Support, Resistance, and Technical Trading

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Abstract

I use coarse Bayesian updating to explain asset price levels at which the price will tend to stop and reverse. With limited attention, traders discretize the space of expected prices. I show that it is ex ante optimal for them to select an equally spaced price grid to minimize financial costs of error. If I assume that traders disagree on their discretization, those who believe the asset is underpriced buy and those who believe the asset is overpriced sell. However, when traders agree on their discretization, support and resistance levels appear in equilibrium prices. As prices near a support (resistance) level, traders collectively believe the asset is underpriced (overpriced), leading to purchases (sales) and ensuing upward (downward) pressure on prices. Given the long-run price distribution generated by such behavior, the ex ante attention problem must minimize *expected* financial costs of error, where expectations are taken with respect to this endogenous price distribution.

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1 Introduction

In asset markets, a support (resistance) level is a price level at which a decreasing (increasing) price will tend to change direction and begin to increase (decrease). This type of language is commonplace in commentary and newsletters published on Wall Street. For example, a quote from a *Market Watch* article written by Ashbaugh (2021) says

> The February peak (3,950) remains an inflection point, and is followed by the 3,915 support.

In fact, according to Brock et al. (1992), trading range break − a strategy involving buying (selling) when prices penetrate a resistance (support) level − is one of the two most commonly used technical trading strategies. These authors famously find that such strategies can, indeed, help predict stock price changes. Support and resistance levels also go by many other names. For example, Shiller (2015) describes quantitative anchors, or "levels of the market that some people use as indications of whether the market is over- or underpriced and whether it is a good time to buy." To make ideas concrete, he mentions the nearest milestone of a prominent index such as the Dow or the nearest round number. As another example, Donaldson and Kim (1993) find that the Dow Jones Industrial Average not only deflects off of price increments of one hundred, but also, having broken through an increment, the index then moves more than otherwise warranted. The authors call such deflection points "price barriers."

In this paper, I develop a theoretical model explaining why support and resistance levels may be an equilibrium outcome of trading strategies when attention is limited. To model limited attention, I combine two existing theories. The first is coarse Bayesian updating from Jakobsen (2021): traders must select the most likely posterior from an exogenous set of posteriors.[1] To make the set of posteriors endogenous, I assume that traders' posterior means can only take on a fixed number of values, similar to the setup of Gul et al. (2017). When traders face uniform risk and convex error costs, a natural result obtains: attention-constrained traders choose an *equally* spaced price grid. Interestingly, both uniform risk (in terms of future prices) and convex costs (in terms of financial cost of rounding errors) are equilibrium objects. That is, these "rule of thumb" traders must generate uniform prices and their trading choices must result in convex financial costs. While uniform prices were introduced here for intuition (and they do appear as a

[1] This is related to categorical thinking from Mullainathan (2002) and hypothesis testing from Ortoleva (2012).

special case in my model), in general, the endogenous future price distribution will not be uniform. Nonetheless, in my main theorem, I find conditions under which the equal spacing result survives.

Consider the resulting trading strategies in the following simple example. Say that traders can only form expectations over two values: a low or a high price. Locally, demand functions are downward-sloping for the usual reasons. However, if higher prices today are an indication of higher prices tomorrow, a price increase may cause the trader's expectation to jump from the low to high value. In anticipation of my results, call this jump point the price barrier. First, consider when the price is below the price barrier. As the price approaches the barrier from below, the trader believes the asset is increasingly overpriced (relative to the low price expectation) and sells. This puts downward pressure on price, and hence the price tends not to cross the barrier. Symmetrically, consider when the price is above the barrier. As the price approaches the barrier from above, the trader believes the asset is increasingly underpriced and buys, putting upward pressure on price. Again, the price tends not to cross the barrier.

Even when I introduce trader heterogeneity in my full model, this main intuition still remains. Heterogeneity in terms of the trader's price grid carries an intuitive interpretation: they disagree on where the bear market (below a resistance level) ends and the bull market (above a support level) begins. When traders disagree, sellers sell because they believe the asset is overpriced in a bear market, while buyers buy because they believe the asset is underpriced in a bull market. First, I use examples to suggest that it is the level of *agreement* across traders that governs the degree to which support and resistance levels appear in equilibrium prices. Then, I characterize the cutoff level of agreement, in terms of the distribution over trader types, that ensures support and resistance levels appear in equilibrium. These conditions guarantee that equilibrium prices are a submartingale (supermartingale) near a support (resistance) level, and that price variance is maximized (minimized) near (away from) support and resistance levels.

The use of coarse reasoning in financial markets is not new. Eyster and Piccione (2013) and Steiner and Stewart (2015) assume that the world evolves according to a Markov process, and model incomplete theories of the world using partitions of the true state space. Unlike my setting, both papers model risk neutral traders in the style of Harrison and Kreps (1978) and assume that traders are correct, on average, within each cell of the partition. Gul et al. (2017) also partition their state space, and do so ex ante optimally given the constraint that a partition can only have k cells. There are some key similarities between our papers: for example, the price space gets optimally partitioned into *intervals*. The first key difference between our papers is that Gul et al. (2017) consider a standard consumption problem, aside from consumers being restricted to consumption plans that are measurable with respect to their partition. In my paper, I consider a speculative setting where only price expectations are coarse; trading choices need not be (because these may depend on today's price). Second, our set of findings is quite different: while they focus on price extremity, I focus on support and resistance levels. A related paper is Gabaix (2014) in the sense that an ex ante attention allocation problem precedes an ex post consumer problem. However, the attention cost function studied there leads to sparsity, meaning that the agent chooses to pay attention to only a few important variables. While not directly related to financial markets, Matejka (2016) derives discrete pricing as a result of a monopolist's problem with standard rational inattention constraints.

I assume that my exchange is facilitated by a market maker, much like the one from Teeple (2023) and Carvajal and Teeple (2023). When traders are limited to placing only market orders, the market maker cannot infer the shape of trader demand functions and clearing markets becomes an arduous task. I set up the market maker's objective according to the three published objectives of the NYSE's designated market maker: prioritize price discovery, lower volatility, and provide liquidity. The result of this program resembles a classical tâtonnement process: the market maker raises (lowers) price when there is excess demand (supply), and injects extra liquidity in order to clear markets. The following quote from Bagehot (1971) provides an excellent summary of my model's microstructure:

> It is well known that market makers of all kinds make surprisingly little use of fundamental information. Instead they observe the relative pressure of buy and sell orders and attempt to find a price that equilibrates these pressures. The resulting market price at any point in time is not merely a consensus of the transactors in the marketplace, it is also a consensus of their mistakes. Under the heading of mistakes we may include errors in computation, errors of judgment, factual oversights and errors in the logic of analysis.

I establish an equilibrium in two steps. Per period, traders begin with an equilibrium conjecture about the distribution of prices. Market orders are submitted based on traders' coarse beliefs, and the market maker adjusts the price based on the relative buy versus sell pressure. I require the resulting distribution of prices to match that of the original conjecture. I call this an *intraperiod* equilibrium. However, this per-period equilibrium concept takes the trader's ex ante attention choice as given. In addition, my overarching *interperiod* equilibrium requires the following. When traders are born, they use the long-run distribution of prices implied by the sequence of intraperiod equilibria and the financial costs of making errors implied by their per-period trading problem to optimally partition the space of price expectations (subject to an exogenous constraint on the number of partitions). Each trader lives for two periods only, and generations overlap.

My paper is tangentially related to a literature, reviewed in Hommes (2001), that uses nonlinear dynamics to model financial markets. The goals of this literature are not so different from those of this paper: to explain technical trading and match realistic price moments. However, the mechanisms at work are very different. Most of these papers use adaptive belief systems, where fundamentalists trade based on news, chartists trade based on price extrapolation, and the choice of strategy (fundamentalist versus chartist) is based on an evolutionary fitness measure. Based on evolutionary forces, these papers show that there can be switching between phases of minor fluctuations, phases of optimism, and phases of pessimism.

The remainder of this paper is organized as follows. In Section 2, I define coarse expectations, describe the model, and define my equilibrium concept. In Section 3, I contrast two examples: one that generates price barriers and another that generates a random walk. In Section 4, I explain my main theoretical findings, discuss equilibrium uniqueness, and run comparative statics. Section 5 concludes.

2 Model

2.1 Market Maker

While it is not the emphasis of my model, I first introduce the market microstructure before discussing the traders who populate my economy. Time is discrete and infinite $t = 0, 1, 2, ...$ Having observed today's price p_t of a single durable asset that pays no dividends, trader j submits a market order, $x^j(p_t)$. If this value is positive (negative), it is interpreted as a long (short) position. Because prices are posted (and market orders are placed), a market maker must step in to take the opposite side of aggregate demand and clear markets. The market maker's objective function is set up according to the three published objectives of the NYSE's designated market maker: prioritize price discovery, lower volatility, and provide liquidity.[2]

$$\max_{p_{t+1}} \; p_{t+1} \sum_j x^j(p_t) - \frac{1}{2c}(p_{t+1} - p_t)^2$$

The first term above corresponds to price discovery, and is precisely the objective of the classical Walrasian auctioneer.[3] The incentive to raise price on days with excess demand and lower prices on days with excess supply is evident in this term. In the classical general equilibrium setting, the fixed point of such a maximization problem is studied; here I do not abstract away from preceding dynamics. The second (quadratic) term above corresponds to the second objective of the designated market maker: lower volatility. The parameter c controls the relative weight allocated between these first two objectives. Such an objective function implies that the market maker *must* provide liquidity (its third goal); it injects extra liquidity into the market by taking the opposite side to excess demand, effectively clearing markets each period.

This objective function leads to the following intuitive first-order condition:

$$p_{t+1} = p_t + c \sum_j x^j(p_t) \tag{1}$$

The market maker maps excess demand into higher prices and excess supply into lower prices at elasticity c. In a standard model, equation (1) would simply generate off-equilibrium dynamics (i.e. tâtonnement) until the fixed point of (1) is attained.[4] At this fixed point, markets clear by construction. Before moving on, I lay out seven reasons why this microstructure is a reasonable one.

1. Equation (1) is consistent with empirical evidence from Chordia et al. (2002), who confirm that excess buy (sell) orders drive up (down) returns, even at lagged time periods.

2. A empirical literature, originating from Tauchen and Pitts (1983), indicates that there is a positive relationship between price volatility and trade volume. This is also consistent with equation (1).[5]

[2] A similar market maker is employed in Teeple (2023) and Carvajal and Teeple (2023).

[3] This market maker does not rely on limit orders to infer the shape of trader demand functions. Instead, she only observes the value of aggregate demand.

[4] Additional conditions would be required to guarantee global stability.

[5] To see this mapping, consider the setup in Teeple (2023). Demand is measurable with respect to an

3. Equation (1) creates a coordination motive between traders to trade in the same direction (buy when others buy so prices rise, and vice versa). This coordination motive is similar in spirit to the beauty contests literature formalized in Morris and Shin (2002) and Allen et al. (2006).

4. Timing assumptions differ from standard market order models like that of Kyle (1985). There, traders face uncertainty about the price at which their orders are executed. Here, traders are able to trade unlimited positions at the fixed price p_t. By ignoring current price impact, I am effectively ignoring the bid-ask spread, which is a reasonable assumption when spreads are small.

5. In Teeple (2023), I microfound the revenue-maximization problem of the market maker based on trader participation concerns. If spreads are set too high, traders do not participate; if spreads are set too low, potential revenues are lost. Importantly, a revenue-maximizing market maker is shown to have the same qualitative properties as the one in (1).

6. Short-lived traders buy (sell) at price p_t then sell (buy) at price p_{t+1}.[6] It follows that the market maker incurs losses on inventory. To see why, prices according to (1) rise (fall) when the market maker takes a short (long) position. This observation is consistent with empirical evidence from Sofianos (1995) that market makers incur positioning losses on their inventory, which are compensated by revenues from spreads (not modeled here).

7. The dynamics of market maker inventories are: When the market maker takes a long (short) position, she buys (sells) at price p_t then sells (buys) at price p_{t+1}. Hence her inventories in any given period are $-\sum_j x^j(p_t)$. When traders are short-lived, she does not accumulate inventories across periods. This is observationally consistent with mean-reversion theories of market maker inventories. For example, see Hasbrouck and Sofianos (1993).

i.i.d. zero-mean signal. Then manipulating (1),

$$\mathbb{E}[(p_{t+1} - p_t)^2] = c^2 n \mathbb{E}[x^j(p_t)^2]$$

where n is the number of traders and the RHS of the equation above is defined as trade volume.

[6] Note that the second half of this round trip is not included in excess demand for period $(t+1)$.

2.2 The Coarse Bayesian

Next I describe the traders in my economy. Before doing so formally, I point out the *necessity* of a particular feature of trader demand in order to generate support and resistance levels. If prices are to “deflect” off of support (resistance) levels from above (below), it must be that traders are buying (selling) to put upward (downward) pressure on prices.[7] Figure 1 graphs such a demand function, where prices deflect off of $p_t = 0$ from above, and off of $p_t = 1$ from below. If there is to be another set of support and resistance levels between $p_t \in [1, 2]$, a similar pattern must repeat.

