For All X

Midwest Collaboration

This is version 0.2 of An Open Introduction to Logic. It is current as of June 6, 2019.

Adam Edwards compiled this edition and wrote original material for it. He takes full responsibility for any mistakes remaining in this version of the text.

Typesetting was carried out entirely in LaTeX2ε. The style for typesetting proofs is based on fitch.sty (v0.4) by Peter Selinger, University of Ottawa.

“When you come to any passage you don’t understand, *read it again*: if you *still* don’t understand it, *read it again*: if you fail, even after *three* readings, very likely your brain is getting a little tired. In that case, put the book away, and take to other occupations, and next day, when you come to it fresh, you will very likely find that it is *quite* easy.”

– Charles Dodgson (Lewis Carroll) *Symbolic Logic* (**?**)

“Few persons care to study logic, because everybody conceives himself to be proficient enough in the art of reasoning already. But I observe that this satisfaction is limited to one’s own ratiocination and does not extend to that of other men. We come to the full possession of our power of drawing inferences the last of all our faculties, for it is not so much a natural gift as a long and difficult art.”

– Charles Sanders Peirce “The Fixation of Belief”, in *Popular Science Monthly*, Vol. 12 (November 1877)”

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About this Book

This book was created by combining two previous books on logic and critical thinking, both made available under a Creative Commons license, and then adding some material so that the coverage matched that of commonly used logic textbooks.

P.D. Magnus' *For All* X (**?**) formed the basis of Part **??**: Formal Logic. I began using *For All* X in my own logic classes in 2009, but I quickly realized I needed to make changes to make it appropriate for the community college students I was teaching. In 2010 I began developing *For All* X *: The Lorain County Remix* and using it in my classes. The main change I made was to separate the discussions of sentential and quantificational logic and to add exercises. It is this remixed version that became the basis for Part **??**: Formal Logic complete version of this text.

Similarly, Part **??**: Critical Thinking and Part **??**: Inductive and Scientific Reasoning grew out of Cathal Woods' *Introduction to Reasoning*. In the Spring of 2011, I began to use an early version of[1] this text ([1]) in my critical thinking courses. I kept up with the updates and changes to the text until the release of [2], [2] all the while gradually merging the material with the work in *For All X*. After that point, my version forks from Woods's.

On May 20, 2016, I posted the combined textbook to Github and all subsequent changes have been tracked there: https://github.com/rob-helpy-chalk/openintroduction

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Acknowledgments

Thanks first of all go to the authors of the textbooks here stitched together: P.D. Magnus for *For All* X and Cathal Woods for *Introduction to Reasoning*. My thanks go to them for writing the excellent textbooks that have been incorporated into this one, for making those publicly available under Creative Commons licenses, and for giving their blessing to this derivative work.

In general, this book would not be possible without a culture of sharing knowledge. The book was typeset using LaTeX2ε developed by Leslie Lamport. Lamport was building on TeXby Donald Knuth. Peter Selinger built on what Lamport made by developing the Fitch typesetting format that the proofs were laid out in. Diagrams were made in PikZ by Till Tantu. All of these are coding systems are not only freely available online, they have extensive user support communities. Add-on packages are designed, manuals are written, questions are answered in discussion forums, all by people who are donating their time and expertise.

The culture of sharing isn't just responsible for the typesetting of this book; it was essential to the content. Essential background information comes from the free online *Stanford Encyclopedia of Philosophy*. Primary sources from the history of logic came from *Project Gutenberg*. Logicians, too, can and should create free knowledge.

Many early adopters of this text provided invaluable feedback, including Jeremy Dolan, Terry Winant, Benjamin Lennertz, Ben Sheredos, and Michael Hartsock. Lennertz, in particular, provided useful edits. Helpful comments were also made by Ben Cordry, John Emerson, Andrew Mills, Nathan Smith, Vera Tobin, Cathal Woods, and many more that I have forgot to mention, but whose emails are probably sitting on my computer somewhere.

I would also like to thank Lorain County Community College for providing the sabbatical leave that allowed me to write the sections of this book on Aristotelian logic. Special thanks goes to all the students at LCCC who had to suffer through earlier versions of this work and provided much helpful feedback. Most importantly, I would like to thank Molly, Caroline and Joey for their incredible love and support.

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Intellectual debts too great to articulate are owed to scholars too many to enumerate. At different points in the work, readers might detect the influence of various works of Aristotle, Toulmin (especially [3]), Fisher and Scriven (?), Walton (especially [4]), Epstein (?), Johnson-Laird (especially (?)), Scriven (?), Giere (?) and the works of the Amsterdam school of pragma-dialectics (?).

Thanks are due to Virginia Wesleyan College for providing me with Summer Faculty Development funding in 2008 and 2010 and a Batten professorship in 2011. These funds, along with some undergraduate research funds (also provided by VWC), allowed me to hire students Gaby Alexander (2008), Ksera Dyette (2009), Mark Jones (2008), Andrea Medrano (2011), Lauren Perry (2009), and Alan Robertson (2010). My thanks to all of them for their hard work and enthusiasm.

For feedback on the text, thanks are due to James Robert (Rob) Loftis (Lorain County Community College) and Bill Roche (Texas Christian University). Answers (to exercises) marked with "(JRL)" are by James Robert Loftis.

Particular thanks are due to my (once) Ohio State colleague Bill Roche. The book began as a collection of lecture notes, combining work by myself and Bill.

Cathal Woods
Norfolk, Virginia, USA
(Taken from *Introduction to Reasoning* (?))

The author would like to thank the people who made this project possible. Notable among these are Cristyn Magnus, who read many early drafts; Aaron Schiller, who was an early adopter and provided considerable, helpful feedback; and Bin Kang, Craig Erb, Nathan Carter, Wes McMichael, and the students of Introduction to Logic, who detected various errors in previous versions of the book.

P.D. Magnus
Albany, New York, USA
(Taken from *For All X* (?))

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Part I

Basic Concepts

1
What Is Logic?

Introduction

Logic is a part of the study of human reasoning—the ability we have to think abstractly, solve problems, explain the things that we know, and infer new knowledge on the basis of evidence. Traditionally, logic has focused on the last of these items, the ability to make inferences on the basis of evidence. This is an activity you engage in every day. Consider, for example, the game of Clue. (For those of you who have never played, Clue is a murder mystery game where players have to decide who committed the murder, what weapon they used, and where they were.) A player in the game might decide that the murder weapon was the candlestick by ruling out the other weapons in the game: the knife, the revolver, the rope, the lead pipe, and the wrench. This evidence lets the player know something they did not know previously, namely, the identity of the murderer.

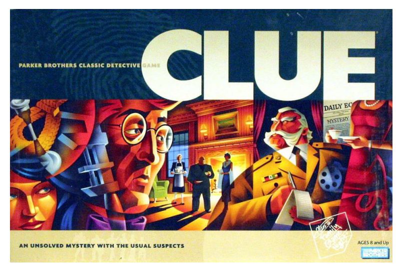

Figure 1.1: The boardgame Clue.

In logic, we use the word "argument" to refer to the attempt to show that certain evidence supports a conclusion. This is very different from the sort of argument you might have when you are mad at someone, which could involve a lot of yelling. We are going to use the word "argument" a lot in this book, so you need to get used to thinking of it as a name for an abstract and rational process, and not a word that describes what happens when people disagree.

A logical argument is structured to give someone a reason to believe some conclusion. Here is the argument about a game of Clue written out in a way that shows its structure.

(A1) In a game of Clue, the possible murder weapons are the knife, the candlestick, the revolver, the rope, the lead pipe, and the wrench.
(A2) The murder weapon was not the knife.
(A3) The murder weapon was also not the revolver, the rope, the lead pipe, or the wrench.
(A4) $\therefore$ Therefore, the murder weapon was the candlestick.

In the argument above, statements $A1$–$A3$ are the evidence. We call these the PREMISES. The word "therefore" indicates that the final statement, marked with a C, is the CONCLUSION of the argument. If you believe the premises, then the argument provides you with a reason to believe the conclusion. You might use reasoning like this purely in your own head, without talking with anyone else. You might wonder what the murder weapon is, and then mentally rule out each item, leaving only the candlestick. On the other hand, you might use reasoning like this while talking to someone else, to convince them that the murder weapon is the candlestick. (Perhaps you are playing as a team.) Either way the structure of the reasoning is the same.

Throughout the book, we will also use the triangle of dots symbol "∴" to indicate that a statement is a conclusion.

We can define LOGIC then more precisely as the part of the study of the structure of good inferences. In more casual situations, we will follow ordinary practice and use the word "logic" to either refer to the business of studying inferences and arguments in general or the thing being studied, that is, inference itself. While logic focuses on inferential reasoning, other disciplines, like decision theory and cognitive science, deal with other aspects of human reasoning, like abstract thinking and problem solving more generally. Logic, as the study of inference, has been pursued for thousands of years by people from civilizations all over the globe. The initial motivation for studying logic is often practical. Given that we use arguments and make inferences all the time, it only makes sense that we would want to learn to do these things better. Once people begin to study logic, however, they often realize that it is a fascinating topic in its own right. Thus, the study of logic quickly moves from being a practical business to a theoretical endeavor people pursue for its own sake.

In order to study reasoning, we have to apply our ability to reason to our reason itself. This reasoning about reasoning is called METAREASONING. It is part of a more general set of processes called METACOGNITION, which is just any kind of thinking about thinking. When we are pursing logic as a practical discipline, one important part of metacognition will be awareness of your own thinking, especially its weakness and biases, as it is occurring. More theoretical metacognition will be about attempting to understand the structure of thought itself.

Whether we are pursuing logical for practical or theoretical reasons, our focus is on argument. The key to studying argument is to set aside the subject being argued about and to focus on the *way* it is argued *for*. The section opened with an example that was about a game of Clue. However, the kind of reasoning used in that example was just the process of elimination. Process of elimination can be applied to any subject. Suppose a group of friends is deciding which restaurant to eat at, and there are six restaurants in town. If you could rule out five of the possibilities, you would use an argument just like the one above to decide where to eat. Because logic sets aside what an argument is about, and just looks at how it works rationally, logic is said to have CONTENT NEUTRALITY. If we say an argument is good, then the same kind of argument applied to a different topic will also be good. If we say an argument is good for solving murders, we will also say that the same kind of argument is good for deciding where to eat, what kind of disease is destroying your crops, or who to vote for.

When logic is studied for theoretical reasons, it typically is pursued as FORMAL LOGIC. In formal logic we get content neutrality by replacing parts of the argument we are studying with abstract symbols. For instance, we could turn the argument above into a formal argument like this:

(B1) There are six possibilities: A, B, C, D, E, and F.
(B2) A is false.
(B3) B, D, E, and F are also false.
(B4) ∴ ∴ The correct answer is C.

Here we have replaced the concrete possibilities in the first argument with abstract letters that could stand for anything. We have also replaced the English word "therefore" with the symbol "∴," which means therefore. This lets us see the formal structure of the argument, which is why it works in any domain you can think of. In fact, we can think of formal logic as the method for studying argument that uses abstract notation to identify the formal structure of argument. Formal logic is closely allied with mathematics, and studying formal logic often has the sort of puzzle-solving character one associates with mathematics. You will see this when we get to Part **??**, which covers formal logic.

When logic is studied for practical reasons, it is typically called critical thinking. We will define CRITICAL THINKING narrowly as the use of metareasoning to improve our reasoning in practical situations. Sometimes we will use the term "critical thinking" more broadly to refer to the results of this effort at self-improvement. You are "thinking critically" when you reason in a way that has been sharpened by reflection and metareasoning. A CRITICAL THINKER someone who has both sharpened their reasoning abilities using metareasoning and deploys those sharpened abilities in real world situations.

Critical thinking is generally pursued as INFORMAL LOGIC, rather than formal logic. This means that we will keep arguments in ordinary language and draw extensively on your knowledge of the world to evaluate them. In contrast to the clarity and rigor of formal logic, informal logic is suffused with ambiguity and vagueness. There are problems with multiple correct answers, or where reasonable people can disagree with what the correct answer is. This is because you will be dealing with reasoning in the real world, which is messy.

You can think of the difference between formal logic and informal logic as the difference between a laboratory science and a field science. If you are studying, say, mice, you could discover things about them by running experiments in a lab, or you can go out into the field where mice live and observe them in their natural habitat. Informal logic is the field science for arguments: you go out and study arguments in their natural habitats, like newspapers, courtrooms, and scientific journal articles. Like studying mice scurrying around a meadow, the process takes patience, and often doesn't yield clear answers but it lets you see how things work in the real world. Formal logic takes arguments out of their natural habitat and performs experiments on them to see what they are capable of. The arguments here are like lab mice. They are pumped full of chemicals and asked to perform strange tasks, as it were. They live lives very different than their wild cousins. Some of the arguments will wind up looking like the "ob/ob mouse", a genetically engineered obese mouse scientists use to study type II diabetes (See Figure 1.2). These arguments will be huge, awkward, and completely unable to survive in the wild. But they will tell us a lot about the limits of logic as a process.

Figure 1.2: The ob/ob mouse (left), a laboratory mouse which has been genetically engineered to be obese, and an ordinary mouse (right). Photo from **?**.

Our main goal in studying arguments is to separate the good ones from the bad ones. The argument about Clue we saw earlier is a good one, based on the process of elimination. It is good because it leads to truth. If I've got all the premises right, the conclusion will also be right. The textbook *Logic: Techniques of Formal Reasoning* (**?**) had a nice way of capturing the meaning of logic: "logic is the study of virtue in argument." This textbook will accept this definition, with the caveat that an argument is virtuous if it helps us get to the truth.

Logic is different from RHETORIC, which is the study of effective persuasion. Rhetoric does not look at virtue in argument. It only looks at the power of arguments, regardless of whether they lead to truth. An advertisement might convince you to buy a new truck by having a gravelly voiced announcer tell you it is "ram tough" and showing you a picture of the truck on top of a mountain, where it no doubt actually had to be airlifted. This sort of persuasion is often more effective at getting people to believe things than logical argument, but it has nothing to do with whether the truck is really the right thing to buy. In this textbook we will only be interested in rhetoric to the extent that we need to learn to defend ourselves against the misleading rhetoric of others. This will not, however, be anything close to a full treatment of the study of rhetoric.

Statement, Argument, Premise, Conclusion

So far we have defined logic as the study of argument and outlined its relationship to related fields. To go any further, we are going to need a more precise definition of what exactly an argument is. We have said that an argument is not simply two people disagreeing; it is an attempt to prove something using evidence. More specifically, an argument is composed of statements. In logic, we define a STATEMENT as a unit of language that can be true or false. That means that statements are TRUTH EVALUABLE . All of the items below are statements.

1. *Tyrannosaurus rex* went extinct 65 million years ago.
2. *Tyrannosaurus rex* went extinct last week.
3. On this exact spot, 100 million years ago, a *T. rex* laid a clutch of eggs.
4. Abraham Lincoln is the king of Mars.
5. Murder is wrong.
6. Abortion is murder.
7. Abortion is a woman's right.
8. Lady Gaga is pretty.
9. Murder is the unjustified killing of a person.
10. The slithy toves did gyre and gimble in the wabe.
11. The murder of logician Richard Montague was never solved.

Because a statement is something that can be true *or* false, statements include truths like 1 and falsehoods like 2. A statement can also be something that that must either be true or false, but we don't know which, like 3. A statement can be something that is completely silly, like 4. Statements in logic include statements about morality, like 5, and things that in other contexts might be called "opinions," like 6 and 7. People disagree strongly about whether 6 or 7 are true, but it is definitely possible for one of them to be true. The same is true about 8, although it is a less important issue than 6 and 7. A statement in logic can also simply give a definition, like 9. This sort of statement announces that we plan to use words a certain way, which is different from statements that describe the world, like 1, or statements about morality, like 6. Statements can include nonsense words like 10, because we don't really need to know what the statement is about to see that it is the sort of thing that can be true or false. All of this relates back to the content neutrality of logic. The statements we study can be about dinosaurs, abortion, Lady Gaga, and even the history of logic itself, as in statement 11, which is true.

We are treating statements primarily as units of language or strings of symbols, and most of the time the statements you will be working with will just be words printed on a page. However, it is important to remember that statements are also what philosophers call "speech acts." They are actions people take when they speak (or write). If someone makes a statement they are typically telling other people that they believe the statement to be true, and will back it up with evidence if asked to. When people make statements, they always do it in a context—they make statements at a place and a time with an audience. Often the context statements are made in will be important for us, so when we give examples, statements, or arguments we will sometimes include a description of the context. When we do that, we will give the context in *italics.* See Figure 1.3 for examples. For the most part, the context for a statement or argument will be important in the chapters on critical thinking, when we are pursing the study of logic for practical reasons. In the chapters on formal logic, context is less important, and we will be more likely to skip it.

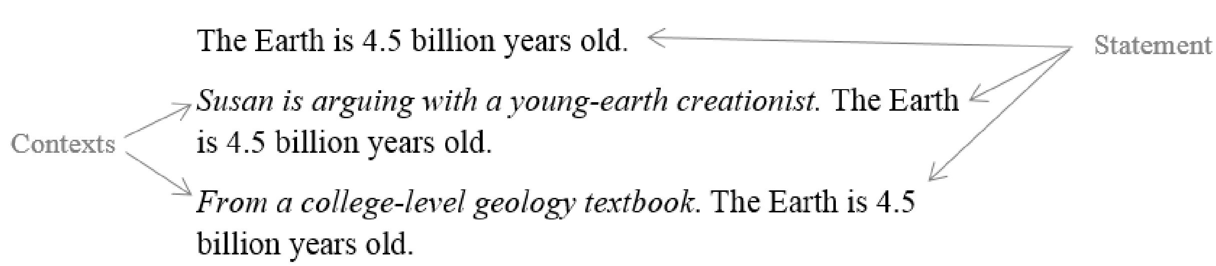

Figure 1.3: A statement in different contexts, or no context.

"Statements" in this text will *not* include questions, commands, exclamations, or sentence fragments. Someone who asks a *question* like "Does the grass need to be mowed?" is typically not claiming that anything is true or false. *Questions* do not count as statements, but *answers* usually will. "What is this course about?" is not a statement. An answer to that question such as "No one knows what this course is about," is a statement.

For the same reason *commands* do not count as statements for us. If someone bellows “Mow the grass, now!” they are not saying whether the grass has been mowed or not. You might infer that they believe the lawn has not been mowed, but then again maybe they think the lawn is fine and just want to see you exercise.

An exclamation like “Ouch!” is also neither true nor false. On its own, it is not a statement. We will treat “Ouch, I hurt my toe!” as meaning the same thing as “I hurt my toe.” The “ouch” does not add anything that could be true or false.

Finally, a lot of possible strings of words will fail to qualify as statements simply because they don’t form a complete sentence. In your composition classes, these were probably referred to as sentence fragments. This includes strings of words that are parts of sentences, such as noun phrases like “The tall man with the hat” and verb phrases, like “ran down the hall.” Phrases like these are missing something they need to make a claim about the world. The class of sentence fragments also includes completely random combinations of words, like “The up if blender route,” which don’t even have the form of a statement about the world.

Other logic textbooks describe the components of argument as “propositions,” or “assertions,” and we will use these terms sometimes as well. There is actually a great deal of disagreement about what the differences between all of these things are and which term is best used to describe parts of arguments. However, none of that makes a difference for this textbook. Some textbooks will also use the term “sentence” here. We will not use the word “sentence” to mean the same thing as “statement.” Instead, we will use “sentence” the way it is used in ordinary grammar, to refer generally to statements, questions, and commands.

Sometimes the outward form of a speech act does not match how it is actually being used. A rhetorical question, for instance, has the outward form of a question, but is really a statement or a command. If someone says “don’t you think the lawn needs to be mowed?” they may actually mean a statement like “the lawn needs to be mowed” or a command like “mow the lawn, now.” Similarly one might disguise a command as a statement. “You will respect my authority” *is* either true or false—either you will or you will not. But the speaker may intend this as an order—”Respect me!”—rather than a prediction of how you will behave.

When we study argument, we need to express things as statements, because arguments are composed of statements. Thus if we encounter a rhetorical question while examining an argument, we need to convert it into a statement. “Don’t you think the lawn needs to be mowed” will become “the lawn needs to be mowed.” Similarly, commands will become should statements. “Mow the lawn, now!” will need to be transformed into “You should mow the lawn.”

The latter kind of change will be important in critical thinking, because critical thinking often studies arguments whose goal is to an get audience to do something. These are called PRACTICAL ARGUMENTS. Most advertising and political speech consists of practical arguments, and these are crucial topics for critical thinking.

Once we have a collection of statements, we can use them to build arguments. An ARGUMENT is a connected series of statements designed to convince an audience of another statement. Here an audience might be a literal audience sitting in front of you at some public speaking engagement. Or it might be the readers of a book or article. The audience might even be yourself as you reason your way through a problem. Let's start with an example of an argument given to an external audience. This passage is from an essay by Peter Singer called "Famine, Affluence, and Morality" in which he tries to convince people in rich nations that they need to do more to help people in poor nations who are experiencing famine.

> *A contemporary philosopher writing in an academic journal.* If it is in our power to prevent something bad from happening, without thereby sacrificing anything of comparable moral importance, we ought, morally, to do so. Famine is something bad, and it can be prevented without sacrificing anything of comparable moral importance. So, we ought to prevent famine. **?**

Singer wants his readers to work to prevent famine. This is represented by the last statement of the passage, "we ought to prevent famine," which is called the conclusion of the passage. The CONCLUSION of an argument is the statement that the argument is trying to convince the audience of. The statements that do the convincing are called the PREMISES. In this case, the argument has three premises: (1) "If it is in our power to prevent something bad from happening, without thereby sacrificing anything of comparable moral importance, we ought, morally, to do so"; (2) "Famine is something bad"; and (3) "it can be prevented without sacrificing anything of comparable moral importance."

Now let's look at an example of internal reasoning.

> *Jack arrives at the track, in bad weather.* There is no one here. I guess the race is not happening.

In the passage above, the words in *italics* explain the context for the reasoning, and the words in regular type represent what Jack is actually thinking to himself. (We will talk more about his way of representing reasoning in section **??**, below.) This passage again has a premise and a conclusion. The premise is that no one is at the track, and the conclusion is that the race was canceled. The context gives another reason why Jack might believe the race has been canceled, the weather is bad. You could view this as another premise—it is very likely a reason Jack has come to believe that the race is canceled. In general, when you are looking at people's internal reasoning, it is often hard to determine what is actually working as a premise and what is just working in the background of their unconscious.

When people give arguments to each other, they typically use words like "therefore" and "because." These are meant to signal to the audience that what is coming is either a premise or a conclusion in an argument. Words and phrases like "because" signal that a premise is coming, so we call these PREMISE INDICATORS. Similarly, words and phrases like "therefore" signal a conclusion and are called CONCLUSION INDICATORS. The argument from Peter Singer (on page 26) uses the conclusion indicator word, "so." Table 1.2 is an incomplete list of indicator words and phrases in English.

Table 1.2: Premise and Conclusion Indicators.

