Classifying time series data via frequency decomposition and manifold techniques

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Objectives

Improve time series classification by:

- Using dynamic models to represent time sereis data as points on a Grassmann manifold
- Using kernel methods on said manifolds to perform classification while taking advantage of the full geometric information

Introduction

The objective of this project is to investigated novel solutions to the problem of time series classification. This problem is widespread in signal processing and machine learning, from identifying the sources of sound or radar signals, to detecting algorithmically-generated "deepfake" video and audio.

We implemented a new method for feature extraction: representing signals as points on a Grassmann manifold via dynamic model parametrisation. This geometric techinque proves to be effective at capturing information from high-dimensional signals, performing well in classification.

Dynamic model parametrisation

The autoregressive-moving-average (ARMA) is a well-known dynamic model for time series data that parametrises a signal $f(t)$ by the equations

$$f(t) = Cz(t) + w(t), \quad w(t) \sim \mathcal{N}(0, R) \qquad (1)$$

$$z(t+1) = Az(t) + v(t), \quad v(t) \sim \mathcal{N}(0, Q) \qquad (2)$$

where $z \in \mathbb{R}^d$ is the hidden state vector, $d \leq p$ is the hidden state dimension [1]. There are widely-used closed form solutions for estimating the parameters A and C. It can be shown that the expected observation sequence is given by

$$\mathbb{E}\begin{bmatrix} f(0) \\ f(1) \\ f(2) \\ \vdots \end{bmatrix} = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \end{bmatrix} z(0) \qquad (3)$$

Representation on Grassmann manifold

From (3), we observe that the expected observations of $f(t)$ lie in the column space of the observability matrix $O_\infty = \left[C^T \ (CA)^T \ (CA^2)^T \cdots\right]^T$.

We approximate the observability matrix by truncating at the mth block to form the finite observability matrix $O_m \in \mathsf{M}_{mp\times d}$. Thus, the ARMA model yields a representation of a signal as a subspace of Euclidean space, and hence a point on the Grassmann manifold. We store each subspace by orthonormalising O_m. [1] recommends setting d and m to be equal, between 5 and 10.

Kernel methods on Grassmann manifold

Many machine learning algorithms, such as support vector machines (SVMs) and deep neural networks, depend on the Euclidean structure of input data, particularly structures such as norms and inner products [2]. One way to address this incongruency is to use kernel methods that are defined on manifolds. In [2] Jayasumana et al. generalise the Gaussian radial basis function (RBF) kernel $\exp\left(-\gamma\|x-y\|^2\right)$ to manifolds by replacing the Euclidean distance with a positive definite distance function, specifically the projection distance on the Grassmann manifold

$$d_P([Y_1],[Y_2]) = 2^{-1/2}\left\|Y_1Y_1^T - Y_2Y_2^T\right\|_F \qquad (4)$$

where $[Y_i]$ is the subspace spanned by the columns of Y_i, and Y_1 and Y_2 are matrices with orthonormal columns. This yields the projection Gaussian kernel on the Grassmann manifold:

$$k_P([Y_1,Y_2]) = \exp(-\gamma d_P^2([Y_1],[Y_2])) \qquad (5)$$

where γ is a hyperparameter. This enables the use of algorithms like SVM to classify time series data in the form of orthonormal matrices representing points on a Grassmann manifold.

Experiments

We computed the representation of each signal as a point on the Grassmann manifold according to (3), and trained an SVM using the kernel function described in (5) to perform classification.

We performed these tests on four sets of data, using 60% of each dataset for training and 40% of testing.

- SUNY EEG Database Data Set, a set of measurements from 64 electrodes placed on the scalp of alcoholic and non-alcoholic subjects [3] $d = m = 9, \gamma = 0.2$
- Epileptologie Bonn EEG data set, a set of single-channel measurements from epileptic and non-epileptic patients [4]
- Audio recordings of different vehicles moving through a parking lot at approximately 15 mph [5]
- Video recordings of people speaking numeric digits 0-9 [6]

Results

Datasets	Accuracy	Feature dim.
Alcohol EEG	99.8%	64
Epilepsy EEG	66%	1
Vehicle audio	63%	2
Video digits	98%	3850

Table 1: Classification accuracies and feature dimensions for each dataset

Conclusions

We conclude that the approach of representing time series data as points on a Grassmann manifold via ARMA parameterisation, then applying a manifold kernel SVM for classification is highly effective for datasets with a large number of features, namely the alcohol EEG and video digit datasets.

The Grassmann SVM approach performs relatively poorly on datasets with fewer features, such as the epilepsy EEG dataset and the vehicle audio dataset, which comprise 1 and 2 features respectively. More sophisticated pre-processing techniques, such Fourier or wavelet transformations and instantaneous frequency decompositions, can help extract more relevant features from each signal and improve the performance of the Grassmann classification algorithm.

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