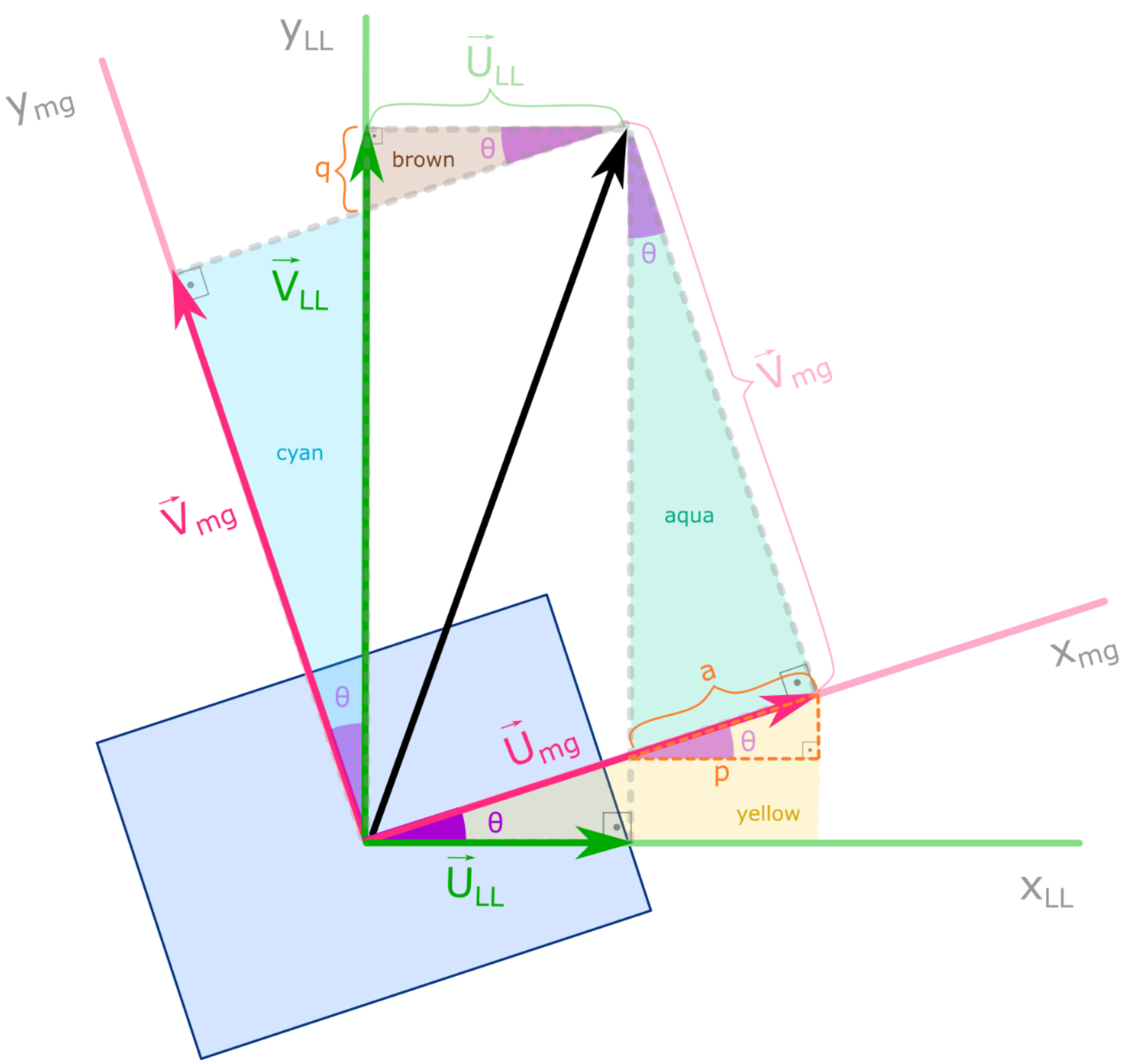
# Rotating U and V components of velocity

# from model grid to a lat-lon grid

The figure below is a schematic of a model grid cell (in dark blue), the velocity vector in black, the model grid x and y orientation ($x_{mg}, y_{mg}$ in magenta), U and V components of velocity in the model grid ($\vec{U}_{mg}, \vec{V}_{mg}$ in magenta), lat-lon grid orientation ($x_{LL}, y_{LL}$ in green), rotated U and V components of the velocity ($\vec{U}_{LL}, \vec{V}_{LL}$ in green, what we want to find), and the angle θ between the model x ($x_{mg}$) and lat-lon x ($x_{LL}$), in dark purple.

Note that below we will only deal with the magnitude of the vectors, where

$$\vec{U}_{LL} = U_{LL} \cdot \hat{\imath}_{LL} \qquad \vec{V}_{LL} = V_{LL} \cdot \hat{\jmath}_{LL}$$
$$\vec{U}_{mg} = U_{mg} \cdot \hat{\imath}_{mg} \qquad \vec{V}_{mg} = V_{mg} \cdot \hat{\jmath}_{mg}$$



First, let's find an expression for $U_{LL}$.

Looking at the yellow triangle, we can see that

$$\cos\theta = \frac{U_{LL}+p}{U_{mg}} \quad \text{or} \quad U_{LL} = U_{mg}\cdot\cos\theta - p \tag{1}$$

To find $p$ we first look at the triangle delineated by the dashed orange lines. From this triangle, we see that

$$\cos\theta = \frac{p}{a} \quad \text{or} \quad p = a\cdot\cos\theta \tag{2}$$

From the aqua triangle,

$$\tan\theta = \frac{a}{V_{mg}} \quad \text{or} \quad a = V_{mg}\cdot\frac{\sin\theta}{\cos\theta} \tag{3}$$

Substituting (3) in (2),

$$p = V_{mg}\cdot\frac{\sin\theta}{\cos\theta}\cdot\cos\theta \quad \text{or} \quad p = V_{mg}\cdot\sin\theta \tag{4}$$

and (4) in (1)

$$U_{LL} = U_{mg}\cdot\cos\theta - V_{mg}\cdot\sin\theta \tag{5}$$

Great! Now let's find an expression for $V_{LL}$.

Looking at the cyan triangle,

$$\cos\theta = \frac{V_{mg}}{V_{LL}-q} \quad \text{or} \quad V_{LL} = \frac{V_{mg}}{\cos\theta} + q \tag{6}$$

Now, from the brown triangle,

$$\tan\theta = \frac{q}{U_{LL}} \quad \text{or} \quad q = U_{LL}\cdot\frac{\sin\theta}{\cos\theta} \tag{7}$$

Substituting the $U_{LL}$ expression (5) into (7),

$$q = \left(U_{mg}\cdot\cos\theta - V_{mg}\cdot\sin\theta\right)\cdot\frac{\sin\theta}{\cos\theta}$$

$$q = U_{mg}\cdot\sin\theta - V_{mg}\cdot\frac{\sin^2\theta}{\cos\theta} \tag{8}$$

Replacing (8) into (6), we have

$$V_{LL} = \frac{V_{mg}}{\cos\theta} + U_{mg}\cdot\sin\theta - V_{mg}\cdot\frac{\sin^2\theta}{\cos\theta}$$

$$V_{LL} = V_{mg}\left(\frac{1-\sin^2\theta}{\cos\theta}\right) + U_{mg}\cdot\sin\theta$$

We know that $\sin^2\theta + \cos^2\theta = 1$ , so replacing $1-\sin^2\theta$ by $\cos^2\theta$, we finally get

$$V_{LL} = V_{mg}\cdot\cos\theta + U_{mg}\cdot\sin\theta \tag{9}$$