Premise Indicators:	because, as, for, since, given that, for the reason that
Conclusion Indicators:	therefore, thus, hence, so, consequently, it follows that, in conclusion, as a result, then, must, accordingly, this implies that, this entails that, we may infer that

The two passages we have looked at in this section so far have been simply presented as quotations. But often it is extremely useful to rewrite arguments in a way that makes their logical structure clear. One way to do this is to use something called "canonical form." An argument written in CANONICAL FORM has each premise numbered and written on a separate line. Indicator words and other unnecessary material should be removed from the premises. Although you can shorten the premises and conclusion, you need to be sure to keep them all complete sentences with the same meaning, so that they can be true or false. The argument from Peter Singer, above, looks like this in canonical form:

(C1) If we can stop something bad from happening, without sacrificing anything of comparable moral importance, we ought to do so.
(C2) Famine is something bad.
(C3) Famine can be prevented without sacrificing anything of comparable moral importance.
(C4) $\therefore$ We ought to prevent famine.

Each statement has been written on its own line and given a number. The statements have been paraphrased slightly, for brevity, and the indicator word "so" has been removed. Also notice that the "it" in the third premise has been replaced by the word "famine," so that statements reads naturally on its own.

Similarly, we can rewrite the argument Jack gives at the racetrack, on page 26, like this:

(D1) There is no one at the race track.
(D2) ∴ The race is not happening.

Notice that we did not include anything from the part of the passage in italics. The italics represent the context, not the argument itself. Also, notice that the "I guess" has been removed. When we write things out in canonical form, we write the content of the statements, ignore information about the speaker's mental state, like "I believe" or "I guess."

One of the first things you have to learn to do in logic is to identify arguments and rewrite them in canonical form. This is a foundational skill for everything else we will be doing in this text, so we are going to run through a few examples now, and there will be more in the exercises. The passage below is paraphrased from the ancient Greek philosopher Aristotle.

> *An ancient philosopher, writing for his students* Again, our observations of the stars make it evident that the earth is round. For quite a small change of position to south or north causes a manifest alteration in the stars which are overhead. ([1], 298a2-10)

[1]

The first thing we need to do to put this argument in canonical form is to identify the conclusion. The indicator words are the best way to do this. The phrase "make it evident that" is a conclusion indicator phrase. He is saying that everything else is *evidence* for what follows. So we know that the conclusion is that the earth is round. "For" is a premise indicator word—it is sort of a weaker version of "because." Thus the premise is that the stars in the sky change if you move north or south. In canonical form, Aristotle's argument that the earth is round looks like this.

(E1) There are different stars overhead in the northern and southern parts of the earth.
(E2) ∴ The earth is spherical in shape.

That one is fairly simple, because it just has one premise. Here's another example of an argument, this time from the book of Ecclesiastes in the Bible. The speaker in this part of the bible is generally referred to as The Preacher, or in Hebrew, Koheleth. In this verse, Koheleth uses both a premise indicator and a conclusion indicator to let you know he is giving reasons for enjoying life.

> *The words of the Preacher, son of David, King of Jerusalem* There is something else meaningless that occurs on earth: the righteous who get what the wicked deserve, and the wicked who get what the righteous deserve. …So I commend the enjoyment of life, because there is nothing better for a person under the sun than to eat and drink and be glad. (Ecclesiastes 8:14-15, New International Version)

Koheleth begins by pointing out that good things happen to bad people and bad things happen to good people. This is his first premise. (Most

Bible teachers provide some context here by pointing that that the ways of God are mysterious and this is an important theme in Ecclesiastes.) Then Koheleth gives his conclusion, that we should enjoy life, which he marks with the word "so." Finally he gives an extra premise, marked with a "because," that there is nothing better for a person than to eat and drink and be glad. In canonical form, the argument would look like this.

(F1) Good things happen to bad people and bad things happen to good people.
(F2) There is nothing better for people than to eat, to drink and to enjoy life.
(F3) ∴ You should enjoy life.

Notice that in the original passages, Aristotle put the conclusion in the first sentence, while Koheleth put it in the middle of the passage, between two premises. In ordinary English, people can put the conclusion of their argument where ever they want. However, when we write the argument in canonical form, the conclusion goes last.

Unfortunately, indicator words aren't a perfect guide to when people are giving an argument. Look at this passage from a newspaper:

> *From the general news section of a national newspaper* The new budget underscores the consistent and paramount importance of tax cuts in the Bush philosophy. His first term cuts affected more money than any other initiative undertaken in his presidency, including the costs thus far of the war in Iraq. All told, including tax incentives for health care programs and the extension of other tax breaks that are likely to be taken up by Congress, the White House budget calls for nearly $300 billion in tax cuts over the next five years, and $1.5 trillion over the next 10 years. (?)

Although there are no indicator words, this is in fact an argument. The writer wants you to believe something about George Bush: tax cuts are his number one priority. The next two sentences in the paragraph give you reasons to believe this. You can write the argument in canonical form like this.

(G1) Bush's first term cuts affected more money than any other initiative undertaken in his presidency, including the costs thus far of the war in Iraq.
(G2) The White House budget calls for nearly $300 billion in tax cuts over the next five years, and $1.5 trillion over the next 10 years.
(G3) ∴ Tax cuts are of consistent and paramount importance of in the Bush philosophy.

The ultimate test of whether something is an argument is simply whether some of the statements provide reason to believe another one of the statements. If some statements support others, you are looking at an argument.

The speakers in these two cases use indicator phrases to let you know they are trying to give an argument.

A final bit of terminology for this section. An INFERENCE is the act of coming to believe a conclusion on the basis of some set of premises. When Jack in the example above saw that no one was at the track, and came to believe that the race was not on, he was making an inference. We also use the term inference to refer to the connection between the premises and the conclusion of an argument. If your mind moves from premises to conclusion, you make an inference, and the premises and the conclusion are said to be linked by an inference. In that way inferences are like argument glue: they hold the premises and conclusion together.

Arguments and Nonarguments

We just saw that arguments are made of statements. However, there are lots of other things you can do with statements. Part of learning what an argument is involves learning what an argument is not, so in this section and the next we are going to look at some other things you can do with statements besides make arguments.

The list below of kinds of nonarguments is not meant to be exhaustive: there are all sorts of things you can do with statements that are not discussed. Nor are the items on this list meant to be exclusive. One passage may function as both, for instance, a narrative and a statement of belief. Right now we are looking at real world reasoning, so you should expect a lot of ambiguity and imperfection.

Simple Statements of Belief

An argument is an attempt to persuade an audience to believe something, using reasons. Often, though, when people try to persuade others to believe something, they skip the reasons, and give a SIMPLE STATEMENT OF BELIEF. This is a kind of nonargumentative passage where the speaker simply asserts what they believe without giving reasons. Sometimes simple statements of belief are prefaced with the words "I believe," and sometimes they are not. A simple statements of belief can be a profoundly inspiring way to change people's hearts and minds. Consider this passage from Dr. Martin Luther King's Nobel acceptance speech.

> I believe that even amid today's mortar bursts and whining bullets, there is still hope for a brighter tomorrow. I believe that wounded justice, lying prostrate on the blood-flowing streets of our nations, can be lifted from this dust of shame to reign supreme among the children of men. I have the audacity to believe that peoples everywhere can have three meals a day for their bodies, education and culture for their minds, and dignity, equality and freedom for their spirits. (**?**)

This actually is a part of a longer passage that consists almost entirely of statements that begin with some variation of "I believe."It is incredibly powerful oration, because the audience, feeling the power of King's beliefs, comes to share in those beliefs. The language King uses to describe how he believes is important, too. He says his belief in freedom and equality requires audacity, making the audience feel his courage and want to share in this courage by believing the same things.

These statements are moving, but they do not form an argument. None of these statements provide evidence for any of the other statements. In fact, they all say roughly the same thing, that good will triumph over evil. So the study of this kind of speech belongs to the discipline of rhetoric, not of logic.

Expository Passages

Perhaps the most basic use of a statement is to convey information. Often if we have a lot of information to convey, we will sometimes organize our statements around a theme or a topic. Information organized in this fashion can often appear like an argument, because all of the statements in the passage relate back to some central statement. However, unless the other statements are given as reasons to believe the central statement, the passage you are looking at is not an argument. Consider this passage:

> *From a college psychology textbook.* Eysenck advocated three major behavior techniques that have been used successfully to treat a variety of phobias. These techniques are modeling, flooding, and systematic desensitization. In **modeling** phobic people watch nonphobics cope successfully with dreaded objects or situations.In **flooding** clients are exposed to dreaded objects or situations for prolonged periods of time in order to extinguish their fear. In contrast to flooding, **systematic desensitization** involves gradual, client-controlled exposure to the anxiety eliciting object or situation. (Adapted from Ryckman [2]Ryckman2007)

[2]

We call this kind of passage an expository passage. In an EXPOSITORY PASSAGE, statements are organized around a central theme or topic statement. The topic statement might look like a conclusion, but the other statements are not meant to be evidence for the topic statement. Instead, they elaborate on the topic statement by providing more details or giving examples. In the passage above, the topic statement is "Eysenck advocated three major behavioral techniques . . . ." The statements describing these techniques elaborate on the topic statement, but they are not evidence for it. Although the audience may not have known this fact about Eysenk before reading the passage, they will typically accept the truth of this statement instantly, based on the textbook's authority. Subsequent statements in the passage merely provide detail.

Deciding whether a passage is an argument or an expository passage is complicated by the fact that sometimes people argue by example:

Steve:	Kenyans are better distance runners than everyone else.
Monica:	Oh come on, that sounds like an exaggeration of a stereotype that isn't even true.
Steve:	What about Dennis Kimetto, the Kenyan who set the world record for running the marathon? And you know who the previous record holder was: Emmanuel Mutai, also Kenyan.

Here Steve has made a general statement about all Kenyans. Monica clearly doubts this claim, so Steve backs it up with some examples that seem to match his generalization. This isn't a very strong way to argue: moving from two examples to statement about all Kenyans is probably going to be a kind of bad argument known as a hasty generalization. (This mistake is covered in the complete version of this text in the chapter on induction Chapter **??** on induction.) The point here however, is that Steve is just offering it as an argument.

The key to telling the difference between expository passages and arguments by example is whether there is a conclusion that they audience needs to be convinced of. In the passage from the psychology textbook, "Eysenck advocated three major behavioral techniques" doesn't really work as a conclusion for an argument. The audience, students in an introductory psychology course, aren't likely to challenge this assertion, the way Monica challenges Steve's overgeneralizing claim.

Context is very important here, too. The Internet is a place where people argue in the ordinary sense of exchanging angry words and insults. In that context, people are likely to actually give some arguments in the logical sense of giving reasons to believe a conclusion.

Narratives

Statements can also be organized into descriptions of events and actions, as in this snippet from book V of *Harry Potter*.

> But she [Hermione] broke off; the morning post was arriving and, as usual, the *Daily Prophet* was soaring toward her in the beak of a screech owl, which landed perilously close to the sugar bowl and held out a leg. Hermione pushed a Knut into its leather pouch, took the newspaper, and scanned the front page critically as the owl took off again. (**?**)

We will use the term NARRATIVE loosely to refer to any passage that gives a sequence of events or actions. A narrative can be fictional or nonfictional. It can be told in regular temporal sequence or it can jump around, forcing the audience to try to reconstruct a temporal sequence. A narrative can describe a short sequence of actions, like Hermione taking a newspaper from an owl, or a grand sweep of events, like this passage about the rise and fall of an empire in the ancient near east:

> The Guti were finally expelled from Mesopotamia by the Sumerians of Erech (*c.* 2100), but it was left to the kings of Ur's famous third dynasty to re-establish the Sargonoid frontiers and write the final chapter of the Sumerian History. The dynasty lasted through the twenty first century at the close of which the armies of Ur were overthrown by the Elamites and Amorites (?).

This passage does not feature individual people performing specific actions, but it is still united by character and action. Instead of Hermione at breakfast, we have the Sumarians in Mesopotamia. Instead of retrieving a message from an owl, the conquer the Guti, but then are conquered by the Elamites and Amorites. The important thing is that the statements in a narrative are not related as premises and conclusion. Instead, they are all events which are united common characters acting in specific times and places.

Key Terms

1. **Argument**
2. **Canonical form**
3. **Conclusion**
4. **Conclusion indicator**
5. **Content neutrality**
6. **Critical thinker**
7. **Critical thinking**
8. **Explanation**
9. **Expository passage**
10. **Formal logic**
11. **Inference**
12. **Informal logic**
13. **Logic**
14. **Metacognition**
15. **Metareasoning**
16. **Narrative**
17. **Practical argument**
18. **Premise**
19. **Premise indicator**
20. **Reason**
21. **Rhetoric**
22. **Simple statement of belief**
23. **Statement**
24. **Target proposition**

2
The Basics of Evaluating Argument

Two Ways an Argument Can Go Wrong

Arguments are supposed to lead us to the truth but they don't always succeed. There are two ways that arguments can fail to lead us to true conclusions. First, they can simply start off with false premises. Consider the following argument:

(A1) It is raining heavily.
(A2) If it is raining heavily, then you should take an umbrella.
(A3) $\therefore$ So, you should take an umbrella.

If premise (1) is false—if it is sunny outside—then the argument gives you no reason to carry an umbrella. The argument has failed its job. Premise (2) could also be false: Even if it is raining outside, you might not need an umbrella. You might wear a rain poncho or keep to covered walkways and still avoid getting soaked. Again, the argument fails because a premise is false. An argument with false premises can not lead us to a true conclusion.

Even if an argument has all true premises, there is still a second way it can fail. Consider another example:

(B1) You are reading this book.
(B2) Most people who read this book are logic students.
(B3) $\therefore$ You are a logic student.

This is not a terrible argument. The premises are true. Most people who read this book *are* logic students. Yet, it is possible for someone besides a logic student to read this book. If your friend who is not currently in a logic class read this book, they would not immediately become a logic student. So the premises of this argument, even though they are true, do not guarantee the truth of the conclusion. The evidential support between premises and conclusion is not a guarantee. This criterion is about the *structure* of the argument, and how the premises and the conclusion are related to one another. There are better and worse ways that the premises of an argument can supply evidential support to its conclusion. Compare the example above with this one:

(C1) You are reading this book.
(C2) At least one person reading this book is a professional surfer.
(C3) $\therefore$ You are a professional surfer.

This argument should strike you as substantially worse than the previous one, even if you really are a professional surfer! Suppose the premises are both true. It still would seem pretty unlikely that the premises are very good reason to think that the conclusion is true. Just because there's at least one professional surfer who read this book, it doesn't follow that that person is you. Even though both of these arguments fail to guarantee their conclusions, one does seem better than the other. We'll be discussing why some arguments that fail this second citerion may still be worthwhile arguments.

To sum up, for any argument there are two ways that it could fail. First, one or more of the premises might be false. Second, the premises might fail to support the conclusion. Even if the premises were true, the form of the argument might be weak, meaning that there is little to no evidential support from premises to conclusion.

Validity and Soundness

In logic, we are mostly concerned with evaluating the quality of inferences, not the truth of the premises. The truth of various premises will be a matter of whatever specific topic we are arguing about and since logic is content neutral we will also remain neutral.

The strongest possible evidential support would be for the premises to somehow force the conclusion to be true. This kind of inference is called VALID. Lets make this notion a bit more precise:

An argument is valid if and only if it is impossible for the premises to be true and the conclusion false.

The important thing to see is that the definition is about what *would* happen if the premises were true. It doesn't state that the premises actually *are* true. This is why our definition is about what is possible or impossible. The argument is valid if, when you imagine the premises are true, you are somehow forced into imagining that the conclusion is also true. Consider the argument in Figure **??**

(D1) Lady Gaga is from Mars.
(D2) $\therefore$ Lady Gaga is from the fourth planet from our sun.

The American pop star Lady Gaga is not from Mars. (She's from New York City.) Nevertheless, if you grant that she is from Mars, you *also* have to grant that she is from the fourth planet from our sun, because Mars simply is the fourth planet form our sun. Therefore this argument is valid.

This way of understanding validity is based on what you can imagine, but not everyone is convinced that the imagination is a reliable tool in logic. That is why definitions like **??** and **??** talk about what is necessary or impossible. If the premises are true, the conclusion necessarily must be true. Alternately, it is impossible for the premises to be true and the conclusion false. The idea here is that instead of talking about the imagination, we will just talk about what can or cannot happen at the same time. The fundamental notion of validity remains the same, however: the truth of the premises would simply guarantee the truth of conclusion.

So, assessing validity means wondering about whether the conclusion would be true *if* the premises were true. This means that valid arguments can have false conclusions. This is important to keep in mind because people naturally tend to think that any argument must be good if they agree with the conclusion. And the more passionately people believe in the conclusion, the more likely we are to think that any argument for it must be brilliant. Conversely, if the conclusion is something we don't believe in, we naturally tend to think the argument is poor. And the more we don't like the conclusion, the less likely we are to like the argument.

But this is not the correct way to evaluate inferences at all. The quality of the inference is entirely independent of the truth of the conclusion. You can have great arguments for false conclusions and horrible arguments for true conclusions. We have trouble seeing this because of biases built deep in the way we think called "cognitive biases." A COGNITIVE BIAS is a habit of reasoning that can be dysfunctional in certain circumstances. Generally these biases developed for a reason, so they serve us well in many or most circumstances. But cognitive biases also systematically distort our reasoning in other circumstances, so we must be on guard against them.

There is a particular cognitive bias that makes it hard for us to recognize when a poor argument is being given for a conclusion we agree with. It is called "confirmation bias" and it is in many ways the mother of all cognitive biases. CONFIRMATION BIAS is the tendency to discount or ignore evidence and arguments that contradict one's current beliefs. It really pervades all of our thinking, right down to our perceptions.

Because of confirmation bias, we need to train ourselves to recognize valid arguments for conclusions we think are false. Remember, an argument is valid if it is impossible for the premises to be true and the conclusion false. This means that you can have valid arguments with false conclusions, they just have to also have false premises. Consider the example in Figure **??**

(E1) Oranges are either fruits or musical instruments.

(E2) Oranges are not fruits.

(E3) ∴ Oranges are musical instruments.

The conclusion of this argument is ridiculous. Nevertheless, it follows validly from the premises. This is a valid argument. *If* both premises were true, *then* the conclusion would necessarily be true.

This shows that a valid argument does not need to have true premises or a true conclusion. Conversely, having true premises and a true conclusion is not enough to make an argument valid. Consider the example in Figure **??**

(F1) London is in England.

(F2) Beijing is in China.

(F3) ∴ Paris is in France.

The premises and conclusion of this argument are, as a matter of fact, all true. This is a terrible argument, however, because the premises have nothing to do with the conclusion. Imagine what would happen if Paris declared independence from the rest of France. Then the conclusion would be false, even though the premises would both still be true. Thus, it is *logically possible* for the premises of this argument to be true and the conclusion false. The argument is not valid. If an argument is not valid, it is called INVALID. As we shall see, this term is a little misleading, because less than perfect arguments can be very useful. But before we do that, we need to look more at the concept of validity.

In general, then, the *actual* truth or falsity of the premises, if known, do not tell you whether or not an inference is valid. There is one exception: when the premises are true and the conclusion is false, the inference cannot be valid, because valid reasoning can only yield a true conclusion when beginning from true premises.

Figure **??** has another invalid argument:

(G1) All dogs are mammals.

(G2) All dogs are animals.

(G3) ∴ All animals are mammals.

In this case, we can see that the argument is invalid by looking at the truth of the premises and conclusion. We know the premises are true. We know that the conclusion is false. This is the one circumstance that a valid argument is supposed to make impossible.

Some invalid arguments are hard to detect because they resemble valid arguments. Consider the one in Figure **??**

(H1) An economic stimulus package will allow the U.S. to avoid a depression.

(H2) There is no economic stimulus package

(H3) ∴ [.3] The U.S. will go into a depression.

This reasoning is not valid since the premises do not *definitively* support the conclusion. To see this, assume that the premises are true and then ask, "Is it possible that the conclusion could be false in such a situation?". There is no inconsistency in taking the premises to be true without taking the conclusion to be true. The first premise says that the stimulus package will allow the U.S. to avoid a depression, but it does not say that a stimulus package is the *only* way to avoid a depression. Thus, the mere fact that there is no stimulus package does not necessarily mean that a depression will occur.

When an argument resembles a good argument but is actually a bad one, we say it is a .FALLACY. Fallacies are similar to cognitive biases, in that they are ways our reasoning can go wrong. Fallacies, however, are always mistakes you can explicitly lay out as arguments in canonical form, as above.

Here is another, trickier, example. I will give it first in ordinary language.

> *A pundit is speaking on a cable news show* If the U.S. economy were in recession and inflation were running at more than 4%, then the value of the U.S. dollar would be falling against other major currencies. But this is not happening — the dollar continues to be strong. So, the U.S. is not in recession.

The conclusion is "The U.S. economy is not in recession." If we put the argument in canonical form, it looks like figure **??**

(I1) If the U.S. were in a recession with more than 4% inflation, then the dollar would be falling.

(I2) The dollar is not falling.

(I3) ∴ The U.S. is not in a recession.

The conclusion does not follow necessarily from the premises. It does follow necessarily from the premises that (i) the U.S. economy is not in recession or (ii) inflation is running at more than 4%, but they do not guarantee (i) in particular, which is the conclusion. For all the premises say, it is possible that the U.S. economy is in recession but inflation is less than 4%. So, the inference does not *necessarily* establish that the U.S. is not in recession. A parallel inference would be "Jack needs eggs and milk to make an omelet. He can't make an omelet. So, he doesn't have eggs.".

If an argument is not only valid, but also has true premises, we call it SOUND. "Sound" is the highest compliment you can pay an argument. If logic is the study of virtue in argument, sound arguments are the most virtuous. We said in Section 2 that there were two ways an argument could go wrong, either by having false premises or weak inferences. Sound arguments have true premises and undeniable inferences. If someone gives a sound argument in a conversation, you have to believe the conclusion, or else you are irrational.

(J1) Socrates is a person.	*(K1)* Socrates is a person.
(J2) All people are carrots.	*(K2)* All people are mortal.
(J3) ∴ Therefore, Socrates is a carrot.	*(K3)* ∴ Therefore, Socrates is mortal.
Valid, but not sound	**Valid and sound**

The argument on the left in Figure 3.1 is valid, but not sound. The argument on the right is both valid and sound.

Both arguments have the exact same form. They say that a thing belongs to a general category and everything in that category has a certain property, so the thing has that property. Because the form is the same, it is the same valid inference each time. The difference in the arguments is not the validity of the inference, but the truth of the second premise. People are not carrots, therefore the argument on the left is not sound. People are mortal, so the argument on the right is sound.

Often it is easy to tell the difference between validity and soundness if you are using completely silly examples. Things become more complicated with false premises that you might be tempted to believe, as in the argument in Figure **??**.

(L1) Every Irishman drinks Guinness.

(L2) Smith is an Irishman.

(L3) ∴ Smith drinks Guinness.

You might have a general sense that the argument in Figure **??** is bad—you shouldn't assume that someone drinks Guinness just because they are Irish. But the argument is completely valid (at least when it is expressed this way.) The inference here is the same as it was in the previous two arguments. The problem is the first premise. Not all Irishmen drink Guinness, but if they did, and Smith was an Irishman, he would drink Guinness.

The important thing to remember is that validity is not about the actual truth or falsity of the statements in the argument. Instead, it is about the way the premises and conclusion are put together. It is really about the *form* of the argument. A valid argument has perfect logical form. The premises and conclusion have been put together so that the truth of the premises is incompatible with the falsity of the conclusion.