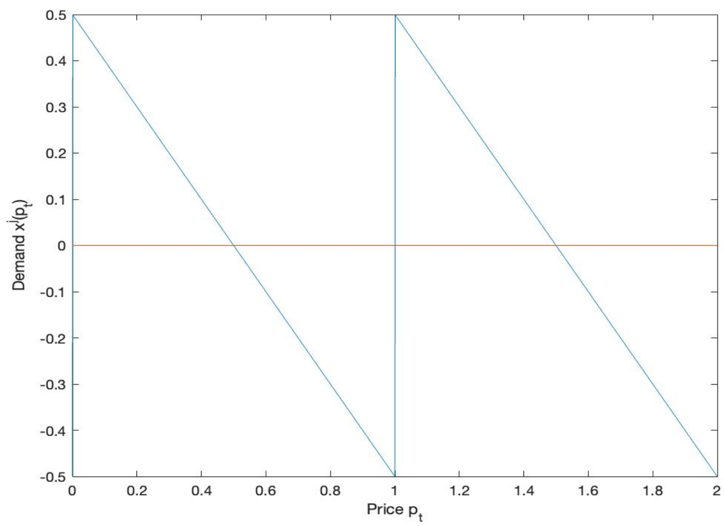

Figure 1: Proposed Demand Function

With this in mind, I formally set up the trader’s problem. A trader observes today’s price p_t of the single durable asset paying no dividends, then forms expectations about tomorrow’s price p_{t+1}.[8] Because there is no exogenous information arrival in this model, this can be viewed as a form of technical trading. Preferences are mean-variance:

$$u(p_t, x_t) = (\mathbb{E}[p_{t+1}|p_t] - p_t)x_t - \frac{\rho \Sigma_t^2 x_t^2}{2}, \tag{2}$$

where holdings of the asset are given by x_t, risk aversion is given by ρ, and the price

[7]This statement assumes the market microstructure from Section 2.1.

[8]I assume that traders are short-lived and, hence, trades are executed myopically.

variance is given by Σ_t^2. The optimal demand is:

$$x(p_t) = \frac{\mathbb{E}[p_{t+1}|p_t] - p_t}{\rho \Sigma_t^2} \tag{3}$$

Traders conjecture that prices are a martingale $\mathbb{E}[p_{t+1}|p_t] = p_t$ with variance Σ_t^2, so there is no expected gain or loss. In equilibrium, these beliefs will need to be consistent with the actual price distribution. If there were no additional assumptions, there would be no trade due to (3) and the martingale conjecture. Instead, I assume that traders form expectations using coarse Bayesian updating. Consistent with the axiomatization of coarse Bayesian updating by Jakobsen (2021), I assume that posterior means are chosen from an exogenous, fixed set $\mathcal{P}$. Much like in Gul et al. (2017), I assume that these posterior means can take on only a fixed number of values, k. This restriction should be interpreted as resulting from limited attention.[9] I assume that traders make this attention decision before they trade.

One complication in my setup, not present in Jakobsen (2021) or Gul et al. (2017), is that prices are unbounded. In particular, I allow the space of prices to be all of $\mathbb{R}$ because the only variable of interest in my setting is the price *difference*, $p_{t+1} - p_t$.[10] To work around this issue, I assume that each interval of prices, $\{..., [-2b, -b], [-b, 0], [0, b], [b, 2b], ...\}$ can contain up to k posterior means. b and k are now important parameters of the model. That each interval has the same width is an assumption; that posterior means are equally spaced within each interval will be a result.

Fix a price interval. It should be intuitive that, if the trader faces uniform price uncertainty, she should optimally select an equally spaced set of posterior means given convex costs of making an error. To see why, say otherwise: uneven spacing gives rise to the possibility of incurring large costs due to convexity. For now, I take this result as given. In Theorem 3, I verify this intuition using the actual price distribution generated by the model (not uniform prices suggested here for intuition). With this result, $\mathcal{P}$ consists of all distributions with a mean in the set $\{\ell\varepsilon + \varepsilon/2\}_{\ell \in \mathbb{Z}}$, where $\mathbb{Z}$ denotes the set of all integers. That is, expectations are restricted to an equally spaced grid with spacing ε. There is a natural mapping between k and ε, which is given by $\varepsilon \equiv \frac{b}{k}$. Figure 2 shows one price interval, where $k = 4$.[11]

[9]See Gul et al. (2017) for a discussion about attention cost functions that are consistent with this type of restriction.

[10]An alternative interpretation is that prices are defined in natural logs (that is, $p_t = ln(\tilde{p}_t)$ for some underlying $\tilde{p}_t$).

[11]While it is not shown in Figure 2, the distance between the last point in the interval $[0, b]$ and the

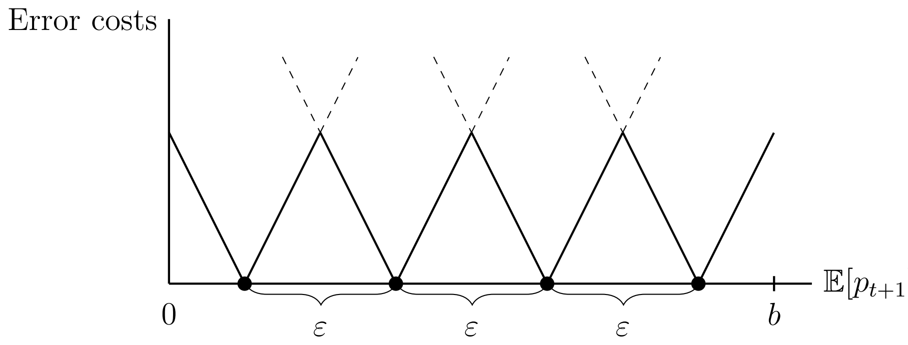

Figure 2: Equal Spacing

If $\varepsilon = 1$, traders think in terms of integers, and if $\varepsilon = 100$, traders think in terms of hundreds. As $\varepsilon \to 0$, the trader becomes Bayesian and considers the entire number line. The coarse expectation is then defined:

$$\begin{aligned}\tilde{\mathbb{E}}[p_{t+1}|p_t] =& \operatorname{argmin}_p |p - \mathbb{E}[p_{t+1}|p_t]| \ \text{ s.t. } p \in \{\ell\varepsilon + \varepsilon/2\}_{\ell\in\mathbb{Z}} \\ =& \operatorname{argmin}_p |p - p_t| \ \text{ s.t. } p \in \{\ell\varepsilon + \varepsilon/2\}_{\ell\in\mathbb{Z}}\end{aligned} \tag{4}$$

The first line of the formula above says that traders pick the closest posterior mean to the true expected value of prices, $\mathbb{E}[p_{t+1}|p_t]$. The second line of the formula follows from the martingale conjecture. Note that (4) is a not a restriction on the *support* of the posterior, only on its mean. Hence traders are not surprised when they see a price p_{t+1} outside their grid; this is simply viewed as a realization not equal to the mean. The interpretation is that traders use rules of thumb and incur rounding errors. Because attention costs do not map well to institutional traders or algorithms, these traders should simply be thought of as retail investors. The previous Figure 1, introduced for intuition only, was a plot of the resulting demand function $x(p_t)$ when $\varepsilon = \rho = \Sigma_t^2 = 1$.

Demand curves slope downwards for the usual reasons. However, at certain prices, the trader uses higher prices today as a signal for higher prices tomorrow and shifts her demand upwards. The fact that she uses prices today to predict prices tomorrow follows from the martingale conjecture. And demand shifts discretely because expectations (the demand intercept) shift discretely. While my model only allows for market orders, if the individual were to submit her entire demand schedule, locally downward sloping intervals would correspond to standard "limit orders," while upward discontinuities in

first point in the interval $[b, 2b]$ is also ε.

demand would correspond to "stop orders."

As noted by Mullainathan (2002), coarse Bayesians can both under- and over-react to news. Here, the news is simply the current price realization. Say $p_{t-1} = 0.5$ in Figure 1, so the prior is $\tilde{\mathbb{E}}[p_{t+1}|p_{t-1}] = 0.5$. For modest price changes, for example $p_t = 0.9$, the posterior remains unchanged, $\tilde{\mathbb{E}}[p_{t+1}|p_t] = 0.5$. This represents an under-reaction with respect to a standard Bayesian. For larger price changes, for example $\tilde{p}_t = 1.1$, the posterior moves more than that of a Bayesian, $\tilde{\mathbb{E}}[p_{t+1}|\tilde{p}_t] = 1.5$. This represents an over-reaction.

With the coarse expectation (4) defined, I can revisit the convex costs incurred by traders when making errors (drawn in Figure 2). In terms of the errors that traders make, by construction they make mistakes of size $|\tilde{\mathbb{E}}[p_{t+1}|p_t] - p_t|$. But because they furthermore *trade* based on these mistakes, a more reasonable error term capturing the financial costs of making errors is

$$\mathcal{E} = \left| \frac{\tilde{\mathbb{E}}[p_{t+1}|p_t] - p_t}{\rho \Sigma_t^2} \right|.$$

If all trade leads to error, why trade at all? The short answer is: because traders do not have access to such commitment technology. This could be for one of two reasons. First, the two problems − rounding and daily trading − are solved using different parts of the brain. In the words of Kahneman (2011), rounding is a System 1 problem; these are automatic, fast, effortless, and subconscious. Trading is a System 2 problem; these are deductive, slow, effortful, and self-aware. Second, the ex ante rounding solution may also be used in additional non-financial problems (not explicitly modeled here). As an example, Lacetera et al. (2012) find evidence of discontinuous price drops in the used car market at 10,000-mile thresholds in odometer mileage.

2.3 Reference Points

In this section, I add a shift parameter r^j to the equally-spaced price grid and assume that this value need not agree across traders.[12] In particular, r^j is drawn independently across traders. Disagreement in this sense leads to disagreement in the expected asset price, and hence a motive for trade.

[12] Heterogeneity in ε (or, equivalently, in the interval length b or coarseness constraint k) is discussed in Section 4.3.

For intuition, consider the case $\varepsilon = 100$. Demand functions look like that of Figure 1, but with larger demand values (due to large ε) and horizontal shifts (due to r^j). At any given price, sellers believe that the asset is overpriced relative to a low discretized value and buyers believe that the asset is underpriced relative to a high discretized value. Instead of low and high discretized values, let me reinterpret each continuous, downward-sloping portion of the demand function as a "regime." Sellers believe that the asset is overpriced in a bear market, and buyers believe that the asset is underpriced in a bull market.[13] Disagreement, then, can be thought of as disagreement about where the bear market ends and the bull market begins.

Incorporating this heterogeneity into the model, my economy is populated by coarse Bayesians indexed by j, who are identical in every way but their idiosyncratic *reference point*. The coarse expectation equation (4) becomes:

$$\tilde{\mathbb{E}}^j[p_{t+1}|p_t] = \text{argmin}_p |p - p_t| \;\; \text{s.t.} \;\; p \in \{r^j + \ell\varepsilon\}_{\ell \in \mathbb{Z}} \tag{5}$$

Equations (3) and (5) together define a coarse demand function, which I denote $x^j(p_t)$. Importantly, traders do not know the value of other traders' reference points; this makes the asset price a random variable. Furthermore, I assume that markets are competitive in the following sense: there are sufficiently many traders that traders ignore their own price impact (although they know their own reference point and, hence, could calculate this number). In contrast, with few traders, each has a non-negligible own price impact. Optimizing over this price impact, even with coarse expectations, could allow traders to do better than the demand functions described here.

To complete the model description, I discuss the dynamic nature of the trader's problem. Each trader only solves a two-period problem: in the first period they are born and solve their ex ante problem, and in the second period they trade. However, I assume that these two periods are sufficiently far apart (in time t) for no other reason than to allow traders to use the long-run price distribution to solve their attention problem.[14] Let me refer to a group of n traders born in each period t as a *generation*. In each period t, a generation is born and solves their ex ante problem. They wait a sufficiently long period of time (i.e. they grow up), then trade. Reference points are drawn independently

[13] Regimes are typically defined at the equilibrium (not individual) level. In Section 3, I show that the degree of *agreement* across traders dictates the degree to which individual regimes manifest in equilibrium prices.

[14] If the two periods were, for example, adjacent, then traders would concentrate posterior means around p_t.

within and across generations. Figure 3 summarizes the timing assumptions. Black circles denote the timing of the ex ante (attention) problem of a generation, and arrows denote the timing of the ex post (trading) problem of that same generation. K, the length of time between ex ante and ex post problems, is assumed to be a sufficiently large number. Note that these timing assumptions are consistent with the myopic nature of the trader's per-period trading problem.

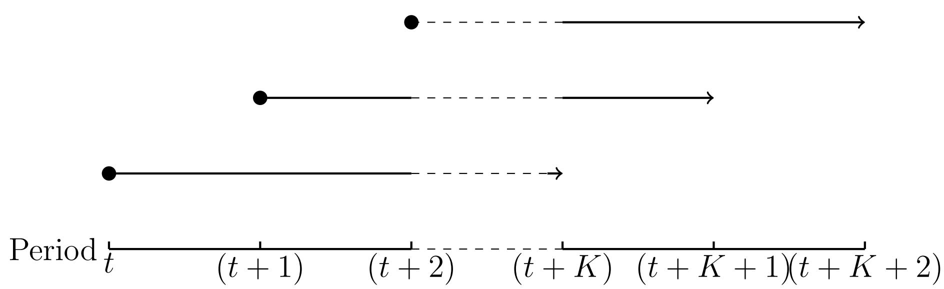

Figure 3: Timing across Generations

2.4 Equilibrium Definition

I now formally define an equilibrium in two steps. First, for each period t there is an *intraperiod* equilibrium given the reference point sequence $\mathcal{R}$, which is a price distribution F_t and demands $(x^j(p_t))_{j=1}^{\infty}$ such that

(a) $|\mathbb{E}_{F_t}[p_{t+1}|p_t] - \tilde{\mathbb{E}}^{j'}[p_{t+1}|p_t]| \leq \varepsilon/2$ where j' denotes the average trader, and $\mathrm{Var}_{F_t}(p_{t+1} - p_t|p_t) = \Sigma_t^2$,

(b) Traders choose $x^j(p_t)$ to maximize (2) given $\tilde{E}^j[p_{t+1}|p_t]$ and Σ_t^2, and

(c) The market maker (1) generates prices with distribution F_t.

