# Lecture 2: Firms, producer theory, and monopoly pricing

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# Producer theory: A review

# Context

- Here, we'll briefly review some material from ECON 6090.
- From the ECON 6090 lecture notes:

  *Assumptions 3.1*:

  (i) $L$ commodities
  (ii) Production plan $y \in \mathbb{R}^L$
     - Net input: good $i$ such that $y_i < 0$
     - Net output: good $j$ such that $y_j > 0$

  (iii) Production possibility set, $Y \subseteq \mathbb{R}^L$ of feasible production plans
  (iv) Prices, $p \geqslant 0$, are unaffected by the activity of the firm.

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  → Today, we'll think about what happens when this assumption holds and when it does not.

# Technological feasibility

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(iv) Prices, $p \geqslant 0$, are unaffected by the activity of the firm.

***Assumptions 3.2*:**

(i) $Y$ is nonempty, closed and (strictly) convex.

(ii) Free disposal: If $y \in Y$ and $y' \leqslant y$, then $y' \in Y$.

**Single-output case:** $f(z) = \max_q q$ s.t. $(-z, q) \in Y$

# Efficiency

**Definition**: A production plan $y \in Y$ is *efficient* if there does not exist a $y' \in Y$ such that $y' \geqslant y$ and $y_i' > y_i$ for some $i$.

# Profit maximization

General case:

$$\pi(p) \equiv \max_{y} p \cdot y \text{ subject to } y \in Y$$

Single-output case:

$$\pi(p, w) \equiv \max_{z \in \mathbb{R}^{L-1}} pf(z) - w \cdot z$$

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FOC:

$$\underbrace{p \nabla f(z)}_{\text{MRP}} = w \;\Rightarrow\; \underbrace{\frac{f_i(z)}{f_{i'}(z)}}_{\text{MRTS}} = \frac{w_i}{w_{i'}}$$

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&= \max_q pq - \left[ \underbrace{\min_{z \in \mathbb{R}^{L-1}} w \cdot z \text{ s.t. } f(z) = q}_{\text{CMP}} \right] \\
&= \max_q pq - C(w, q)
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$$p = C'(q)$$

**Price equals marginal cost. Zero profit on the marginal unit.**

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**Marginal revenue equals marginal cost.**

$$\Rightarrow \;\; p(q^m) = C'(q^m) - \underbrace{p'(q^m)}_{<0} q^m$$

$$> c'(q^m)$$

**Positive profit on the marginal unit.**

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**Positive profit on the marginal unit.** *How much profit?*

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Equivalently, price choice: (Notation: $D = p^{-1}$.)

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"Lerner Index": $\qquad L = -\dfrac{1}{\epsilon}$

**Question**: What does this imply about perfectly elastic demand? Unit elastic demand?

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$$p^m = \left(\frac{1+\epsilon}{\epsilon}\right) C'$$
$$p^m > 0 \Leftrightarrow \frac{1+\epsilon}{\epsilon} > 0$$
$$\Leftrightarrow \epsilon < -1$$

**Elastic part of the demand curve.**

$\rightarrow$ As long as demand is inelastic, $\frac{\partial \pi}{\partial p} > 0$, so increase price (i.e., decrease quantity) until you get to an elastic part of the demand curve.

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- $p^m$ is weakly increasing in marginal cost.

# Proof: $p^m$ is weakly increasing in marginal cost.

- Suppose $C_2'(q) > C_1'(q)$ for all $q > 0$.
- Let $(p_1, q_1)$ and $(p_2, q_2)$ denote the corresponding monopoly prices and quantities.
  - **Key idea**: Both $(p_1, q_1)$ and $(p_2, q_2)$ are points on the demand curve, so both feasible for both monopolists.

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which implies

$$\int_{q_2}^{q_1} \underbrace{[C_2'(x) - C_1'(x)]}_{>0\ \forall x} dx \geqslant 0$$

so $q_1 \geqslant q_2$, which means $p_1 \leqslant p_2$.

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   - Dynamic considerations: Reputation, resale market for durable goods, learning by doing, etc.
   - Multiproduct firm.
     - Supply side: Products' costs are non-separable.
     - Demand side: Cross-elasticities of demand.