A general trick for determining whether an argument is valid is to try to come up with just one way in which the premises could be true but the conclusion false. If you can think of one (just one! anything at all! but no violating the laws of physics!), the reasoning is *invalid.*

Strong, Cogent, Deductive, Inductive

We have just seen that sound arguments are the very best arguments. Unfortunately, sound arguments are really hard to come by, and when you do find them, they often only prove things that were already quite obvious, like that Socrates (a dead man) is mortal. Fortunately, arguments can still be worthwhile, even if they are not sound. Consider this one:

(M1) In January 1997, it rained in San Diego.
(M2) In January 1998, it rained in San Diego.
(M3) In January 1999, it rained in San Diego.
(M4) ∴ It rains every January in San Diego.

This argument is not valid, because the conclusion could be false even though the premises are true. It is possible, although unlikely, that it will fail to rain next January in San Diego. Moreover, we know that the weather can be fickle. No amount of evidence should convince us that it rains there *every* January. Who is to say that some year will not be a freakish year in which there is no rain in January in San Diego? Even a single counterexample is enough to make the conclusion of the argument false.

Still, this argument is pretty good. Certainly, the argument could be made stronger by adding additional premises: In January 2000, it rained in San Diego. In January 2001. . . and so on. Regardless of how many premises we add, however, the argument will still not be deductively valid. Instead of being valid, this argument is strong. An argument is STRONG if the premises would make the conclusion more likely, were they true. In a strong argument, the premises don't guarantee the truth of the conclusion, but they do make it a good bet. If an argument is strong, and it has true premises, we say that it is COGENT Cogency is the equivalent of soundness in strong arguments. If an inference is neither valid, nor strong, we say it is WEAK. In a weak argument, the premises would not even make the conclusion likely, even if they were true.

You may have noticed that the word "likely" is a little vague. How likely do the premises have to make the conclusion before we can count the argument as strong? The answer is a very unsatisfying "it depends." It depends on what is at stake in the decision to believe the conclusion. What happens if you are wrong? What happens if you are right? The phrase "make the conclusion a good bet" is really quite apt. Whether something is a good bet depends a lot on how much money is at stake and how much you are willing to lose. Sometimes people feel comfortable taking a bet that has a 50% chance of doubling their money, sometimes they don't.

The vagueness of the word "likely" brings out an interesting feature of strong arguments: some strong arguments are stronger than others. The argument about rain in San Diego, above, has three premises referring to three previous Januaries. The argument is pretty strong, but it can become stronger if we go back farther into the past, and find more years where it rains in January. The more evidence we have, the better a bet the conclusion is. Validity is not like this. Validity is a black-or-white matter. You either have it, and you're perfect, or you don't, and you're nothing. There is no point in adding premises to an argument that is already valid.

Arguments that are valid, or at least try to be, are called DEDUCTIVE , and people who attempt to argue using valid arguments are said to be arguing *deductively.* The notion of validity we are using here is, in fact, sometimes called *deductive validity.* Deductive argument is difficult, because, as we said, in the real world sound arguments are hard to come by, and people don't always recognize them as sound when they find them. Arguments that purport to merely be strong rather than valid are called INDUCTIVE. The most common kind of inductive argument includes arguments like the one above about rain in San Diego, which generalize from many cases to a conclusion about all cases.

Deduction is possible in only a few contexts. You need to have clear, fixed meanings for all of your terms and rules that are universal and have no exceptions. One can find situations like this if you are dealing with things like legal codes, mathematical systems or logical puzzles. One can also create, as it were, a context where deduction is possible by imagining a universal, exceptionless rule, even if you know that no such rule exists in reality. In the example above about rain in San Diego, we can change the argument from inductive to deductive by adding a universal, exceptionless premise like "It always rains in January in San Diego." This premise is unlikely to be true, but it can make the inference valid. (For more about trade offs between the validity of the inference and the truth of the premise, see the chapter on incomplete arguments in the complete version of this text.

Here is an example in which the context is an artificial code — the tax code:

> *From a the legal code posted on a government website* A tax credit for energy-efficient home improvement is available at 30% of the cost, up to $1,500 total, in 2009 & 2010, ONLY for existing homes, NOT new construction, that are your "principal residence" for Windows and Doors (including sliding glass doors, garage doors, storm doors and storm windows), Insulation, Roofs (Metal and Asphalt), HVAC: Central Air Conditioners, Air Source Heat Pumps, Furnaces and Boilers, Water Heaters: Gas, Oil, & Propane Water Heaters, Electric Heat Pump Water Heaters, Biomass Stoves.

This rule describes the conditions under which a person can or cannot take a certain tax credit. Such a rule can be used to reach a valid conclusion that the tax credit can or cannot be taken.

As another example of an inference in an artificial situation with limited and clearly defined options, consider a Sudoku puzzle. The rules of Sudoku are that each cell contains a single number from 1 to 9, and each row, each column and each 9-cell square contain one occurrence of each number from 1 to 9. Consider the following partially completed board:

5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9

The following inference shows that, in the first column, a 9 must be entered below the 7:

> The 9 in the first column must go in one of the open cells in the column. It cannot go in the third cell in the column, because there is already a 9 in that 9-cell square. It cannot go in the eighth or ninth cell because each of these rows already contains a 9, and a row cannot contain two occurrences of the same number. Therefore, since there must be a 9 somewhere in this column, it must be entered in the seventh cell, below the 7.

The reasoning in this inference is valid: if the premises are true, then the conclusion must be true. Logic puzzles of all sorts operate by artificially restricting the available options in various ways. This then means that each conclusion arrived at (assuming the reasoning is correct) is necessarily true.

One can also create a context where deduction is possible by imagining a rule that holds without exception. This can be done with respect to any subject matter at all. Speakers often exaggerate the connecting premise in order to ensure that the justificatory or explanatory power of the inference is as strong as possible. Consider Smith's words in the following passage:

Smith:	I'm going to have some excellent pizza this evening.
Jones:	I'm glad to hear it. How do you know?
Smith:	I'm going to Adriatico's. They always make a great pizza.

Here, Smith justifies his belief that the pizza will be excellent — it comes from Adriatico's, where the pizza, he claims, is *always* great: in the past, present and future.

As stated by Smith, the inference that the pizza will be great this evening is valid. However, making the inference valid in this way often means making the general premise false: it's not likely that the pizza is great *every single* time; Smith is overstating the case for emphasis. Note that Smith does not need to use a universal proposition in order to convince Jones that the pizza will *very likely* be good. The inference to the conclusion would be strong (though not valid) if he had said that the pizza is "almost always" great, or that the pizza has been great on all of the many occasions he has been at that restaurant in the past. The strength of the inference would fall to some extent—it would not be guaranteed to be great this evening—but a slightly weaker inference seems appropriate, given that sometimes things go contrary to expectation.

Sometimes the laws of nature make constructing contexts for valid arguments more reasonable. Now consider the following passage, which involves a scientific law:

> Jack is about to let go of Jim's leash. The operation of gravity makes all unsupported objects near the Earth's surface fall toward the center of the Earth. Nothing stands in the way. Therefore, Jim's leash will fall.

(Or, as Spock said in a Star Trek episode, "If I let go of a hammer on a planet that has a positive gravity, I need not see it fall to know that it has in fact fallen.") The inference above is represented in canonical form as follows:

(N1) Jack is about to let go of Jim's leash.
(N2) The operation of gravity makes all unsupported objects near the Earth's surface fall toward the center of the Earth.
(N3) Nothing stands in the way of the leash falling.
(N4) $\therefore$ Jim's leash will fall toward the center of the Earth.

As stated, this argument is valid. That is, if you pretend that they are true or accept them "for the sake of argument", you would *necessarily* also accept the conclusion. Or, to put it another way, there is no way in which you could hold the premises to be true and the conclusion false.

Although this argument is valid, it involves idealizing assumptions similar to the ones we saw in the pizza example. P_2 states a physical law which is about as well confirmed as any statement about the world around us you care to name. However, physical laws make assumptions about the situations they apply to—they typically neglect things like wind resistance. In this case, the idealizing assumption is just that nothing stands in the way of the leash falling. This can be checked just by looking, but this check can go wrong. Perhaps there is an invisible pillar underneath Jack's hand? Perhaps a huge gust of wind will come? These events are much less likely than Adriatico's making a lousy pizza, but they are still possible.

Thus we see that using scientific laws to create a context where deductive validity is possible is a much safer bet than simply asserting whatever exceptionless rule pops into your head. However, it still involves improving the quality of the inference by introducing premises that are less likely to be true.

So deduction is possible in artificial contexts like logical puzzles and legal codes. It is also possible in cases where we make idealizing assumptions or imagine exceptionless rules. The rest of the time we are dealing with induction. When we do induction, we try for strong inferences, where the premises, assuming they are true, would make the truth of the conclusion very likely, though not necessary. Consider the two arguments in Figure 2.1

Figure 2.1: Neither argument is valid, but one is strong and one is weak

(O1) 92% of Republicans from Texas voted for Bush in 2000.
(O2) Jack is a Republican from Texas.
(O3) ∴ Jack voted for Bush.

A **strong** argument

(P1) Just over half of drivers are female.
(P2) There's a person driving the car that just cut me off.
(P3) ∴ The person driving the car that just cut me off is female.

A **weak** argument

Note that the premises in neither inference *guarantee* the truth of the conclusion. For all the premises in the first one say, Jack could be one of the 8% of Republicans from Texas who did not vote for Bush; perhaps, for example, Jack soured on Bush, but not on Republicans in general, when Bush served as governor. Likewise for the second; the driver could be one of the 49%.

So, neither inference is valid. But there is a big difference between how much support the premises, if true, would give to the conclusion in the first and how much they would in the second. The premises in the first, assuming they are true, would provide very strong reasons to accept the conclusion. This, however, is not the case with the second: if the premises in it were true then they would give only weak reasons for believing the conclusion. thus, the first is strong while the second is weak.

As we said earlier, there there are only two options with respect to validity—valid or not valid. On the other hand, strength comes in degrees, and sometimes arguments will have percentages that will enable you to exactly quantify their strength, as in the two examples in Figure 2.1.

However, even where the degree of support is made explicit by a percentage there is no firm way to say at what degree of support an inference can be classified as strong and below which it is weak. In other words, it is difficult to say whether or not a conclusion is *very likely* to be true. For example, In the inference about whether Jack, a Texas Republican, voted for Bush. If 92% of Texas Republicans voted for Bush, the conclusion, if the premises are granted, would very probably be true. But what if the number were 85%? Or 75%? Or 65%? Would the conclusion very likely be true? Similarly, the second inference involves a percentage greater than 50%, but this does not seem sufficient. At what point, however, would it be sufficient?

In order to answer this question, go back to basics and ask yourself: "If I accept the truth of the premises, would I then have sufficient reason to believe the conclusion?". If you would not feel safe in adopting the conclusion as a belief as a result of the inference, then you think it is weak, that is, you do not think the premises give sufficient support to the conclusion.

Note that the same inference might be weak in one context but strong in another, because the degree of support needed changes. For example, if you merely have a deposit to make, you might accept that the bank is open on Saturday based on your memory of having gone to the bank on Saturday at some time in the past. If, on the other hand, you have a vital mortgage payment to make, you might not consider your memory sufficient justification. Instead, you will want to call up the bank and increase your level of confidence in the belief that it will be open on Saturday.[1]

[1] This kind of context-sensitivity is called *contextualism* in epistemology. For more information, see e.g. Jason Stanley's book *Knowledge and Practical Interests*.

Most inferences (if successful) are strong rather than valid. This is because they deal with situations which are in some way open-ended or where our knowledge is not precise. In the example of Jack voting for Bush, we know only that 92% of Republicans voted for Bush, and so there is no definitive connection between being a Texas Republican and voting for Bush. Further, we have only statistical information to go on. This statistical information was based on polling or surveying a sample of Texas voters and so is itself subject to error (as is discussed in the chapter on induction in the complete version of this text. A more precise version of the premise might be "92% $\pm$ 3% of Texas Republicans voted for Bush.".

At the risk of redundancy, let's consider a variety of examples of valid, strong and weak inferences, presented in standard form.

(Q1) David Duchovny weighs more than 200 pounds.
(Q2) $\therefore$ David Duchovny weighs more than 150 pounds.

The inference here is valid. It is valid because of the number system (here applied to weight): 200 is more than 150. It might be false, as a matter of fact, that David Duchovny weighs more than 200 pounds, and false, as a matter of fact, that David Duchovny weighs more than 150 pounds. But if you *suppose* or *grant* or *imagine* that David Duchovny weighs more than 200 pounds, it would then *have* to be true that David Duchovny weighs more than 150 pounds. Next:

(R1) Armistice Day is November 11th, each year.
(R2) Halloween is October 31st, each year.
(R3) $\therefore$ Armistice Day is later than Halloween, each year.

This inference is valid. It is valid because of order of the months in the Gregorian calendar and the placement of the New Year in this system. Next:

(S1) All people are mortal.
(S2) Professor Pappas is a person.
(S3) ∴ Professor Pappas is mortal.

As written, this inference is valid. If you accept for the sake of argument that all men are mortal (as the first premise says) and likewise that Professor Pappas is a man (as the second premise says), then you would have to accept also that Professor Pappas is mortal (as the conclusion says). You could not consistently both (i) affirm that all men are mortal and that Professor Pappas is a man and (ii) deny that Professor Pappas is mortal. If a person accepted these premises but denied the conclusion, that person would be making a mistake in logic.

This inference's validity is due to the fact that the first premise uses the word "all". You might, however, wonder whether or not this premise is true, given that we believe it to be true only on our experience of men *in the past*. This might be a case of over-stating a premise, which we mentioned earlier. Next:

(T1) In 1933, it rained in Columbus, Ohio on 175 days.
(T2) In 1934, it rained in Columbus, Ohio on 177 days.
(T3) In 1935, it rained in Columbus, Ohio on 171 days.
(T4) ∴ In 1936, it rained in Columbus, Ohio on at least 150 days.

This inference is strong. The premises establish a record of days of rainfall that is well above 150. It is possible, however, that 1936 was exceptionally dry, and this possibility means that the inference does not achieve validity. Next:

(U1) The Bible says that (male) homosexuality is an abomination.
(U2) ∴ (Male) homosexuality is an abomination.

This inference is an appeal to a source. In brief, you should think about whether the source is reliable, biased, or whether the claim is consistent with what other authorities on the subject say. You should apply all these criteria to this argument for yourself. You should ask what issues, if any, the Bible is reliable on. If you believe humans had any role in writing the Bible, you can ask about what biases and agendas they might have had. And you can think about what other sources—religious texts or moral experts—say on this issue. You can certainly find many who disagree. We can also evaluate the context of the specific verse (Leviticus 18:22) that the premise refers to. It is important to note that here we are not evaluating the inference by taking a position on whether the premise is true (it is) or conclusion is true (it isn't). When evaluating an inference we are evaluating the *degree of evidential support* that the premises confer on the conclusion. Perhaps an appeal to a religious text is very good (if defeasible) evidence for some conclusion, perhaps it isn't.

(V1) Some professional philosophers published books in 2007.
(V2) Some books published in 2007 sold more than 100,000 copies.
(V3) ∴ Some professional philosophers published books in 2007 that sold more than 100,000 copies.

This reasoning is weak. Both premises use the word "some" which doesn't tell you much about how many professional philosophers published books and how many books sold more than 100,000 copies in 2007. This means that you cannot be confident that even one professional philosopher sold more than 100,000 copies. Next:

(W1) Lots of Russians prefer vodka to bourbon.
(W2) ∴ George Bush was the President of the United States in 2006.

No one would sincerely make an inference like this. It is presented here as an example only: it is clearly weak. It's hard to see how the premise justifies the conclusion to any extent at all.

To sum up this section, we have seen that there are two styles of reasoning, deductive and inductive. The former tries to use valid arguments, while the latter contents itself to give arguments that are strong.

Key Terms

1. **Cogent**
2. **Cognitive Bias**
3. **Confirmation bias**
4. **Deductive**
5. **Fallacy**
6. **Inductive**
7. **Invalid**
8. **Sound**
9. **Strong**
10. **Valid**
11. **Weak**

3
What is Formal Logic?

Formal as in Concerned with the Form of Things

??

Many of the other chapters in this book will deal with formal logic. Formal logic is distinguished from other branches of logic by the way it achieves content neutrality. Back on page 20, we said that a distinctive feature of logic is that it is neutral about the content of the argument it evaluates. If a kind of argument is strong—say, a kind of statistical argument—it will be strong whether it is applied to sports, politics, science or whatever. Formal logic takes radical measures to ensure content neutrality: it removes the parts of a statement that tie it to particular objects in the world and replaces them with abstract symbols. (See page 21)

Consider the two arguments from Figure 3.1 again:

(A1)	Socrates is a person.	*(B1)*	Socrates is a person.
(A2)	All persons are mortal.	*(B2)*	All people are carrots.
(A3)	∴ Socrates is mortal.	*(B3)*	∴ Socrates is a carrot.

Figure 3.1: Two examples of valid arguments.

These arguments are both valid. In each case, if the premises were true, the conclusion would have to be true. (In the case of the first argument, the premises are actually true, so the argument is sound, but that is not what we are concerned with right now.) What makes these arguments valid is that they are put together the right way. Another way of thinking about this is to say that they have the same logical form. Both arguments can be written like this:

(C1) S is M.
(C2) All M are P.
(C3) ∴ S is P.

In both arguments S stands for Socrates and M stands for person. In the first argument, P stands for mortal; in the second, P stands for carrot.

The letters 'S', 'M', and 'P' are variables. They are just like the variables you may have learned about in algebra class. In algebra, you had equations like $y = 2x + 3$, where x and y were variables that could stand for any number. Just as x could stand for any number in algebra, 'S' can stand for any name in logic. In fact, this is one of the original uses of variables. Long before variables were used to stand for numbers in algebra, they were used to stand for classes of things, like people or carrots, by Aristotle in his book the (?). Around the same time in India, the grammarian and linguist Pāṇini was also using variables to represent possible sounds that could be used in different forms of a word (?). Both thinkers introduce their variables fairly causally, as if their readers would be familiar with the idea, so it may be that people prior to them actually invented the variable!

Whoever invented it, the variable was one of the most important conceptual innovations in human history, right up there with the invention of zero or alphabetic writing. The importance of the variable for the history of mathematics is obvious. But it was also incredibly important in one of its original fields of application, logic. For one thing, it allows logicians to be more content neutral. We can set aside any associations we have with people, or carrots, or whatever, when we are analyzing an argument. More importantly, once we set aside content in this way, we discover that something incredibly powerful is left over, the logical structure of the sentence itself. This is what we investigate when we study formal logic. In the case of the two arguments above, identifying the logical structure of statements reveals not only that the two arguments have the same logical form, but they have an impeccable logical form. Both arguments are valid, and any other arguments that have this form will be valid.

When Aristotle introduced the variable to the study of logic he used it the way we did in the argument above. His variables stood for names and categories in simple two-premise arguments called syllogisms. The system of logic Aristotle outlined became the dominant logic in the Western world for more than two millennia. It was studied and elaborated on by philosophers and logicians from Baghdad to Paris. The thinkers that carried on Aristotelian tradition were divided by language and religion. They were pagans, Muslims, Jews, and Christians writing typically in Greek, Latin or Arabic. But they were all united by the sense that the project Aristotle had initiated allowed them to see something profound about the nature of reality. They were looking at abstract structures which somehow seemed to be at the foundation of things. As the philosopher and historian of logic Catarina Dutilh Novaes points out, the logic that the thinkers of all these religious traditions were pursuing was formal in that it concerned the *forms* of things (?). As formal logic evolved, however, the idea of being "formal" would take on an additional meaning.

Formal as in Strictly Following Rules

??

Despite its historical importance, Aristotelean logic has largely been superseded. Starting in the 19th century people learned to do more than simply replace categories with variables. They learned to replicate the whole structure of sentences with a formal system that brought out all sorts of features of the logical form of arguments. The result was the creation of entire artificial languages. An ARTIFICIAL LANGUAGE is a language that was consciously developed by identifiable individuals for some purpose. Esperanto, for instance, is an artificial language developed by Ludwig Lazarus Zamenhof in the 19th century with the hope of promoting world peace by creating a common language for all. J.R.R. Tolkien invented several languages to flesh out the fictional world of his fantasy novels, and even created timelines for their evolution. For Tolkien, the creation of languages was an art form in itself, "An art for which life is not long enough, indeed: the construction of imaginary languages in full or outline for amusement, for the pleasure of the constructor or even conceivably of any critic that might occur"[1]. And it is an art that is really beginning to catch on, especially with Hollywood commissioning languages to be constructed for blockbuster films.

1

Artificial languages contrast with NATURAL LANGUAGES, which develop spontaneously and are learned by infants as their first language. Natural languages include all the well-known languages spoken around the world, like English or Japanese or Arabic. It also includes more recently developed languages and evolved spontaneously amongst groups of people. For instance, deaf children will often spontaneously develop their own sign language. This phenomenon was important for the development of American Sign Language (ASL) and is part of why ASL counts as a *natural* language. For similar reasons Nicaraguan Sign Language counts as a natural language, even though it emerged very recently—in the late 1970s and 80s—after the Sandinista government set up schools for the deaf in Nicaragua for the first time. Natural languages can also develop by creolization, when languages merge and children grow up speaking the merged language as their first language. Haitian Creole is one of the most famous examples of this.

The languages developed by logicians are artificial, not natural. Their goal is not to promote global harmony, like Zamenhof's Esperanto. Nor are they creating art for art's sake, as Tolkein was, although logical languages can have a great deal of beauty. When the languages first started being developed in the late 19th and early 20th centuries, the goal was, in fact, to have a logically pure language, free of the irrationalities the plague natural languages. More specifically, they had two distinct goals: first, remove all ambiguity and vagueness, and second, to make the logical structure of the language immediately apparent, so that the language wore its logical structure on its face, as it were. If such a language could be developed, it would help us solve all kinds of problems. The logician and philosopher Rudolf Carnap, for instance, felt that the right artificial language could simply make philosophical problems disappear (**?**).

The languages developed by logicians in the late 19th and early 20th centuries got labeled formal languages, in part because the logicians in question were working in the tradition of formal logic that was already established. A shift began to happen here with the meaning of formal, however, a change which is well documented by Dutilh Novaes ([2]). Logicians began to hope that the languages that were being developed were so logical that everything about them could be characterized by a machine. A machine could be used to create sentences in this language, and then again to identify all the valid arguments in this language. This brings out another sense of the word "formal." As Dutilh Novaes puts it ([3]) instead of being "formal" in the sense of concerning the forms of things, logic was formal in the sense that it followed rules perfectly precisely. You might compare this to the way a "formal hearing" in a court of law follows the rule of law to the letter.

[2]

[3]

For our purposes we will say that the core idea of a FORMAL LANGUAGE is that it is an *artificial language designed to bring out the logical structure of ideas* and remove all the ambiguity and vagueness that plague natural languages like English. We will further add that sometimes, formal languages are languages that can be implemented by a machine. Creating formal languages always involves all kinds of trade offs. On the one hand, we are trying to create a language that makes a logical structure clear and obvious. This will require simplifying things, removing excess baggage from the language. On the other hand, we want to make the language perfectly precise, free of vagueness and ambiguity. This will mean adding complexity to the language. The other thing was that it was very important for the people developing these languages that you be able to prove the all the truths of mathematics in them. This meant that the languages had to have a certain scope and certain limitations.

