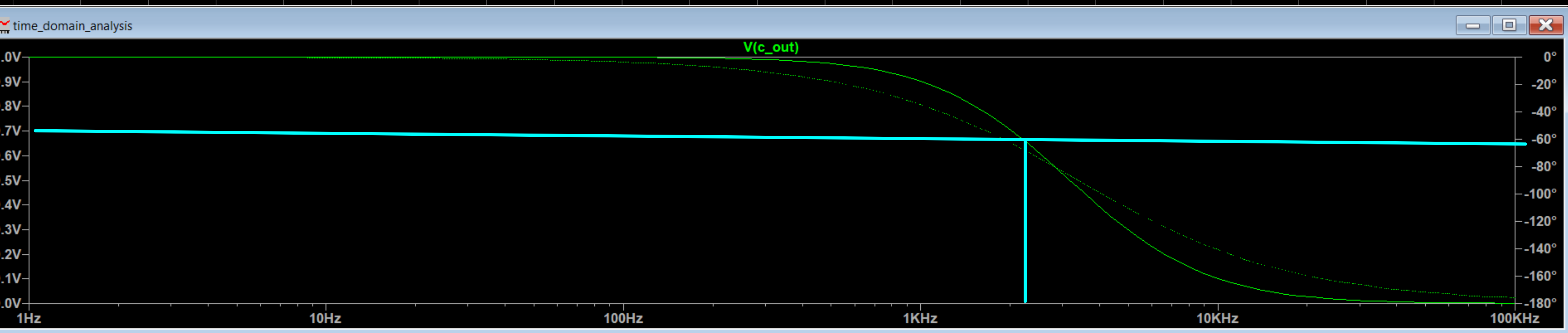
For the second circuit the highest recorded impedance was 500Ohms. This occurred at 1.67kHz which again falls within the acceptable interval after taking into accounts the uncertainty tied to LC component values. Even though the recorded impedance is quite low, after checking a wide range of frequencies this was the highest value. Since the elements are connected in parallel its probable that one of the parasitic impedances is 500Ohms and due to the parallel connection it capped the overall impedance even though the impedance of the elements themselves may have been much greater.

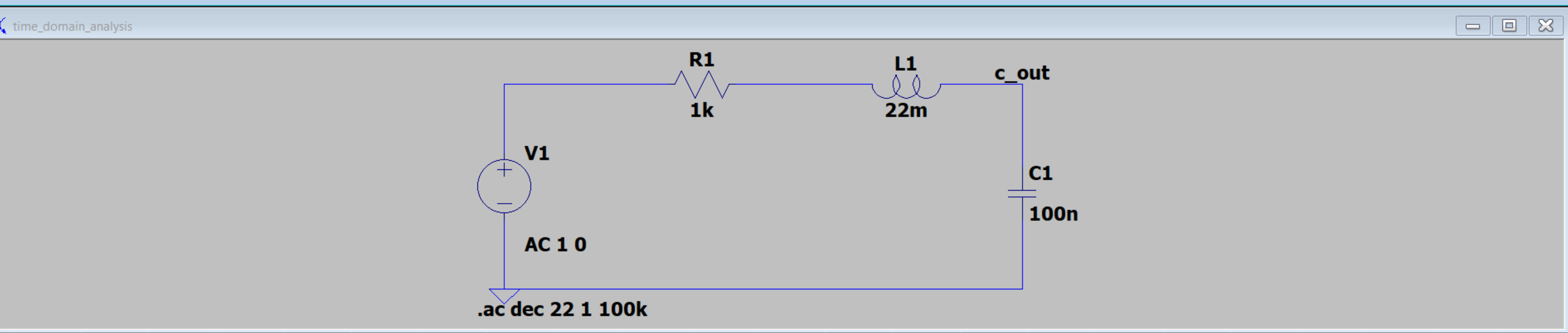
In the RC filter the predictions made on the impedance model closely match up with the measured values. When the frequency is low the capacitor, which has a impedance inversely proportional to frequency acts as a open circuit so the majority of the voltage falls across it. However as we increase the frequency its impedance starts dropping down and since the impedance of the resistor is constant at a certain point it starts overtaking the capctors impedance. This results in the majority of the voltage falling across the resistor at high frequencies.

Challenge;

In a series RLC circuit when you measure the voltage across the capacitor it behaves like a low pass filter meaning that for low ranges of frequencies it doesn't significantly change the amplitude of the signal but for high frequencies it starts attenuating the output amplitude until in the limit it approaches zero.







In general the predicted and actual graphs align very closely. They follow the same trend and have a very similar breaking frequency which confirms the accuracy of the impedance model. However when frequency becomes small on my graph it seems like the function is slightly decreasing. This small measurement inaccuracy may be due to in ideal behaviour of components as well as the fact that the measure probe doesn't have a infinite impedance and pulls some current which decreases the voltage output across the capacitor.