Condition (a) requires that traders are correct in their conjecture about the equilibrium price variance, and within $\varepsilon/2$ of their conjecture about the equilibrium price expectation. First, note the factor $\varepsilon/2$: when traders have a price grid of size ε, errors should be no larger than $\varepsilon/2$. Second, note that the condition only applies to the *average* trader; in general, I will be unable to say much more about the entire population. What makes this a reasonable equilibrium requirement is the following: recall that all traders solve an optimal (homogeneous) grid problem. Then, for reasons outside of this model (these can be thought of as different experiences throughout the trader's lifetime), traders disagree on their grid at the time of trading. The equilibrium condition allows for the fact that such heterogeneity can cause larger error than $\varepsilon/2$, but controlling for this (i.e. considering the average trader), they must have made an optimal grid choice. Otherwise, Conditions (b) and (c) say that traders and the market maker optimize and that resulting prices are consistent with trader beliefs.

Before moving to my overarching equilibrium concept, first consider the following exercise. Say that prices were Uniformly distributed between $[0, b]$. Let $F(x|p) \equiv Pr(p_{t+1} \leq x|p)$. Given all $F_t \equiv F(p_{t+1} - p_t|p_t)$ collected throughout intraperiod equilibria, one could construct such a function $F(x|p)$, its density $f(x|p)$, and the "next iteration,"

$$f^2_{[0,b]}(x) = \int_0^b f(x|p)\frac{1}{b}dp$$

where the subscript in $f^2_{[0,b]}$ indicates that the integral is taken over $[0, b]$. Note that $f^2_{[0,b]}(x)$ is a well-defined density on $(-\infty, \infty)$. Restricting this function back to the interval $[0, b]$ would not be the correct exercise, because this misses the probability that prices from other intervals land in $[0, b]$. Instead, I collect all $\left(f^2_{[b\ell,b(\ell+1)]}\right)_{\ell\in\mathbb{Z}}$ and define $f^2(x)$ as

$$f^2(x) = \sum_{\ell\in\mathbb{Z}} f^2_{[b\ell,b(\ell+1)]}(x), \text{ for } x \in [0, b]$$

Given that prices are distributed according to $f^2(x)$, the next iteration would be

$$f^3_{[0,b]}(x) = \int_0^b f(x|p)f^2(p)dp$$

and so on. A good candidate for where a trader would expect to find prices, from the ex ante perspective, is the fixed point of this iteration, which I call $g(p)$. More succinctly, it can be written as a solution to

$$g(x) = \int_{-\infty}^{\infty} f(x|p)g(p)dp \tag{6}$$

with the understanding that, when restricted to the interval $[0, b]$, $g(x)$ integrates to one. I will refer to $g(p)$ that solves (6) as the *long-run price distribution*. Given the intraperiod equilibrium, I now define the *interperiod* equilibrium, which is a reference point sequence $\mathcal{R}$ such that

(a) For each t, the intraperiod equilibrium given $\mathcal{R}$ is F_t and $(x^j(p_t))_{j=1}^{\infty}$, and

(b) $\mathcal{R}$ minimizes $\mathbb{E}_g\left[\left|\frac{\tilde{\mathbb{E}}[p_{t+1}|p]-p}{\rho\Sigma(p)^2}\right| : p \in [\ell b, (\ell+1)b]\right]$ for each $\ell \in \mathbb{Z}$, where g satisfies the fixed point condition (6).

Condition (a) requires an intraperiod equilibrium for each t. In Condition (b), note that $\Sigma_t^2 \equiv Var(p_{t+1} - p_t|p_t)$ so the notation $\Sigma(p)^2$ denotes $Var(p_{t+1} - p|p)$. By selecting reference points, traders effectively choose $\tilde{\mathbb{E}}[p_{t+1}|p_t]$ in order to minimize the financial costs of trading errors. If traders knew exactly the price at which they will be trading, their problem simplifies greatly. They would simply select reference points at (or around) that known price. However, traders trade sufficiently far out into the future (compared to their ex ante attention problem), and face uncertainty over the price at which they will be trading. The trader uses the frequencies of prices implied by the model to calculate expected costs, conditional on prices being in the range $[\ell b, (\ell+1)b]$, in which they are allotted k posterior means.[15]

3 Examples

3.1 Example: Random Walk

I first analyze an example where support and resistance levels do not appear. This will allow me to contrast properties of this example to those of Example 3.2, where support and resistance levels do appear. In this example, $\rho = \varepsilon = \Sigma^2 = 1$, and hence demands are given by

$$x^j(p_t) = \tilde{\mathbb{E}}^j[p_{t+1}|p_t] - p_t,$$

which look precisely like the one in Figure 1, but shifted horizontally. That Σ^2 is stationary will need to be later confirmed. The distribution of r^j, which is an important object for determining the presence (or lack) of support and resistance levels, is set to the Uniform$[0.5, 1.5]$ distribution. Due to linearity, each demand is distributed Uniform$[-0.5, 0.5]$ for any fixed p_t, which ensures that prices will actually be a martingale. And since no trader makes errors larger than $\varepsilon/2$ by construction of the coarse expectation, the first half of intraperiod equilibrium Condition (a) is satisfied.[16] For no

[15] One might question whether $|\mathcal{E}|$ is a reasonable error term when prices are not a martingale. A subtle distinction that justifies its use is: $|\mathcal{E}|$ captures the trader's *belief* of their error.

[16] It is, in fact, satisfied not just for the average trader but all traders.

other reason than to simplify algebra, instead of solving for an equilibrium variance, I normalize it to one and select a market maker constant, c, consistent with that normalization. By taking the variance of the market maker condition (1), that required constant is $c = \sqrt{12/n}$, where n is the number of traders per period and each demand has variance $\frac{1}{12}$ when it is Uniformly distributed. All intraperiod equilibrium conditions are satisfied, and hence I move on to the interperiod equilibrium.

Prices are not Uniformly distributed, because the sum of n independent Uniform random variables is not Uniform. However, prices do have the property that p_{t+1} is today's price plus some fixed, stationary random variable. Let $F(y)$ denote the distribution of that random variable, so that $Pr(p_{t+1} - p \leq y) = F(y)$ or $Pr(p_{t+1} \leq x) = F(x - p)$. The fixed point (6) is, in fact, solved by the Uniform$[0, b]$ random variable. That is, let $f_{[0,b]}$ solve

$$f_{[0,b]}(x) = \int_0^b f(x-p)\frac{1}{b}dp \tag{7}$$

where $f(x - p)$ is the density of prices generated by the model. Figure 7 shows the construction. Solid lines denote the price densities generated by the model for different values of p. $\sum_{\ell \in \mathbb{Z}} f_{[b\ell, b(\ell+1)]}(x)$ is denoted by the dashed lines. In my first proposition, I show that $\sum_{\ell \in \mathbb{Z}} f_{[b\ell, b(\ell+1)]}(x)$ is, indeed, the density of the Uniform$[0, b]$ random variable when restricted to this interval. All proofs are in Appendix A.

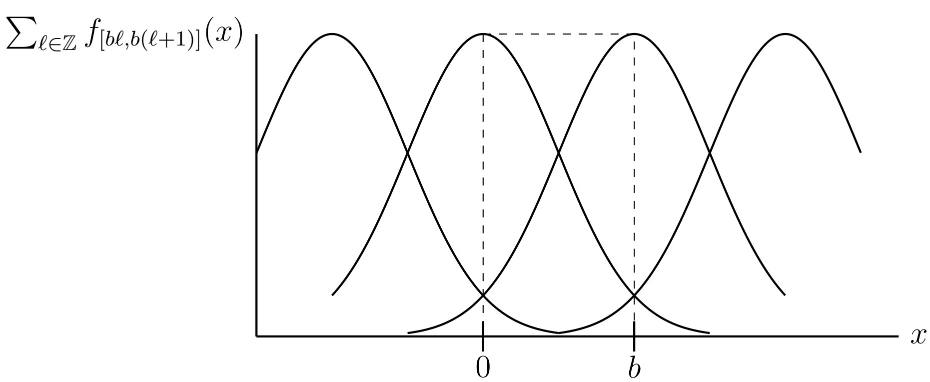

Figure 4: Random Walk Long-Run Distribution

Proposition 1. ***(Limiting Distribution)*** $\sum_{\ell \in \mathbb{Z}} f_{[b\ell, b(\ell+1)]}(x)$*, where* $f_{[0,b]}$ *is defined in* (7)*, is constant throughout* $x \in [0, b]$.

Given that prices will be Uniformly distributed within the interval $[0, b]$, my next proposition pins down the optimal list of reference points.

Proposition 2. ***(Equal Spacing)*** *The unique solution to the program*

$$\min_{r_1,...,r_k} \int_0^b \min\{|r_1 - p|, ..., |r_k - p|\} \frac{1}{b} dp$$

is to select an equally spaced set of points $(r_1, ..., r_k)$*.*

The mapping to my main model is that $(r_1, ..., r_k)$ serves the role of reference points, p denotes the price, and the inner minimum function serves the role of the coarse expectation. That is, the coarse Bayesian selects the closest reference point to the price. Proposition 2 solves a problem similar to the one depicted in Figure 2. While the general problem considered in Theorem 3 will not have a unique solution, the simpler problem considered here will. Necessary conditions for optimality in Proposition 2 can be simplified to a second-order linear difference equation, which has a unique solution with appropriate boundary conditions.

Two characteristics that made this example tractable are the stationary nature of Σ^2 and the Uniform nature of limiting prices. Support and resistance levels do not appear (they will be formally defined in Example 3.2), because the distribution of r^j is characterized by so much disagreement. With a tighter distribution for r^j, reference points will tend to align, leading to less trade near such points and more trade away from them. Importantly, Σ_t^2 will be small near such points and large away from them, which will have implications for the long-run distribution of prices.

3.2 Example: Support and Resistance

I begin by defining the main phenomenon of interest.

Definition 1. ***(Support and Resistance)*** *Prices are characterized by support and resistance levels if there is an interval of size* ε *and a cutoff* $\mathcal{M}$ *within that interval such that*

(a) Prices are a submartingale below $\mathcal{M}$ *and a supermartingale above* $\mathcal{M}$*,*

(b) The price variance has local maxima at both boundaries of the interval, and

(c) Price moments are nonconstant in p_t*.*

The definition requires that there is an interval of size ε with two key characteristics. First, expected returns fluctuate up and down throughout this interval. In particular, prices will be a submartingale near the bottom of the interval, where traders tend to buy, and a supermartingale near the top of the interval, where traders tend to sell. Second, the conditional price variance (as a function of p_t) fluctuates up and down throughout this interval. In the middle of the interval, prices will have low variance because traders trade very little, and at the top of the interval, prices will have high variance because they trade large amounts. Both of these characteristics are consistent with informal descriptions of price barriers, which involve both "deflection" off of barriers (i.e. Definition 1 condition (a)) as well as "more price movement than otherwise warranted" once a barrier has been broken through (i.e. Definition 1 condition (b)). The qualifier that moments be nonconstant rules out equilibria like that of Example 3.1.

Now consider $\rho = \varepsilon = 1$ like in Example 3.1. Demand looks like

$$x^j(p_t) = \frac{\tilde{\mathbb{E}}^j[p_{t+1}|p_t] - p_t}{\Sigma_t^2} \tag{8}$$

and importantly, Σ_t^2 will be nonconstant. For now, I maintain the assumption of equally-spaced posterior means (I will later confirm that this is optimal). While it may seem clear that concentrating probability mass of r^j will lead to less trade near such concentrations and more trade away from, this intuition is incomplete. More trade away from such concentrations leads to a higher Σ_t^2, which lowers trade according to (8). In what follows, I will need to balance these two competing effects.

I maintain the assumption that r^j is Uniformly distributed. However, instead of covering the entire interval of length $\varepsilon = 1$, I assume that the distribution is more concentrated (more agreement amongst traders). In particular, I assume that r^j has a Uniform$[0.25, 0.75]$ distribution. Notice that the center of the distribution in Example 3.1 was 1, while it is 0.5 here. The average trader in Example 3.1 had a posterior mean equal to one (plus any integer), which had a nice interpretation of rounding. However, this meant that the cutoffs (i.e. discontinuities in demand) occurred at $\{0.5, 1.5, 2.5, ...\}$. In contrast, the average trader in Example 3.2 has a posterior mean equal to one half (again, plus any integer). While this loses the nice interpretation of thinking in terms of whole numbers, it has the added benefit that discontinuities in demand (which will correspond to support and resistance levels in equilibrium) now occur at whole number increments.

The first step is to characterize the distribution of $\tilde{\mathbb{E}}^j[p_{t+1}|p_t] - p_t$ (it is a random variable because it is measurable with respect to r^j), and how it depends on p_t. Because this object is the numerator of demand defined in (8), I refer to $\tilde{\mathbb{E}}^j[p_{t+1}|p_t] - p_t$ as non-normalized demand. Figure 5 plots two functions of p_t. The top graphic shows the first moment, $\mathbb{E}[\tilde{\mathbb{E}}^j[p_{t+1}|p_t] - p_t]$, and the bottom graphic shows the second moment, $\mathbb{E}[(\tilde{\mathbb{E}}^j[p_{t+1}|p_t] - p_t)^2]$.