This was a trade off no logician was ever able to get perfectly correct, because, as it turns out, a logically pure language is impossible. No formal language can express everything that a natural language can. Logicians became convinced of this, naturally enough, because of a pair of logical proofs. In 1931, the logician Kurt Gödel showed that you couldn't do all of mathematics in a consistent logical system, which was enough to persuade most of the logicians engaged in the project to drop it. There is a more general problem with the idea of a purely logical language, though, which is that that many of the features logicians were trying to remove from language were actually necessary to make it function. Arika Okrent puts the point quite well. For Okrent, the failure of artificial languages is precisely what illuminates the virtues of natural language.

> [By studying artificial languages we] gain a deeper appreciation of natural language and the messy qualities that give it so much flexibility and power and that a simple communication device. The ambiguity and lack of precision allow to serve as a instrument of thought *formation*, of experimentation and discovery. We don't know exactly what we mean before we speak; we can figure it out as we go along. We can talk just to talk, to be social, to feel connected, to participate. At the same time natural language still works as an instrument of thought transmission, one that can be *made* extremely precise and reliable when we need it to be, or left loose and sloppy when we can't spare the time or effort (**?**)

The languages developed in the late 19th and early 20th centuries had goals that were theoretical, rather than practical. They languages were meant to improve our understanding of the world for the sake of improving our understanding of the world. They failed at this theoretical goal, but they wound up having a practical spin-off of world-historical proportions, which is why formal logic is a thriving discipline to this day. Remember that in this period people started thinking of formal languages as languages that could be implemented mechanically. At first, the idea of a a mechanistic language was a metaphor. The rules that were being followed to the letter were to be followed by a human being actually writing down symbols. This human being was generally referred to as a "computer," because they were computing things. The world changed when a logician named Alan Turing started using literal machines to be computers.

In the 1930s, Turing developed the idea of a reasoning machine that could compute any function. At first, this was just an abstract idea: it involved an infinite stretch of tape. But during World War II, Turing went to work the British code breaking effort at Bletchley Park. The Nazis encoded messages using a device called the Enigma Machine. The Allies had captured one, but since they settings on the machine were reshuffled for each message, it didn't do them much good. Turing, together with people like the mathematicians Gordon Welchman and Joan Clarke, managed to build another machine that could test Enigma settings rapidly to identify the configuration being used. People had made computing machines before, but now the science of logic was so much more advanced that they real power of mechanical computing could be exploited. The human computers became the fully programmable machines we know today, and the formal languages logicians created for theoretical reasons came the computer languages the world of the 21st century depends on. (All of this information, plus lots of fascinating pictures and diagrams, is available at www.turing.org.uk.)

Part **??** of this book begins by exploring the world of Aristotelian logic, where logic is "formal" in the sense of being about the forms of things. Chapter **??** looks at the logical structure of the individual statements studied by the Aristotelian tradition. Chapter **??** then builds these into valid arguments. After we study Aristotelian logic, we will develop two formal languages, called SL and QL. Chapters **??** through **??** develop SL. In SL, the smallest units are individual statements. Simple statements are represented as letters and connected with logical connectives like *and* and *not* to make more complex statements. Chapters **??** through **??** develop QL. In QL, the basic units are objects, properties of objects, and relations between objects. Part III of this book explores the world of Aristotelian logic. Chapter **??** looks at the logical structure of the individual statements studied by the Aristotelian tradition. Chapter **??** then builds these into valid arguments. Part **??** develops a full-blown formal language, called Sentential logic, or SL. In SL Simple statements are represented as letters and connected with logical connectives like *and* and *not* to make more complex statements. In this book we will be developing two formal languages, called SL and QL. Part **??** develops SL.In SL, the smallest units are individual statements. Simple statements are represented as letters and connected with logical connectives like *and* and *not* to make more complex statements. Part **??** develops QL. In QL, the basic units are objects, properties of objects, and relations between objects.

More Logical Notions for Formal Logic

Part I covered the basic concepts you need to study any kind of logic. When we study formal logic, we will be interested in some additional logical concepts, which we will explain here.

Truth values

A truth value is the status of a statement as true or false. Thus the truth value of the sentence "All dogs are mammals" is "True," while the truth value of "All dogs are reptiles" is false. More precisely, a TRUTH VALUE is the status of a statement with relationship to truth. We have to say this, because there are systems of logic that allow for truth values besides "true" and "false," like "maybe true," or "approximately true," or "kinda sorta true." For instance, some philosophers have claimed that the future is not yet determined. If they are right, then statements about *what will be the case* are not yet true or false. Some systems of logic accommodate this by having an additional truth value. Other formal languages, so-called paraconsistent logics, allow for statements that are both true *and* false. We won't be dealing with those in this textbook, however. For our purposes, there are two truth values, "true" and "false," and every statement has exactly one of these. Logical systems like ours are called BIVALENT.

Tautology, contingent statement, contradiction

In considering arguments formally, we care about what would be true *if* the premises were true. Generally, we are not concerned with the actual truth value of any particular statements— whether they are *actually* true or false.

Yet there are some statements that must be true, just as a matter of logic.
Consider these statements:

1. It is raining.
2. Either it is raining, or it is not.
3. It is both raining and not raining.

In order to know if statement 1 is true, you would need to look outside or check the weather channel. Logically speaking, it might be either true or false. Statements like this are called *contingent* statements.

Statement 2 is different. You do not need to look outside to know that it is true. Regardless of what the weather is like, it is either raining or not. If it is drizzling, you might describe it as partly raining or in a way raining and a way not raining. However, our assumption of bivalence means that we have to draw a line, and say at some point that it is raining. And if we have not crossed this line, it is not raining. Thus the statement "either it is raining or it is not" is always going to be true, no matter what is going on outside. A statement that has to be true, as a matter of logic is called a TAUTOLOGY or logical truth.

You do not need to check the weather to know about statement 3, either. It must be false, simply as a matter of logic. It might be raining here and not raining across town, it might be raining now but stop raining even as you read this, but it is impossible for it to be both raining and not raining here at this moment. The third statement is *logically false*; it is false regardless of what the world is like. A logically false statement is called a CONTRADICTION.

We have already said that a contingent statement is one that could be true, or could be false, as far as logic is concerned. To be more precise, we should define a CONTINGENT STATEMENT as a statement that is neither a tautology nor a contradiction. This allows us to avoid worrying about what it means for something to be logically possible. We can just piggyback on the idea of being logically necessary or logically impossible.

A statement might *always* be true and still be contingent. For instance, it may be the case that in no time in the history of the universe was there ever an elephant with tiger stripes. Elephants only ever evolved on Earth, and there was never any reason for them to evolve tiger stripes. The statement "Some elephants have tiger stripes," is therefore always false. It is, however, still a contingent statement. The fact that it is always false is not a matter of logic. There is no contradiction in considering a possible world in which elephants evolved tiger stripes, perhaps to hide in really tall grass. The important question is whether the statement *must* be true, just on account of logic.

When you combine the idea of tautologies and contradictions with the notion of deductive validity, as we have defined it, you get some curious results. For one thing, any argument with a tautology in the conclusion will be valid, even if the premises are not relevant to the conclusion. This argument, for instance, is valid.

(D1) There is coffee in the coffee pot.
(D2) There is a dragon playing bassoon on the armoire.
(D3) ∴ All bachelors are unmarried men.

The statement "All bachelors are unmarried men" is a tautology. No matter what happens in the world, all bachelors have to be unmarried men, because that is how the word "bachelor" is defined. But if the conclusion of the argument is a tautology, then there is no way that the premises could be true and the conclusion false. So the argument must be valid.

Even though it is valid, something seems really wrong with the argument above. The premises are not relevant to the conclusion. Each sentence is about something completely different. This notion of relevance, however, is something that we don't have the ability to capture in the kind of simple logical systems we will be studying. The logical notion of validity we are using here will not capture everything we like about arguments.

Another curious result of our definition of validity is that any argument with a contradiction in the premises will also be valid. In our kind of logic, once you assert a contradiction, you can say anything you want. This is weird, because you wouldn't ordinarily say someone who starts out with contradictory premises is arguing well. Nevertheless, an argument with contradictory premises is valid.

Logically Equivalent and Contradictory Pairs of Sentences

We can also ask about the logical relations *between* two statements. For example:

(a) John went to the store after he washed the dishes.

(b) John washed the dishes before he went to the store.

These two statements are both contingent, since John might not have gone to the store or washed dishes at all. Yet they must have the same truth value. If either of the statements is true, then they both are; if either of the statements is false, then they both are. When two statements necessarily have the same truth value, we say that they are LOGICALLY EQUIVALENT.

On the other hand, if two sentences must have opposite truth values, we say that they are CONTRADICTORIES. Consider these two sentences

(a) Susan is taller than Monica.

(b) Susan is shorter or the same height as Monica.

One of these sentences must be true, and if one of the sentences is true, the other one is false. It is important to remember the difference between a single sentence that is a *contradiction* and a pair of sentences that are *contradictory*. A single sentence that is a contradiction is in conflict with itself, so it is never true. When a pair of sentences is contradictory, one must always be true and the other false.

Consistency

Consider these two statements:

(a) My only brother is taller than I am.

(b) My only brother is shorter than I am.

Logic alone cannot tell us which, if either, of these statements is true. Yet we can say that *if* the first statement (a) is true, *then* the second statement (b) must be false. And if (b) is true, then (a) must be false. It cannot be the case that both of these statements are true. It is possible, however that both statements can be false. My only brother could be the same height as I am.

If a set of statements could not all be true at the same time, they are said to be INCONSISTENT. Otherwise, they are CONSISTENT.

We can ask about the consistency of any number of statements. For example, consider the following list of statements:

(a) There are at least four giraffes at the wild animal park.

(b) There are exactly seven gorillas at the wild animal park.

(c) There are not more than two Martians at the wild animal park.

(d) Every giraffe at the wild animal park is a Martian.

Statements (a) and (d) together imply that there are at least four Martian giraffes at the park. This conflicts with (c), which implies that there are no more than two Martian giraffes there. So the set of statements (a)–(d) is inconsistent. Notice that the inconsistency has nothing at all to do with (b). Statement (b) just happens to be part of an inconsistent set.

Sometimes, people will say that an inconsistent set of statements "contains a contradiction." By this, they mean that it would be logically impossible for all of the statements to be true at once. A set can be inconsistent even when all of the statements in it are either contingent or tautologous. When a single statement is a contradiction, then that statement alone cannot be true.

Key Terms

1. **Artificial language**

2. **Bivalent**
3. **Consistent**
4. **Contingent statement**
5. **Contradiction**
6. **Contradictories**
7. **Formal language**
8. **Formal logic as concern for logical form**
9. **Formal logic as strictly following rules**
10. **Inconsistent**
11. **Logically equivalent**
12. **Natural language**
13. **Tautology**
14. **Truth value**

Part II

Critical Thinking

4

What is Critical Thinking?

5
Informal Fallacies

What are informal fallacies?

A fallacy is simply a mistake in reasoning. Some fallacies are formal and some are informal. In chapter **??**, we saw that we could define validity formally and thus could determine whether an argument was valid or invalid without even having to know or understand what the argument was about. We saw that we could define certain valid rules of inference, such as *modus ponens* and *modus tollens*. These inference patterns are valid in virtue of their form, not their content. That is, any argument that has the same form as *modus ponens* or modus tollens will automatically be valid. A formal fallacy is simply an argument whose form is invalid. Thus, any argument that has that form will automatically be invalid, regardless of the meaning of the sentences. Two formal fallacies that are similar to, but should never be confused with, modus ponens and modus tollens are denying the antecedent and affirming the consequent. Here are the forms of those invalid inferences:

Denying the antecedent $P \rightarrow Q \sim P \quad P \rightarrow Q \; Q \therefore P$
$\therefore \quad \sim Q$ Affirming the consequent

Any argument that has either of these forms is an invalid argument. For example:

1. If Kant was a deontologist, then he was a non-consequentialist.
2. Kant was not a deontologist.
3. Therefore, Kant was a not a non-consequentialist.

The form of this argument is:

1. $D \rightarrow C$
2. $\sim D$

3. $\therefore \sim C$

As you can see, this argument has the form of the fallacy, denying the antecedent. Thus, we know that this argument is invalid even if we don't know what "Kant" or "deontologist" or "non-consequentialist" means. ("Kant" was a famous German philosopher from the early 1800s, whereas "deontology" and "non-consequentialist" are terms that come from ethical theory.) It is mark of a formal fallacy that we can identify it even if we don't really understand the meanings of the sentences in the argument. Consider the following argument. It's an argument which uses silly, made-up words from Lewis Carrol's "Jabberwocky." See if you can determine whether the argument's form is valid or invalid:

1. If toves are brillig then toves are slithy.
2. Toves are slithy
3. Therefore, toves are brillig.

You should be able to see that this argument has the form of affirming the consequent:

1. $B \rightarrow S$
2. S
3. $\therefore B$

As such, we know that the argument is invalid, even though we haven't got a clue what "toves" are or what "slithy" or "brillig" means. The point is that we can identify formal fallacies without having to know what they mean. In contrast, informal fallacies are those which cannot be identified without understanding the concepts involved in the argument. A paradigm example of an informal fallacy is the fallacy of composition. We will consider this fallacy in the next section. In the remaining sections, we will consider a number of other informal logical fallacies.

Fallacy of Composition

Consider the following argument: Each member on the gymnastics team weighs less than 110 lbs. Therefore, the whole gymnastics team weighs less than 110 lbs.

This arguments commits the composition fallacy. In the composition fallacy one argues that since each part of the whole has a certain feature, it follows that the whole has that same feature. However, you cannot generally identify any argument that moves from statements about parts to statements about wholes as committing the composition fallacy because whether or not there is a fallacy depends on what feature we are attributing to the parts and wholes. Here is an example of an argument that moves from claims about the parts possessing a feature to a claim about the whole possessing that same feature, but doesn't commit the composition fallacy: Every part of the car is made of plastic. Therefore, the whole car is made of plastic. This conclusion does follow from the premises; there is no fallacy here. The difference between this argument and the preceding argument (about the gymnastics team) isn't their form. In fact both arguments have the same form: Every part of X has the feature f. Therefore, the whole X has the feature f. And yet one of the arguments is clearly fallacious, while the other isn't. The difference between the two arguments is not their form, but their content. That is, the difference is what feature is being attributed to the parts and wholes. Some features (like weighing a certain amount) are such that if they belong to each part, then it does not follow that they belong to the whole. Other features (such as being made of plastic) are such that if they belong to each part, it follows that they belong to the whole. Here is another example: Every member of the team has been to Paris. Therefore the team has been to Paris. The conclusion of this argument does not follow. Just because each member of the team has been to Paris, it doesn't follow that the whole team has been to Paris, since it may not have been the case that each individual was there at the same time and was there in their capacity as a member of the team. Thus, even though it is plausible to say that the team is composed of every member of the team, it doesn't follow that since every member of the team has been to Paris, the whole team has been to Paris. Contrast that example with this one:

Every member of the team was on the plane. Therefore, the whole team was on the plane. This argument, in contrast to the last one, contains no fallacy. It is true that if every member is on the plane then the whole team is on the plane. And yet these two arguments have almost exactly the same form. The only difference is that the first argument is talking about the property, having been to Paris, whereas the second argument is talking about the property, being on the plane. The only reason we are able to identify the first argument as committing the composition fallacy and the second argument as not committing a fallacy is that we understand the relationship between the concepts involved. In the first case, we understand that it is possible that every member could have been to Paris without the team ever having been; in the second case we understand that as long as every member of the team is on the plane, it has to be true that the whole team is on the plane. The take home point here is that in order to identify whether an argument has committed the composition fallacy, one must understand the concepts involved in the argument. This is the mark of an informal fallacy: we have to rely on our understanding of the meanings of the words or concepts involved, rather than simply being able to identify the fallacy from its form.

Fallacy of Division

The division fallacy is like the composition fallacy and they are easy to confuse. The difference is that the division fallacy argues that since the whole has some feature, each part must also have that feature. The composition fallacy, as we have just seen, goes in the opposite direction: since each part has some feature, the whole must have that same feature. Here is an example of a division fallacy: The house costs 1 million dollars. Therefore, each part of the house costs 1 million dollars. This is clearly a fallacy. Just because the whole house costs 1 million dollars, it doesn't follow that each part of the house costs 1 million dollars. However, here is an argument that has the same form, but that doesn't commit the division fallacy:

The whole team died in the plane crash. Therefore each individual on the team died in the plane crash. In this example, since we seem to be referring to one plane crash in which all the members of the team died ("the" plane crash), it follows that if the whole team died in the crash, then every individual on the team died in the crash. So this argument does not commit the division fallacy. In contrast, the following argument has exactly the same form, but does commit the division fallacy: The team played its worst game ever tonight. Therefore, each individual on the team played their worst game ever tonight. It can be true that the whole team played its worst game ever even if it is true that no individual on the team played their worst game ever. Thus, this argument does commit the fallacy of division even though it has the same form as the previous argument, which doesn't commit the fallacy of division. This shows (again) that in order to identify informal fallacies (like composition and division), we must rely on our understanding of the concepts involved in the argument. Some concepts (like "team" and "dying in a plane crash") are such that if they apply to the whole, they also apply to all the parts. Other concepts (like "team" and "worst game played") are such that they can apply to the whole even if they do not apply to all the parts.

Begging the question

Consider the following argument: Capital punishment is justified for crimes such as rape and murder because it is quite legitimate and appropriate for the state to put to death someone who has committed such heinous and inhuman acts. The premise indicator, "because" denotes the premise and (derivatively) the conclusion of this argument. In standard form, the argument is this: 1. It is legitimate and appropriate for the state to put to death someone who commits rape or murder. 2. Therefore, capital punishment is justified for crimes such as rape and murder.

You should notice something peculiar about this argument: the premise is essentially the same claim as the conclusion. The only difference is that the premise spells out what capital punishment means (the state putting criminals to death) whereas the conclusion just refers to capital punishment by name, and the premise uses terms like "legitimate" and "appropriate" whereas the conclusion uses the related term, "justified." But these differences don't add up to any real differences in meaning. Thus, the premise is essentially saying the same thing as the conclusion. This is a problem: we want our premise to provide a reason for accepting the conclusion. But if the premise is the same claim as the conclusion, then it can't possibly provide a reason for accepting the conclusion! Begging the question occurs when one (either explicitly or implicitly) assumes the truth of the conclusion in one or more of the premises. Begging the question is thus a kind of circular reasoning. One interesting feature of this fallacy is that formally there is nothing wrong with arguments of this form. Here is what I mean. Consider an argument that explicitly commits the fallacy of begging the question. For example, 1. Capital punishment is morally permissible 2. Therefore, capital punishment is morally permissible Now, apply any method of assessing validity to this argument and you will see that it is valid by any method. If we use the informal test (by trying to imagine that the premises are true while the conclusion is false), then the argument passes the test, since any time the premise is true, the conclusion will have to be true as well (since it is the exact same statement). Likewise, the argument is valid by our formal test of validity, truth tables. But while this argument is technically valid, it is still a really bad argument. Why? Because the point of giving an argument in the first place is to provide some reason for thinking the conclusion is true for those who don't already accept the conclusion. But if one doesn't already accept the conclusion, then simply restating the conclusion in a different way isn't going to convince them. Rather, a good argument will provide some reason for accepting the conclusion that is sufficiently independent of that conclusion itself. Begging the question utterly fails to do this and this is why it counts as an informal fallacy. What is interesting about begging the question is that there is absolutely nothing wrong with the argument formally.

Whether or not an argument begs the question is not always an easy matter to sort out. As with all informal fallacies, detecting it requires a careful understanding of the meaning of the statements involved in the argument. Here is an example of an argument where it is not as clear whether there is a fallacy of begging the question: Christian belief is warranted because according to Christianity there exists a being called "the Holy Spirit" which reliably guides Christians towards the truth regarding the central claims of Christianity.

1 This is a much simplified version of the view defended by Christian philosophers such as Alvin Plantinga. Plantinga defends (something like) this claim in: Plantinga, A. 2000. Warranted Christian Belief. Oxford, UK: Oxford University Press.

One might think that there is a kind of circularity (or begging the question) involved in this argument since the argument appears to assume the truth of Christianity in justifying the claim that Christianity is true. But whether or not this argument really does beg the question is something on which there is much debate within the sub-field of philosophy called epistemology ("study of knowledge"). The philosopher Alvin Plantinga argues persuasively that the argument does not beg the question, but being able to assess that argument takes patient years of study in the field of epistemology (not to mention a careful engagement with Plantinga's work). As this example illustrates, the issue of whether an argument begs the question requires us to draw on our general knowledge of the world. This is the mark of an informal, rather than formal, fallacy.

False dichotomy

Suppose I were to argue as follows: Raising taxes on the wealthy will either hurt the economy or it will help it. But it won't help the economy. Therefore it will hurt the economy. The standard form of this argument is: 1. Either raising taxes on the wealthy will hurt the economy or it will help it. 2. Raising taxes on the wealthy won't help the economy. 3. Therefore, raising taxes on the wealthy will hurt the economy.

This argument contains a fallacy called a "false dichotomy." A false dichotomy is simply a disjunction that does not exhaust all of the possible options. In this case, the problematic disjunction is the first premise: either raising the taxes on the wealthy will hurt the economy or it will help it. But these aren't the only options. Another option is that raising taxes on the wealthy will have no effect on the economy. Notice that the argument above has the form of a disjunctive syllogism:

1. $A \vee B$
2. $\sim A$
3. $\therefore B$

However, since the first premise presents two options as if they were the only two options, when in fact they aren't, the first premise is false and the argument fails. Notice that the form of the argument is perfectly good—the argument is valid. The problem is that this argument isn't sound because the first premise of the argument commits the false dichotomy fallacy. False dichotomies are commonly encountered in the context of a disjunctive syllogism or constructive dilemma (see chapter 2). In a speech made on April 5, 2004, President Bush made the following remarks about the causes of the Iraq war: Saddam Hussein once again defied the demands of the world. And so I had a choice: Do I take the word of a madman, do I trust a person who had used weapons of mass destruction on his own people, plus people in the neighborhood, or do I take the steps necessary to defend the country? Given that choice, I will defend America every time. The false dichotomy here is the claim that: Either I trust the word of a madman or I defend America (by going to war against Saddam Hussein's regime). The problem is that these aren't the only options. Other options include ongoing diplomacy and economic sanctions. Thus, even if it true that Bush shouldn't have trusted the word of Hussein, it doesn't follow that the only other

option is going to war against Hussein's regime. (Furthermore, it isn't clear in what sense this was needed to defend America.) That is a false dichotomy. As with all the previous informal fallacies we've considered, the false dichotomy fallacy requires an understanding of the concepts involved. Thus, we have to use our understanding of world in order to assess whether a false dichotomy fallacy is being committed or not.

Equivocation

Consider the following argument: Children are a headache. Aspirin will make headaches go away. Therefore, aspirin will make children go away. This is a silly argument, but it illustrates the fallacy of equivocation. The problem is that the word "headache" is used equivocally—that is, in two different senses. In the first premise, "headache" is used figuratively, whereas in the second premise "headache" is used literally. The argument is only successful if the meaning of "headache" is the same in both premises. But it isn't and this is what makes this argument an instance of the fallacy of equivocation. Here's another example: Taking a logic class helps you learn how to argue. But there is already too much hostility in the world today, and the fewer arguments the better. Therefore, you shouldn't take a logic class. In this example, the word "argue" and "argument" are used equivocally. Hopefully, at this point in the text, you recognize the difference. (If not, go back and reread section 1.1.) The fallacy of equivocation is not always so easy to spot. Here is a trickier example: The existence of laws depends on the existence of intelligent beings like humans who create the laws. However, some laws existed before there were any humans (e.g., laws of physics). Therefore, there must be some non-human, intelligent being that created these laws of nature.

