1. Určte, ktoré z rovníc majú v množine reálnych čísel jedno riešenie, viac riešení, nekonečne veľa riešení alebo žiadne riešenie.

R:
   a. $4x - 3 - 3x - 4 = x - 7$ $x \in \mathbb{R}$

   b. $x^4 + 4 = 0$ $x \in \emptyset$ $(x^4) \neq (-4)$

   c. $\frac{x+1}{x-1} = \frac{x-5}{x-3}$ $(*)$ $x = \{2\}$

   d. $5(x + 2) = 5x - 10$ $10 \neq -10$; $x \in \emptyset$; $x = \emptyset$

   e. $x^2 + 2x = 0$ $(\triangle)$ $x = \{0, -2\}$

   f. $\frac{x-3}{6} + x = \frac{2x-1}{2} \ominus \frac{4-x}{2} \Big| \cdot 6$ $x = \{6\}$

   g. $x^2 = 9$ $(\triangle)$ $x^2 = 9 \Big| \sqrt{}$ $x = \{0, -3\}$ $|x| = 3$; $x = \{3, -3\}$

$(*)$ $\frac{x+1}{x-1} = \frac{x-5}{x-3} \Big| \cdot (x-1)(x-3)$; $x \neq 1$, $x \neq 3$
$(x+1)(x-3) = (x-5)(x-1)$
$x^2 - 2x - 3 = x^2 - 6x + 5$
$4x = 8$
$x = 2$

$(\triangle)$ $x^2 + 2x = 0$
$x \cdot (x + 2) = 0 \Leftrightarrow x = 0 \lor x + 2 = 0$
$x_2 = -2$

$x - 9 = 0$
$(x+3)(x-3) = 0 \Leftrightarrow x + 3 = 0 \lor x - 3 = 0$
$x = -3$ $x = 3$

2. https://www.skolasnadhledem.cz/game/97

3. Vyriešte rovnice R:

podielovom tvare $\frac{*}{0} = 0$

   a. $\frac{4-x}{x^2-16} = 0 \Leftrightarrow 4 - x = 0 \land x^2 - 16 \neq 0$! $(x+4)(x-4) \neq 0 \Leftrightarrow x_1 \neq -4 \land x_2 \neq 4 \Rightarrow x = \emptyset$ $\boxed{4 = x}$

   b. $\frac{x^2-9}{x^2+2x-3} = 0 \Leftrightarrow x - 9 = 0 \land x^2 + 2x - 3 \neq 0$
   $(x+3)(x-3) = 0$
   $x_1 = 3$; $x_2 = 3$ $(x-1)(x+3) \neq 0 \Leftrightarrow x \neq 1 \land x \neq -3$ $x = \{3\}$

rovnice s neznámou v menovateli

   c. $\frac{x^2-5x+6}{x^2-4} = 0$

   d. $\frac{3x-2}{x+5} = 2$ $(*)$
   $(*)$ $\frac{3x-2}{x+5} = 2 \Big| \cdot (x+5)$; $\boxed{x \neq -5}$ $(*)$ $\frac{3x-2}{x+5} - 2 = 0$
   $3x - 2 = 2x + 10$ $\frac{3x-2-2x-10}{x+5} = 0$
   $x = 12$ $x = \{12\}$ $\frac{x-12}{x+5} = 0 \Leftrightarrow x - 12 = 0 \land x + 5 \neq 0$
   $x = 12$ $x \neq -5$
   $x = \{12\}$

   e. $\frac{2x+6}{x+3} = 3$

   f. $\frac{x-3}{x+2} + \frac{2x-3}{x-1} = 3 \Big|_{(x+2)(x-1)}$ $\boxed{x \neq -2, x \neq 1}$
   $(x-3)(x-1) + (2x-3)(x+2) = 3 \cdot (x+2)(x-1)$
   $x^2 - 4x + 3 + 2x^2 + x - 6 = 3x^2 + 3x - 6$ $-3x - 3 = 3x - 6$
   $3 = 6x$ $\boxed{\frac{1}{2} = x}$ $x = \{\frac{1}{2}\}$

