Pricing Dynamics of Oil Futures with Tail Risk

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KEY FINDINGS

- A new model is developed to study jump tail risks in the oil market.
- Empirical evidence indicates that tail risk significantly matters in the pricing dynamics of oil futures.
- The use of the tail factor, which is orthogonal to the variance factor, improves performance in forecasting future oil returns, particularly at the short horizon.

ABSTRACT

Oil is one of the most important commodities in the global economy. We study jump tail risks in the oil market by developing a new model for depicting commodity prices. This model combines the cost of carry, stochastic volatility, and the tail factor. An application of this model shows that tail risk significantly matters in the pricing dynamics of oil futures. The use of the tail factor, which is orthogonal to the variance factor, improves performance in forecasting future oil returns, particularly at the short horizon.

Does the jump tail risk matter in the oil market? The importance of tail risk is widely addressed in the literature on asset pricing (e.g., Bollerslev and Todorov 2011; Kelly and Jiang 2014; Fousseni, Ruenzi, and Weigert 2018; Schreindorfer 2020). In the model proposed by Andersen, Fusari, and Todorov (2015), tail risk is captured by a novel factor. These researchers' empirical evidence reveals that this new factor plays a prominent role in forecasting equity risk premiums. In this study, we aim to explore tail risk in the oil market. The motivation for studying this market is that oil is the most important commodity globally, and it has a significant impact on economic activity (Kilian 2009; Singleton 2014; Heath 2019).

We conduct our analysis in two steps. In the first step, we develop a parametric model with tail risk to describe dynamics of oil spot prices. This model is established by generalizing the commodity price dynamics in Trolle and Schwartz (2009) and Li (2019) with the tail factor specification of Andersen, Fusari, and Todorov (2015, 2017). This model is equipped with time-varying upward and downward jumps. The model also captures the self-exciting jump phenomenon emphasized by Aït-Sahalia, Cacho-Diaz, and Laeven (2015); Carr and Wu (2017); and Bates (2019). We then derive the dynamics of futures returns by using the spot process. Insight into the dynamics of oil prices can help decision makers perceive risks and reduce uncertainty in the oil market. To the best of our knowledge, this is the first study to price dynamics of oil futures with jump tail risks.

We apply the particle filter method to estimate the model parameters and to extract state variables from oil futures. Futures are an important type of financial derivatives, as they convey forward-looking information concerning the risks perceived by investors. Empirical evidence indicates that futures prices have predictive power for future spot returns (e.g., Fama 1984a, 1984b; Fama and French 1987). Under our model framework, this predictive power is reflected in two state variables—the variance and the tail risk factors. The empirical results of our study indicate that the tail risk significantly exists in the oil market and the tail factor owns distinctive dynamic properties, compared with the variance factor. The state variables extracted from futures with differing terms also show some differential features.

In the second part of our study, we explore the predictive power that futures-implied risk factors have for future spot returns. The predictive regression proceeds over a horizon from 1 to 24 months. The tail factor is newly discovered by Bollerslev and Todorov (2011), who use the nonparametric technique, and by Andersen, Fusari, and Todorov (2015), who use the parametric method. To analyze the incremental role of the tail factor, we project this factor onto the variance and then use the residual to replace the tail factor in the predictive regression. The *t*-statistics of the predictive coefficients show that the tail factor contributes to the accuracy of return forecasting and has a dominant explanatory power in short-horizon regressions.

To further demonstrate the vital role of the tail factor, we compare the adjusted R^2 between regressions with all factors combined and regressions that involve only the variance factor. We make this comparison across different forecasting horizons. At the short horizon, the variance factor adds nothing in terms of return prediction. The R^2 comes almost entirely from the tail factor. However, at the long horizon, the variance factor shows strong predictive power. Stambaugh (1999); Sizova (2016); and Andersen, Fusari, and Todorov (2017) demonstrate that a persistent component is embedded in the risk premium. Our results reveal that the variance factor is strongly correlated with this persistent component.

Many studies focus on analyzing the term structures of financial assets, for example, returns on bonds (Cochrane and Piazzesi 2005; Ang, Bekaert, and Wei 2008), equity returns (Van Binsbergen and Koijen 2017), variance swaps (Dew-Becker et al. 2017), currency carry returns (Zviadadze 2017), and many other assets. In addition, Mishkin (1990a, 1990b) and De Roon, Nijman, and Veld (1998) confirm that different terms convey different types of information. To build on this insight, we explore the forecasting performance of the state variables extracted from futures with a variety of terms. These variables yield different performance predictions because the risk premium that is embedded in futures varies over the term.

Our study makes two main contributions. First, we present a new model for depicting commodity prices that captures time-varying upward and downward jump risks and incorporates a tail factor. We empirically examine this model in terms of the pricing dynamics of oil futures. This model specification contributes to the commodity valuation literature (e.g., Gibson and Schwartz 1990; Garbade 1993; Schwartz 1997; Trolle and Schwartz 2009; Hamilton and Wu 2014; Li 2019; Ames et al. 2020). Our second contribution is to provide evidence that the tail factor implied from oil futures has predictive power for future spot returns. This finding provides supplementary evidence that the information contained in futures is helpful in forecasting spot returns. Our study also contributes to the growing literature on the predictive power of the tail factor (e.g., Bollerslev and Todorov 2011; Andersen, Fusari, and Todorov 2015; Ellwanger 2017).

The studies most closely related to our research are Ellwanger (2017) and Andersen, Fusari, and Todorov (2015). Using the setup of Bollerslev and Todorov (2011), Ellwanger (2017) studies tail risks in the oil market by applying a nonparametric technique. Our study differs from that approach in that we use a parametric model. This difference is similar to that between the approaches of Bollerslev and Todorov (2011) and Andersen, Fusari, and Todorov (2015). Cochrane and Piazzesi

(2008) and Li and Zinna (2018) highlight the stability of the parametric model, and they show that the parametric model can help to avoid specification uncertainty. Our model is established on the research of Andersen, Fusari, and Todorov (2015). As we focus on the oil market, we extend their model framework by additionally specifying the cost of carry.

The rest of the study is organized as follows. The next section presents the dynamics of commodity spot and futures prices, augmented with jump tail risks. The following section describes the estimation methodology and the results. The subsequent section explores the predictability of future oil returns using the futures-implied factors. The final section offers our conclusions and proposals for further research.

MODEL

Spot Price Dynamics

In this section, we extend the commodity price dynamics specified in Trolle and Schwartz (2009) and Li (2019) by including consideration of the tail risk factor, as proposed by Andersen, Fusari, and Todorov (2015, 2020). The spot price dynamics under the risk-neutral measure are given by

$$\frac{dS_t}{S_{t^-}} = \delta_t dt + \sqrt{V_t}\, dW_{s,t} + \int_R (e^x - 1)[\pi(dx, dt) - \nu_t(dx)dt], \tag{1}$$

where S_t is the commodity price at time t, which has an instantaneous spot cost of carry, δ_t. δ_t is also known as the convenience yield (Gibson and Schwartz 1990; Schwartz 1997; Cortazar and Schwartz 2003; Casassus and Collin-Dufresne 2005). Furthermore, we follow Trolle and Schwartz (2009) by using $y(t, T)$ to depict the evolution of the entire forward cost of carry curve. $y(t, T)$ is the time-t instantaneous forward cost of carry at time T, and its stochastic process is

$$dy(t,T) = \mu_y(t,T)dt + \sigma_y(t,T)\sqrt{V_t}\, dW_{y,t}, \tag{2}$$

$$y(t,t) = \delta_t. \tag{3}$$

The increments of Brownian motions in Equations (1) and (2) are correlated, as $dW_{s,t} dW_{y,t} = \rho dt$. ρ is the pairwise correlation coefficient. The volatility of the instantaneous forward cost of carry is set as

$$\sigma_y(t,T) = \alpha e^{-\gamma\tau}, \tag{4}$$

where $\tau = T - t$ denotes the days to maturity. α and γ are two positive parameters that have been confirmed by Trolle and Schwartz (2009). In making this specification, we assume that the short-term forward cost of carry rates should be more volatile than the long-term forward cost of carry rates.

V_t in Equation (1) is a variance factor, the evolution of which is described by the Cox–Ingersoll–Ross process,

$$dV_t = k_v(\bar{V} - V_t)dt + \sigma_v \sqrt{V_t}\, dW_{v,t}, \tag{5}$$

where we allow $W_{s,t}$ and $W_{v,t}$ to be correlated with ρ_{sv} to capture the leverage effect (Black 1976).