The term "law" is used equivocally here. In the first premise it is used to refer to societal laws, such as criminal law; in the second premise it is used to refer to laws of nature. Although we use the term "law" to apply to both cases, they are importantly different. Societal laws, such as the criminal law of a society, are enforced by people and there are punishments for breaking the laws. Natural laws, such as laws of physics, cannot be broken and thus there are no punishments for breaking them. (Does it make sense to scold the electron for not doing what the law says it will do?) As with every informal fallacy we have examined in this section, equivocation can only be identified by understanding the meanings of the words involved. In fact, the definition of the fallacy of equivocation refers to this very fact: the same word is being used in two different senses (i.e., with two different meanings). So, unlike formal fallacies, identifying the fallacy of equivocation requires that we draw on our understanding of the meaning of words and of our understanding of the world, generally.

Slippery slope fallacies

Slippery slope fallacies depend on the concept of vagueness. When a concept or claim is vague, it means that we don't know precisely what claim is being made, or what the boundaries of the concept are. The classic example used to illustrate vagueness is the "sorites paradox." The term "sorites" is the Greek term for "heap" and the paradox comes from ancient Greek philosophy. Here is the paradox. I will give you two claims that each sound very plausible, but in fact lead to a paradox. Here are the two claims: 1. One grain of sand is not a heap of sand. 2. If I start with something that is not a heap of sand, then adding one grain of sand to that will not create a heap of sand.

For example, two grains of sand is not a heap, thus (by the second claim) neither is three grains of sand. But since three grains of sand is not a heap then (by the second claim again) neither is four grains of sand. You can probably see where this is going. By continuing to add one grain of sand over and over, I will eventually end up with something that is clearly a heap of sand, but that won't be counted as a heap of sand if we accept both claims 1 and 2 above.

Philosophers continue to argue and debate about how to resolve the sorites paradox, but the point for us is just to illustrate the concept of vagueness. The concept "heap" is a vague concept in this example. But so are so many other concepts, such a color concepts (red, yellow, green, etc.), moral concepts (right, wrong, good, bad), and just about any other concept you can think of. The one domain that seems to be unaffected by vagueness is mathematical and logical concepts. There are two fallacies related to vagueness: the causal slippery slope and the conceptual slippery slope. We'll cover the conceptual slippery slope first since it relates most closely to the concept of vagueness I've explained above.

Conceptual slippery slope

It may be true that there is no essential difference between 499 grains of sand and 500 grains of sand. But even if that is so, it doesn't follow that there is no difference between 1 grain of sand and 5 billion grains of sand. In general, just because we cannot draw a distinction between A and B, and we cannot draw a distinction between B and C, it doesn't mean we cannot draw a distinction between A and C. Here is an example of a conceptual slippery slope fallacy. It is illegal for anyone under 21 to drink alcohol. But there is no difference between someone who is 21 and someone who is 20 years 11 months old. So there is nothing wrong with someone who is 20 years and 11 months old drinking. But since there is no real distinction between being one month older and one month younger, there shouldn't be anything wrong with drinking at any age. Therefore, there is nothing wrong with allowing a 10 year old to drink alcohol. Imagine the life of an individual in stages of 1 month intervals. Even if it is true that there is no distinction in kind between any one of those stages, it doesn't follow that there isn't a distinction to be drawn at the extremes of either end. Clearly there is a difference between a 5 year old and a 25 year old—a distinction in kind that is relevant to whether they should be allowed to drink alcohol. The conceptual slippery slope fallacy assumes that because we cannot draw a distinction between adjacent stages, we cannot draw a distinction at all between any stages. One clear way of illustrating this is with color. Think of a color spectrum from purple to red to orange to yellow to green to blue. Each color grades into the next without there being any distinguishable boundaries

between the colors—a continuous spectrum. Even if it is true that for any two adjacent hues on the color wheel, we cannot distinguish between the two, it doesn't follow from this that there is no distinction to be drawn between any two portions of the color wheel, because then we'd be committed to saying that there is no distinguishable difference between purple and yellow! The example of the color spectrum illustrates the general point that just because the boundaries between very similar things on a spectrum are vague, it doesn't follow that there are no differences between any two things on that spectrum. Whether or not one will identify an argument as committing a conceptual slippery slope fallacy, depends on the other things one believes about the world. Thus, whether or not a conceptual slippery slope fallacy has been committed will often be a matter of some debate. It will itself be vague. Here is a good example that illustrates this point. People are found not guilty by reason of insanity when they cannot avoid breaking the law. But people who are brought up in certain deprived social circumstances are not much more able than the legally insane to avoid breaking the law. So we should not find such individuals guilty any more than those who are legally insane. Whether there is conceptual slippery slope fallacy here depends on what you think about a host of other things, including individual responsibility, free will, the psychological and social effects of deprived social circumstances such as poverty, lack of opportunity, abuse, etc. Some people may think that there are big differences between those who are legally insane and those who grow up in deprived social circumstances. Others may not think the differences are so great. The issues here are subtle, sensitive, and complex, which is why it is difficult to determine whether there is any fallacy here or not. If the differences between those who are insane and those who are the product of deprived social circumstances turn out to be like the differences between one shade of yellow and an adjacent shade of yellow, then there is no fallacy here. But if the differences turn out to be analogous to those between yellow and green (i.e., with many distinguishable stages of difference between) then there would indeed be a conceptual slippery slope fallacy here. The difficulty of distinguishing instances of the conceptual slippery slope fallacy, and the fact that distinguishing it requires us to draw on our knowledge about the world, shows that the conceptual slippery slope fallacy is an informal fallacy.

Causal slippery slope fallacy

The causal slippery slope fallacy is committed when one event is said to lead to some other (usually disastrous) event via a chain of intermediary events. If you have ever seen Direct TV's "get rid of cable" commercials, you will know exactly what I'm talking about. (If you don't know what I'm talking about you should Google it right now and find out. They're quite funny.) Here is an example of a causal slippery slope fallacy (it is adapted from one of the Direct TV commercials): If you use cable, your cable will probably go on the fritz. If your cable is on the fritz, you will probably get frustrated. When you get frustrated you will probably hit the table. When you hit the table, your young daughter will probably imitate you. When your daughter imitates you, she will probably get thrown out of school. When she gets thrown out of school, she will probably meet undesirables. When she meets undesirables, she will probably marry undesirables. When she marries undesirables, you will probably have a grandson with a dog collar. Therefore, if you use cable, you will probably have a grandson with dog collar. This example is silly and absurd, yes. But it illustrates the causal slippery slope fallacy. Slippery slope fallacies are always made up of a series of conjunctions of probabilistic conditional statements that link the first event to the last event. A causal slippery slope fallacy is committed when one assumes that just because each individual conditional statement is probable, the conditional that links the first event to the last event is also probable. Even if we grant that each "link" in the chain is individually probable, it doesn't follow that the whole chain (or the conditional that links the first event to the last event) is probable. Suppose, for the sake of the argument, we assign probabilities to each "link" or conditional statement, like this. (I have italicized the consequents of the conditionals and assigned high conditional probabilities to them. The high probability is for the sake of the argument; I don't actually think these things are as probable as I've assumed here.)

- If you use cable, then your cable will probably go on the fritz (.9)

- If your cable is on the fritz, then you will probably get angry (.9)
- If you get angry, then you will probably hit the table (.9)
- If you hit the table, your daughter will probably imitate you (.8)
- If your daughter imitates you, she will probably be kicked out of school (.8)
- If she is kicked out of school, she will probably meet undesirables (.9)
- If she meets undesirables, she will probably marry undesirables (.8)
- If she marries undesirables, you will probably have a grandson with a dog collar (.8)

However, even if we grant the probabilities of each link in the chain is high (80- 90% probable), the conclusion doesn't even reach a probability higher than chance. Recall that in order to figure the probability of a conjunction, we must multiply the probability of each conjunct: $(.9) \times (.9) \times (.9) \times (.8) \times (.8) \times (.9) \times (.8) \times (.8) = .27$ That means the probability of the conclusion (i.e., that if you use cable, you will have a grandson with a dog collar) is only 27%, despite the fact that each conditional has a relatively high probability! The causal slippery slope fallacy is actually a formal probabilistic fallacy and so could have been discussed in chapter 3 with the other formal probabilistic fallacies. What makes it a formal rather than informal fallacy is that we can identify it without even having to know what the sentences of the argument mean. I could just have easily written out a nonsense argument comprised of series of probabilistic conditional statements. But I would still have been able to identify the causal slippery slope fallacy because I would have seen that there was a series of probabilistic conditional statements leading to a claim that the conclusion of the series was also probable. That is enough to tell me that there is a causal slippery slope fallacy, even if I don't really understand the meanings of the conditional statements. It is helpful to contrast the causal slippery slope fallacy with the valid form of inference, hypothetical syllogism. Recall that a hypothetical syllogism has the following kind of form:

- $A \rightarrow B$
- $B \rightarrow C$
- $C \rightarrow D$
- $D \rightarrow E$
- $\therefore A \rightarrow E$

The only difference between this and the causal slippery slope fallacy is that whereas in the hypothetical syllogism, the link between each component is certain, in a causal slippery slope fallacy, the link between each event is probabilistic. It is the fact that each link is probabilistic that accounts for the fallacy. One way of putting this is point is that probability is not transitive. Just because A makes B probable and B makes C probable and C makes X probable, it doesn't follow that A makes X probable. In contrast, when the links are certain rather than probable, then if A always leads to B and B always leads to C and C always leads to X, then it has to be the case that A always leads to X.

Fallacies of relevance

What all fallacies of relevance have in common is that they make an argument or response to an argument that is irrelevant. Fallacies of relevance can be compelling psychologically, but it is important to distinguish between rhetorical techniques that are psychologically compelling, on the one hand, and rationally compelling arguments, on the other. What makes something a fallacy is that it fails to be rationally compelling, once we have carefully considered it. That said, arguments that fail to be rationally compelling may still be psychologically or emotionally compelling. The first fallacy of relevance that we will consider, the ad hominem fallacy, is an excellent example a fallacy that can be psychologically compelling.

Ad hominem

"Ad hominem" is a Latin phrase that can be translated into English as the phrase, "against the man." In an ad hominem fallacy, instead of responding to (or attacking) the argument a person has made, one attacks the person him or herself. In short, one attacks the person making the argument rather than the argument itself. Here is an anecdote that reveals an ad hominem fallacy (and that has actually occurred in my ethics class before). A philosopher named Peter Singer had made an argument that it is morally wrong to spend money on luxuries for oneself rather than give all of your money that you don't strictly need away to charity. The argument is actually an argument from analogy (whose details I discussed in section 3.3), but the essence of the argument is that there are every day in this world children who die preventable deaths, and there are charities who

could save the lives of these children if they are funded by individuals from wealthy countries like our own. Since there are things that we all regularly buy that we don't need (e.g., Starbuck's lattes, beer, movie tickets, or extra clothes or shoes we don't really need), if we continue to purchase those things rather than using that money to save the lives of children, then we are essentially contributing to the deaths of those children if we choose to continue to live our lifestyle of buying things we don't need, rather than donating the money to a charity that will save lives of children in need. In response to Singer's argument, one student in the class asked: "Does Peter Singer give his money to charity? Does he do what he says we are all morally required to do?" The implication of this student's question (which I confirmed by following up with her) was that if Peter Singer himself doesn't donate all his extra money to charities, then his argument isn't any good and can be dismissed. But that would be to commit an ad hominem fallacy. Instead of responding to the argument that Singer had made, this student attacked Singer himself. That is, they wanted to know how Singer lived and whether he was a hypocrite or not. Was he the kind of person who would tell us all that we had to live a certain way but fail to live that way himself? But all of this is irrelevant to assessing Singer's argument. Suppose that Singer didn't donate his excess money to charity and instead spent it on luxurious things for himself. Still, the argument that Singer has given can be assessed on its own merits. Even if it were true that Peter Singer was a total hypocrite, his argument may nevertheless be rationally compelling. And it is the quality of the argument that we are interested in, not Peter Singer's personal life and whether or not he is hypocritical. Whether Singer is or isn't a hypocrite, is irrelevant to whether the argument he has put forward is strong or weak, valid or invalid. The argument stands on its own and it is that argument rather than Peter Singer himself that we need to assess. Nonetheless, there is something psychologically compelling about the question: Does Peter Singer practice what he preaches? I think what makes this question seem compelling is that humans are very interested in finding "cheaters" or hypocrites—those who say one thing and then do another. Evolutionarily, our concern with cheaters makes sense because cheaters can't be trusted and it is essential for us (as a group) to be able to pick out those who can't be trusted. That said, whether or not a person giving an argument is a hypocrite is irrelevant to whether that person's argument is good or bad. So there may be psychological reasons why humans are prone to find certain kinds of ad

hominem fallacies psychologically compelling, even though ad hominem fallacies are not rationally compelling. Not every instance in which someone attacks a person's character is an ad hominem fallacy. Suppose a witness is on the stand testifying against a defendant in a court of law. When the witness is cross examined by the defense lawyer, the defense lawyer tries to go for the witness's credibility, perhaps by digging up things about the witness's past. For example, the defense lawyer may find out that the witness cheated on her taxes five years ago or that the witness failed to pay her parking tickets. The reason this isn't an ad hominem fallacy is that in this case the lawyer is trying to establish whether what the witness is saying is true or false and in order to determine that we have to know whether the witness is trustworthy. These facts about the witness's past may be relevant to determining whether we can trust the witness's word. In this case, the witness is making claims that are either true or false rather than giving an argument. In contrast, when we are assessing someone's argument, the argument stands on its own in a way the witness's testimony doesn't. In assessing an argument, we want to know whether the argument is strong or weak and we can evaluate the argument using the logical techniques surveyed in this text. In contrast, when a witness is giving testimony, they aren't trying to argue anything. Rather, they are simply making a claim about what did or didn't happen. So although it may seem that a lawyer is committing an ad hominem fallacy in bringing up things about the witness's past, these things are actually relevant to establishing the witness's credibility. In contrast, when considering an argument that has been given, we don't have to establish the arguer's credibility because we can assess the argument they have given on its own merits. The arguer's personal life is irrelevant.

Straw Man

Suppose that my opponent has argued for a position, call it position A, and in response to his argument, I give a rationally compelling argument against position B, which is related to position A, but is much less plausible (and thus much easier to refute). What I have just done is attacked a straw man—a position that "looks like" the target position, but is actually not that position. When one attacks a straw man, one commits the straw man fallacy. The straw man fallacy misrepresents one's opponent's argument and is thus a kind of irrelevance. Here is an example.

Two candidates for political office in Colorado, Tom and Fred, are having an exchange in a debate in which Tom has laid out his plan for putting more money into health care and education and Fred has laid out his plan which includes earmarking more state money for building more prisons which will create more jobs and, thus, strengthen Colorado's economy. Fred responds to Tom's argument that we need to increase funding to health care and education as follows: "I am surprised, Tom, that you are willing to put our state's economic future at risk by sinking money into these programs that do not help to create jobs. You see, folks, Tom's plan will risk sending our economy into a tailspin, risking harm to thousands of Coloradans. On the other hand, my plan supports a healthy and strong Colorado and would never bet our state's economic security on idealistic notions that simply don't work when the rubber meets the road." Fred has committed the straw man fallacy. Just because Tom wants to increase funding to health care and education does not mean he does not want to help the economy. Furthermore, increasing funding to health care and education does not entail that fewer jobs will be created. Fred has attacked a position that is not the position that Tom holds, but is in fact a much less plausible, easier to refute position. However, it would be silly for any political candidate to run on a platform that included "harming the economy." Presumably no political candidate would run on such a platform. Nonetheless, this exact kind of straw man is ubiquitous in political discourse in our country. Here is another example. Nancy has just argued that we should provide middle schoolers with sex education classes, including how to use contraceptives so that they can practice safe sex should they end up in the situation where they are having sex. Fran responds: "proponents of sex education try to encourage our children to a sex-with-no-strings-attached mentality, which is harmful to our children and to our society." Fran has committed the straw man (or straw woman) fallacy by misrepresenting Nancy's position. Nancy's position is not that we should encourage children to have sex, but that we should make sure that they are fully informed about sex so that if they do have sex, they go into it at least a little less blindly and are able to make better decision regarding sex.

As with other fallacies of relevance, straw man fallacies can be compelling on some level, even though they are irrelevant. It may be that part of the reason we are taken in by straw man fallacies is that humans are prone to "demonize" the "other"—including those who hold a moral or political position different from our own. It is easy to think bad things about those with whom we do not regularly interact. And it is easy to forget that people who are different than us are still people just like us in all the important respects. Many years ago, atheists were commonly thought of as highly immoral people and stories about the horrible things that atheists did in secret circulated widely. People believed that these strange "others" were capable of the most horrible savagery. After all, they may have reasoned, if you don't believe there is a God holding us accountable, why be moral? The Jewish philosopher, Baruch Spinoza, was an atheist who lived in the Netherlands in the 17th century. He was accused of all sorts of things that were commonly believed about atheists. But he was in fact as upstanding and moral as any person you could imagine. The people who knew Spinoza knew better, but how could so many people be so wrong about Spinoza? I suspect that part of the reason is that since at that time there were very few atheists (or at least very few people actually admitted to it), very few people ever knowingly encountered an atheist. Because of this, the stories about atheists could proliferate without being put in check by the facts. I suspect the same kind of phenomenon explains why certain kinds of straw man fallacies proliferate. If you are a conservative and mostly only interact with other conservatives, you might be prone to holding lots of false beliefs about liberals. And so maybe you are less prone to notice straw man fallacies targeted at liberals because the false beliefs you hold about them incline you to see the straw man fallacies as true.

Tu quoque

"Tu quoque" is a Latin phrase that can be translated into English as "you too" or "you, also." The tu quoque fallacy is a way of avoiding answering a criticism by bringing up a criticism of your opponent rather than answer the criticism. For example, suppose that two political candidates, A and B, are discussing their policies and A brings up a criticism of B's policy. In response, B brings up her own criticism of A's policy rather than respond to A's criticism of her policy. B has here committed the tu quoque fallacy. The fallacy is best understood as a way of avoiding having to answer a tough criticism that one may not have a good answer to. This kind of thing happens all the time in political discourse.

Tu quoque, as I have presented it, is fallacious when the criticism one raises is simply in order to avoid having to answer a difficult objection to one's argument or view. However, there are circumstances in which a tu quoque kind of response is not fallacious. If the criticism that A brings toward B is a criticism that equally applies not only to A's position but to any position, then B is right to point this fact out. For example, suppose that A criticizes B for taking money from special interest groups. In this case, B would be totally right (and there would be no tu quoque fallacy committed) to respond that not only does A take money from special interest groups, but every political candidate running for office does. That is just a fact of life in American politics today. So A really has no criticism at all to B since everyone does what B is doing and it is in many ways unavoidable. Thus, B could (and should) respond with a "you too" rebuttal and in this case that rebuttal is not a tu quoque fallacy.

Genetic fallacy

The genetic fallacy occurs when one argues (or, more commonly, implies) that the origin of something (e.g., a theory, idea, policy, etc.) is a reason for rejecting (or accepting) it. For example, suppose that Jack is arguing that we should allow physician assisted suicide and Jill responds that that idea first was used in Nazi Germany. Jill has just committed a genetic fallacy because she is implying that because the idea is associated with Nazi Germany, there must be something wrong with the idea itself. What she should have done instead is explain what, exactly, is wrong with the idea rather than simply assuming that there must be something wrong with it since it has a negative origin. The origin of an idea has nothing inherently to do with its truth or plausibility. Suppose that Hitler constructed a mathematical proof in his early adulthood (he didn't, but just suppose). The validity of that mathematical proof stands on its own; the fact that Hitler was a horrible person has nothing to do with whether the proof is good. Likewise with any other idea: ideas must be assessed on their own merits and the origin of an idea is neither a merit nor demerit of the idea. Although genetic fallacies are most often committed when one associates an idea with a negative origin, it can also go the other way: one can imply that because the idea has a positive origin, the idea must be true or more plausible. For example, suppose that Jill argues that the Golden Rule is a good way to live one's life because the Golden Rule originated with Jesus in the Sermon on the Mount (it didn't, actually, even though Jesus does state a version of the Golden Rule). Jill has committed the genetic fallacy in assuming that the (presumed)

fact that Jesus is the origin of the Golden Rule has anything to do with whether the Golden Rule is a good idea. I'll end with an example from William James's seminal work, The Varieties of Religious Experience. In that book (originally a set of lectures), James considers the idea that if religious experiences could be explained in terms of neurological causes, then the legitimacy of the religious experience is undermined. James, being a materialist who thinks that all mental states are physical states—ultimately a matter of complex brain chemistry, says that the fact that any religious experience has a physical cause does not undermine that veracity of that experience. Although he doesn't use the term explicitly, James claims that the claim that the physical origin of some experience undermines the veracity of that experience is a genetic fallacy. Origin is irrelevant for assessing the veracity of an experience, James thinks. In fact, he thinks that religious dogmatists who take the origin of the Bible to be the word of God are making exactly the same mistake as those who think that a physical explanation of a religious experience would undermine its veracity. We must assess ideas for their merits, James thinks, not their origins.

Appeal to consequences

The appeal to consequences fallacy is like the reverse of the genetic fallacy: whereas the genetic fallacy consists in the mistake of trying to assess the truth or reasonableness of an idea based on the origin of the idea, the appeal to consequences fallacy consists in the mistake of trying to assess the truth or reasonableness of an idea based on the (typically negative) consequences of accepting that idea. For example, suppose that the results of a study revealed that there are IQ differences between different races (this is a fictitious example, there is no such study that I know of). In debating the results of this study, one researcher claims that if we were to accept these results, it would lead to increased racism in our society, which is not tolerable. Therefore, these results must not be right since if they were accepted, it would lead to increased racism. The researcher who responded in this way has committed the appeal to consequences fallacy. Again, we must assess the study on its own merits. If there is something wrong with the study, some flaw in its design, for example, then that would be a relevant criticism of the study. However, the fact that the results of the study, if widely circulated, would have a negative effect on society is not a reason for rejecting these results as false. The consequences of some idea (good or bad) are irrelevant to the truth or reasonableness of that idea.

Notice that the researchers, being convinced of the negative consequences of the study on society, might rationally choose not to publish the study (for fear of the negative consequences). This is totally fine and is not a fallacy. The fallacy consists not in choosing not to publish something that could have adverse consequences, but in claiming that the results themselves are undermined by the negative consequences they could have. The fact is, sometimes truth can have negative consequences and falsehoods can have positive consequences. This just goes to show that the consequences of an idea are irrelevant to the truth or reasonableness of an idea.

Appeal to authority

In a society like ours, we have to rely on authorities to get on in life. For example, the things I believe about electrons are not things that I have ever verified for myself. Rather, I have to rely on the testimony and authority of physicists to tell me what electrons are like. Likewise, when there is something wrong with my car, I have to rely on a mechanic (since I lack that expertise) to tell me what is wrong with it. Such is modern life. So there is nothing wrong with needing to rely on authority figures in certain fields (people with the relevant expertise in that field)—it is inescapable. The fallacy comes when we invoke someone whose expertise is not relevant to the issue for which we are invoking it.