4. Riešte v množine reálnych čísel

   a. $\frac{1}{x+4} + \frac{x^2-20}{x^2-16} = 1$
   $\frac{1}{x+4} + \frac{x^2-20}{(x+4)(x-4)} = 1 \Big| \cdot (x+4)(x-4)$ $\boxed{x \neq -4, x \neq 4}$
   $x - 4 + x^2 - 20 = x^2 - 16$
   $x = 8$ $x = \{8\}$

DÚ

   b. $\frac{\frac{x}{3} - \frac{x-1}{2}}{\frac{x}{3} - \frac{x+1}{4}} = x$

   c. $\frac{2}{2x+3} - \frac{2}{2x-3} = \frac{4x^2-21}{4x^2-9}$ $(2x+3)(2x-3)$

   d. $\frac{1}{x+1} + \frac{4}{x^2-x+1} = \frac{3}{x^3+1} \Big| \cdot \frac{x+1}{(x+1)(x^2-x+1)}$ $(*)$; $\boxed{x \neq -1}$
   $x^2 - x + 1 + 4x + 4 = 3$
   $x^2 + 3x + 2 = 0$
   $(x+1)(x+2) = 0 \Leftrightarrow x = -1 \lor x = -2$ $x = \{-2\}$

   $x^2 - x + 1 = (x - \frac{1}{2})^2 - \frac{1}{4} + 1$
   $(*)$ $(x - \frac{1}{2})^2 + \frac{3}{4} \neq 0$
   $(*)$ $x^3 + 1 = (x+1)(x^2-x+1)$
   $a^3 + b^3 = (a+b)(a^2-ab+b^2)$

   e. $\left(\frac{x-2}{x+3} - 3\right)\left(\frac{x+3}{x-2} - 1\right) = \frac{2-x}{x+3}$ Substitúcia
   $\left(\frac{x-2-3x-9}{x+3}\right)\left(\frac{x+3-x+2}{x-2}\right) = \frac{2-x}{x+3}$
   $\left(\frac{-2x-11}{x+3}\right) \cdot \left(\frac{5}{x-2}\right) = \frac{2-x}{x+3}$
   $\frac{-10x-55}{(x+3)(x-2)} = \frac{2-x}{x+3} \Big| \cdot (x+3)(x-2)$ $\boxed{x \neq -3, x \neq 2}$

   $(a-3)(\frac{1}{a}-1) = -a$
   $(a-3)(\frac{1-a}{a}) = -a$
   $\frac{(a-3)(1-a)}{a} = -a \Big| \cdot a$; $\boxed{a \neq 0}$
   $a - a^2 - 3 + 3a = -a^2$
   $4a = 3$
   $\boxed{a = \frac{3}{4}}$ ✓

   $(*)$ $a = \frac{x-2}{x+3}$
   $\frac{3}{4} = \frac{x-2}{x+3} \Big| \cdot 4(x+3)$ $x \neq -3$
   $3x + 9 = 4x - 8$
   $\boxed{17 = x}$
   $x = \{17\}$

   $x = \{17\}$

f. $\left(\dfrac{x-3}{x+2}-5\right)\left(\dfrac{x-3}{x+2}+3\right)-9=0$ $\longrightarrow$ $\dfrac{x-3}{x+2}=b\longrightarrow$ $b_1=6$ ···· $\dfrac{x-3}{x+2}=6 \mid \cdot x+2$ $\boxed{x\neq-2}$

$(b-5)(b+3)-9=0$

$b^2-2b-24=0$

$(b-6)(b+4)=0 \Leftrightarrow b_1=6 \lor b_2=-4$

$x-3=6x+12$
$-15=5x$
$-3=x$ ✓

$b_2=-4$ ··· $\dfrac{x-3}{x+2}=-4$ $\boxed{x\neq-2}$

$x-3=-4x-8$
$5x=-5$
$x=-1$ ✓

$\mathbb{L}=\{-3;-1\}$