Jumps in crude oil prices are nontrivial to price oil derivatives (Christoffersen, Jacobs, and Li 2016; Kyriakou, Pouliasis, and Papapostolou 2016). The jump size in Equation (1) is defined in the real line R. The martingale jump measure is constituted by $\pi(dx, dt) - \nu_t(dx)dt$. $\pi(dx, dt)$ is a random counting measure, and $\nu_t(dx)dt$ acts as its compensator. Following Kou (2002) and Andersen, Fusari, and Todorov (2015), the conditional jump distribution takes the following form:

$$\frac{\nu_t(dx)}{dx} = c_t^- \cdot 1_{\{x<0\}} \lambda_- e^{-\lambda_-|x|} + c_t^+ \cdot 1_{\{x>0\}} \lambda_+ e^{-\lambda_+ x}. \tag{6}$$

The first term on the right-hand side refers to the negative price jumps. The second term on the right-hand side is the positive jump component with a separate decay parameter λ_+. The negative jump size follows the exponential distribution with the tail decay parameter, λ_-. c_t^- and c_t^+ denote the left and right jump intensities, respectively. Building on the empirical findings presented by Andersen, Fusari, and Todorov (2015), we separately specify the dynamics for the negative and positive jump intensities as

$$c_t^- = c_1^- V_t + U_t,\ c_t^+ = c_0^+ + c_1^+ V_t, \tag{7}$$

where c_1^-, c_0^+, and c_1^+ are parameters determined by estimation.[1] U_t is a tail risk factor. To capture the tail risk factor in the oil market, we specify its stochastic process as follows:

$$dU_t = -k_u U_t dt + \eta \int_R x^2 1_{x<0} \pi(dx, dt). \tag{8}$$

In Equation (8), the (squared) negative jumps in the spot price are directly modeled to affect the dynamics of the tail risk factor. This specification enables our model to capture the self-exciting jump clustering (as the occurrence of a jump is likely to cause more jumps to follow). The importance of such clustering is emphasized by Aït-Sahalia, Cacho-Diaz, and Laeven (2015); Fulop, Li, and Yu (2015); Carr and Wu (2017); and Du and Luo (2019).

Futures Price Dynamics

Let $F_{t,T}$ be the time-t price of a futures contract that expires at time T. Based on this definition, this contract's pricing formula can be written as

$$F_{t,T} = S_t \exp\left(\int_t^T y_{t,u} du\right). \tag{9}$$

After defining $Y_{t,T} = \int_t^T y_{t,u} du$, the stochastic process of $Y_{t,T}$ can be derived based on Equation (2),

$$dY_{t,T} = \left(-\delta_t + \int_t^T \mu_y(t,u) du\right) dt - \sqrt{V_t}\, \eta dW_{y,t}, \tag{10}$$

where $\eta = \frac{\alpha}{\gamma}(e^{-\gamma\tau} - 1)$.

Applying Ito's lemma to the pricing formula in Equation (9) gives the dynamics of the futures price

[1] Andersen, Fusari, and Todorov (2015) show that the left jump intensity for negative jumps is governed exclusively by the variance and tail risk factors and that the right intensity for positive jumps is unrelated to the tail risk factor.

$$\frac{dF_{t,T}}{F_{t^-,T}} = \left(\int_t^T \mu_y(t,u)du + \frac{1}{2}V_t\eta^2 - \rho\eta V_t\right)dt$$
$$+ \sqrt{V_t}dW_{s,t} - \sqrt{V_t}\eta dW_{y,t} + \int_R (e^x - 1)[\pi(dx,dt) - \nu_t(dx)dt]. \tag{11}$$

The no-arbitrage restriction implies that the futures price must be a martingale under the risk-neutral measure. Hence, we have

$$\frac{dF_{t,T}}{F_{t^-,T}} = \sqrt{V_t}dW_{s,t} - \sqrt{V_t}\eta dW_{y,t} + \int_R (e^x - 1)[\pi(dx,dt) - \nu_t(dx)dt]. \tag{12}$$

In addition, Li (2019) shows that the stochastic cost of carry, α, is hard to identify and that modeling the stochastic cost of carry adds marginal improvements in fitting performance. Following Li, we assume that the forward cost of carry is deterministic by setting $\alpha = 0$. Correspondingly, η is equal to zero. Finally, the stochastic process for the log futures price, $\log F_{t,T}$, can be simplified under the Q-measure as

$$d\log F_{t,T} = -\frac{1}{2}V_t dt - \int_R (e^x - 1)\nu_t(dx)dt + \sqrt{V_t}dW_{s,t} + \int_R x\pi(dx,dt). \tag{13}$$

EMPIRICAL ANALYSIS

In this section, we first present the crude oil spot and futures data. Then we introduce the estimation method and report the estimation results.

Data

Exhibit 1 summarizes the statistics of the logarithm returns for West Texas Intermediate oil spot and futures prices. To analyze the dynamic properties, we also plot the time series of oil and futures returns in Exhibit 2. The sample period is from January 12, 1990, to August 31, 2021. The spot price is obtained from the US Energy Information Administration website.[2] The oil futures contracts are downloaded from Bloomberg. We use six expiry contracts with maturities of 2, 4, 6, 9, 12, and 17 months. These data not only are adequately liquid but also cover the short-, middle-, and long-term futures prices.

Looking across the columns of Exhibit 1, we can see that the average of the spot returns is –0.0005% and that the average of the futures returns ranges from 0.0138% to 0.0146%. The minimum spot return is –302% on April 20, 2020, and the maximum value is 124% on April 21, 2020, as the spot price turned negative on April 20, 2020.[3] The mean values of the futures returns are larger than those of the spot returns. A further comparison shows that the futures returns tend be less volatile, skewed, and leptokurtic than the spot returns. The standard deviation, skewness (in absolute terms), and kurtosis values for the futures progressively decrease with the increase in maturity terms.

The first row of Exhibit 2 gives the daily spot returns. The two horizontal lines represent a ±2 standard deviations around the mean of the spot returns. In this graph, we can see that the spot return dynamics vary drastically. Several large jump-like

[2] https://www.eia.gov/.

[3] The spot returns on April 20 and April 21, 2020, are computed by $R_{t+1} = \frac{S_{t+1} - S_t}{|S_t|}$ because of the negative prices.

EXHIBIT 1

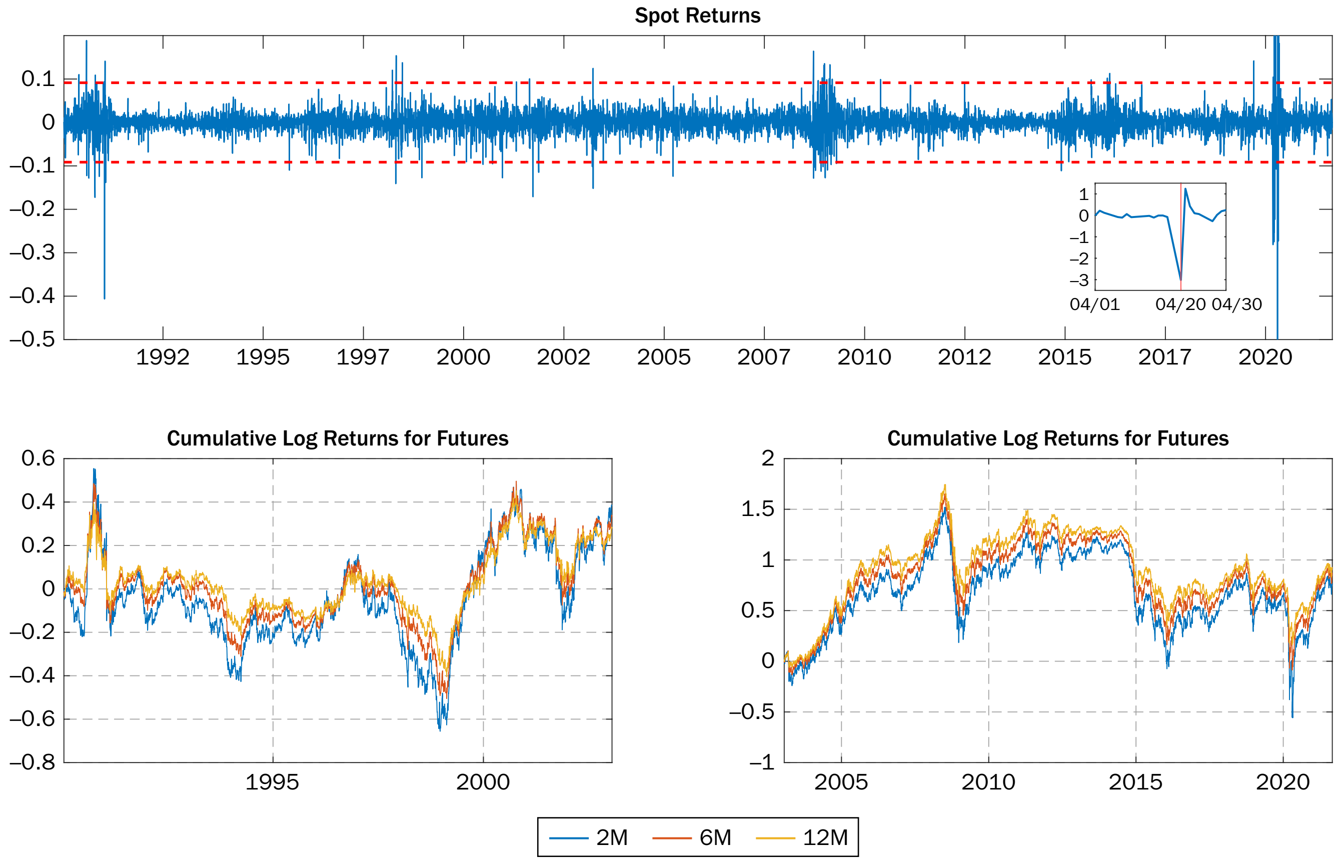
Summary Statistics for Oil Spot and Futures Returns