# Multi-product monopoly

# Setup

- Cost function: $C(q_1, \ldots, q_n)$
- Demand: $D_1(\mathbf{p}), \ldots, D_n(\mathbf{p})$

$$\pi(\mathbf{p}) = \sum_i p_i D_i(\mathbf{p}) - C\left(D_1(\mathbf{p}), \ldots, D_n(\mathbf{p})\right)$$

# First order condition

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**Questions**:

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**Questions**:

1. What's new here relative to single-product monopoly pricing?
2. What's the interpretation of each term?

# Case 1: Separable costs, inseparable demands

Suppose $C(q_1, \ldots, q_n) = \sum_i C_i(q_i)$. Can we derive an expression for $L_i$ for each $i$?

A definition that will prove useful:

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# Case 2: Separable demands, inseparable costs

Suppose $q_i = D_i(p_i)$ for each $i$.

*Example*: Learning by doing in a two-period game.

- At $t = 1$, cost is $C_1(q_1)$.
- At $t = 2$, cost is $C_2(q_2, q_1)$, where $\partial C_2 / \partial q_1 < 0$.

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- **Question**: What do you predict happens here?
- Profit is $p_1 D_1(p_1) - C_1(D_1(p_1)) + \delta [p_2 D_2(p_2) - C_2(D_2(p_2), D_1(p_1))]$.

# Case 2: Separable demands, inseparable costs

Suppose $q_i = D_i(p_i)$ for each $i$.

*Example*: Learning by doing in a two-period game.

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- FOC wrt $p_1$:

$$D_1 + p_1 D_1' = C_1' D_1' + \delta (\partial C_2 / \partial q_1) D_1'$$
$$L_1 = -\frac{1}{\epsilon_1} + \delta \frac{1}{p_1} \frac{\partial C_2}{\partial q_1} < -\frac{1}{\epsilon_1}$$

# Durable goods monopolist

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- **Question**: What might consumers do? If consumers anticipate that $p_2 = \frac{1}{4}$, they may decide to wait. And if the monopolist were not myopic, she’d anticipate this waiting....

# Durable goods with commitment

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- Can't do better than $p_t = p^m$ for all $t$. [Bulow (1982)]
- **Question**: How likely is it that the monopolist actually has commitment power?

# Coase Conjecture

Consider an extreme commitment problem. Suppose the monopolist has no commitment power, can set prices at $t = 0, \Delta T, 2\Delta T, 3\Delta T, \ldots$, and consumers are fully rational and get gross utility $\theta e^{-rt}$ if they purchase at $t$.

**Coase Conjecture**: Under some conditions the monopolist's profits go to zero as $\Delta T \to 0$.

Intuition:

- Suppose that $\theta \sim U(0,1)$ and $c \in [0,1]$
- Suppose that in the limiting time path of prices is not an immediate drop to zero.
- Hence, prices must be dropping linearly in $T$ at some point and quantities must also be proportional to $\Delta T$.
- Suppose the monopolist jumps ahead in its price sequence and charges $p_{r+\Delta T}$ instead of $p_r$.
- The gain from having all sales occur $\Delta T$ earlier is first order in $\Delta T$.
- The loss from earning less on the sales at time $t$ is second order in $\Delta T$ the price difference and quantity on which you get the lower price are both of order $\Delta T$. Hence, it is better to jump ahead and cut prices faster.

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# If Coase is right, then monopolies are fine I guess?

- The required conditions are actually somewhat narrow: Whether the result is true depends on whether the lower-bound of the value distribution is above or below $c$.
- Durable goods producers don't seem to earn zero profit. **Why? What was Coase missing?**
  - Delays/costs of changing prices (unlikely to be explanation)
  - Reputation
  - Inflows of new high willingness-to-pay customers
  - Per-period fixed costs
  - Strategic actions:
    - Rent rather than sell (but can run into moral hazard or antitrust problems)
    - Most favored customer contracts or money back guarantee
    - Destroy/limit ability to produce
    - Convince consumers that marginal cost is higher than it really is

# Product quality

# Monopoly and Product Quality

- Suppose the monopolist also chooses the quality $s \in \mathbb{R}$ of its good.
- Suppose it has a constant marginal cost $c(s)$ with $c'(s) > 0$, i.e. $C(q,s) = qc(s)$.
- Suppose a unit mass of consumers with types $\theta \sim U(0,1)$ have unit demands: Utility from one unit is $v(s;\theta) - p$ where $v_s(s;\theta) > 0$ and $v_\theta(s;\theta) > 0$.

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The marginal cost of increasing $s$ equals the marginal benefit to the marginal consumer.

# Product Quality: Monopoly versus First-Best

**Monopolist FOC:** (Chooses optimal quality for the *marginal* consumer)

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**Takeaway**: Quality will almost certainly be distorted, but we have no general prediction on the direction of the distortion.