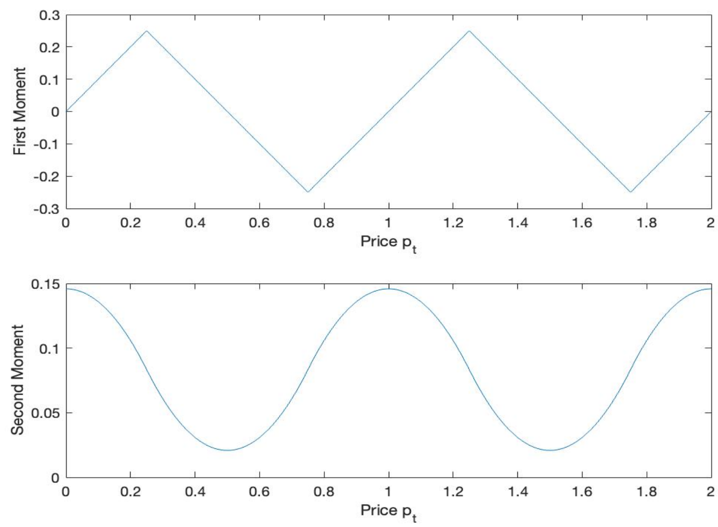

Figure 5: Non-Normalized Demand Moments

Non-normalized demand is a martingale when $p_t = 0.5$, which is the expected value of the reference point. To the left, it is a submartingale because the average trader tends to buy. Expected prices have a peak at $p_t = 0.25$, because all traders buy at that price (some traders with $r^j = 0.25$ buy zero and others with $r^j = 0.75$ buy 0.5, with an average of 0.25). Symmetrically, in the range $p_t \in [0.5, 1]$, prices are a supermartingale because the average trader tends to sell. Expected prices have a trough at $p_t = 0.75$ because all traders sell at that price. The second moment graph from Figure 5 (bottom panel) is more straightforward: the least amount of trade occurs around the center of the reference point distribution, and the most amount of trade occurs away from the center of the distribution. Not surprisingly, both functions in the two panels of Figure 5 are periodic.

The second step is to characterize the intraperiod equilibrium mean and variance, which solve two conditions

$$\begin{aligned}\mathbb{E}[p_{t+1}|p_t] - p_t &= cn\frac{\mathbb{E}[\tilde{\mathbb{E}}^j[p_{t+1}|p_t] - p_t]}{\Sigma_t^2}, \text{ and} \\ \Sigma_t^2 &= c^2 n \frac{\mathbb{E}[(\tilde{\mathbb{E}}^j[p_{t+1}|p_t] - p_t)^2] - (\mathbb{E}[\tilde{\mathbb{E}}^j[p_{t+1}|p_t] - p_t])^2}{\Sigma_t^4}\end{aligned} \tag{9}$$

which are the mean and variance of the market maker's rule (1). Due to the periodic nature of non-normalized demand moments, both price returns and variance in (9) are periodic. While they are not plotted here, price moments solving (9) inherit the qualitative properties of Figure 5. Therefore, my economy is characterized by support and resistance levels.

Figure 6 plots a simulation of this example economy with a starting price of $p_0 = 1000$. The number of traders is set to $n = 20$ and $c = 0.0014$.[17] As previously discussed, because reference points have an expected value of 0.5, support and resistance levels occur at whole number increments. I will refer to the horizontal white strip between a support and resistance level in Figure 6 as a regime. As prices near the upper (lower) limit of a regime, selling (buying) pressure tends to keep prices within that regime. There are relatively few prices where all traders' demands "line up," generating sufficient demand to push prices into the next regime.

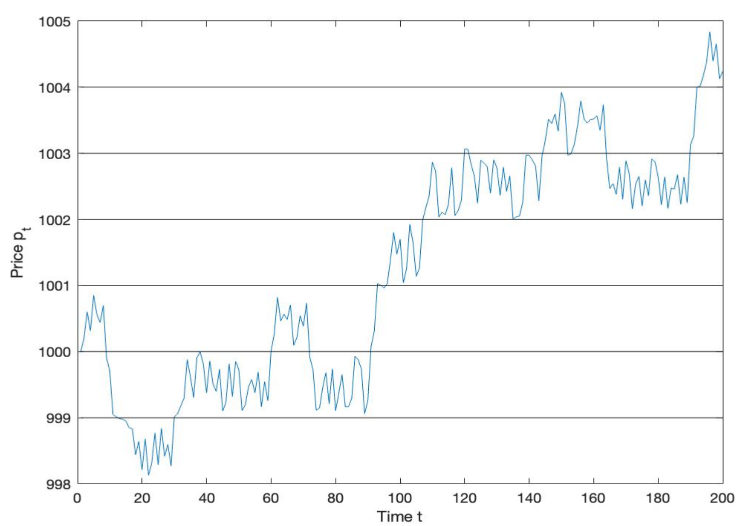

Figure 6: Support and Resistance

[17]This value of c is below the required cutoff from upcoming Theorem 1. When c is too small, prices never break through support and resistance levels.

What remains to be shown are two things. First, I have not guaranteed that price expectations are within $\varepsilon/2$ of the average trader's reference point. Second, I have not confirmed whether equal spacing is part of some interperiod equilibrium. It turns out that the structure afforded by this example does not help in proving either fact. In the next section, I prove more general results, which will cover the special case of this example.

4 Generalizations

4.1 Equilibrium

First I focus on generalizing results related to intraperiod equilibria. The goal is to extend beyond the Uniform distributions which have been employed for tractability. For all upcoming results, I restrict the distributions of r^j to have support of size ε. The support condition is without loss of generality if posterior means are equally spaced, because probability mass outside of the size-ε support can be redistributed (modulo ε) into the size-ε support without changing the nature of the demand.

Theorem 1. ***(Intraperiod Equilibrium)*** *Assume that posterior means are equally spaced, and that the distribution of r^j is nondegenerate. Then*

(a) All price moments are periodic,

(b) $|\mathbb{E}[p_{t+1}|p_t] - \tilde{\mathbb{E}}^{j'}[p_{t+1}|p_t]| \leq \varepsilon/2$ when c is small enough, j' denotes the average trader, and the distribution of r^j is symmetric and satisfies

$$\begin{aligned} p/\varepsilon &\geq Pr(r \leq p), \text{ for } p \in [0, \varepsilon/2] \text{ and} \\ p/\varepsilon &\leq Pr(r \leq p), \text{ for } p \in [\varepsilon/2, \varepsilon], \end{aligned} \tag{10}$$

and

(c) Price moments are constant only if r^j is Uniform with support ε.

Condition (a) shows that my examples generalize: price moments are, indeed, periodic in the general case. This fact will be needed in later arguments related to interperiod equilibria. Condition (b) is shown in the following way. Recall that any trader, even the average one, can make mistakes up to size $\varepsilon/2$ versus their martingale conjecture. That is, $|\tilde{\mathbb{E}}^{j'}[p_{t+1}|p_t] - p_t| \leq \varepsilon/2$ by construction of the coarse expectation (5). By the triangle inequality, I could bound the desired difference in Condition (b), but the bound would always be larger than $\varepsilon/2$ when prices are not a martingale. Hence, the only way that Condition (b) can be satisfied is if the average trader makes mistakes (due to coarseness) that *cancel out* with mistakes due to prices not being a martingale. But it is intuitive why this will be true: the average trader's reference point is at the center of the regime. If prices are at the top (bottom) of the regime, their mistake (due to coarseness) can be up to size $\varepsilon/2$ ($-\varepsilon/2$). But because prices are a supermartingale (submartingale), actual price expectations are lower (higher) than p_t. This diminishes the size of their original error.

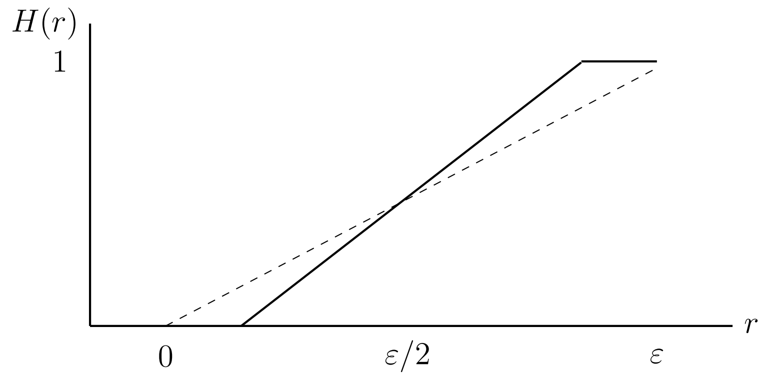

Figure 7: Restrictions on Reference Point Distribution $H(r)$

Next consider the conditions required for this above intuition to hold. Symmetry of the distribution is *not* necessary, and is imposed for two reasons. First, it simplifies expressions and second, it is required in upcoming Theorem 3. Given symmetry, condition (10) is both necessary and sufficient for a submartingale (supermartingale) in the bottom half (top half) of the regime.[18] In Figure 7, I draw condition (10). In the bottom half of the regime, where prices must be a submartingale, the distribution must first-order stochastically dominate the Uniform$[0, \varepsilon]$ distribution. In the top half of the regime, where prices must be a supermartingale, the distribution must be first-order stochastically dominated by the Uniform$[0, \varepsilon]$ distribution.

One way to guarantee that these conditions hold is to reduce the support of the Uniform distribution (i.e. introduce more agreement in the economy), drawn as the black curve in Figure 7 and explored in detail in Example 3.2. Importantly, the Uniform

[18]Note that the overlapping intervals in (10) imply that $Pr(r \leq \varepsilon/2) = \frac{1}{2}$.

distribution (with support ε) drawn in dashed lines in Figure 7 can be thought of as the agreement "cutoff." There are many other ways to build distributions with more agreement than the Uniform (i.e. any distribution satisfying (10)), but also there are ways to build distributions with less agreement than the Uniform (i.e. consider bimodal, symmetric distributions). In these latter cases, the super- and sub-martingale results break down. Finally, the market maker cannot act too aggressively (i.e. c large), or else prices always jump into neighboring regimes.

Condition (c) states that Example 3.1 was very special; in general, price moments will not be constant. Because Theorem 1 required an investigation into sub- and super-martingales, for the sake of completeness, my next theorem characterizes maxima and minima of the equilibrium price variance. Because Theorem 1 already showed that price moments are periodic, it suffices to consider the range $[0, \varepsilon]$.

Theorem 2. ***(Support and Resistance)*** *Assume everything assumed in Theorem 1 part (b). Then*

(a) $\mathbb{E}[p_{t+1}|p] - p \geq 0$ *for* $p \in [0, \varepsilon/2]$ *and* $\mathbb{E}[p_{t+1}|p] - p \leq 0$ *for* $p \in [\varepsilon/2, \varepsilon]$*, and*

(b) If the distribution of r^j *is absolutely continuous, then* $\Sigma(p)^2$ *has a local maximum at* $p = 0$ *and a local minimum at* $p = \varepsilon/2$*.*

Theorem 2 confirms that support and resistance levels appear in the general case. Part (a) formally restates the sub- and super-martingale result discussed after Theorem 1. Part (b) is new. Like the top panel of Figure 5 suggests, differentiability of price moments may be an issue but I explicitly verify (twice) differentiability of $\Sigma(p)^2$ at these two prices. Then in the proof, I check first- and second-order conditions. It turns out that I always have a local minimum at the center of the regime, assuming a symmetric distribution. The explicit condition I derive to get a local maximum at the edges of the regime is $h(\varepsilon/2) \geq 1/\varepsilon$, where $h(r)$ denotes the reference point density. It is notable that, like in Theorem 1, the cutoff amount of "agreement" in the economy is given by the Uniform distribution (with support ε). The condition says that the reference point density must put more mass on the center point than would the density of the Uniform distribution. Condition (10) is must stronger than what is required, and is sufficient (but not necessary) for Theorem 2 part (b). Graphically, in Figure 7, Theorem 2 requires that the slope of the distribution be greater than $1/\varepsilon$ at the center point, $p = \varepsilon/2$.

My main theorem, next, characterizes the interperiod equilibrium, and, in particular, justifies the use of equal spacing in Example 3.2.

Theorem 3. ***(Interperiod Equilibrium)*** *Assume that all price moments are periodic. Furthermore, assume that the distribution of r^j is absolutely continuous. Then*

(a) The long-run density of prices, g, satisfies $g(p+\varepsilon) = g(p)$ for all p, and

(b) If the distribution of r^j is symmetric, one argmin of $\mathbb{E}_g[|\mathcal{E}| : p \in [\ell b, (\ell+1)b]]$ is given by an equally spaced $\mathcal{R}$.

Part (a) of the theorem is an existence proof. The trick is to think of the fixed point problem, $g(x) = \int_{-\infty}^{\infty} f(x|p)g(p)$, as a linear operator from the space of ε-periodic densities (non-negative and integrate to one on $[0, b]$) to itself. Then, compactness properties of the image of the operator can be deduced from standard results in probability. Nothing in the proof guarantees uniqueness of this fixed point, nor can I rule out other non-periodic fixed points.

While the reader may conjecture that (a) is sufficient to conclude (b), it is not. Additional symmetry properties must be imposed on the reference point distribution, which I show are inherited first by non-normalized demand, then demand, then prices $f(x|p)$, and finally by the fixed point $g(p)$.[19] To see why symmetry is needed, consider an asymmetric $g(p)$. There is an insurmountable tension between two forces: first, $g(p)$ is ε-periodic so regimes must be set to $\{[0, \varepsilon], [\varepsilon, 2\varepsilon], ...\}$. Second, the coarse Bayesian switches from one reference point to the next at precisely the halfway point between the two. It follows that reference points must be placed at the center of these regimes, which cannot be optimal when $g(p)$ is asymmetric.