	Mean	Median	Std. Dev.	Max.	Min.	Skewness	Kurtosis
Panel A: Spot Returns (%)							
Spot	–0.0005	0.0650	4.5732	124.0941	–301.9661	–34.1421	2497.0094
Panel B: Futures Returns (%)							
2M	0.0138	0.0434	2.1631	20.2599	–36.5790	–1.4154	28.8559
4M	0.0142	0.0396	1.9427	17.5284	–26.1762	–0.9898	18.4377
6M	0.0145	0.0387	1.8139	14.7105	–23.0687	–0.8615	15.1865
9M	0.0146	0.0353	1.6782	11.5449	–20.0895	–0.7052	12.0870
12M	0.0146	0.0287	1.5826	9.9631	–17.7045	–0.6031	10.3516
17M	0.0143	0.0211	1.4755	9.6600	–14.5160	–0.5214	9.0862

NOTES: Panel A reports the summary statistics for the oil spot returns. Panel B summarizes the statistics of the oil futures returns for the 2-, 4-, 6-, 9-, 12-, and 17-month expiry contracts. The sample period is from January 12, 1990, to August 31, 2021. M = month.

EXHIBIT 2

Daily Returns



NOTES: The first row plots the dynamics of spot returns. The two horizontal lines indicate ±2 standard deviations around the mean of spot returns. The inset shows the spot returns in April 2020. The second row shows the cumulative futures returns with 2-, 6-, and 12-month maturities. The sample period covers from January 12, 1990, to August 31, 2021.

movements and jump clustering phenomena are clearly visible during the extreme periods, for example, the war shock in 1990, the financial crisis in 2008, and the demand shock in 2020. The inset plots the time series of spot returns in April 2020.

The second row in Exhibit 2 reports the time series of cumulative futures returns for the 2-, 6-, and 12-month terms. For the sake of clarity, we split the full sample period into two parts and display the dynamic properties of futures returns. The average of the futures returns during the first period is smaller than that during the second period. As shown in Heath (2019), oil futures have similar dynamic properties across different terms. This intriguing fact motivate us to estimate the state variables by using the middle-term futures (i.e., the 6-month expired futures contracts).

Estimation Approach

In this section, we filter the stochastic volatility model by using the particle filter method, which is commonly applied to estimate stochastic volatility. To implement the particle filter method, we first discretize the futures return process in Equation (13) by using the Euler discretization:

$$\log(F((t+1)\Delta,T)) = \log(F(t\Delta,T)) - \frac{1}{2}V_{t\Delta}\Delta - \varpi^{-}c^{-}_{t\Delta}\Delta - \varpi^{+}c^{+}_{t\Delta}\Delta + \sqrt{\Delta V_{t\Delta}}Z_{s,(t+1)\Delta} + J^{-}_{(t+1)\Delta} + J^{+}_{(t+1)\Delta}, \tag{14}$$

where $\varpi^{-} = \frac{\lambda_{-}}{\lambda_{-}+1} - 1$ and $\varpi^{+} = \frac{\lambda_{+}}{\lambda_{+}-1} - 1$. $\Delta = 1/252$ represents the daily sampling frequency. Z_s is a standard normal random variable. J^{-} and J^{+} follow the compound Poisson process and represent the negative and positive jump components, respectively. By applying the standard Euler scheme to the state variables in Equations (5) and (8), we have

$$V_{(t+1)\Delta} = V_{t\Delta} + k_v(\bar{V} - V_{t\Delta})\Delta + \sigma_v\sqrt{\Delta V_{t\Delta}}\left(\rho_{sv}Z_{s,(t+1)\Delta} + \sqrt{1-\rho_{sv}^2}Z_{v,(t+1)\Delta}\right), \tag{15}$$

$$U_{(t+1)\Delta} = U_{t\Delta} - k_u U_{t\Delta}\Delta + \eta\tilde{J}^{-}_{(t+1)\Delta}. \tag{16}$$

Here, Z_s and Z_v are mutually independent, and $\tilde{J}^{-}$ denotes the jump component in the tail factor.

Following the procedure in Elkamhi, Jeon, and Zhang (2016), we then simulate m particles for the negative jumps $J^{-,(i)}$ and the positive jumps $J^{+,(i)}$, where $i = 1, 2, \ldots, m$ is a sequence of particles. These simulated price jump particles are generated over the period $\Delta t = 1/252$. Accordingly, the particles for the diffusive return innovations can be derived as follows:

$$Z^{(i)}_{s,(t+1)\Delta} = \Big(\log(F((t+1)\Delta,T)) - \log(F(t\Delta,T)) + \tfrac{1}{2}V^{(i)}_{t\Delta}\Delta + \varpi^{-}c^{-,(i)}_{t\Delta}\Delta + \varpi^{+}c^{+,(i)}_{t\Delta}\Delta - J^{-,(i)}_{(t+1)\Delta} - J^{+,(i)}_{(t+1)\Delta}\Big)\Big/\sqrt{\Delta V^{(i)}_{t\Delta}}. \tag{17}$$

Combining the filtered innovations in Equation (17) and simulating another m normally distributed random variables, $Z^{(i)}_{v,(t+1)\Delta}$ can give the conditional variance $V^{(i)}_{(t+1)\Delta}$ for the next period on the basis of the variance dynamics in Equation (15). We construct the tail jump component $\tilde{J}^{-,(i)}_{(t+1)\Delta}$ by squaring the negative price jump size and then adding them all, according to the negative jump numbers that occur. Then we derive the m simulated tail factor based on Equation (16).

Given the state particles $\{V_{t\Delta}^{(i)}, U_{t\Delta}^{(i)}\}$, the weight of each particle can be computed as the likelihood of observing futures returns. We establish the likelihood of futures returns by using the quasi-maximum-likelihood method. The consistency of this method estimation is demonstrated in Gourieroux, Monfort, and Trognon (1984) and Bollerslev and Wooldridge (1992). The quasi-likelihood of $R_{t+1} = \log(F_{(t+1)\Delta,T}/F_{t\Delta,T})$, on the condition that the information known at time t, can be written as

$$\tilde{W}_t^{(i)} = \frac{1}{\sqrt{2\pi \mathrm{var}_t[R_{t+1}^{(i)}]}} \exp\left(-\frac{(R_{t+1} - E_t[R_{t+1}^{(i)}])^2}{2\mathrm{var}_t[R_{t+1}^{(i)}]}\right), \tag{18}$$

with

$$E_t[R_{t+1}^{(i)}] = \left(-\frac{1}{2}V_{t\Delta}^{(i)} - (\varpi^- + \frac{1}{\lambda_-})c_{t\Delta}^{-,(i)} - (\varpi^+ - \frac{1}{\lambda_+})c_{t\Delta}^{+,(i)}\right)\Delta, \tag{19}$$

$$\mathrm{var}_t[R_{t+1}^{(i)}] = V_{t\Delta}^{(i)}\Delta + \frac{2}{\lambda_-^2}c_{t\Delta}^{-,(i)}\Delta + \frac{2}{\lambda_+^2}c_{t\Delta}^{+,(i)}\Delta. \tag{20}$$

To define the proper probability distribution, the weights are then normalized as follows:

$$\hat{W}_t^{(i)} = \frac{\tilde{W}_t^{(i)}}{\sum_{i=1}^{m}\tilde{W}_t^{(i)}}. \tag{21}$$

Following Pitt (2002) and Ornthanalai (2014), we resample the particles based on the discrete probability distribution in Equation (21). Using the resampled state particles $\{\bar{V}_t^{(i)}\}_{i=1}^m$ and $\{\bar{U}_t^{(i)}\}_{i=1}^m$, we update the filtered state variables as follows:

$$V_t = \sum_{i=1}^{m}\hat{W}_t^{(i)}\bar{V}_t^{(i)},\ U_t = \sum_{i=1}^{m}\hat{W}_t^{(i)}\bar{U}_t^{(i)}. \tag{22}$$

Recursively repeating the same step yields a time series of state variables $\{V_t, U_t\}$ and probability weights $\tilde{W}_t^{(i)}$. Finally, the log-likelihood for futures returns under the risk-neutral measure is given by

$$L(\Theta) = \sum_{t=1}^{T}\log\left(\frac{1}{m}\sum_{i=1}^{m}\tilde{W}_t^{(i)}\right), \tag{23}$$

where Θ is the parameter vector, and T represents the total number of days in our sample period. The parameters are estimated by optimizing the total log-likelihood in Equation (23).

