IIA PROJECT

GF3 Audio Modem

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DFT Discrete Fourier Transform

- $$X_n = \sum_{k=0}^{N-1} x_k e^{-j\frac{2\pi}{N}kn}$$

- $$x_k = \frac{1}{N}\sum_{n=0}^{N-1} X_n e^{j\frac{2\pi}{N}kn}$$

DFT Matrix

$$\alpha := e^{-j\frac{2\pi}{N}}$$

$$\underline{X} = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & \alpha & \alpha^2 & \cdots & \alpha^{N-1} \\ 1 & \alpha^2 & \alpha^4 & \cdots & \alpha^{2(N-1)} \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & \alpha^{N-1} & \alpha^{2(N-1)} & \cdots & \alpha^{(N-1)^2} \end{bmatrix} \underline{x}$$

DFT fast implementation

- Fast Fourier Transform (FFT)
- Various algorithms exist (Cooley Tukey, Good Thomas, etc.)

Convolution Property

if $\underline{z} = \underline{x} *_N \underline{y}$ where "$*_N$" is the cyclic convolution $z_k = \sum_{n=0}^{N-1} x_n y_{R_N(k-n)}$

and $R_N(x)$ is the remainder when x is divided by N

then $Z_k = X_k Y_k$ for all k

Frequency Shift Property

if $z_k = x_k e^{j\frac{2\pi}{N}k\ell}$ for some ℓ and $k = 0 \ldots N-1$

then $Z_n = X_{R_N(n+\ell)}$

Relation to physical spectrum

if $\underline{x}$ = N samples from a signal sampled at f_s

then X_n for $n = 1, 2, \ldots \frac{N}{2} - 1$ corresponds to frequency $\frac{n}{N} f_s$

for $n = \frac{N}{2} + 1, \ldots, N-1$, frequency $-\frac{N-n}{N} f_s$

X_0 is frequency 0 (bias)