For example, suppose that a group of doctors sign a petition to prohibit abortions, claiming that abortions are morally wrong. If Bob cites that fact that these doctors are against abortion, therefore abortion must be morally wrong, then Bob has committed the appeal to authority fallacy. The problem is that doctors are not authorities on what is morally right or wrong. Even if they are authorities on how the body works and how to perform certain procedures (such as abortion), it doesn't follow that they are authorities on whether or not these procedures ought to be performed—they are not experts on the ethical status of these procedures. It would be just as much an appeal to consequences fallacy if Melissa were to argue that since some other group of doctors supported abortion, that shows that it must be morally acceptable. In either case, since doctors are not authorities on moral issues, their opinions on a moral issue like abortion is irrelevant. In general, an appeal to authority fallacy occurs when someone takes what an individual says as evidence for some claim, when that individual has no particular expertise in the relevant domain (even if they do have expertise in some other, unrelated, domain).

Part III

Categorical Logic

6
Categorical Statements

Quantified Categorical Statements

Back in Chapter **??**, we saw that a statement was a unit of language that could be true or false. In this chapter and the next we are going to look at a particular kind of statement, called a quantified categorical statement, and begin to develop a formal theory of how to create arguments using these statements. This kind of logic is generally called "categorical" or "Aristotelian" logic, because it was originally invented by the great logician and philosopher Aristotle in the fourth century BCE. This kind of logic dominated the European and Islamic worlds for 20 centuries afterward, and was expanded in all kinds of fascinating ways, some of which we will look at here.

Consider the following propositions:

(a) All dogs are mammals.

(b) Most physicists are smart.

(c) Few teachers are rock climbers.

(d) No dogs are cats.

(e) Some Americans are doctors.

(f) Some adults are not logicians.

(g) Thirty percent of Canadians speak French.

(h) One chair is missing.

These are all examples of quantified categorical statements. A QUANTIFIED CATEGORICAL STATEMENT is a statement that makes a claim about a certain quantity of the members of a class or group. (Sometimes we will just call these "categorical statements") Statement (a), for example, is about the class of dogs and the class of mammals. These statements make no mention of any particular members of the categories or classes or types they are about.

The propositions are also *quantified* in that they state *how many* of the things in one class are also members of the other. For instance, statement (b) talks about *most* physicists, while statement (c) talks about *few* teachers.

Categorical statements can be broken down into four parts: the quantifier, the subject term, the predicate term, and the copula. The QUANTIFIER is the part of a categorical sentence that specifies a portion of a class. It is the "how many" term. The quantifiers in the sentences above are all, most, few, no, some, thirty percent, and one. Notice that the "no" in sentence (d) counts as a quantifier, the same way zero counts as a number. The subject and predicate terms are the two classes the statement talks about. The SUBJECT CLASS is the first class mentioned in a quantified categorical statement, and the PREDICATE CLASS is the second. In sentence (e), for instance, the subject class is the class of Americans and the predicate class is the class of doctors. The COPULA is simply the form of the verb "to be" that links subject and predicate. Notice that the quantifier is always referring to the subject. The statement "Thirty percent of Canadians speak French" is saying something about a portion of Canadians, not about a portion of French speakers.

Sentence (g) is a little different than the others. In sentence (g) the subject is the class of Canadians and the predicate is the class of people who speak French. That's not quite the way it is written, however. There is no explicit copula, and instead of giving a noun phrase for the predicate term, like "people who speak French," it has a verb phrase, "speak French." If you are asked to identify the copula and predicate for a sentence like this, you should say that the copula is implicit and transform the verb phrase into a noun phrase. You would do something similar for sentence (h): the subject term is "chair," and the predicate term is "things that are missing." We will go into more detail about these issues in Section 6.

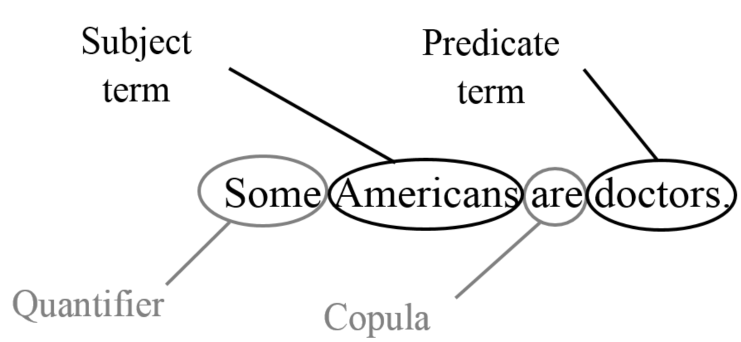

Figure 6.1: Parts of a quantified categorical statement.

In the previous chapter we noted that formal logic achieves content neutrality by replacing some or all of the ordinary words in a statement with symbols. For categorical logic, we are only going to be making one such substitution. Sometimes we will replace the classes referred to in a quantified categorical statement with capital letters that act as variables. Typically we will use the letter S when referring to the class in the subject term and P when referring to the predicate term, although sometimes more letters will be needed. Thus the sentence "Some Americans are doctors," above, will sometimes become "Some S are P." The sentence "No dogs are cats" will sometimes become"No S is P."

Quantity, Quality, Distribution, and Venn Diagrams

Ordinary English contains all kinds of quantifiers, including the counting numbers themselves. In this chapter and the next, however, we are only going to deal with two quantifiers: "all," and "some." We are restricting ourselves to the quantifiers "all" and "some" because they are the ones that can easily be combined to create valid arguments using the system of logic that was invented by Aristotle. We will deal with other quantifiers in Chapter **??**, on induction. There we will talk about much more specific quantified statements, like "Thirty percent of Canadians speak French," and do a little bit of work with modern statistical methods.

The quantifier used in a statement is said to give the QUANTITY of the statement. Statements with the quantifier "All" are said to be "UNIVERSAL" and those with the quantifier "some" are said to be "PARTICULAR."

Here "some" will just mean "at least one." So, "some people in the room are standing" will be true even if there is only one person standing. Also, because "some" means "at least one," it is compatible with "all" statements. If I say "some people in the room are standing" it might actually be that *all* people in the room are standing, because if all people are standing, then at least one person is standing. This can sound a little weird, because in ordinary circumstances, you wouldn't bother to point out that something applies to some members of a class when, in fact, it applies to all of them. It sounds odd to say "*some* dogs are mammals," when in fact they *all* are. Nevertheless, when "some" means "at least one" it is perfectly true that some dogs are mammals.

In addition to talking about the quantity of statements, we will talk about their QUALITY. The quality of a statement refers to whether the statement is negated. Statements that include the words "no" or "not" are NEGATIVE, and other statements are AFFIRMATIVE. Combining quantity and quality gives us four basic types of quantified categorical statements, which we call the STATEMENT MOODS or just "moods." The four moods are labeled with the letters A, E, I, and O. Statements that are universal and affirmative are MOOD-A STATEMENTS. Statements that are universal and negative are MOOD-E STATEMENTS. Particular and affirmative statements are MOOD-I STATEMENTS, and particular and negative statements are MOOD-O

Mood	Form	Example
A	All S are P	All dogs are mammals.
E	No S are P	No dogs are reptiles.
I	Some S are P	Some birds can fly.
O	Some S are not P	Some birds cannot fly.

Table 6.1: The four moods of a categorical statement

STATEMENTS. (See Table 6.1.)

Aristotle didn't actually use those letters to name the kinds of categorical propositions. His later followers writing in Latin came up with the idea. They remembered the labels because the "A" and the "I" were in the Latin word "**a**ff**i**rmo," ("I affirm") and the "E" and the "O" were in the Latin word "n**e**g**o**" ("I deny").

The DISTRIBUTION of a categorical statement refers to how the statement describes its subject and predicate class. A term in a sentence is said to be distributed if a claim is being made about the whole class. In the sentence "All dogs are mammals," the subject class, dogs, is distributed, because the quantifier "All" refers to the subject. The sentence is asserting that every dog out there is a mammal. On the other hand, the predicate class, mammals, is not distributed, because the sentence isn't making a claim about all the mammals. We can infer that at least some of them are dogs, but we can't infer that all of them are dogs. So in mood-A statements, only the subject is distributed.

On the other hand, in an I sentence like "Some birds can fly" the subject is not distributed. The quantifier "some" refers to the subject, and indicates that we are not saying something about all of that subject. We also aren't saying anything about all flying things, either. So in mood-I statements, neither subject nor predicate is distributed.

Even though the quantifier always refers to the subject, the predicate class can be distributed as well. This happens when the statement is negative. The sentence "No dogs are reptiles" is making a claim about all dogs: they are all not reptiles. It is also making a claim about all reptiles: they are all not dogs. So mood-E statements distribute both subject and predicate. Finally, negative particular statements (mood-O) have only the predicate class distributed. The statement "some birds cannot fly" does not say anything about all birds. It does, however say something about all flying things: the class of all flying things excludes some birds.

The quantity, quality, and distribution of the four forms of a categorical statement are given in Table 6.2. The general rule to remember here is that universal statements distribute the subject, and negative statements distribute the predicate.

In 1880 English logician John Venn published two essays on the use of diagrams with circles to represent categorical propositions (Venn [1]Venn1880a, [2]Venn1880b). Venn noted that the best use of such diagrams so far had

[1]

[2]

come from the brilliant Swiss mathematician Leonhard Euler, but they still had many problems, which Venn felt could be solved by bringing in some ideas about logic from his fellow English logician George Boole. Although Venn only claimed to be building on the long logical tradition he traced, since his time these kinds of circle diagrams have been known as VENN DIAGRAMS.

In this section we are going to learn to use Venn diagrams to represent our four basic types of categorical statement. Later in this chapter, we will find them useful in evaluating arguments. Let us start with a statement in mood A: "All *S* are *P*." We are going to use one circle to represent *S* and another to represent *P*. There are a couple of different ways we could draw the circles if we wanted to represent "All *S* are *P*." One option would be to draw the circle for *S* entirely inside the circle for *P*, as in Figure **??**

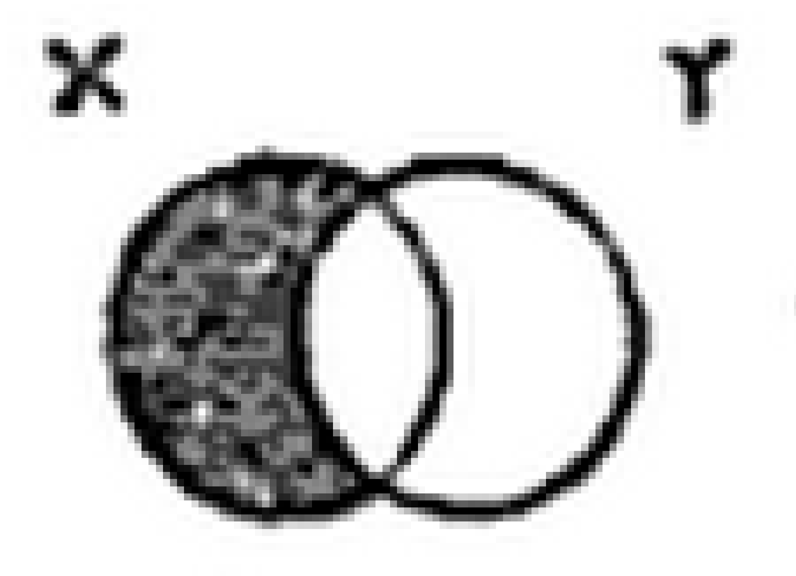

Figure 6.2: Venn's original diagram for an mood-A statement (Venn (?)). Screencap from Google Books by J. Robert Loftis.

It is clear from Figure **??** that all *S* are in fact *P*. And outside of college logic classes, you may have seen people use a diagram like this to represent a situation where one group is a subclass of another. You may have even

Mood	Form	Quantity	Quality	Terms Distributed
A	All *S* are *P*	Universal	Affirmative	S
E	No *S* are *P*	Universal	Negative	S and P
I	Some *S* are *P*	Particular	Affirmative	None
O	Some *S* are not *P*	Particular	Negative	P

Table 6.2: Quantity, quality, and distribution.

seen people call concentric circles like this a Venn diagram. But Venn did not think we should put one circle entirely inside the other if we just want to represent "All S is P." Technically speaking Figure **??** shows Euler circles.

Venn pointed out that the circles in Figure **??** don't just say that "All S are P." They also says that "All P are S" is false. But we don't necessarily know that if we have only asserted "All S are P." The statement "All S are P" leaves it open whether the S circle should be smaller than or the same size as the P circle.

Venn suggested that to represent just the content of a single proposition, we should always begin by drawing partially overlapping circles. This means that we always have spaces available to represent the four possible ways the terms can combine:

If we want to say that something does exist in a region, we put an "x" in it. This is the diagram for "Some S are P":

If a region of a Venn diagram is blank, if it is neither shaded nor has an x in it, it could go either way. Maybe such things exist, maybe they do not.

The Venn diagrams for all four basic forms of categorical statements are in Figure **??**. Notice that when we draw diagrams for the two universal forms, A and E, we do not draw any x's. For these forms we are only ruling out possibilities, not asserting that things actually exist. This is part of what Venn learned from Boole, and we will see its importance in Section 6.

Finally, notice that so far, we have only been talking about categorical statements involving the variables S and P. Sometimes, though, we will want to represent statements in regular English. To do this, we will include a key saying what the variables S and P represent in this case. We will call a list that assigns English phrases or sentences to variable names a TRANSLATION KEY. These are sometimes also called "symbolization keys" or simply just "dictionaries." As our logical systems get more complicated, the symbolization keys will get more complicated. For now, though, they just consist of a note saying what the S and P stand for. For instance, this is the diagram for "No dogs are reptiles."

Transforming English into Logically Structured English

Because the four basic forms are stated using variables, they have a great deal of generality. We can expand on that generality by showing how many different kinds of English sentences can be represented as sentences in our four basic forms. We already touched on this a little in section 6, when we look at sentences like "Thirty percent of Canadians speak French." There we saw that the predicate was not explicitly a class. We needed to change "speak French" to "people who speak French." In this section, we are going to expand on that to show how ordinary English sentences can be transformed into something we will call "logically structured English." LOGICALLY STRUCTURED ENGLISH is English that has been put into a standardized form that allows us to see its logical structure more clearly and removes ambiguity. Doing this is a step towards the creation of formal languages, which we will start doing in Chapter **??**.

Transforming English sentences into logically structured English is fundamentally a matter of understanding the meaning of the English sentence and then finding the logically structured English statements with the same or similar meaning. Sometimes this will require judgment calls. English, like any natural language, is fraught with ambiguity. One of our goals with logically structured English is to reduce the amount of ambiguity. Clarifying ambiguous sentences will always require making judgments that can be questioned. Things will only get harder when we start using full blown formal languages in Chapter **??**, which are supposed to be completely free of ambiguity.

To transform a quantified categorical statement into logically structured English, we have to put all of its elements in a fixed order and be sure they are all of the right type. All statements must begin with the quantifies "All" or "Some" or the negated quantifier "No." Next comes the subject term, which must be a plural noun, a noun phrase, or a variable that stands for any plural noun or noun phrase. Then comes the copula "are" or the negated copula "are not." Last is the predicate term, which must also be a plural noun or noun phrase. We also specify that you can only say "are not" with the quantifier "some," that way the universal negative statement is always phrased "No S are P," not "All S are not P." Taken together, these criteria define the STANDARD FORM FOR A CATEGORICAL STATEMENT in logically structured English.

The subsections below identify different kinds of changes you might need to make to put a statement into logically structured English. Sometimes translating a sentence will require using multiple changes.

Change the Predicate into a Noun Phrase

In section 6 we saw that "Some Canadians speak French" has a verb phrase "speaks French" instead of a copula and a plural noun phrase. To transform these sentences into logically structured English, you need to add the copula and turn all the terms into plural nouns or plural noun phrases. Adding a plural noun phrase means you have to come up with some category, like "people" or "animals." When in doubt, you can always use the most general category, "things." Below are some examples

English	Logically Structured English
No cats bark.	No cats are animals that bark.
All birds can fly.	All birds are animals that can fly.
Some thoughts should be left unsaid.	Some thoughts are things that should be left unsaid.

Sometimes English sentences will have a copula and an adjective or adjective phrase as the predicate. These need to be changed to noun phrases, just as the verb phrases did.

English	Logically Structured English
Some roses are red.	Some roses are red flowers.
Football players are strong.	All football players are strong persons.
Some names are hurtful.	Some names are hurtful things.

Again, you will have to come up with a category for the predicate, and when it doubt, you can just use "things."

Standardize the Quantifier

English has a wide variety of ways to express quantity. We need to reduce all of these to either "all" or "some," plus negations. Here are some examples

English	Logically Structured English
Most people with a PhD in psychology are female.	Some people with a PhD in psychology are female.
Among the things that Sylvia inherited was a large mirror	Some things that Sylvia inherited were large mirrors
There are Americans that are doctors.	Some Americans are doctors.
At least a few Americans are doctors.	Some Americans are doctors.
A man is walking down the street.	Some men are things that are walking down the street.
Every day is a blessing.	All days are blessings.
Whatever is a dog is not a cat.	No dogs are cats.
Not a single dog is a cat.	No dogs are cats.
Take nothing for granted	No things are things that should be taken for granted
Something is rotten in Denmark	Some things are things that are rotten in Denmark
Everything is coming up roses	All things are things that are coming up roses
"What does not destroy me, makes me stronger." –Friedrich Nietzsche	All things that do not destroy me are things that make me stronger.

Notice in the last case we are losing quite a bit of information when we transform the sentence into logically structured English. "Most" means more that fifty percent, while "some" could be any percentage less than a hundred. This is simply a price we have to pay in creating a standard logical form. As we will see when we move to constructing artificial languages in Chapter **??**, no logical language has the expressive richness of a natural language.

Sometimes universal statements in English don't have an explicit quantifier. Instead they use a plural noun or indefinite article to express generality.

English	Logically Structured English
Boots are footwear.	All boots are footwear.
Giraffes are tall.	All giraffes are tall things.
A dog is not a cat.	No dogs are cats.
A lion is a fierce creature.	All lions are fierce creatures.

Notice that in the second sentence we had to make two changes, adding both the words "All" and "things."

In the last two sentences, the indefinite article "a" is being used to create a kind of generic sentence. Not all sentences using the indefinite article work this way. The list before this one included the example "A man is walking down the street." This sentence is not talking about all men generically. It is talking about a specific man whose identity is unknown. Here the indefinite article is being used like a nonstandard version of the quantifier "some," which is why it appeared in the earlier list. You will have to use your good judgment and understanding of context to know when the indefinite article is being used like the word "all" and when it is being used like the word "some."

English also uses specialized adverbial phrases as quantifiers for people, places and times. If we want to talk about all people, we use a specialized quantifier like "everyone," "someone" or "no one." We use "everywhere," "somehwere," and "nowhere" for places, and "always," "sometimes," and "never" for times. All of these need to be transformed into using the simple quantifiers "all" or "some," plus negations.

English	Logically Structured English
Someone in America is a doctor.	Some Americans are doctors.
Not everyone who is an adult is a logician.	Some adults are not logicians.
"Whenever you need me, I'll be there." – Michael Jackson	All times that you need me are times that I will be there.
"We are never, ever, ever getting back together." – Taylor Swift	No times are times when we will get back together.
"Whoever fights with monsters should be careful lest he thereby become a monster." –Friedrich Nietzsche	All persons who fight with monsters are persons who should be careful lest they become a monster.

Standardize Alternative Universal Forms

Many constructions in English can be represented as universal statements in Logically Structured English, either affirmative (A) or negative (E)

For instance, it turns out that statements about individual people or specific objects can be represented by A or E statements. This is not something Aristotle originally noticed. For him a statement like "Socrates is mortal," for Aristotle, were neither universal nor particular. They were a third class he called "singular." The power of categorical logic was expanded considerably when it was realized singular statements can converted into universal statements. The trick is to add a phrase like "All things identical to…" to our singular sentence. Essentially we are adding a universal quantifier that only picks out one specific object.

English	Logically Structured English
Socrates is mortal.	All persons identical with Socrates are mortal.
The Empire State Building is tall.	All things identical to The Empire State Building are tall things.
Ludwig was not happy.	No people identical with Ludwig are happy people.

Another kind of statement that can be transformed into a universal statement is a conditional. A conditional is a statement of the form "If …then …." They will become a big focus of our attention starting in Chapter **??** when we begin introducing modern formal languages. They are not given special treatment in the Aristotelian tradition, however. Instead, where we can, we just treat them as categorical generalizations:

English	Logically Structured English
If something is a cat, then it is a felines.	All cats are feline.
If something is a dog, then it's not a cat.	No dogs are cats.

The word "only" is used in a couple of different constructions in English that can be represented as universal statements. The first kind are called "exclusive propositions." These are statements that say the subject excludes everything except what is in the predicate. For instance the sentence "Only people over 21 may drink" says that the class of people who may drink excludes everyone except those who are over 21. In English exclusive propositions are created using the words "only," "none but," or "none except." These statements become A statements when translated into logically structured English. So “Only people over 21 may drink” becomes “If you may drink, you are over 21.” It is important to see that in each case these words are used to introduce the predicate, not the subject. In the sentence “Only people over 21 may drink,” the term “people over 21” is actually the predicate, and “people who may drink” is the subject.

English	Logically Structured English
Only people over 21 may drink.	All people who drink are over 21.
No one, except those with a ticket, may enter the theater.	All people who enter the theater have a ticket.
None but the strong survive.	All people who survive are strong people.

Sentences with “The only” are a little different than sentences regular exclusive propositions, which just have “only” in them. The sentence “Humans are the only animals that talk on cell phones” should be translated as “All animals who talk on cell phones are humans.” In this sentence, “the only” introduces the subject, rather than the predicate. The statement still asserts that the subject excludes everything except what is in the predicate, and we still represent them using mood A statements.

English	Logically Structured English
Humans are the only animals who talk on cell phones.	All animals who talk on cell phones are human.
Shrews are the only venomous mammal in North America.	All venomous mammals in North America are shrews.

Transforming sentences into Logically Structured English requires judgment and attention to the nuances of meaning in English. You must be able to recognize which of the transformations describe above needs to be applied and apply it correctly. One frequent mistake by people starting out is to overgeneralize. We saw at the start of the subsection on alternative universal forms that singular propositions can be turned into universal propositions by adding the phrase “Things identical to …” Once you get in the habit of doing this, it becomes tempting to add the phrase “things identical to …” to everything, even when it isn’t necessary or doesn’t make sense. The sentence “Fido is a dog” should become “all things identical to Fido are dogs” in logically structured English, because “Fido” is a singluar term referring to an individual dog. But with the sentence “dogs are mammals,” you do not need to add the phrase “All things identical to…”, because “dogs” is already a collective noun, not an individual.

The same is true for the phrases we use to transform adjective and verb phrases into noun phrases. The sentence “No cats bark” has to be changed, because “bark” is a verb, so it becomes “No cats are animals that bark” in Logically Structured English. But the sentence “No cats are reptiles” already has a noun, “reptiles,” for a predicate, so you do not need to transform it into “No cats are animals that are reptiles.” The key is not only knowing when o use the transformations we describe, but knowing when not to use them.