Several options are available for estimating the pricing of financial derivatives. The first option involves joint estimation of the physical and risk-neutral measures' information (e.g., Chernov and Ghysels 2000; Pan 2002; Santa-Clara and Yan 2010; Christoffersen, Jacobs, and Ornthanalai 2012; Yang, Wang, and Chen 2019). The second option involves sequential estimation (Broadie, Chernov, and Johannes 2007; Elkamhi, Jeon, and Zhang 2016; Babaoğlu et al. 2018). The third option is to conduct only risk-neutral estimation (Andersen, Fusari, and Todorov 2015, 2020). In this study,

EXHIBIT 3

Parameter Estimates for Oil Futures

Futures (6M)

k_v	$\bar{V}$	σ_v	ρ_{sv}	k_u	η
1.0444	0.0143	0.2255	–0.9433	1.9357	120.1588
(0.0047)	(0.0001)	(0.0016)	(0.0043)	(0.0265)	(6.8680)

λ_-	λ_+	c_0^+	c_1^-	c_1^+
14.2377	54.2718	1.1160	65.1370	23.9382
(0.0934)	(0.3170)	(0.0052)	(10.4370)	(2.7903)

NOTES: This exhibit reports parameter estimates and their standard deviations for the 6-month (6M) oil futures. The standard errors are calculated by using the outer product of the gradient.

we adhere to the risk-neutral estimation approach, in which the state variables are extracted from the oil futures under the risk-neutral measure. We do this for two reasons. First, the risk-neutral estimation method avoids the misspecification of the pricing kernel. Second, this estimation approach does not incorporate information from physical return dynamics. As emphasized by Andersen, Fusari, and Todorov (2015), it is important to appraise the forecasting power of the risk factor regarding the spot returns, as is explained in the Return Predictability section.

Estimation Results

We report the parameter estimates for the 6-month futures in Exhibit 3. The standard errors in brackets are calculated by using the outer product of the gradient.

Exhibit 3 presents the model estimates for the futures with 6-month maturity. All of the estimated parameter values are significant. The mean-reversion speed for the instantaneous variance is 1.0444, which implies a half-life of 168 days. Here, the half-life is defined as the number of days for an autocorrelation of the process to decay to half of its daily autocorrelation level, as in Egloff, Leippold, and Wu (2010).[4] The tail factor has a faster mean-reversion speed than the variance factor. The tail factor's half-life is almost 91 days. The correlation coefficient ($\rho_{sv} = -0.9433$) is significant and negative, which confirms the existence of the leverage effect between the spot returns and their variance factors.

$\bar{V}$ and σ_v denote the mean variance and the volatility of the variance, and both are statistically significant. The parameter η measures the degree of jump self-excitation. The significance of this parameter suggests that the channel of the jump itself works in capturing the jump propagation phenomenon. Simultaneously, the jump propagation, captured by the variance channel, is also clearly identified (e.g., c_1^- and c_1^+, used to describe the jump intensities, are significantly estimated). The negative jump size $1/\lambda_-$ and the positive jump size $1/\lambda_+$ are 0.0702 and 0.0184, respectively.

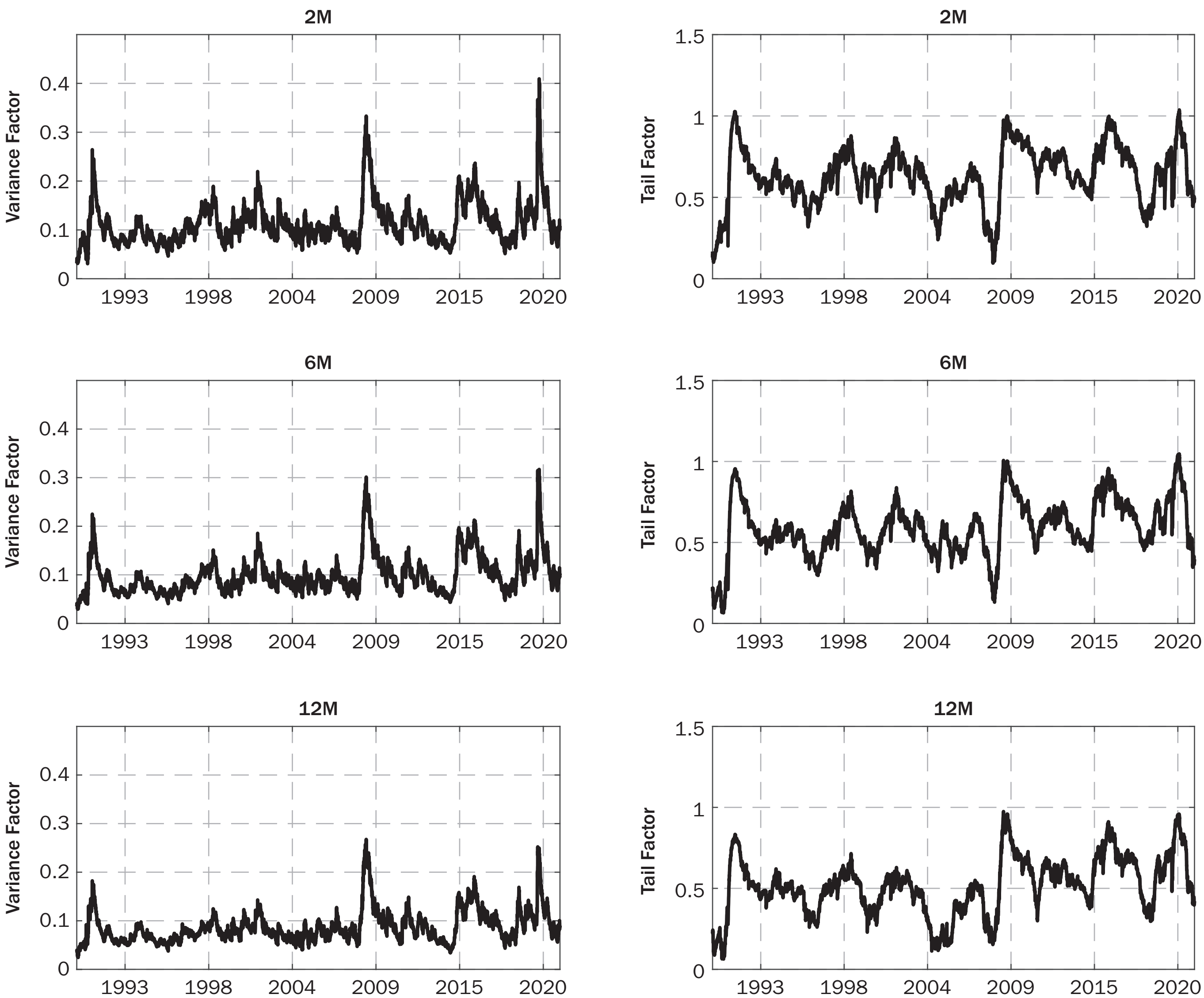
Using the parameter estimates in Exhibit 3, Exhibit 4 plots the daily state variables $\{V_t, U_t\}$ implied from the 2-, 6-, and 12-month terms. As in Ornthanalai (2014), the results are obtained based on the particle filter method, using 10,000 particles.

The left column of Exhibit 4 displays the time series of the variance factor. The paths of the series look quite similar. However, the variance dynamics derived from the 2-month term show the most drastic shifts, followed by those derived from the 6-month and 12-month terms. The variations are particularly great during extreme periods. The right column reports the results of the tail risk factor. Similar

[4] half-life = $1 - \log(0.5)/k_v \times 252 \approx 168$ days.