44 000 Hz

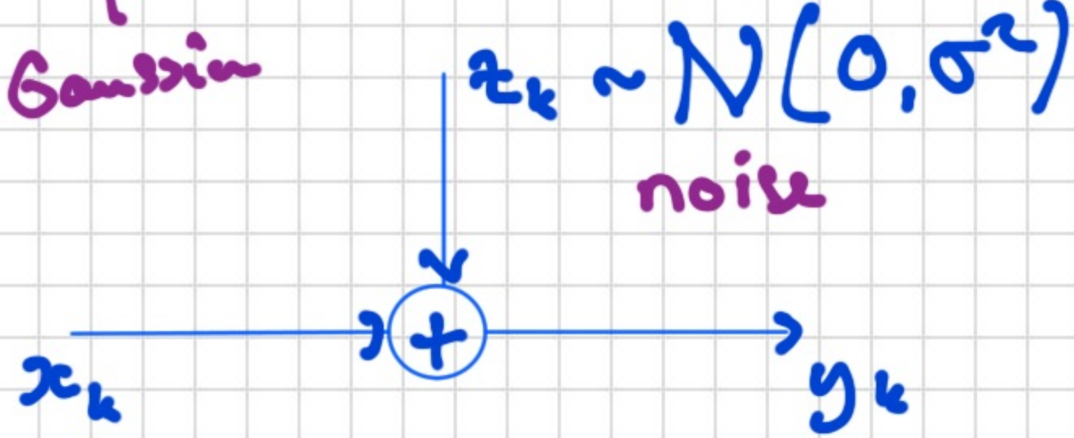
$2^{10} = 1024$ sample FFT

$1 \ldots 511 \qquad \frac{44000}{1024} \cdot (1..511)$

$1023, \ldots 513 \qquad -\frac{44000}{1024}(1..511)$

additive white Gaussian noise

AWGN channel



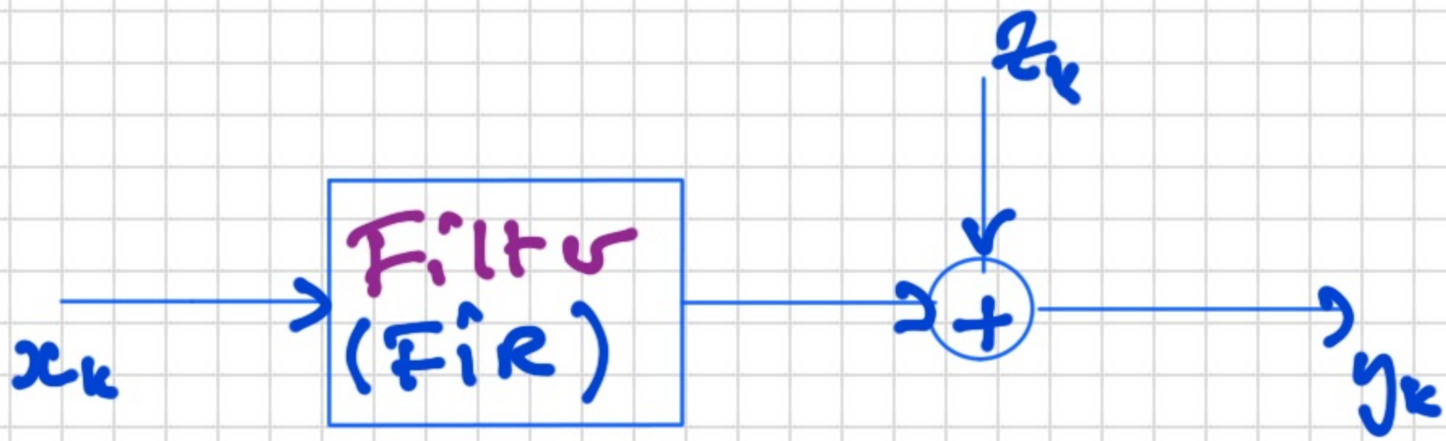
"white" = "i.i.d." = power spectral density is constant.

Echo

- occurs on cables (guided wave reflection)
- wireless transmission (reflection off mountains, clouds, buildings, objects)
- in audio (acoustic wave propagation)

Echo = inter-symbol interference (isi)