Conversion, Obversion, and Contraposition

Now that we have shown the wide range of statements that can be represented in our four standard logical forms A, E, I, and O, it is time to begin constructing arguments with them. The arguments we are going to look at are sometimes called “immediate inferences” because they only have one premise. We are going to learn to identify some valid forms of these one-premise arguments by looking at ways you can transform so that a true sentence will stay true and a false sentence will stay false. Remember that on page 54 we said that the truth value of a sentence is simply whether the sentence is true or false. So we can say that the transformations we will be looking at here preserve the truth values of the sentences.

Consider the statements, “No dogs are reptiles” and “No reptiles are dogs.” They have the same truth value and basically mean the same thing. On the other hand if you change “All dogs are mammals” into “All mammals are dogs” you turn a true sentence into a false one. In this section we are going to look at three ways of transforming categorical statements—conversion, obversion, and contraposition—and use Venn diagrams to determine whether these transformations also lead to a change in truth value. From there we can identify valid argument forms.

Conversion

The two examples in the last paragraph are examples of conversion. CONVERSION is the process of transforming a categorical statement by switching the subject and the predicate. When you convert a statement, it keeps its form—an A statement remains an A statement, an E statement remains an E statement—however it might change its truth value. The Venn diagrams in Figure **??** illustrate this.

As you can see, the Venn diagram for the converse of an E statement is exactly the same as the original E statement, and likewise for I statements. This means that the two statements are logically equivalent. Recall that two statements are logically equivalent if they always have the same truth value. (See page 3). In this case, that means that if an E statement is true, then its converse is also true, and if an E statement is false, then its converse is also false. For instance, “No dogs are reptiles” is true, and so is “No reptiles are dogs." On the other hand "No dogs are mammals" is false, and so is "No mammals are dogs."

Likewise, if an I statement is true, its converse is true, and if an I statement is false, than its converse is false. "Some dogs are pets" is true, and so is "Some pets are dogs." On the other hand "Some dogs can fly" is false and so is "Some flying things are dogs."

The converses of A and O statements are not so illuminating. As you can see from the Venn diagrams, these statements are not identical to their converses. They also don't contradict their converses. If we know that an A or O statement is true, we still don't know anything about their converses. We say their truth value is undetermined.

Because E and I statements are logically equivalent to their converses, we can use them to construct valid arguments. Recall from Chapter 2 (page **??**) that an argument is valid if it is impossible for its conclusion to be false whenever its premises are true. Because E and I are logically equivalent to their converses, the two argument forms in Figure **??** are valid.

(A1) No S are P.

(A2) $\therefore$ No P are S.

(B1) Some S are P.

(B2) $\therefore$ Some P are S.

Notice that these are argument forms, with variables in the place of the key terms. This means that these arguments will be valid no matter what; S and P could be people, or squirrels, or the Gross Domestic Product of industrialized nations, or anything, and the arguments are still valid. While these particular argument forms may seem trivial and obvious, we are beginning to see some of the power of formal logic here. We have uncovered a very general truth about the nature of validity with these two argument forms.

The truth value of the converses of A and O statements, on the other hand, are undetermined by the truth value of the original statements. This means we cannot construct valid arguments from them. Imagine you have an argument with an A or O statement as its premise and the converse of that statement as the conclusion. Even if the premise is true, we know nothing about the truth of the conclusion. So there are no valid argument forms to be found here.

Obversion

Obversion is a more complex process. To understand what an obverse is, we first need to define the complement of a class. The COMPLEMENT of a class is everything that is not in the class. So the complement of the class of dogs is everything that is not a dog, including not just cats, but battleships, pop songs, and black holes. In English we can easily create a name for the complement of any class using the prefix "non-". So the complement of the class of dogs is the class of non-dogs. We will use complements in defining both obversion and contraposition.

The OBVERSION of a categorical proposition is a new proposition created by changing the quality of the original proposition and switching its predicate to its complement. Obversion is thus a two step process. Take, again, the proposition "All dogs are mammals." For step 1, we change its quality, in this case going from affirmative to negative. That gives us "No dogs are mammals." For step 2, we take the complement of the predicate. The predicate in this case is "mammals" so the complement is "non-mammals." That gives us the obverse "No dogs are non-mammals."

We can map this process out using Venn diagrams. Let's start with an A statement.

Changing the quality turns it into an E statement.

Now what happens when we take the complement of P? That means we will shade in all the parts of S that are non-P, which puts us back where we started. We still have an E statement, but it is now equivalent to the A statement.

The final statement is logically equivalent to the original A statement. It has the same form as an E statement, but because we have changed the predicate, it is not logically equivalent to an A statement. As you can see from Figure **??** this is true for all four forms of categorical statement. This in turn gives us four valid argument forms, which are shown in Figure **??**

One further note on complements. We don't just use complements to describe sentences that come out of obversion and contraposition. We can also perform these operations on statements that already have complements in them. Consider the sentence "Some S are non-P." This is its Venn diagram.

How would we take the obverse of this statement? Step 1 is to change the quality, making it "Some S are not non-P." Now how do we take the complement of the predicate? We could write "non-non-P," but if we think about it for a second, we'd realize that this is the same thing as P. So we can just write "Some S is not P." This is logically equivalent to the original statement, which is what we wanted.

Taking the converse of "Some S are non-P" also takes a moment of thought. We are supposed to reverse subject and predicate. But does that mean that the "non-" moves to the subject position along with the "P"? Or does the "non-" now attach to the S? We saw that E and I statements kept their truth value after conversion, and we want this to still be true when the statements start out referring to the complement of some class. This means that the "non-" has to travel with the predicate, because "Some S are non-P" will always have the same truth value as "Some non-P are S." Another way of thinking about this is that the "non-" is part of the name of the class that forms the predicate of "Some S are non-P." The statement is making a claim about a class, and that class happens to be defined as the complement of another class. So, the bottom line is when you take the converse of a statement where one of the terms is a complement, move the "non-" with that term.

Contraposition

CONTRAPOSITION is a two-step process, like obversion, but it doesn't always lead to results that are logically equivalent to the original sentence. The contrapositive of a categorical sentence is the sentence that results from reversing subject and predicate and then replacing them with their complements. Thus "All S are P" becomes "All non-P are non-S."

Figure **??** shows the corresponding Venn diagrams. In this case, the shading around the outside of the two circles in the contraposed form of E is meant to indicate that nothing can lie outside the two circles. Everything must be S or P or both. Like conversion, applying contraposition to two of the forms gives us statements that are logically equivalent to the original. This time, though, it is forms A and O that come through the process without changing their truth value.

This then gives us two valid argument forms, shown in Figure **??**. If you have an argument with an A or O statement as its premise and the contraposition of that statement as the conclusion, you know it must be valid. Whenever the premise is true, the conclusion must be true, because the two statements are logically equivalent. On the other hand, if you had an E or an I statement as the premise, the truth of the conclusion is undetermined, so these arguments would not be valid.

Evaluating Short Arguments

So far we have seen eight valid forms of argument with one premise: two arguments that are valid by conversion, four that are valid by obversion, and two that are valid by contraposition. As we said, short arguments like these are sometimes called "immediate inferences," because your brain just flits automatically from the truth of the premises to the truth of the conclusion. Now that we have identified these valid forms of inference, we can use this knowledge to see whether some of the arguments we encounter in ordinary language are valid. We can now tell in a few cases if our brain is right to flit so seamlessly from the premise to the conclusion.

In the real world, the inferences we make are messy and hard to classify. Much of the complexity of this issue is tackled in the parts of the complete version of this text that cover critical thinking. Part **??** of this text, on critical thinking. Right now we are just going to deal with a limited subset of inferences: immediate inferences that might be based on conversion, obversion, or contraposition. Let's start start with the uncontroversial premise "All dogs are mammals." Can we infer from this that all non-mammals are non-dogs? In canonical form, the argument would look like this.

(C1) All dogs are mammals.

(C2) ∴ All non-mammals are non-dogs.

Evaluating an immediate inference like this is a four step process. First, identify the subject and predicate classes. Second, draw the Venn diagram for the premise. Third, see if the Venn diagram shows that the conclusion must be true. If it must be, then the argument is valid. Finally, if the argument is valid, identify the process that makes it valid. (You can skip this step if the argument is invalid.)

For the argument above, the result of the first two steps would look like this:

The Venn diagram for the premise shades out the possibility that there are dogs that aren't mammals. For step three, we ask, does this mean the conclusion must be true? In this case, it does. The same shading implies that everything that is not a mammal must also not be a dog. In fact, the Venn diagram for the premise and the Venn diagram for the conclusion are the same. So the argument is valid. This means that we must go on to step four and identify the process that makes it valid. In this case, the conclusion is created by reversing subject and predicate and taking their complements, which means that this is a valid argument by contraposition.

Now, remember what it means for an argument to be valid. As we said on page **??**, an argument is valid if it is impossible for the premises to be true and the conclusion false. This means that we can have a valid argument with false premises, so long as it is the case that *if* the premises were true, the conclusion would have to be true. So if the argument above is valid, then so is this one:

(D1) All dogs are reptiles.

(D2) ∴ All non-reptiles are non-dogs.

The premise is now false: all dogs are not reptiles. However, *if* all dogs were reptiles, then it would also have to be true that all non-reptiles are non-dogs. The Venn diagram works the same way.

The Venn diagram for the premise still matches the Venn diagram for the conclusion. Only the labels have changed. The fact that this argument form remains true even with a false premise is just a variation on a theme we saw in Figure 3.1 when we saw a valid argument (with false premises) for the conclusion "Socrates is a carrot." So arguments by transposition, just like any argument, can be valid even if they have false premises. The same is true for arguments by conversion and obversion.

Arguments like these can also be invalid, even if they have true premises and a true conclusion. Remember that A statements are not logically equivalent to their converse. So this is an invalid argument with a true premise and a false conclusion:

(E1) All dogs are mammals.
(E2) ∴ All mammals are dogs.

Our Venn diagram test shows that this is invalid. Steps one and two give us this for the premise:

But this is the Venn diagram for the conclusion:

This is an argument by conversion on an mood-A statement, which is invalid. The argument remains invalid, even if we substitute in a predicate where the conclusion happens to be true. For instance this argument is invalid.

(F1) All dogs are *Canis familiaris.*
(F2) ∴ All *Canis familiaris* are dogs.

The Venn diagrams for the premise and conclusion of this argument will be just like the ones for the previous argument, just with different labels. So even though the argument has a true premise and a true conclusion, it is still invalid, because it is possible for an argument of this form to have a true premise and a false conclusion. This is an unreliable argument form that just happened, in this instance, not to lead to a false conclusion. This again is just a variation on a theme we saw in Chapter **??**, in Figure **??**, when we saw an invalid argument for the conclusion that Paris was in France.

The Traditional Square of Opposition

We have seen that conversion, obversion, and contraposition allow us to identify some valid one-premise arguments. There are actually more we can find out there, but investigating them is a bit more complicated. The original investigation made by the Aristotelian philosophers made an assumption that logicians no longer make. To help you understand all sides of the issue, we will begin by looking at things in the traditional Aristotelian fashion, and then in the next section move on to the modern way of looking at things.

When Aristotle was first investigating these four kinds of categorical statements, he noticed they they conflicted with each other in different ways. If you are just thinking casually about it, you might say that “No *S* is *P*” is somehow “the opposite” of “All *S* is *P*.” But isn’t the real “opposite” of “All *S* is *P*” actually “Some *S* is not *P*”?

Aristotle, in his book [3], notes that the real opposite of A is O, because one must always be true and the other false. If we know that “All dogs are mammals” is true, then we know “some dog is not a mammal” is false.

[3]

On the other hand, if “All dogs are mammals” is false then “some dog is not a mammal” must be true. Back on page 56 we said that when two propositions must have opposite truth values they are called contradictories. Aristotle noted that A and O sentences are contradictory in this way. Forms E and I also form a contradictory pair. If “Some dogs are mammals” then “No dogs are mammals” is false, and if “Some dogs are mammals” is false, then “No dogs are mammals” is true.

Mood-A and mood-E statements are opposed to each other in a different way. Aristotle claimed that they can’t both be true, but could both be false. Take the statements “All dogs are strays” and “No dogs are strays.” We know that they are both false, because some dogs are strays and others aren’t. However, it is also clear that they could not both be true. When a pair of statements cannot both be true, but might both be false, the Aristotelian tradition says they are CONTRARIES. Aristotle’s idea of a pair of contraries is really just a specific case of a set of sentences that are *inconsistent*, an idea that we looked at in Chapter **??**. (See page 3)

These distinctions, plus a few other comments from Aristotle, were developed by his later followers into an idea that came to be known as the SQUARE OF OPPOSITION . The square of opposition is simply the diagram you see in Figure **??**. It is a way of representing the four basic propositions and the ways they relate to one another. As we said before, this way of picturing the proposition turned out to make a problematic assumption. To emphasize that this is no longer the way logicians view things, we will call this diagram the traditional square of opposition.

The traditional square of opposition begins by picturing a square with A, E, I, and O at the four corners. The lines between the corners then represent the ways that the kinds of propositions can be opposed to each other. The diagonal lines between A and O and between E and I represent contradiction. These are pairs of propositions where one has to be true and the other false. The line across the top represents contraries. These are propositions that Aristotle thought could not both be true, although they might both be false.

In Figure **??**, we have actually drawn each relationship as a pair of lines, representing the kinds of inferences you can make in that relationship. Contraries cannot both be true. So we know that if one is true, the other must be false. This is represented by the two lines going from a T to an F. Notice that there aren’t any lines here that point from an F to something else. This is because you can’t infer anything about contrary statements if you just know that one is false. For the contradictory statements, on the other hand, we have drawn double-headed arrows. This is because we know both that the truth of one statement implies that the other is false and that the falsity of one statement implies the truth of the other.

Contraries and contradictories just give us the diagonal lines and the top line of the square. There are still three other sides to investigate. Form I and form O are called SUBCONTRARIES. In the traditional square of opposition, their situation is reversed from that of A and E. Statements of forms A and E cannot both be true, but they can both be false. Statements of forms I and O cannot both be false, but they can both be true. Consider the sentences "Some people in the classroom are paying attention" and "Some people in the classroom are not paying attention." It is possible for them both to be true. Some people are paying attention and some aren't. But the two sentences couldn't both be false. That would mean that everyone in the room was neither paying attention nor not paying attention. But they have to be doing one or the other!

This means that there are two inferences we can make about subcontraries. We know that if I is false, O must be true, and vice versa. This is represented in Figure **??** by arrows going from Fs on one side to Ts on the other. This is reversed from the way things were on the top of the square with the contraries. Notice that this time there are no arrows going away from a T. This is because we can't infer anything about subcontraries if all we know is that one is true

The trickiest relationship is the one between universal statements and their corresponding particulars. We call this SUBALTERNATION. Both of the statements in these pairs could be true, or they could both be false. However, in the traditional square of opposition, if the universal statement is true, its corresponding particular statement must also be true. For instance, "All dogs are mammals" implies that some dogs are mammals. Also, if the particular statement is false, then the universal statement must also be false. Consider the statement "Some dinosaurs had feathers." If that statement is false, if no dinosaurs had feathers, then "All dinosaurs have feathers" must also be false. Something like this seems to be true on the negative side of the diagram as well. If "No dinosaurs have feathers" is true, then you would think that "some dinosaurs do not have feathers" is true. Similarly, if "some dinosaurs do not have feathers" is false, then "No dinosaurs have feathers" cannot be true either.

In our diagram for the traditional square of opposition, we represent subalternation by a downward arrow for truth and an upward arrow for falsity. We can infer something here if we know the top is true, or if we know the bottom is false. In other situations, there is nothing we can infer.

Note, by the way, that the language of subalternation works a little differently than the other relationships. With contradiction, we say that each sentence is the "contradictory" of the other. The relationship is symmetrical. With subalternation, we say that the particular sentence is the "subaltern" of the universal one, but not the other way around.

People started using diagrams like this as early as the second century CE to explain Aristotle's ideas in *On Interpretation* (See Parsons [4]Parsons1997). Figure 6.3 shows one of the earliest surviving versions of the square of opposition, from a 9th century manuscript of a commentary on Aristotle at-

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tributed to the Roman writer Apuleius of Madaura. Although this particular manuscript dates from the 9th century, the commentary itself was written in the 2nd century, and copied by hand many times over before this one was made. Figure 6.4 shows a later illustration of the square, from a 16th century book by the Scottish philosopher and logician Johannes de Magistris.

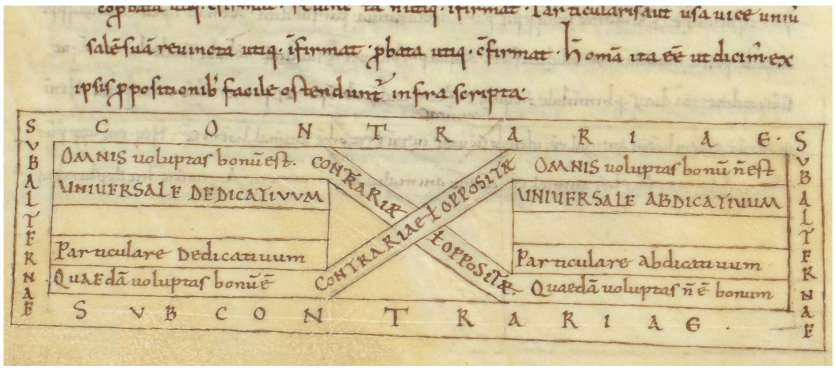

Figure 6.3: One of the earliest surviving versions of the square of opposition, from a 9th century manuscript that includes a commentary on Aristotle by the African writer Apuleius of Madaura ([a]). The manuscript is held Lawrence J. Schoenberg collection (LJS 101) at the University of Pennsylvania, who have kindly put a facsimile online (`http://dla.library.upenn.edu/dla/medren/detail.html?id=MEDREN_5186550`.) Screencap by J. Robert Loftis.

[a] [

As with the processes of conversion, obversion, and contraposition, we can use the traditional square of opposition to evaluate arguments written in canonical form. It will help us here to introduce the phrase "It is false that" to some of our statements, so that we can make inferences from the truth of one proposition to the falsity of another. This, for instance, is a valid argument, because A and O statements are contradictories.

(G1) All humans are mortal.
(G2) $\therefore$ It is false that some human is not mortal.

The argument above is an immediate inference, like the arguments we saw in the previous section, because it only has one premise. It is also similar to those arguments in that the conclusion is actually logically equivalent to the premise. This will not be the case for all immediate inferences based on the square of opposition, however. This is a valid argument, based on the subaltern relationship, but the premise and the conclusion are not logically equivalent.

(H1) It is false that some humans are dinosaurs.
(H2) $\therefore$ It is false that all humans are dinosaurs.

Existential Import and the Modern Square of Opposition

The traditional square of opposition seems straightforward and fairly clever. Aristotle made an interesting distinction between contraries and contradictories, and subsequent logicians developed it into a nifty little diagram. So why did we have to keep saying things like "Aristotle thought" and "according to the traditional square of opposition." What is wrong here?

The traditional square of opposition goes awry because it makes assumptions about the existence of the things being talked about. Remember that when we drew the Venn diagram for "All S are P," we shaded out the area of S that did not overlap with P to show that nothing could exist there. We pointed out, though, that we did not put a little x in the intersection between S and P. Statements of the form A ruled out the existence of one kind of thing, but they did not assert the existence of another. The A proposition, "All dogs are mammals," denies the existence of any dog that is not a mammal, but it does not assert the existence of some dog that is a mammal. But why not? Dogs obviously do exist.

The problem comes when you start to consider categorical statements about things that don't exist, for instance "All unicorns have one horn." This seems like a true statement, but unicorns don't exist. Perhaps what we mean by "All unicorns have one horn" is that *if* a unicorn existed, *then* is would have one horn. But if we interpret the statement about unicorns that way, shouldn't we also interpret the statement about dogs that way? Really all we mean when we say "All dogs are mammals" is that if there were dogs, then they would be mammals. It takes an extra assertion to point out that dogs do, in fact, exist.

The issue we are discussing here is called existential import. A sentence is said to have EXISTENTIAL IMPORT if it asserts the existence of the things it is talking about. Figure **??** shows the two ways you could draw Venn diagrams for an A statement, with the x, as in the traditional interpretation, and without, as in our interpretation. If you interpret A statements in the traditional way, they are always false when you are talking about things that don't exist. So, "All unicorns have one horn" is false in the traditional interpretation. On the other hand, in the modern interpretation all statements about things that don't exist are true. "All unicorns have one horn" simply asserts that there are no multi-horned unicorns, and this is true because there are no unicorns at all. We call this VACUOUS TRUTH. Something is vacuously true if it is true simply because it is about things that don't exist. Note that *all* statements about nonexistent things become vacuously true if you assume they have no existential import, even a statement like "All unicorns have more than one horn." A statement like this simply rules out the existence of unicorns with one horn or fewer, and these don't exist because unicorns don't exist. This is a complicated issue that will come up again starting in Chapter **??** when we consider conditional statements. For now just assume that this makes sense because you can make up any stories you want about unicorns.

Any statement can be read with or without existential import, even the particular ones. Consider the statements "Some unicorns are rainbow colored" and "Some unicorns are not rainbow colored." You can argue that both of these statements are true, in the sense that if unicorns existed, they could come in many colors. If you say these statements are true, however, you are assuming that particular statements do not have existential import. As Terence Parsons ([5]Parsons1997) points out, you can change the wording

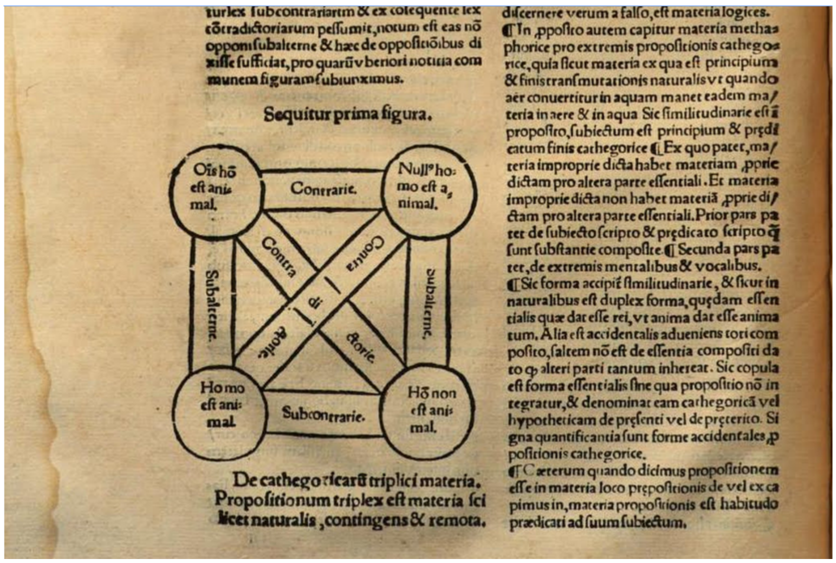

Figure 6.4: A 16th century illustration of the square of opposition from John Major's *Introductorium Perutile in Aristotelicam Dialecticen* ([a]][fol.L]Major1527). Screencap from Google Books by J. Robert Loftis.

[a] [

of particular categorical statements in English to make them seem like they do or do not have existential import. "Some unicorns are not rainbow colored" might have existential import, but "not every unicorn is rainbow colored" doesn't seem to.