EXHIBIT 4

Daily State Variables from 2-, 6-, and 12-Month Terms



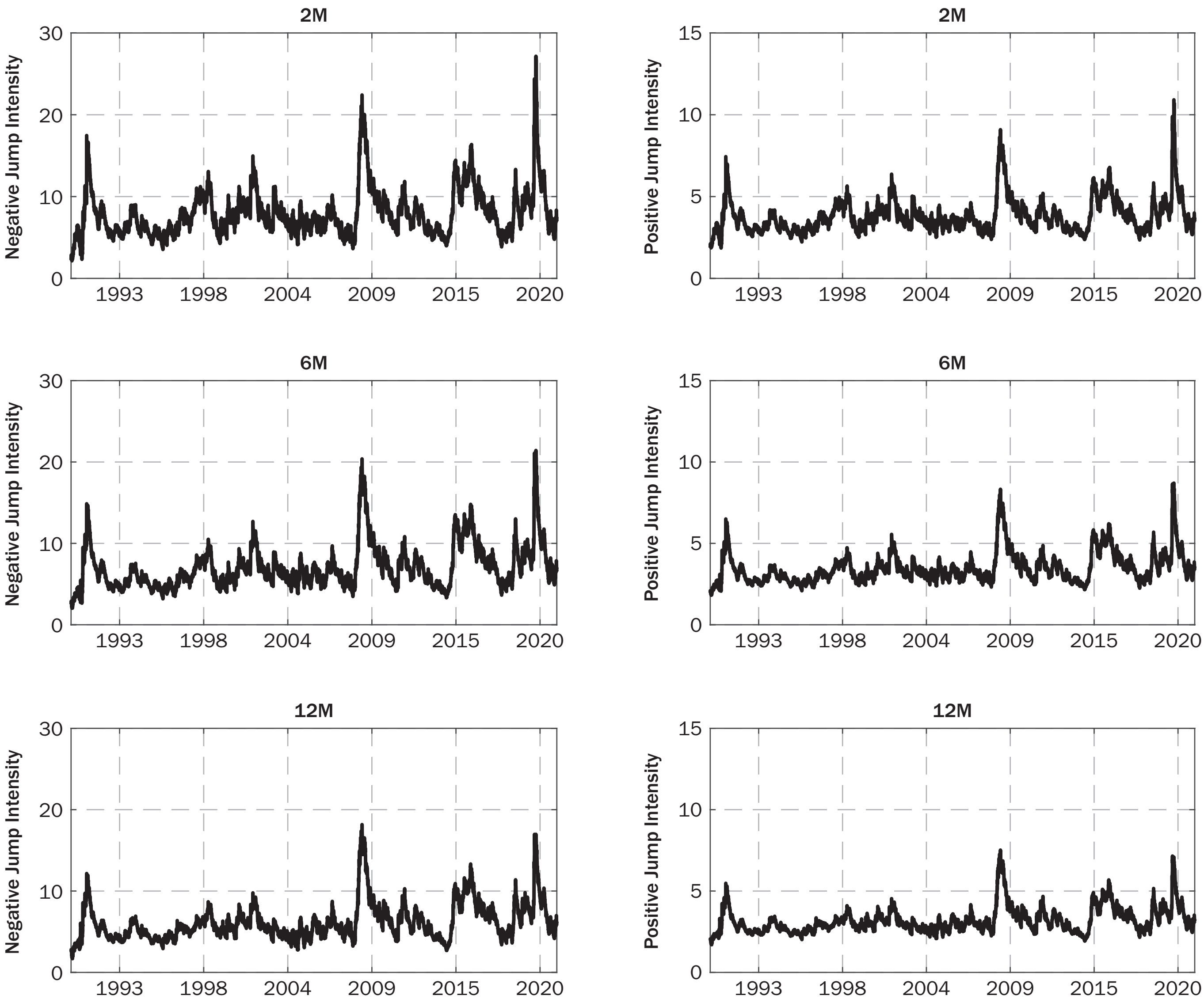
NOTES: The left column displays the variance factor, and the right column plots the tail risk factor. The results are based on the parameter estimates in Exhibit 3. The sample period covers from January 12, 1990, to August 31, 2021. M = month.

dynamic properties are also found for this factor. However, the tail factor's path evolves differently than that of the variance factor and has unique properties. This pattern suggests that the two factors provide different types of forward-looking information on oil futures.

Exhibit 5 plots the intensity dynamics defined in Equation (7). The top panels show the intensity of the negative and positive jumps for the 2-month term. For our sample period, the mean value of the filtered left jump intensity is 7.96, which suggests that negative jumps occur at a rate of 7.96 per year. The positive jumps are less frequent than the negative jumps, with 3.81 positive jumps per year. The middle panels display the intensity derived from the 6-month term. The frequency of the negative and positive jumps is 7.03 and 3.48, respectively. At the bottom of Exhibit 5, we report the left and right jump intensities derived from the 12-month term. The long term

EXHIBIT 5

Time Series of Jump Intensity for Negative and Positive Jumps



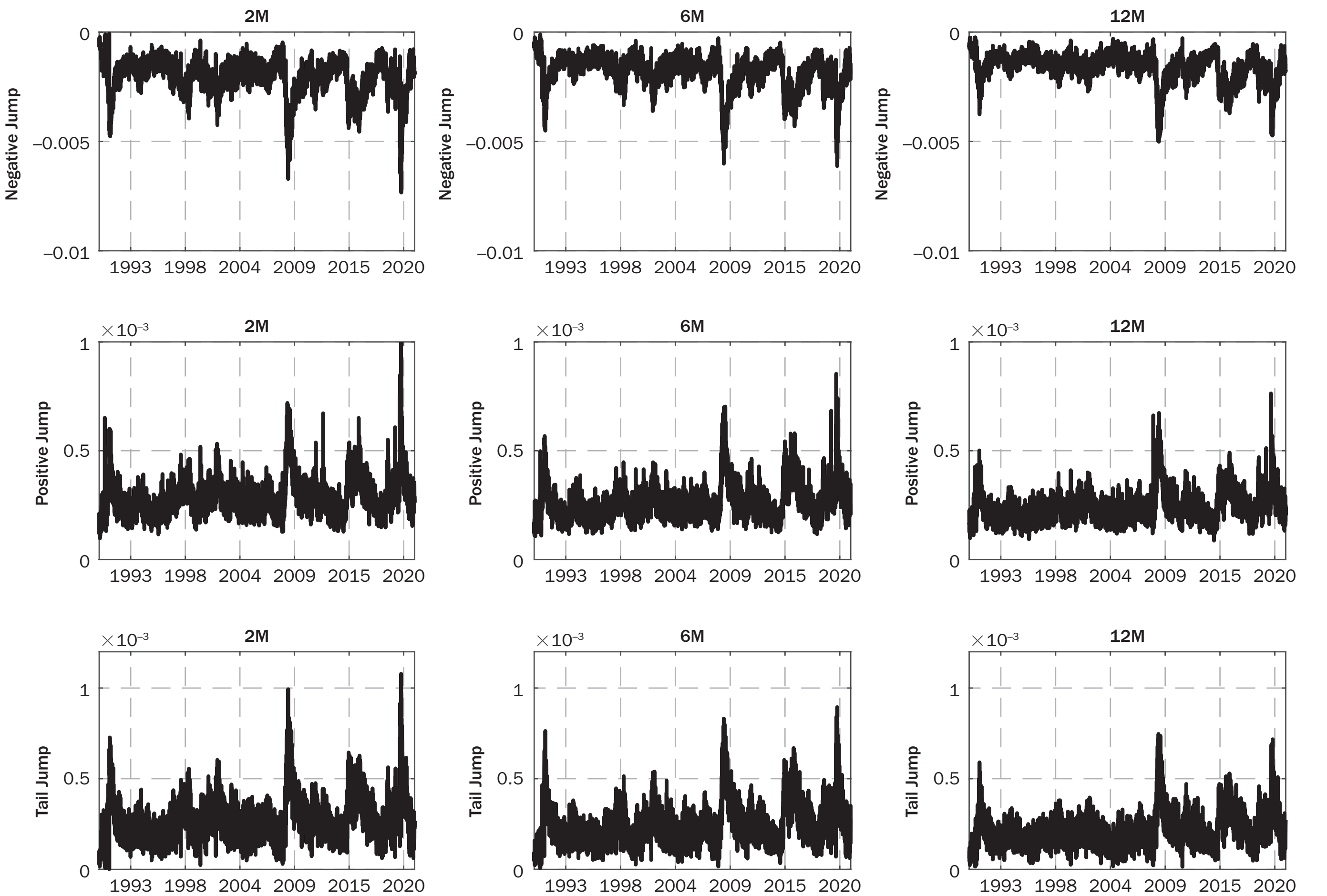
NOTES: The first row shows the intensity dynamics implied from 2-month (2M) futures. The second row plots the intensity of 6-month (6M) futures. The third row displays the intensity implied from 12-month (12M) term. The results are based on the parameter estimates in Exhibit 3. The sample period is from January 12, 1990, to August 31, 2021.

has the lowest average of jump arrivals, with a mean value for negative and positive jump intensities of 6.17 and 3.19, respectively.

