

Zjednodušení lomených výrazů

1. Zjednodušte výrazy

a. $\frac{\frac{4x^2y}{6a^2b^3}}{\frac{8xy^2}{12a^2b^4}}$

b. $\frac{\frac{(a+b)^2}{a^2-b^2}}{\frac{a+b}{a-b}} = \frac{(a+b)^2}{a^2-b^2} : \frac{a+b}{a-b} = \frac{(a+b)^2}{(a+b)(a-b)} \cdot \frac{a-b}{a+b} = 1$; $a \neq -b$

c. $\frac{\frac{x^2-9}{x^2+4x+3}}{\frac{x^2-4x+3}{x^2-1}} = \frac{x^2-9}{x^2+4x+3} : \frac{x^2-4x+3}{x^2-1} = \frac{(x+3)(x-3)}{(x+3)(x+3)} \cdot \frac{(x-1)(x+1)}{(x-3)(x+1)} = 1$; $x \neq 1$; $x \neq -1$
 $x \neq 3$
 $x \neq -3$

d. $\frac{1-\frac{1}{x}}{\frac{x-1}{3}} = \frac{3}{x}$; $x \neq 0$; $x \neq 1$

e. $(x - \frac{1}{x}) : (x + \frac{3x+1}{x-1}) = \frac{x^2-1}{x} : \frac{x^2-x+3x+1}{x-1} = \frac{(x+1)(x-1)}{x} \cdot \frac{(x-1)}{x^2+2x+1} = \frac{(x+1)(x-1)}{x} \cdot \frac{(x-1)}{(x+1)^2} = \frac{(x-1)^2}{x(x+1)}$; $x \neq 0$; $x \neq -1$

f. $(t - \frac{t-1}{t+1}) : (1 + \frac{t(t-1)}{t+1}) = 1$; $t \neq -1$

g. $(\frac{\frac{1}{x}}{1+\frac{1}{x}} + \frac{1-\frac{1}{x}}{\frac{1}{x}}) : (\frac{x^{-1}}{1+x^{-1}} - \frac{1-x^{-1}}{x^{-1}}) = (\frac{\frac{1}{x}}{\frac{x+1}{x}} + \frac{1-\frac{1}{x}}{\frac{1}{x}}) : (\frac{\frac{1}{x}}{\frac{x+1}{x}} - \frac{1-\frac{1}{x}}{\frac{1}{x}}) = (\frac{x}{x+1} + \frac{x(x-1)}{x}) : (\frac{x}{x+1} - \frac{x(x-1)}{x}) = (\frac{x}{x+1} + \frac{x(x-1)}{x}) : (\frac{x}{x+1} - \frac{x(x-1)}{x})$
 $= (\frac{1}{x+1} + x^{-1}) : (\frac{1}{x+1} - x + 1) = \frac{x^2+x^2+x-x-1}{x+1} : \frac{1-x^2-x+x+1}{x+1} = \frac{x^2}{x+1} \cdot \frac{x+1}{2-x^2} = \frac{x^2}{2-x^2}$

h. $\frac{3ab}{a^2-ab} + \frac{5a}{a+b} - 2\frac{b^2+2a^2}{a^2-b^2} = \frac{3ab}{a(a-b)} + \frac{5a}{a+b} - 2\frac{b^2+2a^2}{(a+b)(a-b)} = \frac{3b(a+b) + 5a(a-b) - 2b^2-4a^2}{(a+b)(a-b)}$
 $= \frac{3ab+3b^2+5a^2-5ab-2b^2-4a^2}{(a+b)(a-b)} = \frac{a^2+b^2-2ab}{(a+b)(a-b)} = \frac{(a-b)^2}{(a+b)(a-b)} = \frac{a-b}{a+b}$; $a \neq -b$
 $a \neq b$
 $a \neq 0$

i. $(\frac{1}{2x-y} + \frac{3y}{y^2-4x^2} - \frac{2}{2x+y}) : (\frac{4x^2+y^2}{4x^2-y^2} + 1) = (\frac{-1}{2x+y} + \frac{3y}{(y+2x)(y-2x)} - \frac{2}{2x+y}) : \frac{4x^2+y^2+4x^2-y^2}{4x^2-y^2}$
 $= \frac{-y-2x+3y-2y+4x}{(y+2x)(y-2x)} \cdot \frac{(2x+y)(2x-y)}{8x^2} = \frac{2x}{8x^2} \cdot \frac{(2x+y)(2x-y)}{8x^2} = \frac{1}{4x}$
 $x \neq 0$
 $2x+y \neq 0 \Rightarrow x \neq -\frac{y}{2}$
 $2x-y \neq 0 \Rightarrow x \neq \frac{y}{2}$

$$\begin{aligned}
 \text{j. } 6a + \underbrace{\left(\frac{a}{a-2} - \frac{a}{a+2}\right)} &: \frac{4a}{\underbrace{a^4 - 2a^3 + 8a - 16}} = 6a + \left(\frac{\cancel{a^2} + 2a - \cancel{a^2} + 2a}{(a-2)(a+2)}\right) \cdot \frac{\overbrace{a^3(a-2) + 8(a-2)}^{(a-2)(a^3+8)}}{4a} = \\
 = 6a + \frac{\cancel{4a}}{\cancel{(a-2)}\cancel{(a+2)}} \cdot \frac{\cancel{(a-2)} \cdot \cancel{(a+2)} (\cancel{a^2} - 2a + 4)}{\cancel{4a}} &= 6a + \cancel{a^2} - 2a + 4 = \cancel{a^2} + 4a + 4 = (a+2)^2 \Rightarrow
 \end{aligned}$$

$$a \neq 0, a \neq \pm 2$$