So what does this have to do with the square of opposition? A lot of the claims made in the traditional square of opposition depend on assumptions about which statements have existential import. For instance, Aristotle's claim that contrary statements cannot both be true requires that A statements have existential import. Think about the sentences "All dragons breathe fire" and "no dragons breathe fire." If the first sentence has no existential import, then both sentences could actually be true. They are both ruling out the existence of certain kinds of dragons and are correct because no dragons exist.

In fact, the entire traditional square of opposition falls apart if you assume that all four forms of a categorical statement have existential import. Parsons ((**?**)) shows how we can derive a contradiction in this situation. Consider the I statement "Some dragons breathe fire." If you interpret it as having existential import, it is false, because dragons don't exist. But then its contradictory statement, the E statement "No dragons breathe fire" must be true. And if that statement is true, and has existential import, then its subaltern, "Some dragon does not breathe fire" is true. But if it has existential import, it can't be true, because dragons don't exist. In logic, the worst thing you can ever do is contradict yourself, but that is what we have just done. So we have to change the traditional square of opposition.

The way some textbooks talk about the problem, you'd think that for two thousand years logicians were simply ignorant about the problem of existential import and thus woefully confused about the square of opposition, until finally George Boole wrote *The Laws of Thought*[6] and 5
found the one true solution to the problem. In fact, there was an extensive discussion of existential import from the 12th to the 16th centuries, mostly under the heading of the "supposition" of a term. Very roughly, we can say that the supposition of a term is the way it refers to objects, or what we now call the "denotation" of the term.[7] So in "All people are mortal" the 6
supposition of the subject term is all of the people out there in the world. Or, as the medievals sometimes put it, the subject term "supposits" all the people in the world.

At least some medieval thinkers had a theory of supposition that made the traditional square of opposition work. Terrance Parsons ((?), (?)) has argued for the importance of one solution, found most clearly in the writings of William of Ockham. Under this theory, affirmative forms A and I had existential import, but the negative forms E and O did not. We would say that a statement has existential import if it would be false whenever the subject or predicate terms refer to things that don't exist. To put the matter more precisely, we would say that the statement would be false whenever the subject or predicate terms "fail to refer." Linguistic philosophers these days prefer say that a term "fails to refer" rather than saying that it "refers to something that doesn't exist," because referring to things that don't exist seems impossible.

In any case, Ockham describes the supposition of affirmative propositions the same way we would describe the reference of terms in those propositions. Again, if the proposition supposes the existence of something in the world, the medievals would say it "supposits." Ockham says "In affirmative propositions a term is always asserted to supposit for something. Thus, if it supposits for nothing the proposition is false" ((?), 206). On the other hand, failure to refer or to supposit actually supports the truth of negative propositions: "in negative propositions the assertion is either that the term does not supposit for something or that it supposits for something of which the predicate is truly denied. Thus a negative proposition has two causes of truth" ((?), 206).

So, for Ockham, affirmative statements about nonexistent objects are false. "All unicorns have one horn" and "Some unicorns are rainbow colored" are false, because there are no unicorns. Negative statements, on the other hand, are vacuously true. "No unicorns are rainbow colored" and "No unicorns have one horn" are both true. There are no rainbow colored unicorns out there, and no one horned unicorns out there, because there are no unicorns out there. The O statement “Some unicorns are not rainbow colored” is also vacuously true. This might be harder to see, but it helps to think of the statement as saying “It is not the case that every unicorn is rainbow colored.”

This way of thinking about existential import leaves the traditional square of opposition intact, even in cases where you are referring to nonexistent objects. Contraries still cannot both be true when you are talking about nonexistent objects, because the A proposition will be false, and the E vacuously true. “All dragons breathe fire” is false, because dragons don’t exist, and “No dragons breathe fire” is vacuously true for the same reason. Similarly, subcontraries cannot both be false when talking about dragons and whatnot, because the I will always be false and the O will always be true. You can go through the rest of the relationships and show that similar arguments hold.

Boole proposed a different solution, which is now taken as the standard way to do things. Instead of looking at the division between positive and negative statements, Boole looked at the division between singular and universal propositions. The universal statements A and E do not have existential import, but the particular statements I and O do have existential import. Thus all particular statements about nonexistent things are false and all universal statements about nonexistent things are vacuously true.

John Venn was building on the work of George Boole. His diagrams avoided the problems that Euler had by using a Boolean interpretation of mood-A statements, where they really just assert that something is impossible. In fact, the whole system of Venn diagrams embodies Boole’s assumptions about existential import, as you can see in Figure **??**. The particular forms I and O have you draw an x, indicating that something exists. The other two forms just have us shade in regions to indicate that certain combinations of subject and predicate are impossible. Thus A and E statements like “All dragons breathe fire” or “No dragons are friendly” can be true, even though no dragons exist.

Venn diagrams doesn’t even have the capacity to represent Ockham’s understanding of existential import. We can represent A statements as having existential import by adding an x, as we did on the right hand side of Figure **??**. However, we have no way to represent the O form without existential import. We have to draw the x, indicating existence. We don’t have a way of representing O form statements about nonexistent objects as vacuously true.

The Boolean solution to the the question of existential import leaves us with a greatly restricted form of the square of opposition. Contrary statements are both vacuously true when you refer to nonexistent objects, because neither have existential import. Subcontrary statements are both false when you refer to nonexistent objects, because they do have existential import. Finally, the subalterns of vacuously true statements are false, while on the traditional square of opposition they had to be true. The only thing remaining from the traditional square of opposition is the relationship of contradiction, as you can see in Figure **??**.

Key Terms

1. **Affirmative**
2. **Complement**
3. **Contradictories**
4. **Contraposition**
5. **Contraries**
6. **Converse**
7. **Copula**
8. **Distribution**
9. **Existential import**
10. **Logically structured English**
11. **Mood-A statement**
12. **Mood-E statement**
13. **Mood-I statement**
14. **Mood-O statement**
15. **Negative**
16. **Obverse**
17. **Particular**
18. **Predicate class**
19. **Quality**
20. **Quantified categorical statement**
21. **Quantifier**
22. **Quantity**
23. **Square of opposition**
24. **Standard form categorical statement**
25. **Subalternation**
26. **Subcontraries**
27. **Subject class**
28. **Translation key**
29. **Truth value**
30. **Universal**
31. **Vacuous truth**
32. **Venn diagram**

Part IV

Propositional Logic

Part V

Appendices

A

Other Symbolic Notation

In the history of formal logic, different symbols have been used at different times and by different authors. Often, authors were forced to use notation that their printers could typeset.

In one sense, the symbols used for various logical constants is arbitrary. There is nothing written in heaven that says that $\sim$ must be the symbol for truth-functional negation. We might have specified a different symbol to play that part. Once we have given definitions for well-formed formulae (wff) and for truth in our logic languages, however, using $\sim$ is no longer arbitrary. That is the symbol for negation in this textbook, and so it is the symbol for negation when writing sentences in our languages SL or QL.

This appendix presents some common symbols, so that you can recognize them if you encounter them in an article or in another book.

Summary of Symbols	
negation	$\sim$, $\neg$
conjunction	&, $\wedge$, $\bullet$
disjunction	$\vee$
conditional	$\rightarrow$, $\supset$
biconditional	$\leftrightarrow$, $\equiv$

Negation Two commonly used symbols are the *hoe*, '$\neg$', and the *swung dash*, '$\sim$.' In some more advanced formal systems it is necessary to distinguish between two kinds of negation; the distinction is sometimes represented by using both '$\neg$' and '$\sim$.'

Disjunction The symbol '$\vee$' is typically used to symbolize inclusive disjunction.

Conjunction Conjunction is often symbolized with the *ampersand*, '&.' The ampersand is actually a decorative form of the Latin word 'et' which means 'and'; it is commonly used in English writing. As a symbol in a formal system, the ampersand is not the word 'and'; its meaning is given by the formal semantics for the language. Perhaps to avoid this confusion, some systems use a different symbol for conjunction. For example, '$\wedge$' is a counterpart to the symbol used for disjunction. Sometimes a single dot, '$\bullet$', is used. In some older texts, there is no symbol for conjunction at all; 'A and B' is simply written 'AB.'

Material Conditional There are two common symbols for the material conditional: the *arrow*, '$\rightarrow$', and the *hook*, '$\supset$.'

Material Biconditional The *double-headed arrow*, '$\leftrightarrow$', is used in systems that use the arrow to represent the material conditional. Systems that use the hook for the conditional typically use the *triple bar*, '$\equiv$', for the biconditional.

Quantifiers The universal quantifier is typically symbolized as an upside-down A, '$\forall$', and the existential quantifier as a backwards E, '$\exists$.' In some texts, there is no separate symbol for the universal quantifier. Instead, the variable is just written in parentheses in front of the formula that it binds. For example, 'all x are P' is written $(x)Px$.

In some systems, the quantifiers are symbolized with larger versions of the symbols used for conjunction and disjunction. Although quantified expressions cannot be translated into expressions without quantifiers, there is a conceptual connection between the universal quantifier and conjunction and between the existential quantifier and disjunction. Consider the sentence $\exists xPx$, for example. It means that *either* the first member of the UD is a P, *or* the second one is, *or* the third one is, ... Such a system uses the symbol '$\bigvee$' instead of '$\exists$.'

Polish notation

This section briefly discusses sentential logic in Polish notation, a system of notation introduced in the late 1920s by the Polish logician Jan Łukasiewicz.

Lower case letters are used as sentence letters. The capital letter N is used for negation. A is used for disjunction, K for conjunction, C for the conditional, E for the biconditional. ('A' is for alternation, another name for logical disjunction. 'E' is for equivalence.)

notation of SL	Polish notation
$\sim$	N
$\wedge$	K
$\vee$	A
$\rightarrow$	C
$\leftrightarrow$	E

In Polish notation, a binary connective is written *before* the two sentences that it connects. For example, the sentence $A \wedge B$ of SL would be written Kab in Polish notation.

The sentences $\sim A \rightarrow B$ and $\sim(A \rightarrow B)$ are very different; the main logical operator of the first is the conditional, but the main connective of the second is negation. In SL, we show this by putting parentheses around the conditional in the second sentence. In Polish notation, parentheses are never required. The left-most connective is always the main connective. The first sentence would simply be written $CNab$ and the second $NCab$.

This feature of Polish notation means that it is possible to evaluate sentences simply by working through the symbols from right to left. If you were constructing a truth table for $NKab$, for example, you would first consider the truth-values assigned to b and a, then consider their conjunction, and then negate the result. The general rule for what to evaluate next in SL is not nearly so simple. In SL, the truth table for $\sim(A \wedge B)$ requires looking at A and B, then looking in the middle of the sentence at the conjunction, and then at the beginning of the sentence at the negation. Because the order of operations can be specified more mechanically in Polish notation, variants of Polish notation are used as the internal structure for many computer programming languages.

B
Glossary

Affirmative The quality of a statement without a “not” or a “no.” 85

Argument A collection of statements, called the premises, that provides evidential support to another statement, called the conclusion. 26

Artificial language A language that was consciously developed by identifiable individuals for some purpose. 51

Bivalent A property of logical systems which is present when the system only has two truth values, generally “true” and “false.” 54

Canonical form A method for representing arguments where each premise is numbered and written on a separate line. The premises are followed by a horizontal bar and then the conclusion. Statements in the argument may be paraphrased for brevity and indicator words are removed. 27

Cogent A property of arguments that holds when the argument is strong and the premises are true. 41

Cognitive bias a habit of reasoning that can become dysfunctional in certain circumstances. Often these biases are not a matter of explicit belief. See also *fallacy* 37

Complement The class of everything that is not in a given class. 95

Conclusion The part of an argument that is being provided evidence. 20, 26

Conclusion Indicator A word or phrase such as “therefore” used to indicate that what follows is the conclusion of an argument. 27

Confirmation bias The tendency to discount or ignore evidence and arguments that contradict one’s current beliefs. 37

Consistency A property possessed by a set of sentences when they can all be true at the same time, but are not necessarily so. 57

Content neutrality the feature of the study of logic that makes it indifferent to the topic being argued about. If a method of argument is considered rational in one domain, it should be considered rational in any other domain, all other things being equal. 20

Contingent statement A statement that is neither a tautology nor a contradiction. 55

Contradiction A statement that must be false, as a matter of logic. 55

Contradictories Two statements that must have opposite truth values, so that one must true and the other false. 56

Contraposition The process of transforming a categorical statement by reversing subject and predicate and replacing them with their complements. 97

Contraries Two statements that can't both be true, but can both be false. A set two inconsistent sentences. 100

Conversion The process of changing a sentence by reversing the subject and predicate. 94

Copula The form of the verb "to be" that links subject and predicate. 84

Critical thinker A person who has both sharpened their reasoning abilities using metareasoning and deploys those sharpened abilities in real world situations.. 21

Critical thinking The use of metareasoning to improve our reasoning in practical situations. Sometimes the term is also used to refer to the results of this effort at self improvement, that is, reasoning in practical situations that has been sharpened by reflection and metareasoning. 21

Deductive A style of arguing where one attempts to use valid arguments. 42

Distribution A property of the terms of a categorical statement that is present when the statement makes a claim about the whole term. 86

Existential import An aspect of the meaning of a statement that which is present if the statement can only be true when the objects it describes exist. 103

Expository passage A nonargumentative passage that organizes statements around a central theme or topic statement. 31

Fallacy A common mistake in reasoning. Fallacies are generally conceived of as mistake forms of inference and are generally explained by arguments represented in canonical form. See also *cognitive bias*. 39

Formal language An artificial language designed to bring out the logical structure of ideas and remove all the ambiguity and vagueness that plague natural languages like English. Sometimes, formal languages are also said to be languages that can be implemented by a machine. 52

Formal logic A way of studying logic that achieves content neutrality by replacing parts of the arguments being studied with abstract symbols. Often this will involve the construction of full formal languages. 21

Inconsistency A property possessed by a set of sentences when they cannot all be true at the same time, but they may all be false at the same time. 57

Inductive A style of arguing where one attempts to use strong arguments. 42

Inference the act of coming to believe a conclusion on the basis of some set of premises. 30

Informal logic The study of arguments given in ordinary language. 21

Invalid A property of arguments that holds when the premises do not force the truth of the conclusion. The opposite of valid. 38

Logic The part of the study of reasoning that focuses on argument. 20

Logical equivalence A property held by a pair of sentences that must always have the same truth value. 56

Logically structured English English that has been regimented into a standard form to make its logical structure clear and to remove ambiguity. A stepping stone to full-fledged formal languages. 88

Metacognition Thought processes that are applied to other thought processes See also *metareasoning*. 20

Metareasoning Using reasoning to study reasoning. See also *metacognition*. 20

Mood-A statement A quantified categorical statement of the form "All S are P." 85

Mood-E statement A quantified categorical statement of the form "No S are P." 85

Mood-I statement A quantified categorical statement of the form "Some S are P." 85

Mood-O statement A quantified categorical statement of the form "Some S are not P." 85

Narrative A nonargumentative passage that describes a sequence of events or actions. 32

Natural language A language that develops spontaneously and learned by infants as their first language. 51

Negative The quality of a statement containing a "not" or "no." 85

Obversion The process of transforming a categorical statement by changing its quality and replacing the predicate with its complement. 96

Particular The quantity of a statement that uses the quantifier "some." 85

Practical argument An argument whose conclusion is a statement that someone should do something. 26

Predicate class The second class named in a quantified categorical statement. 84

Premise A statement in an argument that provides evidence for the conclusion. 20, 26

Premise Indicator A word or phrase such as "because" used to indicate that what follows is the premise of an argument. 27

Quality The status of a categorical statement as affirmative or negative. 85

Quantified categorical statement A statement that makes a claim about a certain quantity of the members of a class or group. 83

Quantifier The part of a categorical sentence that specifies a portion of a class. 84

Quantity The portion of the subject class described by a categorical statement. Generally "some" or "none." 85

Rhetoric The study of effective persuasion. 22

Simple statement of belief A kind of nonargumentative passage where the speaker simply asserts what they believe without giving reasons. 30

Sound A property of arguments that holds if the argument is valid and has all true premises. 39

Square of opposition A way of representing the four basic propositions and the ways they relate to one another. 100

Standard form for a categorical statement A categorical statement that has been put into logically structured English, with the following elements in the following order: (1) The quantifiers "all," "some," or "no"; (2) the subject term; (3) the copula "are" or "are not"; and (4) the predicate term. 89

Statement A unit of language that can be true or false. 23

Statement mood The classification of a categorical statement based on its quantity and quality. 85

Strong A property of arguments which holds when the premises, if true, mean the conclusion must be likely to be true. 41

Subalternation The relationship between a universal categorical statement and the particular statement with the same quality. 101

Subcontraries Two categorical statements that cannot both be false, but might both be true. 101

Subject class The first class named in a quantified categorical statement. 84

Tautology A statement that must be true, as a matter of logic. 55

Translation key A list that assigns English phrases or sentences to variable names. Also called a "symbolization key" or simply a "dictionary." 88

Truth evaluable A property of some objects (such as bits of language, maps, or diagrams) that means they can be appropriately assessed as either true or false. 23

Truth value The status of a statement with relationship to truth. For this textbook, this means the status of a statement as true or false. 54, 94

Universal The quantity of a statement that uses the quantifier "all." 85

Vacuous truth The kind of truth possessed by statements that do not have existential import and refer to objects that do not exist. 103

Valid An argument is valid just in case its premises entail its conclusion. In other words, it is impossible for the premises to be true and the conclusion false. 36

Venn diagram A diagram that represents categorical statements using circles that stand for classes. 87

Weak A property of arguments that are neither valid nor strong. In a weak argument, the premises would not even make the conclusion likely, even if they were true. 41

C

Quick Reference

Characteristic Truth Tables

$\mathcal{P}$	$\sim\mathcal{P}$
T	F
F	T

$\mathcal{P}$	$\mathcal{Q}$	$\mathcal{P} \wedge \mathcal{Q}$	$\mathcal{P} \vee \mathcal{Q}$	$\mathcal{P} \rightarrow \mathcal{Q}$	$\mathcal{P} \leftrightarrow \mathcal{Q}$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	F	T	T	F
F	F	F	F	T	T

Symbolization

Sentential Connectives (chapter **??**)

It is not the case that P.	$\sim P$
Either P, or Q.	$(P \vee Q)$
Neither P, nor Q.	$\sim(P \vee Q)$ or $(\sim P \wedge \sim Q)$
Both P, and Q.	$(P \wedge Q)$
If P, then Q.	$(P \rightarrow Q)$
P only if Q.	$(P \rightarrow Q)$
P if and only if Q.	$(P \leftrightarrow Q)$
Unless P, Q.	$(P \vee Q)$
P unless Q.	$(P \vee Q)$

Predicates (chapter **??**)

All Fs are Gs.	$\forall x(Fx \rightarrow Gx)$

Some Fs are Gs.	$\exists x(Fx \land Gx)$
Not all Fs are Gs.	$\sim\forall x(Fx \to Gx)$ or $\exists x(Fx \land \sim Gx)$
No Fs are Gs.	$\forall x(Fx \to \sim Gx)$ or $\sim\exists x(Fx \land Gx)$

Identity (section **??***)*

Only j is G.	$\forall x(Gx \leftrightarrow x = j)$
Everything besides j is G.	$\forall x(x \neq j \to Gx)$
j is more R than anyone else.	$\forall x(x \neq j \to Rjx)$
The F is G.	$\exists x(Fx \land \forall y(Fy \to x = y) \land Gx)$

'The F is not G' can be translated two ways:

It is not the case that the F is G. (wide)	$\sim\exists x(Fx \land \forall y(Fy \to x = y) \land Gx)$
The F is non-G. (narrow)	$\exists x(Fx \land \forall y(Fy \to x = y) \land \sim Gx)$

Using identity to symbolize quantities

There are at least n Fs.

one	$\exists x Fx$
two	$\exists x_1 \exists x_2 (Fx_1 \land Fx_2 \land x_1 \neq x_2)$
three	$\exists x_1 \exists x_2 \exists x_3 (Fx_1 \land Fx_2 \land Fx_3 \land x_1 \neq x_2 \land x_1 \neq x_3 \land x_2 \neq x_3)$
four	$\exists x_1 \exists x_2 \exists x_3 \exists x_4 (Fx_1 \land Fx_2 \land Fx_3 \land Fx_4 \land x_1 \neq x_2 \land x_1 \neq x_3 \land x_1 \neq x_4 \land x_2 \neq x_3 \land x_2 \neq x_4 \land x_3 \neq x_4)$
n	$\exists x_1 \cdots \exists x_n (Fx_1 \land \cdots \land Fx_n \land x_1 \neq x_2 \land \cdots \land x_{n-1} \neq x_n)$

There are at most n Fs.

One way to say 'at most n things are F' is to put a negation sign in front of one of the symbolizations above and say $\sim$ 'at least $n+1$ things are F.' Equivalently:

one	$\forall x_1 \forall x_2 \big[(Fx_1 \land Fx_2) \to x_1 = x_2\big]$

two $\forall x_1 \forall x_2 \forall x_3 \big[(Fx_1 \wedge Fx_2 \wedge Fx_3) \to (x_1 = x_2 \vee x_1 = x_3 \vee x_2 = x_3)\big]$

three $\forall x_1 \forall x_2 \forall x_3 \forall x_4 \big[(Fx_1 \wedge Fx_2 \wedge Fx_3 \wedge Fx_4) \to (x_1 = x_2 \vee x_1 = x_3 \vee x_1 = x_4 \vee x_2 = x_3 \vee x_2 = x_4 \vee x_3 = x_4)\big]$

n $\forall x_1 \cdots \forall x_{n+1} \big[(Fx_1 \wedge \cdots \wedge Fx_{n+1}) \to (x_1 = x_2 \vee \cdots \vee x_n = x_{n+1})\big]$

There are exactly n Fs.

One way to say 'exactly n things are F' is to conjoin two of the symbolizations above and say 'at least n things are F' $\wedge$ 'at most n things are F.' The following equivalent formulae are shorter:

zero $\forall x \sim Fx$

one $\exists x \big[Fx \wedge \sim \exists y(Fy \wedge x \neq y)\big]$

two $\exists x_1 \exists x_2 \big[Fx_1 \wedge Fx_2 \wedge x_1 \neq x_2 \wedge \sim \exists y \big(Fy \wedge y \neq x_1 \wedge y \neq x_2\big)\big]$

three $\exists x_1 \exists x_2 \exists x_3 \big[Fx_1 \wedge Fx_2 \wedge Fx_3 \wedge x_1 \neq x_2 \wedge x_1 \neq x_3 \wedge x_2 \neq x_3 \wedge \sim \exists y(Fy \wedge y \neq x_1 \wedge y \neq x_2 \wedge y \neq x_3)\big]$

n $\exists x_1 \cdots \exists x_n \big[Fx_1 \wedge \cdots \wedge Fx_n \wedge x_1 \neq x_2 \wedge \cdots \wedge x_{n-1} \neq x_n \wedge \sim \exists y(Fy \wedge y \neq x_1 \wedge \cdots \wedge y \neq x_n)\big]$

Specifying the size of the UD

Removing F from the symbolizations above produces sentences that talk about the size of the UD. For instance, 'there are at least 2 things (in the UD)' may be symbolized as $\exists x \exists y(x \neq y)$.

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