Using the estimates given in Exhibit 3, we plot the filtered jump components for the 2-, 6-, and 12-month futures in Exhibit 6. The top panels report the filtered negative jumps. From this graph, we can see that the 2-month futures give the largest negative jump components, followed by the 6-month futures, and then the 12-month futures. The middle panels present the positive jump components. The bottom panel of Exhibit 6 shows the squared negative jump components that are used to depict self-exciting jump clustering. Overall, we find that the filtered jump components shoot up to their peaks during extreme periods, and they have declining patterns over the terms. These findings are in line with reality.

EXHIBIT 6

Filtered Jump Components



NOTES: The first row plots the negative jump components for 2-, 6-, and 12-month terms. The second row plots the positive jump components. The third row reports the filtered tail jump components. These results are based on the particle filter methodology with 10,000 particles. The sample period is from January 12, 1990, to August 31, 2021. M = month.

RETURN PREDICTABILITY

Predictive Regression

In this section, we explore the predictability of oil risk premiums by using factors derived from oil futures. The predictive regression is conducted as follows:

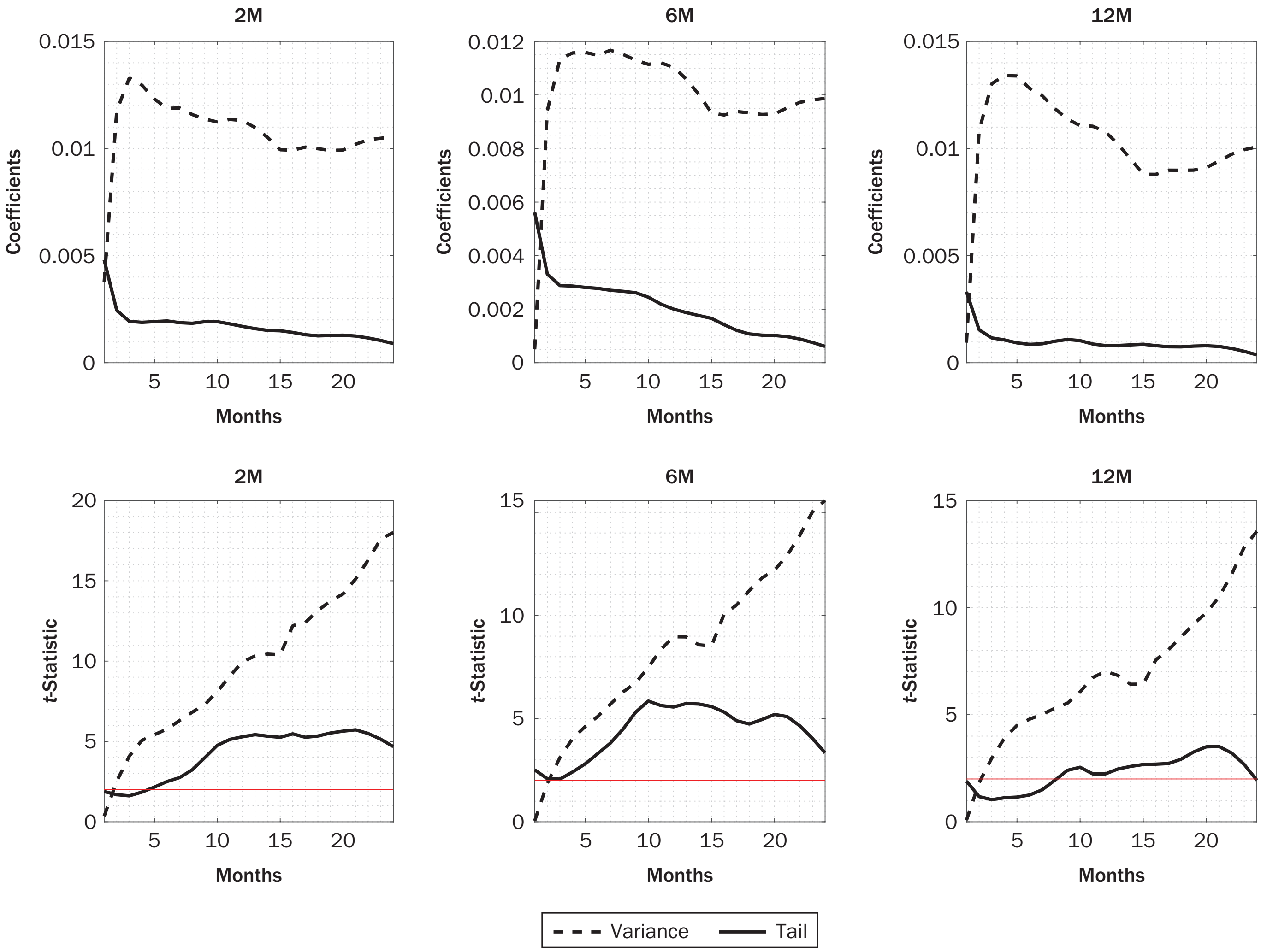
$$y_{t,t+k} = \alpha + \beta_1 V_t + \beta_2 \tilde{U}_t + \epsilon_{t,t+k}, \tag{24}$$

where $y_{t,t+k}$ is the average returns for the oil spot price from day t to day $t + k$. On the right side of Equation (24), the predictors $\{V_t, \tilde{U}_t\}$ are the state variables that are derived from oil futures. As we aim to study the nontrivial role of the tail factor in predicting returns, we initially regress the tail factor U_t on the variance factor V_t and obtain the residual term $\tilde{U}_t$, as in Andersen, Fusari, and Todorov (2015). $\tilde{U}_t$ is orthogonal to V_t and thus provides additional information that is not related to the volatility factor.

Exhibit 7 summarizes the predictive results based on the state variables as they are derived from different futures terms. The forecasting horizon ranges from 1 to 24 months. To correct for autocorrelation and heteroskedasticity in the regression

EXHIBIT 7

Predictive Coefficients and Their *t*-Statistics



NOTES: The dotted line represents the predictive results of variance factor. The solid line denotes the results of tail factor. The horizontal red line indicates a *t*-Statistic value of 2. The left, middle, and right columns are the results based on the 2-, 6-, and 12-month futures terms, respectively. The forecasting horizon is from 1 to 24 months. M = month.

residuals, we compute the *t*-statistics based on the Newey and West (1987) standard errors. The number of lags is chosen as the integer portion of $T^{1/4}$, where T denotes the sample observations.

The top panels of Exhibit 7 show the regression coefficients for the state variables under the different forecasting horizons. The dotted and solid lines correspond to the predictive variables V_t and $\tilde{U}_t$, respectively. For the variance factor, we see that the regression coefficients have similar hump-shaped patterns across the 2-, 6-, and 12-month futures terms. For example, the coefficient β_1 under the 2-month futures term initially increases sharply in the first 3 months. Next, this coefficient slightly decreases from 0.0133 for the 3-month returns to 0.0099 for the 15-month returns. At the long forecasting horizon, the predictive coefficient is stable at around 0.01.

The coefficient β_2 pertains to the tail risk factor. We get the expected sign for this coefficient, as a high tail factor is associated with positive oil returns. The value of this coefficient shows a monotonically declining pattern over these forecasting horizons. This coefficient under the 2-month term decreases sharply from 0.0048 (for the 1-month forecasting horizon) to 0.0019 for the 3-month forecasting horizon. The predictive coefficient of the tail factor is relatively more stable across different horizons than the coefficient of the variance factor. A consistent conclusion can be reached by analyzing the 6-month and 12-month expiry futures contracts.

In the bottom panel of Exhibit 7, we plot the corresponding *t*-statistics of these predictive coefficients for horizons of up to 2 years. The dotted and solid black lines represent the *t*-ratios for the variance and tail factors. The horizontal red line indicates that the *t*-statistic is 2. Looking across the panels, we can see that the *t*-statistic of β_1 increases substantially from the 1-month horizon to the 3-month horizon and stays high at the long end. However, the variance factor adds nothing to short-term return forecasting, that is, the predictive coefficient is not significant. This observation aligns with the findings of Andersen, Fusari, and Todorov (2015), who also find a nonsignificant regression coefficient for predicting the stock excess returns. Only the tail factor has predictive power on future oil returns at the short horizon. At the 1-month forecasting horizon, the *t*-statistics of β_2 are 1.88, 2.52, and 1.90 under the 2-, 6-, and 12-month expiry futures contracts, respectively.

