Classifying time series data via frequency decomposition and manifold techniques

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Objectives

Improve time series classification by:

- Using dynamic models to represent time series data as points on a Grassmann manifold
- Using kernel methods on said manifolds to perform classification while taking advantage of the full geometric information

Introduction

- Time series classification is a widespread problem in signal processing and machine learning
- Identifying sources of sound or radar signals, detecting algorithmically-generated "deepfake" video and audio, and many other applications
- We investigate a novel method for feature extraction: representing signals as points on a Grassmann manifold via dynamic model parametrisation

Dynamic model parametrisation

The autoregressive-moving-average (ARMA) is a well-known dynamic model for time series data that parametrises a signal $f(t)$ by the equations

$$f(t) = Cz(t) + w(t), \quad w(t) \sim \mathcal{N}(0, R) \tag{1}$$

$$z(t+1) = Az(t) + v(t), \quad v(t) \sim \mathcal{N}(0, Q) \tag{2}$$

where $z \in \mathbb{R}^d$ is the hidden state vector, $d \le p$ is the hidden state dimension [1]. There are widely-used closed form solutions for estimating the parameters A and C. It can be shown that the expected observation sequence is given by

$$O_\infty = \mathbb{E}\begin{bmatrix} f(0) \\ f(1) \\ f(2) \\ \vdots \end{bmatrix} = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \end{bmatrix} z(0) \tag{3}$$

Representation on Grassmann manifold

- (3) shows that the expected observations of $f(t)$ lie in the column space of the observability matrix O_∞
- Approximate O_∞ by truncating at the mth block, to form $O_m \in \mathsf{M}_{mp\times d}$
- Hence, ARMA model yields a representation of a signal as a Euclidean subspace (given by the column space of O_m), and thus a point on the Grassmann manifold

Kernel methods on Grassmann manifold

- Machine learning algorithms often assume that the data lies in Euclidean space
- Kernel methods defined on manifolds can help generalise algorithms to non-Euclidean space
- Jayasumana et al. [2] extends the Gaussian RBF kernel to the Grassmann manifold:

$$d_P([Y_1],[Y_2]) = 2^{-1/2}\left\|Y_1Y_1^T - Y_2Y_2^T\right\|_F \tag{4}$$

$$k_P([Y_1, Y_2]) = \exp(-\gamma d_P^2([Y_1],[Y_2])) \tag{5}$$

where $[Y_i]$ is the subspace spanned by the columns of Y_i, Y_1 and Y_2 are matrices with orthonormal columns, and γ is a hyperparameter

- Allows use of kernel-based algorithms like SVM to classify time series data

Algorithm

Input: list of train signals $\{X_i\}_{i=1}^n$, list of train labels $\{y_i\}_{i=1}^n$, list of test signals $\{Y_i\}_{i=1}^m$
Output: list of predicted test labels

- For $i = 1, 2, \ldots, n$:
  - Compute parameters C and A for X_i
  - Compute O_m for X_i
  - Orthonormalise O_m and store as U_i
- Using kernel k_P, fit SVM on $\{U_i\}_{i=1}^n, \{y_i\}_{i=1}^n$
- Predict SVM on $\{Y_i\}_{i=1}^m$, return predicted labels

Experiments

Experiments were performed on four sets of data:

- SUNY EEG Database Data Set: EEG tests of alcoholic and non-alcoholic subjects [3]
- Epileptologie Bonn EEG data set, EEG tests of epileptic and non-epileptic subjects [4]
- Audio recordings of different vehicles moving through a parking lot at approximately 15 mph [5]
- Video recordings of people speaking digits 0-9 [6]

Figure 1: Test setup of car audio dataset

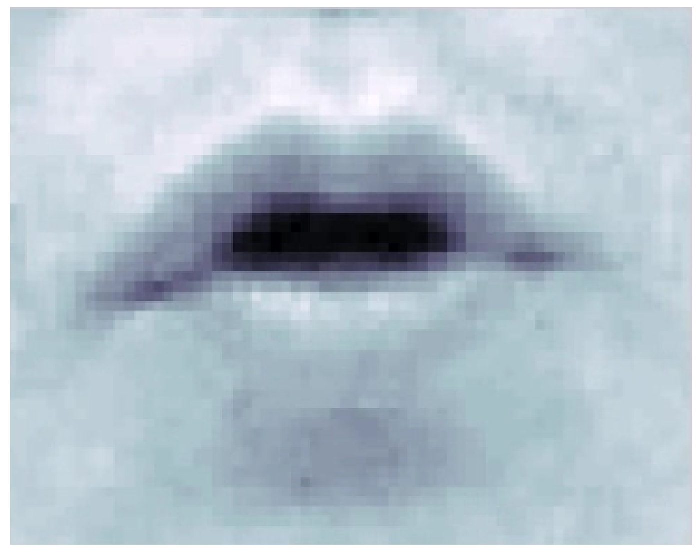

Figure 2: Sample frame from lip video dataset

Results

Datasets	Accuracy	Benchmark	Feature dim.
Alcohol EEG	99.82%	97.08% [7]	64
Epilepsy EEG	66%	99.66% [8]	1
Vehicle audio	63%	88.2% [5]	2
Video digits	86.3%	94.7% [6]	3850

Table 1: Classification accuracies and feature dimensions for each dataset

Conclusions

- Classification is effective for datasets with high-dimensional features (alcohol EEG, video digits)
- Ineffective for datasets with few channels (epilepsy EEG, vehicle audio)
- Preprocessing techniques (e.g. Fourier or wavelet transform, instantaneous frequency decompositions [9]) can help extract more relevant features from each signal, improving classification performance

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