Notice that I have lost the uniqueness of the solution to the attention problem, which was there in Proposition 2. The reason is that $g(p)$ many not have full support. Then multiplicity, and even non-equal spacing, can arise as a solution to minimizing expected costs of inattention due to shifts in reference points along price intervals for which $g(p) = 0$.

Theorem 3 part (b) is consistent with how I have been discussing trader heterogeneity. All traders solve the homogeneous problem in Theorem 3 part (b) and agree on equal spacing. After this, heterogeneity is introduced in the form of the reference point r^j. Appendix B contains an alternative microfoundation for reference points that relies on heterogeneous information. Traders receive i.i.d. signals about the price mean centered around their martingale guess. Ex ante, before many signals have been observed,

[19] It is unclear what periodicity and symmetry mean for $f(x|p)$. I show that $f(x|p) = f(x+\varepsilon|p+\varepsilon)$, which I call *conditional periodicity*, and $f(x|\varepsilon/2-\delta) = f(\varepsilon-x|\varepsilon/2+\delta)$, which I call *conditional symmetry*.

this generates trader heterogeneity and leads to an endogenous choice of reference points shifted by r^j. Ex post, after many signals have been observed, traders recover their martingale guess from the baseline setting. In this extension, it is still true that the average trader makes mistakes no larger than $\varepsilon/2$. However, there is a new interpretation of the added error attributed to heterogeneity: it arises from the imprecision of ex ante signals.

To put everything together, a reference point makes a trader trade less within a neighborhood of that reference point. The long-run density of prices from part (a) of Theorem 3 puts more probability mass around reference points, and, furthermore, it inherits both periodic and symmetric properties from price moments. Part (b) of Theorem 3 says that reference points should, indeed, be placed at such high probability mass points to minimize errors. While this explanation is put forward for intuition, note that Theorem 3 does not explicitly rely on any sort of "agreement" conditions like those described in condition (10).

4.2 Uniqueness

Next I discuss uniqueness, or lack thereof. In fact, uniqueness fails at both the intra- and inter-period level. Within period, consider Example 3.1, which was characterized by an absence of support and resistance levels, but with $\varepsilon \neq 1$. I maintain the assumption that reference points are drawn from a Uniform distribution with support ε. The martingale conjecture can lead to multiplicity in the following way. Say that all traders believe prices will rise each period by ε. This particular belief was chosen so that a coarse Bayesian always trades $\varepsilon/(\rho\Sigma^2)$ more than in the baseline (martingale-guess) economy.

Recall that prices will be an actual martingale in the baseline economy. Hence, for an equilibrium in the non-martingale economy, I need

$$\varepsilon = \frac{cn\varepsilon}{\rho\Sigma^2} \tag{11}$$

which says that the additional price increase (LHS) is confirmed by the additional trade (RHS). The equilibrium price variance solves

$$\Sigma^2 = \frac{c^2 n\varepsilon^2}{12\Sigma^4\rho^2} \tag{12}$$

which is just the variance of the market maker's rule (1). These two equations, (11) and (12), can be solved for Σ^2 and ε to find an economy where the first equilibrium is $(\mathbb{E}[p_{t+1}|p_t] - p_t, \Sigma_t^2) = (0, \Sigma^2)$ and the second equilibrium is (ε, Σ^2), with the same $\Sigma^2 = \frac{cn}{\rho}$. That is, both martingale and ε-drift constitute intraperiod equilibria. Note that the long-run price distribution will remain Uniform, confirming the equal spacing assumption. While I do not formally prove this next claim, as the previous construction suggests, economies with multiplicity in this sense are rare.

Unfortunately, interperiod equilibria may also be non-unique. I split the multiplicity problems into two cases, and discuss each only informally. Case one refers to the technical problem of multiplicity arising from the long-run density fixed point problem, and case two refers to multiplicity of the expected inattention cost minimization problem *for fixed* g. Either types of multiplicity can give rise to some other equally spaced reference points, for which all of my results still apply, so instead I discuss the case of non-periodic multiplicity. Could it be that a different choice of grid leads to prices that confirm this choice? I cannot fully rule out this kind of multiplicity for the following reason. Say that traders form an equally-spaced grid, however miss one grid point. This creates regimes like those in Figure 6, but with one large regime. Traders make large mistakes in this range. This tends to push next-period-prices out of this range, confirming the original decision to pay less attention to this range.

4.3 Comparative Statics

The key parameters of my model are ρ, the reference point distribution, ε, c and n. I de-emphasize ρ, c, and n because they are amplifiers of the equilibrium price variance, Σ_t^2. Taking the variance of the market maker's rule (1) makes this evident:

$$\Sigma_t^2 = c^2 n \frac{\mathbb{E}[(\tilde{\mathbb{E}}^j[p_{t+1}|p_t] - p_t)^2] - (\mathbb{E}[\tilde{\mathbb{E}}^j[p_{t+1}|p_t] - p_t])^2}{\rho^2 \Sigma_t^4}$$

This leaves me with the reference point distribution and ε. Examples 3.1 and 3.2 should have already hinted that changing the reference point distribution changes the degree of *agreement* in the economy. Figure 8 "fills the gaps" between Examples 3.1 and 3.2, by plotting the second moment of non-normalized demand for different levels of agreement. In the figure, $\rho = \varepsilon = 1$ and the different curves correspond to different Uniform distributions with varying support (from relative agreement at $[0.4, 0.6]$ to relative disagreement at $[0, 1]$). Consistent with results from Example 3.1, moments are constant when the distribution is Uniform$[0, 1]$.

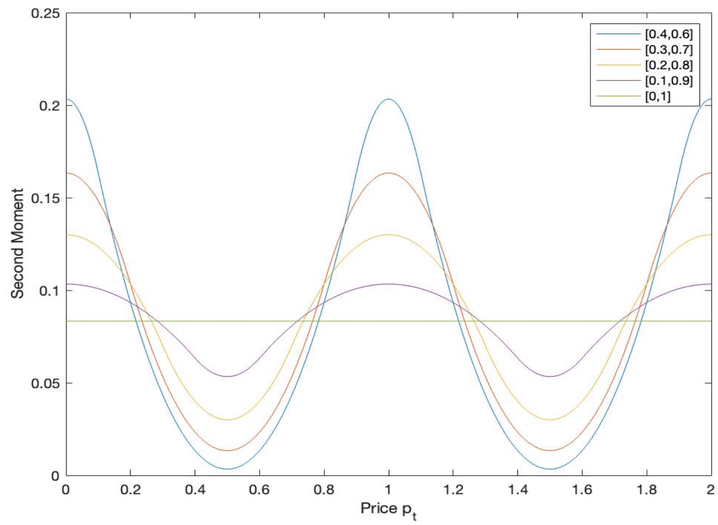

Figure 8: Comparative Statics

To further investigate the implications of agreement in my economy, I tie the degree of agreement back to profits made by traders and show that profits (generally) increase in agreement. Given the market microstructure, this finding should not be surprising. When prices are formed according to (1), traders all want to trade in the same direction (buy when others buy so prices rise, and vice versa); agreement on reference points is just one way to achieve this goal. For intuition, I rewrite expected profits as

$$\mathbb{E}[(p_{t+1} - p_t)x^j(p_t)] = c\frac{\mathbb{E}[(\tilde{\mathbb{E}}^j[p_{t+1}|p_t] - p_t)^2] + (n-1)(\mathbb{E}[\tilde{\mathbb{E}}^j[p_{t+1}|p_t] - p_t])^2}{\rho^2\Sigma_t^4} \tag{13}$$

which shows that profits can be decomposed into two factors. The first term on the RHS captures the average trader's own price impact implying that, even if they are the only trader in the economy, their purchases (sales) move prices up (down).[20] The second term on the RHS capturers the aforementioned coordination motive. The average trader tends to buy (sell) when prices are a submartingale (supermartingale) and others are buying (selling).

Figure 9 plots profits for three levels of agreement: Uniform$[0, 1]$, Uniform$[0.05, 0.95]$, and Uniform$[0.1, 0.9]$, and additional simulations suggest that patterns observed here are robust. Like in Example 3.2, parameters are set to $\rho = \varepsilon = 1$, $c = 0.0014$, and $n = 20$. With the maximum level of disagreement (i.e. Uniform $[0, 1]$), prices are everywhere a

[20]Recall that traders do not realize that they have own price impact.

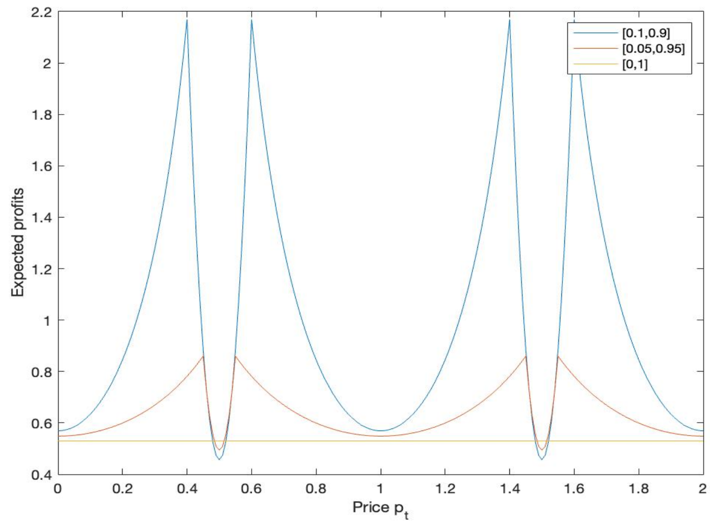

Figure 9: Expected Profit Levels

martingale, and hence the (second) coordination term in (13) disappears. However, profits are nonzero due to the trader's own price impact. While intuition might suggest that more agreement increases profits everywhere, this is not true (i.e. see prices near $p_t = 0.5$).

To understand why, note that, near the center and edges of the distribution, prices are a martingale (see top panel of Figure 5 for an example).[21] At these extremes, the coordination term in (13) is zero, leaving only the own-price impact term. According to Figure 8, the second moment increases near the edges and decreases near the center of the regime with respect to the level of agreement. This manifests in Figure 9 with slightly higher (lower) profits near the edge (center) of the regime. As n is decreased, the coordination motive decreases and, hence, the relative magnitude of the own-price impact term increases.

Next, consider the peaks in Figure 9, for example $p_t \in \{0.4, 0.6\}$ corresponding to the Uniform$[0.1, 0.9]$ distribution. A price of $p_t = 0.4$ ($p_t = 0.6$) is where the largest proportion of the population is buying (selling). To see why, at $p_t = 0.4$, everyone with a reference point in $[0.4, 0.9]$ is buying, resulting in a maximal five-eights of the population buying (because the distribution has support of size 0.8, the proportion is $\frac{0.9-0.4}{0.8}$). Symmetrically, at $p_t = 0.6$, everyone with a reference point in $[0.1, 0.6]$ is selling, resulting in a maximal five-eights of the population selling. Due to the zero-sum nature

[21]The proof of Theorem 2 shows that this property holds for all symmetric distributions.

of the model, the market maker makes losses equal to all such gains.

Finally, consider different values of ε. With a fixed reference point distribution, the previous discussion suggests that changing ε will change the level of agreement in the economy. To control for this, the appropriate exercise should be, for example, to consider varying ε under a Uniform reference point distribution with support of size $a\varepsilon$ (and $a < 1$). Importantly, the frequency of moments is affected by this hypothetical exercise. So while changing the level of agreement (reference point distribution) affected the *amplitude* of non-normalized demand moments, changing the attention parameter (ε) affects the *frequency* of non-normalized demand moments.

This discussion has important implications for an economy with heterogeneous ε^i. It is very natural (perhaps more so than varying reference points) to consider heterogeneity in the attention parameter, ε. The potential exercise I have in mind is an economy with reference point heterogeneity like before, but furthermore, I groups with proportions β^i such that each group has attention ε^i. These proportions and attention values are known to all traders. While it is a striking negative result, it is important to discuss why my model cannot sustain such heterogeneity. Consider $\varepsilon^1 = 0.5$ and $\varepsilon^2 = 2.5$. Equilibrium price moments will, indeed be periodic in $\varepsilon^2 = 2.5$, but not in $\varepsilon^1 = 0.5$. So while I could apply Theorem 3 to the inattentive group, it will importantly fail to apply to the attentive group. They will, in some sense, take advantage of the inattentive group and focus their posterior means near the inattentive group's reference point (where long-run prices will tend to stay).[22] In this paper, I refrain from this interesting investigation and restrict my analysis only to the equally spaced reference points, $\mathcal{R}$.

5 Conclusion

One might ask: Why this model? Many assumptions went into the result, including coarse Bayesian updating, reference point heterogeneity, and a nonstandard market maker. My first answer to this question is: To my knowledge, this is the *only* model that endogenizes support and resistance levels. But perhaps a better answer is the following. Take the market maker as given and see Teeple (2023), which is an entire paper dedicated to justifying this market maker. Given this microstructure, the only way to generate support and resistance levels is with a demand function that looks like Figure 1. At the top

[22] Perhaps this failure is not surprising when recast from the perspective of long-run survival of inattentive traders.

(bottom) of a regime, traders must sell (buy) to keep prices within that regime. My other assumptions are one way, but certainly not the only, to generate such demand.