A comparison of the predictive results for the different futures terms shows several differences. First, at the forecasting horizons from 2 to 24 months, the short-term futures (2-month) generate the largest *t*-statistics for the predictive coefficient of the variance factor, followed by the middle-term futures (6-month) and then the long-term futures (12-month). Second, all *t*-statistics for the tail factor extracted from 6-month futures are greater than 2, whereas *t*-statistics of β_2 for 12-month futures ranges from 1.03 to 3.52. These differences confirm our intuition that different futures terms show different types of forward-looking information and that it is important to appraise their performance in return forecasting.

To further analyze the incremental role of the tail factor, we report the adjusted R^2 statistics in Exhibit 8. The solid line denotes the R^2 based on the predictive regression with both the variance and tail factors. The dashed line corresponds to the restricted regression that uses only the variance factor.

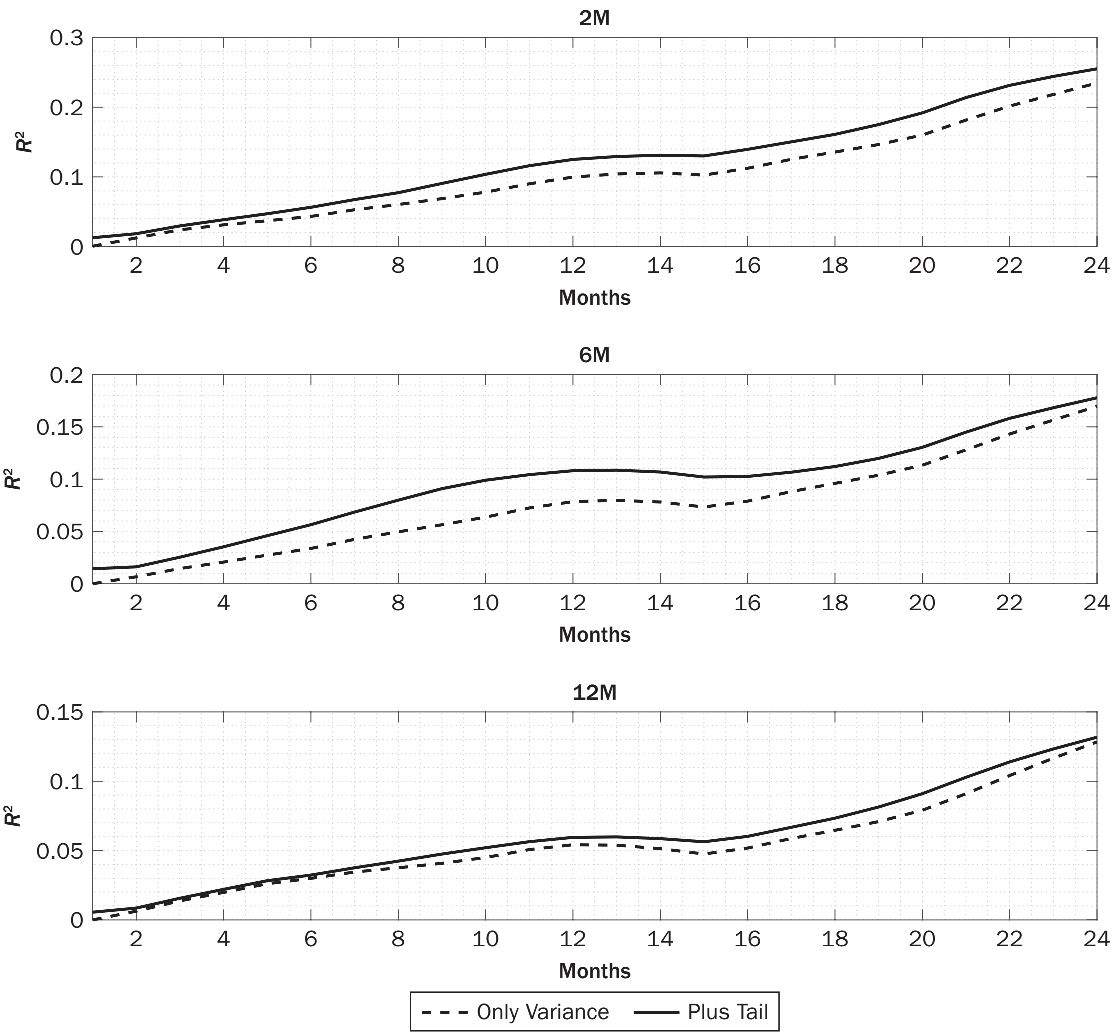
From the graphs in Exhibit 8, we can clearly see that the predictive equation defined in Equation (24) has relatively low explanatory power at the short horizon. When we exclude the tail factor from the regression, the R^2 of the short horizon becomes approximately zero, which suggests that the predictive power comes almost exclusively from the tail factor. Meanwhile, we can see that the value of R^2 increases apparently after the short horizon.

For example, the top panel shows the R^2 statistics that are based on the factors derived from 2-month expiry futures. The R^2 in the solid line increases from 1.27% at the 1-month horizon to 25.50% at the 24-month horizon. A comparison between the dotted and the solid lines shows that the increase in R^2 over the horizon is mainly driven by a rise in the explanatory power of the variance factor. The solid line is always above the dotted line, and it maintains a significant distance, which indicates that the tail factor helps to forecast oil returns for all time horizons.

The middle and bottom panels of Exhibit 8 show the adjusted R^2 derived from the 6-month and 12-month expiry futures, respectively. Similar qualitative features of the R^2 statistics are also found in the middle and long futures terms. However, noteworthy differences are seen in these longer-term futures projections. First, the explanatory power of all the factors together becomes lower when we adopt the factors extracted from futures with longer maturity. Second, the solid and dotted lines under the 12-month futures are closest, compared with those under the 2-month and 6-month futures, which indicates that the incremental predictive power of the tail factor decreases for long-term futures.

We also explore the robustness of oil return predictability by considering the 4-, 9-, and 17-month expiry futures contracts. The forecasting results are shown in Exhibit 9.

EXHIBIT 8
Regression R^2



NOTES: The top, middle, and bottom graphs plot the adjusted R^2 under the 2-, 6-, and 12-month expiry futures, respectively. The dashed line corresponds to the predictive regression only with variance factor. The solid line corresponds to the predictive regression with both variance and tail factors. The forecasting horizon ranges from 1 to 24 months. M = month.

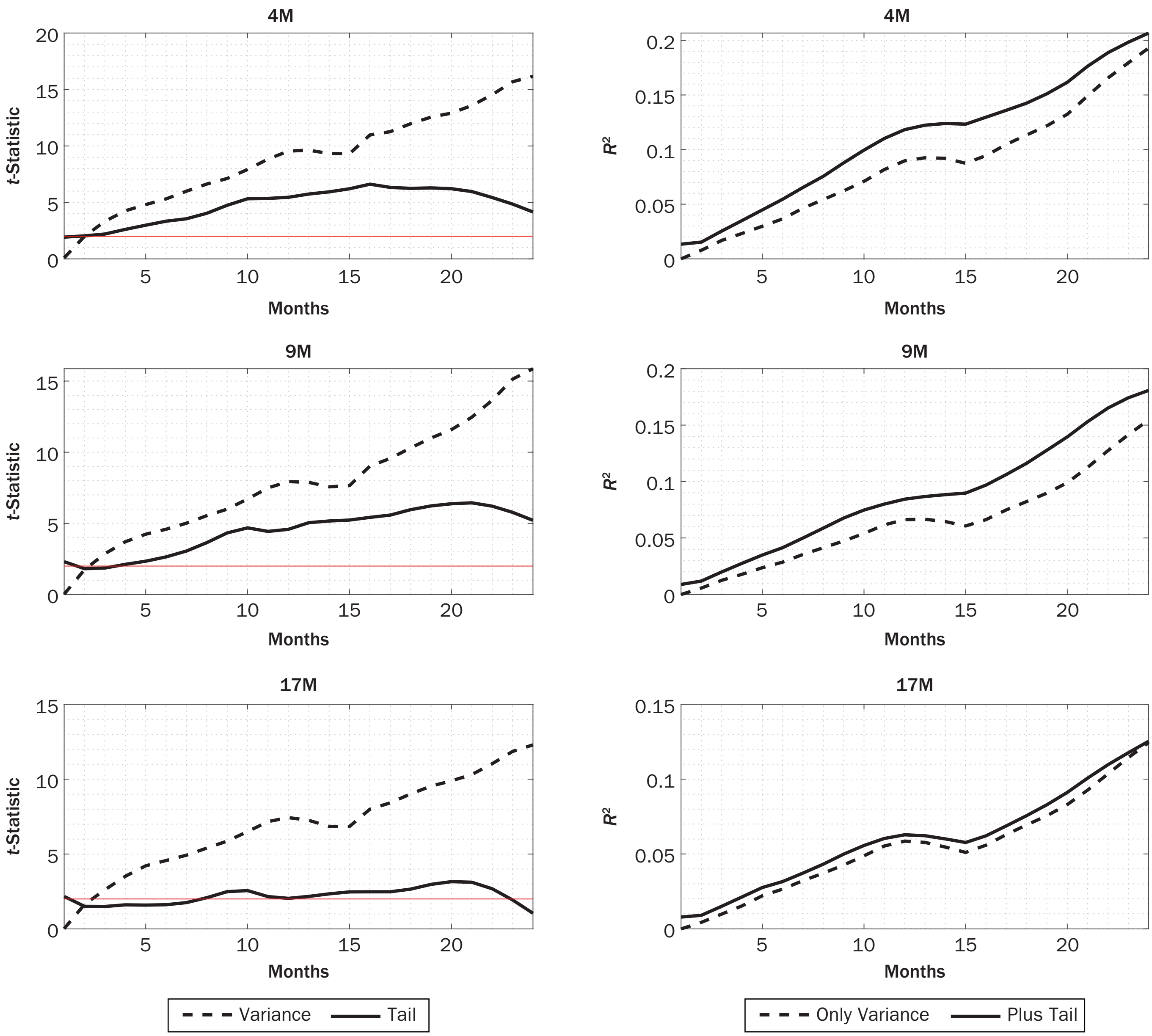
The left column of Exhibit 9 shows the *t*-statistics of the regression coefficients on the variance and tail factors, and the right column shows the adjusted R^2 of two predictive regressions. This robustness check yield similar conclusions. First, the tail factor provides robust predictive power at the short horizons. Second, the variance factor adds nothing to the performance of return forecasting at the short horizon but performs particularly well at the long horizon. Third, the explanatory power of all the factors together tends to decrease as the futures' maturity terms increase.