= frequency selective channel



Deconvolution before noise

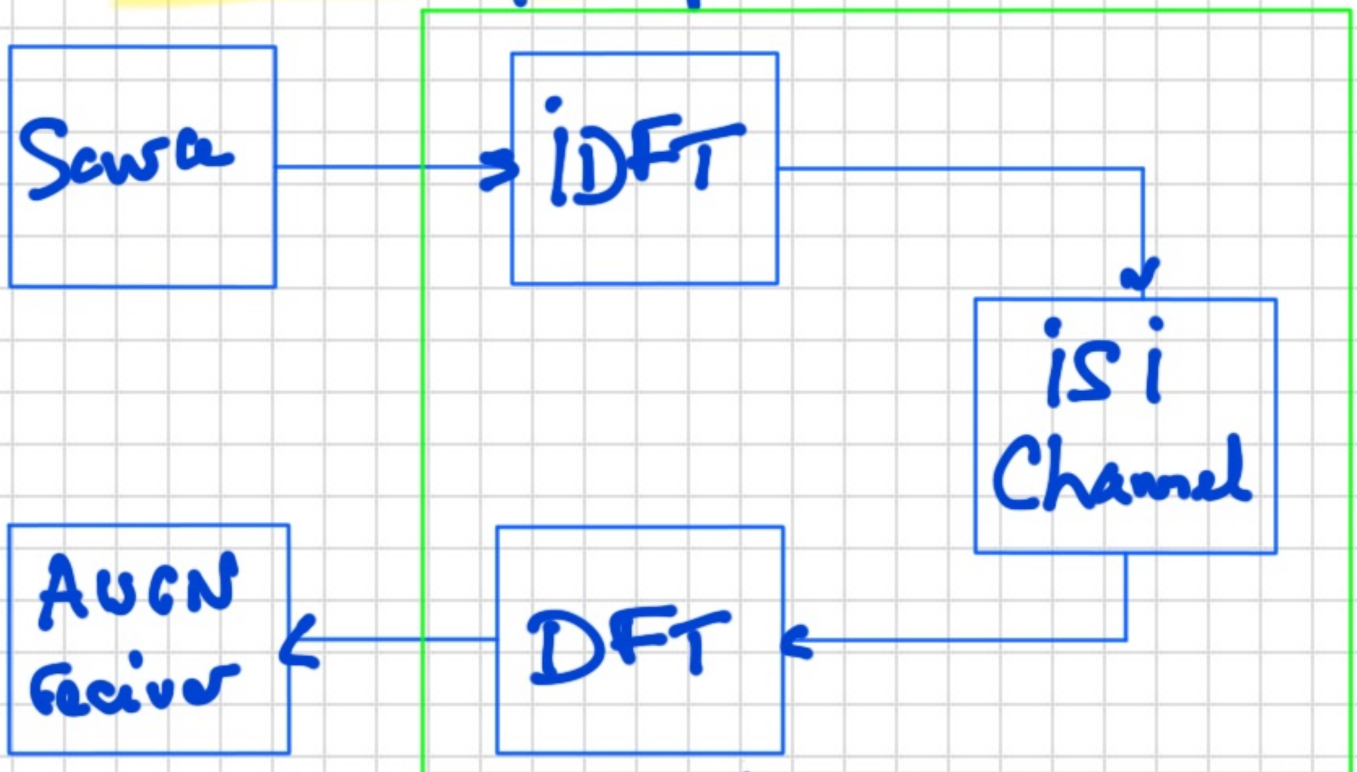
- inverse filter → noise amplification
- matched filter
- Nyquist criteria

Convolution Property Idea

- filter does convolution in time domain
- convolution in time domain
  = multiplication in frequency domain
- in frequency domain, channel on each frequency bin is plain AWGN

⇒ let's prepare our signal in freq. domain

Conv property scheme



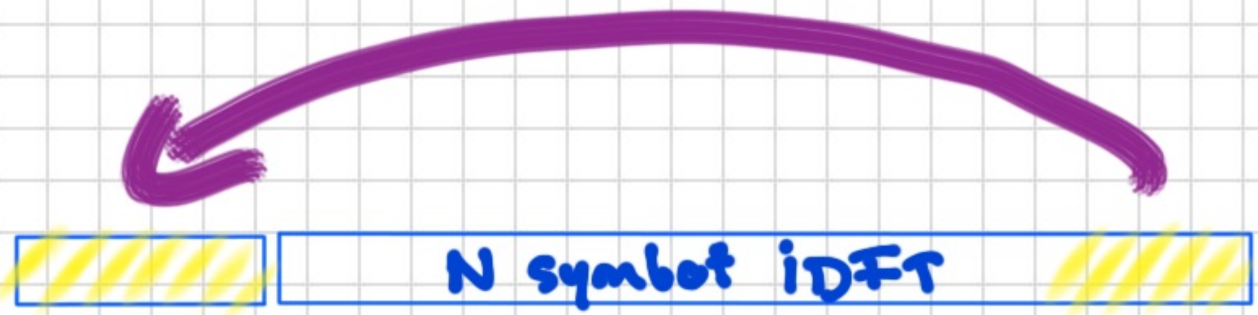
nice idea, but...

The channel operates a linear convolution

The DFT Convolution property is for the cyclic convolution

OFDM Trick

(Orthogonal Frequency Division Modulation)



k point "cyclic prefix"

if the channel = FIR with less than k taps it will do a cyclic convolution

Equivalent Channel in one freq. bin

$$Y_n = cX_n + Z_n$$

$$\tilde{Y}_n = \frac{1}{c}Y_n = X_n + \frac{1}{c}Z_n$$