When traders faced a time-zero problem of choosing their optimal price grid, they chose an equally-spaced grid to minimize trading errors. This led to demand functions with both a conventional downward-sloping component, as well as a speculative upward-sloping component (where higher prices today were used as an indication of higher prices tomorrow). Finally, when restricted to placing only market orders, the market maker was forced to inject liquidity in order to clear markets. The resulting trial-and-error, or tâtonnement, process, generated price barriers, or price levels where prices tend to deflect (but move more so than otherwise warranted if not). A key parameter was the reference point distribution, which captured the degree of agreement across traders. With less agreement, buyers believed the asset was underpriced in a bull market and sellers believed the asset was overpriced in a bear market. With more agreement, when prices rose (fell) to the top (bottom) of a regime, traders collectively believed that the asset was overpriced (underpriced); this created downward (upward) price pressure, pushing prices back into said regime.

While I have only alluded to applications of my model in equity markets, there are others. There is a well-established literature focused on support and resistance in foreign exchange markets, and also growing evidence that support and resistance levels are prevalent in cryptocurrency markets. For example, Hudson and Urquhart (2021) find significant predictability and profitability in several cryptocurrencies using support-resistance trading rules. As another example, Gerritsen et al. (2020) find significant forecasting power trading range break in the Bitcoin market.

A Proofs

A.1 Proof of Proposition 1

Call the long-run price density $g(x)$. Consider any $y \neq x$. Then

$$\begin{aligned} g(y) &= \int_{-\infty}^{\infty} f(y-p)\frac{dp}{b} \\ &= \int_{-\infty}^{\infty} f(x-u)\frac{du}{b} \\ &= g(x) \end{aligned}$$

where the second equality follows from the change of variable $y - p = x - u$. This line of reasoning shows that $g(x)$ is a constant function. $\square$

A.2 Proof of Proposition 2

The problem is

$$\min_{r_1,\ldots,r_k} \int_0^b \min\{|r_1 - p|, ..., |r_k - p|\}\, dp = \min_{r_1,\ldots,r_k} \sum_{i=1}^{k} \int_{(r_i+r_{i-1})/2}^{(r_i+r_{i+1})/2} |r_i - p| dp$$

with the understanding that $(r_0 + r_1)/2 = 0$ and $(r_k + r_{k+1})/2 = b$. Take a first-order condition with respect to r_i:

$$\begin{aligned} &\frac{1_{\{i\neq 1\}}}{2}\left|\frac{r_{i-1}-r_i}{2}\right| - \frac{1_{\{i\neq 1\}}}{2}\left|\frac{r_i - r_{i-1}}{2}\right| + \frac{1_{\{i\neq k\}}}{2}\left|\frac{r_i - r_{i+1}}{2}\right| \\ &+ \int_{(r_i+r_{i-1})/2}^{r_i} dp - \int_{r_i}^{(r_i+r_{i+1})/2} dp - \frac{1_{\{i\neq k\}}}{2}\left|\frac{r_{i+1}-r_i}{2}\right| = 0 \end{aligned}$$

which simplifies to

$$r_i = \frac{r_{i-1} + r_{i+1}}{2}$$

when $i < 1 < k$, $r_1 = r_2/3$, and $r_k = (2b + r_{k-1})/3$. This resulting system is uniquely solved by $r_i^* = \varepsilon/2 + \varepsilon(i-1)$. $\square$

A.3 Proof of Theorem 1

First consider statement (a). Non-normalized demand moments, which can be written

$$D_m(p) = \int_0^{\varepsilon} [\text{mod}(r - p - \varepsilon/2, \varepsilon) - \varepsilon/2]^m dH(r)dr,$$

are periodic by construction. Note that $H(r)$ denotes the distribution of the reference point, which I restrict to distributions with support of size ε (see discussion immediately before Theorem 1). Then, $\Sigma(p)^2$ must also be periodic by taking the variance of the market maker's rule (1). By the multinomial theorem, price moments $\mathbb{E}[(p_{t+1} - p)^m | p]$ are given by a linear combination of $D_m(p)$ and, hence, are periodic as well.

Next consider statement (b). Due to symmetry, the regime center is given by $\mathbb{E}[r] = \varepsilon/2$. Based on the discussion after Theorem 1, I need to show that prices are a submartingale for $p_t \in [0, \varepsilon/2]$. The first moment of the market maker's rule (1) yields

$$\mathbb{E}[p_{t+1}|p_t] - p_t = \frac{cn}{\rho} \frac{\mathbb{E}[\tilde{\mathbb{E}}^j[p_{t+1}|p_t] - p_t]}{\Sigma_t^2}$$

And because I care only about signing this object, it suffices to analyze the numerator on the RHS. The mathematical requirement for a price submartingale becomes

$$\int_0^{\varepsilon} [\text{mod}(r - p - \varepsilon/2, \varepsilon) - \varepsilon/2] dH(r)dr \geq 0, \text{ for } p \in [0, \varepsilon/2]$$

The modulo can be rewritten as

$$\begin{aligned} &\int_0^{p+\varepsilon/2} [r - p] dH(r)dr + \int_{p+\varepsilon/2}^{\varepsilon} [r - p - \varepsilon] dH(r)dr \geq 0 \\ \iff &\mathbb{E}[r] - p - \varepsilon \int_{p+\varepsilon/2}^{\varepsilon} dH(r)dr \geq 0 \end{aligned}$$

Simplifying further,

$$\mathbb{E}[r] - p \geq \varepsilon[1 - H(p + \varepsilon/2)]$$

or

$$p + \varepsilon/2 \leq \varepsilon Pr(r \leq p + \varepsilon/2) \tag{14}$$

Symmetrically, I need to show that prices are a supermartingale for $p_t \in [\varepsilon/2, \varepsilon]$. The mathematical requirement for a price supermartingale becomes

$$\int_0^{\varepsilon} [\mathrm{mod}(r - p - \varepsilon/2, \varepsilon) - \varepsilon/2] dH(r) dr \leq 0, \text{ for } p \in [\varepsilon/2, \varepsilon]$$

Steps mirror those of the submartingale, and I simplify to an analogous condition,

$$\mathbb{E}[r] - p \leq -\varepsilon H(p - \varepsilon/2)$$

or

$$p - \varepsilon/2 \geq \varepsilon Pr(r \leq p - \varepsilon/2) \tag{15}$$

for $p \in [\varepsilon/2, \varepsilon]$. Written more concisely, (14) and (15) reduce to

$$p/\varepsilon \geq Pr(r \leq p), \text{ for } p \in [0, \varepsilon/2] \text{ and}$$
$$p/\varepsilon \leq Pr(r \leq p), \text{ for } p \in [\varepsilon/2, \varepsilon]$$

The last part of the argument for part (b) is to show that, for c small enough, $|\mathbb{E}[p_{t+1}|p_t] - p_t| \leq \varepsilon/2$. If this is true, then for $p_t \in [0, \varepsilon/2]$,

$$-\varepsilon/2 \leq p_t - r^{j'} \leq 0, \text{ and } 0 \leq \mathbb{E}[p_{t+1}|p_t] - p_t \leq \varepsilon/2$$

where the first two inequalities follow from the fact that $r^{j'} = \varepsilon/2$, and the second two inequalities follow from the submartingale argument combined with a sufficiently small c. Add these two conditions together to get the desired condition (b):

$$-\varepsilon/2 \leq \mathbb{E}[p_{t+1}|p_t] - r^{j'} \leq \varepsilon/2$$

A similar argument applies for the case when $p_t \in [\varepsilon/2, \varepsilon]$. Then

$$0 \leq p_t - r^{j'} \leq \varepsilon/2, \text{ and } -\varepsilon/2 \leq \mathbb{E}[p_{t+1}|p_t] - p_t \leq 0$$

which can be added together to get condition (b). To show that $|\mathbb{E}[p_{t+1}|p_t] - p_t| \leq \varepsilon/2$ for small c, combine the first and second moment of the market maker's rule (1) to get

$$\mathbb{E}[p_{t+1}|p_t] - p_t = \frac{cn}{\rho} \frac{\mathbb{E}[\tilde{\mathbb{E}}^j[p_{t+1}|p_t] - p_t]}{(\frac{c^2 n}{\rho^2}\{\mathbb{E}[(\tilde{\mathbb{E}}^j[p_{t+1}|p_t] - p_t)^2] - (\mathbb{E}[\tilde{\mathbb{E}}^j[p_{t+1}|p_t] - p_t])^2\})^{1/3}},$$

where the term raised to the $\frac{1}{3}$ power is precisely Σ_t^2. Taking absolute values and cancelling terms,

$$|\mathbb{E}[p_{t+1}|p_t] - p_t| = \frac{c^{1/3}n^{2/3}}{\rho^{1/3}} \frac{|\mathbb{E}[\tilde{\mathbb{E}}^j[p_{t+1}|p_t] - p_t]|}{(\mathbb{E}[(\tilde{\mathbb{E}}^j[p_{t+1}|p_t] - p_t)^2] - (\mathbb{E}[\tilde{\mathbb{E}}^j[p_{t+1}|p_t] - p_t])^2)^{1/3}},$$

If the large fraction on the RHS is bounded, then my argument is complete because the LHS gets small as c gets small. But note that $\tilde{\mathbb{E}}^j[p_{t+1}|p_t] - p_t$ is always bounded from above and below by $\varepsilon/2$, and a bounded random variable has finite moments. The only case that still needs to be ruled out is that of zero variance. For this, it is sufficient to assume that reference points are nondegenerate.

Finally, consider statement (c). Say that all price moments are constant and, in particular, the first price moment is constant. Then

$$\int_0^{\varepsilon} [\mathrm{mod}(r - p - \varepsilon/2, \varepsilon) - \varepsilon/2] dH(r)dr = \int_0^{\varepsilon} [\mathrm{mod}(r - p' - \varepsilon/2, \varepsilon) - \varepsilon/2] dH(r)dr$$

for $p \neq p'$. By explicitly considering the modulo function, this equality can be written, for some integers i and ℓ and real numbers $c \leq d$ in $[0, \varepsilon]$,

$$\begin{aligned} &\int_0^c [(r - p - i\varepsilon) - (r - p' - \ell\varepsilon)] dH(r)dr \\ &+ \int_c^d [(r - p - [i+1]\varepsilon) - (r - p' - \ell\varepsilon)] dH(r)dr \\ &+ \int_d^{\varepsilon} [(r - p - [i+1]\varepsilon) - (r - p' - [\ell+1]\varepsilon)] dH(r)dr = 0 \end{aligned}$$

With some algebra, all terms cancel except

$$p' - p + \ell\varepsilon - i\varepsilon = \varepsilon \int_c^d dH(r)dr \tag{16}$$

But consider the cutoffs c and d. c is precisely the point where $r - p - i\varepsilon = \varepsilon$, and d is precisely the point where $r - p' - \ell\varepsilon = \varepsilon$. Their difference, $d - c$, must then equal $p' + \ell\varepsilon - p - i\varepsilon$. Using this observation, I can rewrite my condition (16) as

$$d - c = \varepsilon \int_c^d dH(r)dr \iff \frac{H(d) - H(c)}{d - c} = \frac{1}{\varepsilon}$$

which shows that the slope of the distribution function is always $\frac{1}{\varepsilon}$. □

A.4 Proof of Theorem 2

For the proof of part (a), see Theorem 1. The following is the proof of part (b). The equilibrium price variance solves

$$\Sigma(p)^2 = \left(\frac{c^2 n}{\rho^2}\{\mathbb{E}[(\tilde{\mathbb{E}}^j[p_{t+1}|p] - p)^2] - (\mathbb{E}[\tilde{\mathbb{E}}^j[p_{t+1}|p] - p])^2\}\right)^{1/3}$$

Removing positive transformations and explicitly writing moments of non-normalized demand, it suffices to consider the FOC of the following expression

$$\int_0^\varepsilon [\mathrm{mod}(r - p - \varepsilon/2, \varepsilon) - \varepsilon/2]^2 h(r)dr - \left(\int_0^\varepsilon [\mathrm{mod}(r - p - \varepsilon/2, \varepsilon) - \varepsilon/2]h(r)dr\right)^2$$