DISCUSSION

The preceding analysis indicates that the predictive properties of the variance and tail factors change across different forecasting horizons and under futures contracts of different expiry periods. In this section, we seek to evaluate these findings and to gain deeper insights into their implications.

EXHIBIT 9

Predictive Results Based on the 4-, 9-, and 17-Month Expiry Futures



NOTES: The left column displays *t*-statistics for the coefficients on the variance factor (dashed line) and on the tail factor (solid line). The right column plots the R^2 over the horizons from 1 to 24 months. The dashed line corresponds to the predictive regression only with the variance factor. The solid line corresponds to the predictive regression with both variance and tail factors. M = month.

How are the state variables extracted from futures linked to the spot returns? Equation (1) specifies the oil spot dynamics under the risk-neutral measure. To build on insights from the literature on affine asset pricing (e.g., Egloff, Leippold, and Wu 2010; Andersen, Fusari, and Todorov 2015; Aït-Sahalia, Karaman, and Mancini 2020), we define the Brownian motion under the physical measure (P-measure) as $dW^P_{s,t} = dW_{s,t} - \gamma_s \sqrt{V_t}\, dt$. The jump compensator under the P-measure is given by $\nu^P_t(dx)dt$. Following Pan (2002) and Boswijk, Laeven, and Lalu (2015), our mapping from $\nu^P_t(dx)$ to $\nu_t(dx)$ is defined for the jump size, and hence this method treats the jump risk premiums as based only on the differences in jump size distributions between the Q-measure and the P-measure.

Therefore, the spot dynamics under the P-measure can be written as

$$\frac{dS_t}{S_{t^-}} = \delta_t dt + \gamma_s V_t dt + \int_R (e^x - 1)\nu_t^P(dx)dt - \int_R (e^x - 1)\nu_t(dx)dt$$
$$+ \int_R (e^x - 1)[\pi(dx, dt) - \nu_t(dx)^P dt]. \tag{25}$$

Based on Equations (1) and (25), the average spot risk premium over the horizon k is

$$ERP_t^k = \frac{1}{k} E_t\left(\int_t^{t+k} \left((\gamma_s + \gamma_j^- c_1^- + \gamma_j^+ c_1^+)V_u + \gamma_j^- U_u + \gamma_j^+ c_0^+\right)du\right), \tag{26}$$

where the parameters γ_j^- and γ_j^+ are used to capture the change in jump size distribution between the P- and Q-measures. Equation (26) connects risk factors and risk premiums, and thus rationalizes our exploration of the link between future returns and state variables. Therefore, oil futures contain forward-looking information that is critical for return forecasting.

Why does the predictive power of the variance factor perform poorly in the short horizon regression but perform well in the long horizon regression? To explain these findings, we consider a simple predictive regression, as in Stambaugh (1999) and Sizova (2016):

$$y_{t,t+k} = \alpha + \beta x_t + \varepsilon_{t,t+k}, \tag{27}$$

$$x_t = \rho_x x_{t-1} + u_t, \tag{28}$$

where ε_t and u_t denote a sequence of shocks; ε_t is serially correlated, and u_t is assumed to be i.i.d. For the variance factor (i.e., $x_t = V_t$), the coefficient $\rho_x = 1 - k_v\Delta$ is less than 1. The increasing forecasting power for longer return horizons, as shown in Exhibit 8, indicates that the variance factor is correlated with the persistent component of future returns. βx_t in Equation (27) captures this relationship. At the short horizon, the stochastic component ε_t is dominant and is uncorrelated with V_t, which causes the inefficacy of the variance factor. Concerning the tail factor, the persistent coefficient is $\rho_x = 1 - k_u\Delta$, which is less than that for the variance factor. Unlike the variance factor, the tail factor provides relatively stable and strong forecasting power at the short horizon, which suggests that ε_t is predictable by the prior values of U_t.

Why do the factors embedded in different expiry futures contracts generate different levels of forecasting performance? By using the regression approach in Fama (1984a, 1984b), Fama and French (1987) provide compelling evidence that the information embedded in commodity futures prices has predictive power for future spot prices. In our study, the summary statistics on futures returns and the filtered state variables indicate apparent divergences between the terms. These divergences are also addressed in Mishkin (1990a, 1990b); Bakshi, Cao, and Chen (2000); and Schwartz and Smith (2000).

Different futures terms convey different types of information. One explanation for these differences is the variation in risk premiums. The risk premiums under different terms are not constant, and this fact suggests that the expectations hypothesis is rejected, as indicated by Cochrane and Piazzesi (2005) and Johnson (2017). Another explanation is that each term of maturity has its own dynamic properties. For oil futures, the short and middle terms are more sensitive than long terms to shifts in the economic environment. This insight can be used to build a model that more accurately estimates the risk factors and more adequately captures the spot risk premiums. Using a similar approach, Andersen, Fusari, and Todorov (2017) use short-dated options to analyze tail risks. In contrast, the variations in long-term futures returns are relatively stable, as summarized in Exhibit 1. Long-term futures provide information about the equilibrium price levels (Schwartz and Smith 2000).

CONCLUSION

In this study, we examine the time-varying jump tail risks in the oil market. First, we extend the exploration of commodity spot price dynamics in Trolle and Schwartz (2009) and Li (2019) with a novel tail factor developed by Andersen, Fusari, and Todorov (2015). On the basis of this model specification, we derive a pricing formula for oil futures and then estimate the dynamics of oil futures. Our empirical results provide strong evidence concerning the patterns of time-varying upward and downward jump risks in the oil market. We find that the tail factor is significantly helpful for pricing dynamics of futures.

Understanding the stochastic behavior of oil prices is crucial for market investors in risk management and asset pricing (e.g., Gibson and Schwartz 1990; Schwartz 1997; Brooks and Prokopczuk 2013). The significant existence of tail risks means that the tail factor is nontrivial in oil futures pricing. Meanwhile, our results also suggest that this tail factor contributes to depicting time-varying jump intensities and thus occupies a prominent role in capturing the extreme shocks in oil market.

Next, we consider the tail factor extracted from futures and examine this factor's predictive power for future oil spot returns. Our empirical findings demonstrate that this tail factor conveys important information concerning future risk premiums and is not spanned by the traditional variance factor. A further comparison between the tail and variance factors shows that the tail factor can relatively provide strong predictive power in the short horizon regression.

These predictive findings also yield some important implications. First, our results provide supplementary evidence on spot price predictability using futures prices. Under the incomplete market, the oil futures are nonredundant assets and contain important forward-looking information. Second, we identify a tail risk factor in the oil market using a parametric model. The forecasting performance suggests that modeling the tail factor is important for forecasting future returns. Third, our findings reveal that different futures terms convey different information. The short-term futures are more sensitive to the risk of shifts in the economic environment than the long-term futures.

Our study can be extended in the following ways. First, it would be interesting to apply the tail factor to price other commodity derivatives such as options. Second, it would be valuable to test the hedging performance of the model specification with jump tail risks as in Kaeck (2013) and Kyriakou et al. (2016). We leave these topics for future research.

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