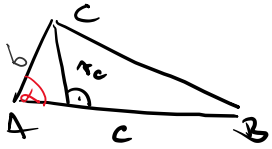
Obsah trojuholníka

$$S = \frac{c.v_c}{2}$$



$\sin\alpha = \frac{r_c}{b}$

$r_c = b \cdot \sin\alpha$

$$S = \frac{1}{2}a.b.\sin\gamma = \frac{1}{2}b.c.\sin\alpha = \frac{1}{2}a.c.\sin\beta$$

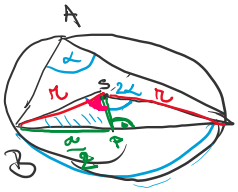
$$S = \frac{abc}{4r}$$

$\triangle BSC$ - rovnoramenný

$P = B = C$

$|\sphericalangle BSP| = \alpha$

P - $\triangle BPS$: $\sin\alpha = \frac{\frac{a}{2}}{r} = \frac{a}{2r}$



$$S = \frac{1}{2} b.c.\frac{a}{2r} = S = \frac{a.b.c}{4r}$$

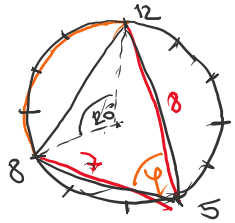
$$S = \rho.s$$

$S = S_{\triangle ABC} + S_{\triangle ACS} + S_{\triangle ABS} = \frac{\rho.c}{2} + \frac{\rho.a}{2} + \frac{\rho.b}{2} = \rho\left(\frac{a+b+c}{2}\right) = \rho.s$

$$S = \sqrt{s(s-a)(s-b)(s-c)};\ s = \frac{a+b+c}{2};$$ Herónov vzorec

Úloha

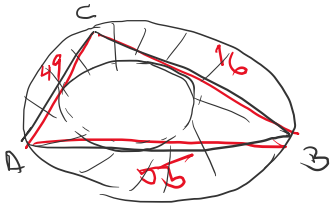
1. Vypočítajte obsah trojuholníka, ktorý vznikne na ciferníku hodiniek spojením bodov 12, 5, 8, pričom $d(12;5) = 8cm$, $d(5;8) = 7\ cm$.



$S = \frac{8.7.\sin\varphi}{2} = \frac{8.7.\sin 60}{2} = 28.\frac{\sqrt{3}}{2} = 14\sqrt{3}\ cm^2$

$\varphi = 60°$

2. Vypočítajte obsah plochy ohraničenej opísanou a vpísanou kružnicou trojuholníku, ktorého strany sú $a = 16\ cm, b = 49\ cm, c = 55\ cm$.



$S = S_{op} - S_{vp} = \pi r^2 - \pi\rho^2 = \pi(r^2 - \rho^2)$

$S_\Delta = \sqrt{s(s-a)(s-b)(s-c)}$ $\quad s = \frac{a+b+c}{2} = \frac{120}{2} = 60$

$S_\Delta = \sqrt{60.(60-16)(60-49)(60-55)} = \sqrt{60.44.11.5}$

$S_\Delta = \sqrt{4^2.11^2.5^2.3} = 4.11.5\sqrt{3} = 220\sqrt{3}\ cm^2$

$\rho \rightarrow S = \rho.s \rightarrow \rho = \frac{S}{s} = \frac{220\sqrt{3}}{60} = \frac{11\sqrt{3}}{3}\ cm$

$r \rightarrow S = \frac{abc}{4r} \rightarrow r = \frac{abc}{4S} = \frac{16.49.55}{4.220\sqrt{3}} = \frac{16.49.55}{4.4.11.5\sqrt{3}} = \frac{49}{\sqrt{3}} = \frac{49\sqrt{3}}{3}\ cm$

DÚ $S = \pi(r^2 - \rho^2) = \pi\left(\frac{49^2.3}{9.3} - \frac{11^2.3}{9.3}\right) = \pi\frac{49^2 - 11^2}{3} = \frac{2280}{3}\pi\ cm^2$

3. Aká je strana rovnostranného trojuholníka, ktorého obsah sa rovná obsahu trojuholníka so stranami 7, 10, 11.