I focus on two values, $p \in \{0, \varepsilon/2\}$ for which the second (squared) term above equals zero. If it is zero, then its FOC will also equal zero by the chain rule. I begin with $p = 0$:

$$\begin{aligned}\int_0^\varepsilon [\mathrm{mod}(r - \varepsilon/2, \varepsilon) - \varepsilon/2]h(r)dr &= \int_0^{\varepsilon/2} rh(r)dr + \int_{\varepsilon/2}^{\varepsilon} (r - \varepsilon)h(r)dr \\ &= \int_0^{\varepsilon/2} rh(r)dr + \int_{\varepsilon/2}^{0} uh(-u + \varepsilon)du \\ &= \int_0^{\varepsilon/2} rh(r)dr - \int_0^{\varepsilon/2} uh(-u + \varepsilon)du \\ &= 0\end{aligned} \tag{17}$$

where the second equality follows from the change of variable, $u = -(r - \varepsilon)$, and the last equality follows from symmetry of $h(r)$. Similar logic applies when $p = \varepsilon/2$:

$$\begin{aligned}\int_0^\varepsilon [\mathrm{mod}(r - \varepsilon, \varepsilon) - \varepsilon/2]h(r)dr &= \int_0^\varepsilon [r - \varepsilon/2]h(r)dr \\ &= \int_0^{\varepsilon/2} [r - \varepsilon/2]h(r)dr + \int_{\varepsilon/2}^{\varepsilon} [r - \varepsilon/2]h(r)dr \\ &= \int_{-\varepsilon/2}^{0} vh(v + \varepsilon/2)dv + \int_0^{-\varepsilon/2} uh(\varepsilon/2 - u)du \\ &= -\int_0^{-\varepsilon/2} vh(v + \varepsilon/2)dv + \int_0^{-\varepsilon/2} uh(\varepsilon/2 - u)du \\ &= 0\end{aligned} \tag{18}$$

where the third equality follows from two changes of variable, $v = r - \varepsilon/2$ in the first integral and $u = -(r - \varepsilon/2)$ in the second, and the final equality follows from symmetry of $h(r)$. I now take my FOC, ignoring the second term based on this previous line of reasoning. At $p = 0$,

$$
\begin{aligned}
&\frac{\partial}{\partial p}\left[\int_0^{\varepsilon}[\mathrm{mod}(r-p-\varepsilon/2,\varepsilon)-\varepsilon/2]^2 h(r)dr\right] \\
=&\frac{\partial}{\partial p}\left[\int_0^{p+\varepsilon/2}(r-p)^2 h(r)dr + \int_{p+\varepsilon/2}^{\varepsilon}(r-p-\varepsilon)^2 h(r)dr\right] \\
=&(\varepsilon/2)^2 h(\varepsilon/2) - 2\int_0^{\varepsilon/2} rh(r)dr - (-\varepsilon/2)^2 h(\varepsilon/2) - 2\int_{\varepsilon/2}^{\varepsilon}(r-\varepsilon)h(r)dr \\
=&-2\int_0^{\varepsilon/2} rh(r)dr - 2\int_{\varepsilon/2}^{\varepsilon}(r-\varepsilon)h(r)dr
\end{aligned}
$$

But this last expression must equal zero based on my line of reasoning in (17). Similarly, at $p = \varepsilon/2$,

$$
\begin{aligned}
\frac{\partial}{\partial p}\left[\int_0^{\varepsilon}[\mathrm{mod}(r-p-\varepsilon/2,\varepsilon)-\varepsilon/2]^2 h(r)dr\right] =&\frac{\partial}{\partial p}\left[\int_0^{\varepsilon}(r-p)^2 h(r)dr\right] \\
=&-2\int_0^{\varepsilon}(r-\varepsilon/2)h(r)dr
\end{aligned}
$$

But this last expression equals zero by my line of reasoning in (18). To verify whether these critical points are maxima or minima, I check second-order conditions. At $p = 0$ and considering the second moment only,

$$
\begin{aligned}
&\frac{\partial^2}{\partial p^2}\left[\int_0^{p+\varepsilon/2}(r-p)^2 h(r)dr + \int_{p+\varepsilon/2}^{\varepsilon}(r-p-\varepsilon)^2 h(r)dr\right] \\
=&\frac{\partial}{\partial p}\left[(\varepsilon/2)^2 h(p+\varepsilon/2) - 2\int_0^{p+\varepsilon/2}(r-p)h(r)dr - (-\varepsilon/2)^2 h(p+\varepsilon/2) - 2\int_{p+\varepsilon/2}^{\varepsilon}(r-p-\varepsilon)h(r)dr\right] \\
=&-2\left[(\varepsilon/2)h(\varepsilon/2) - \int_0^{\varepsilon/2} h(r)dr - (-\varepsilon/2)h(\varepsilon/2) - \int_{\varepsilon/2}^{\varepsilon} h(r)dr\right] \\
=&-2\left[\varepsilon h(\varepsilon/2) - 1\right]
\end{aligned}
$$

Next I consider the first moment squared. To save on notation, I call $\Omega(p) \equiv \int_0^\varepsilon [\text{mod}(r - p - \varepsilon/2, \varepsilon) - \varepsilon/2]h(r)dr$, so that

$$\frac{\partial^2}{\partial p^2}\left[(\Omega(p))^2\right] = \frac{\partial}{\partial p}\left[2\Omega(p)\Omega'(p)\right] = 2\Omega'(p)^2 + 2\Omega(p)\Omega''(p)$$

But I have already shown that $\Omega(p) = 0$ at $p \in \{0, \varepsilon/2\}$. Hence I focus on $2\Omega'(p)^2$. At $p = 0$,

$$\begin{aligned}
&2\left(\frac{\partial}{\partial p}\left[\int_0^\varepsilon [\text{mod}(r - p - \varepsilon/2, \varepsilon) - \varepsilon/2]h(r)dr\right]\right)^2 \\
=&2\left(\frac{\partial}{\partial p}\left[\int_0^{p+\varepsilon/2} (r-p)h(r)dr + \int_{p+\varepsilon/2}^\varepsilon (r - p - \varepsilon)h(r)dr\right]\right)^2 \\
=&2\left((\varepsilon/2)h(\varepsilon/2) - \int_0^{\varepsilon/2} h(r)dr - (-\varepsilon/2)h(\varepsilon/2) - \int_{\varepsilon/2}^\varepsilon h(r)dr\right)^2 \\
=&2\left(\varepsilon h(\varepsilon/2) - 1\right)^2
\end{aligned}$$

Hence the entire second-order condition, at $p = 0$, equals

$$\begin{aligned}
-2\left[\varepsilon h(\varepsilon/2) - 1\right] - 2\left(\varepsilon h(\varepsilon/2) - 1\right)^2 =& -2\varepsilon h(\varepsilon/2) - 2\varepsilon^2 h(\varepsilon/2)^2 + 4\varepsilon h(\varepsilon/2) \\
=& 2\varepsilon h(\varepsilon/2)[1 - \varepsilon h(\varepsilon/2)]
\end{aligned}$$

A local maximum requires this term to be nonpositive, or $h(\varepsilon/2) \geq 1/\varepsilon$, which is guaranteed by Theorem 1 condition (b). Similarly, at $p = \varepsilon/2$, the second-order condition of the second moment is

$$\begin{aligned}
\frac{\partial^2}{\partial p^2}\left[\int_0^\varepsilon [\text{mod}(r - p - \varepsilon/2, \varepsilon) - \varepsilon/2]^2 h(r)dr\right] =& \frac{\partial^2}{\partial p^2}\left[\int_0^\varepsilon (r-p)^2 h(r)dr\right] \\
=& \frac{\partial}{\partial p}\left[-2\int_0^\varepsilon (r-p)h(r)dr\right] \\
=& 2\int_0^\varepsilon h(r)dr \\
=& 2
\end{aligned}$$

The second-order condition of the first moment squared is still $2\Omega'(p)^2$, which, at $p = \varepsilon/2$, equals

$$
\begin{aligned}
2\left(\frac{\partial}{\partial p}\left[\int_0^\varepsilon [\mathrm{mod}(r-p-\varepsilon/2,\varepsilon)-\varepsilon/2]h(r)dr\right]\right)^2 =& 2\left(\frac{\partial}{\partial p}\left[\int_0^\varepsilon (r-p)h(r)dr\right]\right)^2 \\
=& 2\left(-\int_0^\varepsilon h(r)dr\right)^2 \\
=& 2
\end{aligned}
$$

Hence, a second derivative of zero at $p = \varepsilon/2$ is consistent with a local minimum.[23] □

A.5 Proof of Theorem 3

First consider statement (a). I know that all price moments are periodic, and I must consider the impact of this on the density $f(x|p)$, where x denotes price tomorrow and p denotes price today. This object exists for two reasons. First, the sum of independent absolutely continuous random variables is absolutely continuous (its density is given by the convolution). This statement is relevant here because prices consist of the sum of independent demands. Second, demand (understood as the random variable) is absolutely continuous for all p when reference points are absolutely continuous. Because

$$\int_{-\infty}^{\infty}(x-p)^m f(x|p)dx$$

is periodic in p, with the substitution $u = x - \varepsilon$,

$$\int_{-\infty}^{\infty}(x-p)^m f(x|p)dx = \int_{-\infty}^{\infty}(x-p-\varepsilon)^m f(x|p+\varepsilon)dx = \int_{-\infty}^{\infty}(u-p)^m f(u+\varepsilon|p+\varepsilon)du$$

Because this holds for all moments, I conclude that $f(x|p) = f(x+\varepsilon|p+\varepsilon)$.[24] I refer to this condition as *conditional periodicity*. Next I claim that, if the reference point distribution is symmetric, then so is $\Sigma(p)^2$.[25] To prove this, I begin with moments of

[23] Simulations confirm this flat portion of the variance function, $\Sigma(p)^2$, in a neighborhood around $p = \varepsilon/2$. Intuitively, all traders believe they are in the same regime so prices simply inherit the reference point variance.

[24] If two random variables have bounded support, their distributions equal if and only if all moments are equal. Note that prices have bounded support.

[25] Symmetry is defined in the standard way: $\Sigma(\delta)^2 = \Sigma(\varepsilon-\delta)^2$ for all $0 \leq \delta \leq \varepsilon$.

non-normalized demand evaluated at $p = \varepsilon/2 - \delta$, for $0 \le \delta \le \varepsilon/2$:

$$
\begin{aligned}
&\int_0^{\varepsilon} [\mathrm{mod}(r - p - \varepsilon/2, \varepsilon) - \varepsilon/2]^m h(r) dr \\
&= \int_0^{\varepsilon} [\mathrm{mod}(r + \delta, \varepsilon) - \varepsilon/2]^m h(r) dr \\
&= \int_0^{\varepsilon-\delta} (r + \delta - \varepsilon/2)^m h(r) dr + \int_{\varepsilon-\delta}^{\varepsilon} (r + \delta - 3\varepsilon/2)^m h(r) dr \\
&= \int_{\delta-\varepsilon/2}^{\varepsilon/2} u^m h(u - \delta + \varepsilon/2) du + \int_{\varepsilon/2}^{\delta+\varepsilon/2} (u - \varepsilon)^m h(u - \delta + \varepsilon/2) du
\end{aligned}
$$

where the last equality follows from the change of variable, $r = u - \delta + \varepsilon/2$. In order to make arguments related to symmetry, I evaluate the same expression at $p = \varepsilon/2 + \delta$:

$$
\begin{aligned}
&\int_0^{\varepsilon} [\mathrm{mod}(r - p - \varepsilon/2, \varepsilon) - \varepsilon/2]^m h(r) dr \\
&= \int_0^{\varepsilon} [\mathrm{mod}(r - \delta, \varepsilon) - \varepsilon/2]^m h(r) dr \\
&= \int_0^{\delta} (r - \delta + \varepsilon/2)^m h(r) dr + \int_{\delta}^{\varepsilon} (r - \delta - \varepsilon/2)^m h(r) dr \\
&= -\int_{\delta+\varepsilon/2}^{\varepsilon/2} (\varepsilon - v)^m h(-v + \delta + \varepsilon/2) dv - \int_{\varepsilon/2}^{\delta-\varepsilon/2} (-v)^m h(-v + \delta + \varepsilon/2) dv \\
&= \int_{\varepsilon/2}^{\delta+\varepsilon/2} (-1)^m (v - \varepsilon)^m h(-v + \delta + \varepsilon/2) dv + \int_{\delta-\varepsilon/2}^{\varepsilon/2} (-1)^m v^m h(-v + \delta + \varepsilon/2) dv \\
&= \int_{\varepsilon/2}^{\delta+\varepsilon/2} (-1)^m (v - \varepsilon)^m h(v - \delta + \varepsilon/2) dv + \int_{\delta-\varepsilon/2}^{\varepsilon/2} (-1)^m v^m h(v - \delta + \varepsilon/2) dv
\end{aligned}
$$

where the third equality follows from the change of variable, $r = -v + \delta + \varepsilon/2$, and the last equality follows from symmetry of $h(r)$. Comparing moments when $p = \varepsilon/2 - \delta$ to moments when $p = \varepsilon/2 + \delta$, I have shown that even moments agree and odd moments agree up to a factor of (-1). An immediate corollary is that the variance of non-normalized demand is symmetric, and hence $\Sigma(p)^2$ is symmetric.

Next consider $(p_{t+1} - p_t)$, which consists of the sum of n independent non-normalized demands, normalized by the symmetric $\Sigma(p)^2$. I argue that this property − even moments agree and odd moments agree up to a factor of (-1) − is inherited by prices as well. To show this, I induct over the number of traders, n. My base case is $n = 2$. Then

$$\begin{aligned}
\mathbb{E}\left[\left(c\sum_{j=1}^{2}x^j(\varepsilon/2-\delta)\right)^m\right] =& c^m\mathbb{E}\left[\sum_{k=0}^{m}\binom{m}{k}x^1(\varepsilon/2-\delta)^k x^2(\varepsilon/2-\delta)^{m-k}\right] \\
=& c^m\sum_{k=0}^{m}\binom{m}{k}\mathbb{E}\left[x^1(\varepsilon/2-\delta)^k\right]\mathbb{E}\left[x^2(\varepsilon/2-\delta)^{m-k}\right] \\
=& c^m(-1)^m\sum_{k=0}^{m}\binom{m}{k}\mathbb{E}\left[x^1(\varepsilon/2+\delta)^k\right]\mathbb{E}\left[x^2(\varepsilon/2+\delta)^{m-k}\right] \\
=& (-1)^m\mathbb{E}\left[\left(c\sum_{j=1}^{2}x^j(\varepsilon/2+\delta)\right)^m\right]
\end{aligned}$$

where the first equality follows from the binomial theorem, the second equality follows from independence, and the third equality follows from what what just proved above. Now assume that my desired property holds for n traders. For the $(n+1)$ case:

$$\begin{aligned}
\mathbb{E}\left[\left(c\sum_{j=1}^{n+1}x^j(\varepsilon/2-\delta)\right)^m\right] =& c^m\mathbb{E}\left[\sum_{k=0}^{m}\binom{m}{k}\left(\sum_{j=1}^{n}x^1(\varepsilon/2-\delta)\right)^k x^{n+1}(\varepsilon/2-\delta)^{m-k}\right] \\
=& c^m\sum_{k=0}^{m}\binom{m}{k}\mathbb{E}\left[\left(\sum_{j=1}^{n}x^1(\varepsilon/2-\delta)\right)^k\right]\mathbb{E}\left[x^{n+1}(\varepsilon/2-\delta)^{m-k}\right] \\
=& c^m(-1)^m\sum_{k=0}^{m}\binom{m}{k}\mathbb{E}\left[\left(\sum_{j=1}^{n}x^1(\varepsilon/2+\delta)\right)^k\right]\mathbb{E}\left[x^{n+1}(\varepsilon/2+\delta)^{m-k}\right] \\
=& (-1)^m\mathbb{E}\left[\left(c\sum_{j=1}^{n+1}x^j(\varepsilon/2+\delta)\right)^m\right]
\end{aligned}$$

where the first equality again follows from the binomial theorem, the second equality follows from independence, and the third equality follows from the induction assumption. Using the price density $f(x|p)$, the moment agreement shown above (up to a factor of -1) can be written as

$$\int_{-\infty}^{\infty}(x-[\varepsilon/2-\delta])^m f(x|\varepsilon/2-\delta)dx = (-1)^m\int_{-\infty}^{\infty}(x-[\varepsilon/2+\delta])^m f(x|\varepsilon/2+\delta)dx \quad (19)$$

Manipulating this expression, the RHS equals

$$-\int_{\infty}^{-\infty}(u-[\varepsilon/2-\delta])^m f(\varepsilon-u|\varepsilon/2+\delta)du=\int_{-\infty}^{\infty}(u-[\varepsilon/2-\delta])^m f(\varepsilon-u|\varepsilon/2+\delta)du$$

due the change of variable, $(u-[\varepsilon/2-\delta])^m=(-1)^m(x-[\varepsilon/2+\delta])^m$. But now compare this expression to the LHS of (19). Since all moments match, it must be that the densities match,

$$f(x|\varepsilon/2-\delta)=f(\varepsilon-x|\varepsilon/2+\delta) \tag{20}$$

I will refer to condition (20) as *conditional symmetry* of $f(x|p)$.

Next I investigate whether the long-run distribution, $g(x)$, inherits symmetry and periodicity properties. Recall that $g(x)$ satisfies

$$g(x)=\int_{-\infty}^{\infty}f(x|p)g(p)dp$$

I consider this condition as a fixed point of a linear operator $\mathcal{T}(g)$, mapping from the space of ε-periodic, symmetric, non-negative functions that integrate to one on $[0,b]$ to itself. Call this space K. Using the properties of $f(x|p)$ that I just found, I rewrite $\mathcal{T}(g)$ as

$$\begin{aligned}\int_{-\infty}^{\infty}f(x|p)g(p)dp=&\sum_{\ell\in\mathbb{Z}}\int_{\ell\varepsilon}^{(\ell+1)\varepsilon}f(x|p)g(p)dp\\=&\sum_{\ell\in\mathbb{Z}}\int_0^{\varepsilon}f(x|u+\ell\varepsilon)g(u+\ell\varepsilon)du\\=&\sum_{\ell\in\mathbb{Z}}\int_0^{\varepsilon}f(x-\ell\varepsilon|u)g(u)du\end{aligned}$$

where the second equality follows from the change of variable $u=p-\ell\varepsilon$, and the third equality follows from conditionally periodic $f(x|p)$ and periodic domain, $g(p)$. To see that the image of $\mathcal{T}(g)$, indeed, integrates to one on $[0,b]$,

$$\begin{aligned}\int_0^b\sum_{\ell\in\mathbb{Z}}\int_0^{\varepsilon}f(x-\ell\varepsilon|u)g(u)dudx=&\int_0^{\varepsilon}\sum_{\ell\in\mathbb{Z}}\int_0^b f(x-\ell\varepsilon|u)dxg(u)du\\=&\int_0^{\varepsilon}kg(u)du\\=&1\end{aligned}$$

where the first equality follows from Fubini's Theorem, the second equality follows from integrating the function $f(x|u)$ over $(-\infty,\infty)$ k-times, and the final equality follows from the fact that the domain is the space of ε-periodic functions that integrate to one on $[0,b]$. To see that the image of $\mathcal{T}(g)$ is ε-periodic,

$$\begin{aligned}\sum_{\ell\in\mathbb{Z}}\int_0^{\varepsilon} f(x+\varepsilon-\ell\varepsilon|u)g(u)du =& \sum_{\ell\in\mathbb{Z}}\int_0^{\varepsilon} f(x-[\ell-1]\varepsilon|u)g(u)du \\ =& \sum_{\ell\in\mathbb{Z}}\int_0^{\varepsilon} f(x-\ell\varepsilon|u)g(u)du\end{aligned}$$

where the second equality follows from the fact that the sum is over all $\ell\in\mathbb{Z}$ (and, hence, a shift of the index by one makes no difference). Finally, I must show that $\mathcal{T}(g)$ is symmetric. That is, I must show that

$$\sum_{\ell\in\mathbb{Z}}\int_0^{\varepsilon} f(\delta-\ell\varepsilon|u)g(u)du = \sum_{\ell\in\mathbb{Z}}\int_0^{\varepsilon} f(\varepsilon-\delta-\ell\varepsilon|u)g(u)du \tag{21}$$

for $0\le\delta\le\varepsilon/2$. Manipulating the LHS,

$$\begin{aligned}\sum_{\ell\in\mathbb{Z}}\int_0^{\varepsilon} f(\delta-\ell\varepsilon|u)g(u)du =& \sum_{\ell\in\mathbb{Z}}\int_0^{\varepsilon} f(\varepsilon-\delta+\ell\varepsilon|\varepsilon-u)g(\varepsilon-u)du \\ =& -\sum_{\ell\in\mathbb{Z}}\int_{\varepsilon}^{0} f(\varepsilon-\delta+\ell\varepsilon|v)g(v)dv \\ =& \sum_{\ell\in\mathbb{Z}}\int_0^{\varepsilon} f(\varepsilon-\delta+\ell\varepsilon|v)g(v)dv \\ =& \sum_{\ell\in\mathbb{Z}}\int_0^{\varepsilon} f(\varepsilon-\delta-\ell\varepsilon|v)g(v)dv\end{aligned}$$

where the first equality follows from conditional symmetry of $f(x|p)$ and symmetry of the domain $g(p)$, the second equality follows from the change of variable, $v=\varepsilon-u$, and the final equality follows from the fact that the sum is over all integers. This last expression equals the desired RHS of (21). To conclude so far, I have shown that $\mathcal{T}:K\to K$. The goal is to apply the Schauder fixed point theorem to this linear operator.

The theorem requires that the space, K, be convex, closed, and nonempty. These properties can be explicitly verified. The theorem also requires that the operator be continuous, which is equivalent to bounded when it is linear. But this was just shown above: $||\mathcal{T}(f)||_1=||f||_1$. Finally, the theorem requires that the image of $\mathcal{T}$ be contained in a compact subset of K. By Prokhorov's theorem, a collection of probability measures is tight if and only if its closure is compact (with the topology of weak convergence). But because the image of $\mathcal{T}$ consists of densities compactly supported in $[0, b]$, tightness is immediate.

Next consider statement (b). The problem is

$$\min_{r_1,\ldots,r_k} \int_0^b \min\left\{\frac{|r_1 - p|}{\Sigma(p)^2}, \ldots, \frac{|r_k - p|}{\Sigma(p)^2}\right\} g(p)dp = \min_{r_1,\ldots,r_k} \sum_{i=1}^k \int_{(r_i+r_{i-1})/2}^{(r_i+r_{i+1})/2} |r_i - p| \frac{g(p)}{\Sigma(p)^2} dp$$

with the understanding that $(r_0 + r_1)/2 = 0$ and $(r_k + r_{k+1})/2 = b$. Take a first-order condition with respect to r_i:

$$\begin{aligned}
&\frac{1_{\{i \neq 1\}}}{2}\left|\frac{r_{i-1} - r_i}{2}\right| \frac{g([r_{i-1} + r_i]/2)}{\Sigma([r_{i-1} + r_i]/2)^2} - \frac{1_{\{i \neq 1\}}}{2}\left|\frac{r_i - r_{i-1}}{2}\right| \frac{g([r_i + r_{i-1}]/2)}{\Sigma([r_i + r_{i-1}]/2)^2} \\
&+ \frac{1_{\{i \neq k\}}}{2}\left|\frac{r_i - r_{i+1}}{2}\right| \frac{g([r_i + r_{i+1}]/2)}{\Sigma([r_i + r_{i+1}]/2)^2} + \int_{(r_i+r_{i-1})/2}^{r_i} \frac{g(p)}{\Sigma(p)^2} dp \\
&- \int_{r_i}^{(r_i+r_{i+1})/2} \frac{g(p)}{\Sigma(p)^2} dp - \frac{1_{\{i \neq k\}}}{2}\left|\frac{r_{i+1} - r_i}{2}\right| \frac{g([r_{i+1} + r_i]/2)}{\Sigma([r_{i+1} + r_i]/2)^2} = 0
\end{aligned}$$

which simplifies to

$$\int_{(r_i+r_{i-1})/2}^{r_i} \frac{g(p)}{\Sigma(p)^2} dp - \int_{r_i}^{(r_i+r_{i+1})/2} \frac{g(p)}{\Sigma(p)^2} dp = 0$$

Finally, I verify that $r_i^* = \varepsilon/2 + \varepsilon(i-1)$ is one solution.

$$\begin{aligned}
&\int_{\varepsilon(i-1)}^{\varepsilon/2+\varepsilon(i-1)} \frac{g(p)}{\Sigma(p)^2} dp - \int_{\varepsilon/2+\varepsilon(i-1)}^{\varepsilon i} \frac{g(p)}{\Sigma(p)^2} dp = 0 \\
\iff &\int_{-\varepsilon/2}^{0} \frac{g(u + \varepsilon[i-1] + \varepsilon/2)}{\Sigma(u + \varepsilon[i-1] + \varepsilon/2)^2} du + \int_{0}^{-\varepsilon/2} \frac{g(\varepsilon i - \varepsilon/2 - v)}{\Sigma(\varepsilon i - \varepsilon/2 - v)^2} dv = 0 \\
\iff &\int_{-\varepsilon/2}^{0} \frac{g(u + \varepsilon[i-1] + \varepsilon/2)}{\Sigma(u + \varepsilon[i-1] + \varepsilon/2)^2} du - \int_{-\varepsilon/2}^{0} \frac{g(\varepsilon i - \varepsilon/2 - v)}{\Sigma(\varepsilon i - \varepsilon/2 - v)^2} dv = 0 \\
\iff &\int_{-\varepsilon/2}^{0} \frac{g(u + \varepsilon/2)}{\Sigma(u + \varepsilon/2)^2} du - \int_{-\varepsilon/2}^{0} \frac{g(\varepsilon/2 - v)}{\Sigma(\varepsilon/2 - v)^2} dv = 0
\end{aligned}$$

where the first equivalence is due to two changes of variable, $u = p - \varepsilon(i-1) - \varepsilon/2$ in the first integral and $v = -[p - \varepsilon i + \varepsilon/2]$ in the second, and the final equivalence follows from periodicity. The final equality holds due to symmetry of both $\Sigma(p)^2$ and $g(p)$. $\square$

B Microfoundation for r^j

In this appendix, I consider the reference point r^j as a reduced form way of modeling trader informational disagreement.[26] To model this, reconsider the martingale conjecture and say that, instead, each trader receives an i.i.d. signal from a symmetric, absolutely continuous distribution each period that prices will rise by s^j with $\mathbb{E}[s^j] = 0$. First, this impacts the ex ante rounding problem:

$$\min_{r_1,\ldots,r_k} \int_0^b \min\left\{\frac{|r_1 - p - s^j|}{\Sigma(p)^2}, \ldots, \frac{|r_k - p - s^j|}{\Sigma(p)^2}\right\} g(p)dp$$

Directly applying the results of Theorem 3, an optimal sequence is $r_i - s^j = \varepsilon/2 + \varepsilon(i-1)$, or $r_i = \varepsilon/2 + \varepsilon(i-1) + s^j$. Hence, I obtain my shifted reference points. Second, all of my intraperiod equilibrium results go through. By the time her trading period arrives, each trader asymptotically knows the average signal, $\mathbb{E}[s^j] = 0$.[27] With the resulting martingale guess, the coarse expectation becomes

$$\tilde{\mathbb{E}}^j[p_{t+1}|p_t] = \text{argmin}_p |p - p_t| \;\text{ s.t. }\; p \in \{s^j + \varepsilon/2 + \ell\varepsilon\}_{\ell\in\mathbb{Z}}$$

which is analogous to (5). Like in the baseline setting, it is only true that expected prices are within $\varepsilon/2$ of $\tilde{E}^{j'}[p_{t+1}|p_t]$ for the average trader, j'. However, now there is an intuitive interpretation of this additional error: it is due to imprecise information at the time of the ex ante problem.

[26] Because traders agree to disagree, information should be interpreted as a subjective prior distribution.

[27] All subjective uncertainty about the price mean is dispelled by this time. However, objective price uncertainty remains.

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