

# Lecture 4: Static Competition

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ECON 7510

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Slides draw upon lecture materials from Glenn & Sara Ellison (MIT).

# Introduction

- Strategic interaction among firms involves many decision variables. They differ in the longevity of their effects.
- Today, we'll discuss the most short-run of these, focusing on the determination of prices and markups holding technologies and market structure fixed.
- We focus on markups because deadweight loss is an important welfare concern, but will also highlight other welfare considerations.
- We'll start with a quick review of classic models and then spend more time on the differentiated product demand models that are now most commonly used.

## Competition with undifferentiated goods

# Two models of competition for undifferentiated goods

1. Cournot competition: Quantity choice
2. Bertrand competition: Price choice

# Cournot Competition (1838)

- $N$  firms
- Inverse demand  $P(X)$  for homogeneous good
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- **Question:** Does this seem like a good representation of firms' choices? Is there a way we can interpret it that makes it more relevant?
- Literal interpretation might be appropriate for commodities like wheat, natural gas, iron ore.
- A more widely applicable interpretation: reduced form for a situation where firms choose capacities of factories that will always run at full capacity.
- Not a go-to empirical model these days, but still potentially useful for thinking about the effects of competition/market power.



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Nash equilibrium FOC: If  $(x_1^*, \dots, x_N^*)$  is a NE then

$$\left[ P \left( \sum_i x_i^* \right) - c_i'(x_i^*) \right] + P' \left( \sum_i x_i^* \right) x_i^* = 0 \text{ for all } i \text{ with } x_i^* > 0$$

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A useful interpretation:  $i$  is a monopolist, facing the *residual demand curve*:

- Let  $X_{-i} = \sum_{j \neq i} x_j$ .
- Then, let  $\tilde{P}(x_i) = P(x_i + X_{-i})$ .
- Firm  $i$ 's problem:

$$\max_{x_i} [\tilde{P}(x_i) - c_i(x_i)] x_i$$

- FOC:

$$\frac{\tilde{P}(x_i) - c'(x_i)}{\tilde{P}(x_i)} = -\tilde{P}'(x_i) \frac{x_i}{\tilde{P}(x_i)} = -\frac{1}{\tilde{\epsilon}}$$

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3. The industry-wide Lerner index is  $\frac{P - \sum_i \frac{x_i^*}{X^*} c_i'(x_i^*)}{P} = -\frac{H}{\epsilon}$ , where  $H = \sum_i \left( \frac{x_i^*}{X^*} \right)^2$  is the industry "Herfindahl Index".

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6. Firm outputs are *usually* "strategic substitutes":  $\frac{\partial BR_i}{\partial x_{-i}} < 0$  where  $BR_i = x_i^*(X_{-i})$ . **Question:** How would we prove this? What conditions on  $P''$  and  $C''$  are required?

## Bertrand Competition (1883)

- 2 firms (could be  $N$ )
- $X(p)$  is market demand function. Assume  $X(p)$  is weakly decreasing and  $pX(p)$  is bounded.
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Bertrand is an “exemplifying theory.” It illustrates forces using extreme assumptions that we would not see in practice.

- No product differentiation creates infinitely elastic firm-level demand
- Constant returns to scale with no capacity constraints
- One-shot interaction

With asymmetric costs,  $c_1 < c_2$ , an equilibrium is  $p_1^* = p_2^* = c_2$  with all consumers purchasing from firm 1.



## Competition with differentiated goods

# Hotelling Competition (1929)

- Continuum of consumers with types  $\theta \sim U[0, 1]$  have unit demands
- Utility is  $v - t\theta - p_1$  if buy from firm 1,  $v - t(1 - \theta) - p_2$  if buy from firm 2, and 0 if they don't buy.
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Let's start by deriving demand function.

If  $v$  is sufficiently large relative to  $p_1, p_2$ , then all consumers will purchase from one firm or the other. The indifferent type has

$$v - t\theta - p_1 = v - t(1 - \theta) - p_2 \Rightarrow \theta = \frac{1}{2} + \frac{p_2 - p_1}{2t}$$

## Hotelling Competition (1929)

Assuming that equilibrium prices are sufficiently low relative to  $v$  so that this case applies:

$$BR_i(p_j) = \arg \max_p (p - c) \left( \frac{1}{2} + \frac{p_j - p}{2t} \right)$$

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- **Question:** What if both firms observe each consumer's  $\theta$ ?
- In an  $N$ -firm circular version markups decline like  $1/N$  as in Cournot.
- Actions are “strategic complements”: firms increase prices when rivals increase prices.

## Vertical Differentiation

- Firms L and H produce goods of quality  $s_L$  and  $s_H$ , respectively, with  $s_L < s_H$ .
- Consumers with types  $\theta \sim U[\underline{\theta}, \bar{\theta}]$  have unit demands with utility  $u_i(\theta) = \theta s_i - p_i$  if buy from  $i$  and 0 from outside good. For simplicity assume mass  $\bar{\theta} - \underline{\theta}$  of consumers.
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Given prices  $p_L, p_H$ , let  $\hat{\theta}_{LH}$  be the solution to  $u_L(\hat{\theta}_{LH}) = u_H(\hat{\theta}_{LH})$ , and let  $\theta_{0L}$  be the solution to  $u_L(\theta_{0L}) = 0$ .

When  $\theta_{0L} < \underline{\theta} < \hat{\theta}_{LH}$ , demands are given by

$$D_H(p_L, p_H) = \bar{\theta} - \hat{\theta}_{LH} = \bar{\theta} - \frac{p_H - p_L}{s_H - s_L}$$

$$D_L(p_L, p_H) = \hat{\theta}_{LH} - \underline{\theta} = \frac{p_H - p_L}{s_H - s_L} - \underline{\theta}$$

## Vertical Differentiation

Again, finding BRs and NE is easy with linear demand curves:

$$BR_H(p_L) = \arg \max_p (p - c) \left( \bar{\theta} - \frac{p - p_L}{s_H - s_L} \right) = \frac{1}{2} (p_L + c + \bar{\theta}(s_H - s_L))$$

$$BR_L(p_H) = \frac{1}{2} (p_H + c - \underline{\theta}(s_H - s_L))$$

The solution to these is the NE provided  $\bar{\theta} \geq 2\underline{\theta}$  and  $\frac{c + \bar{\theta} - 2\underline{\theta}}{3(s_H - s_L)} < \underline{\theta}s_L$ :

$$p_L^* = c + \frac{\bar{\theta} - 2\underline{\theta}}{3(s_H - s_L)} \quad p_H^* = c + \frac{2\bar{\theta} - \underline{\theta}}{3(s_H - s_L)}$$

Notes:

1. Vertical differentiation also creates finite elasticities and positive markups.
2. Firm H sets a higher price and earns higher profits.
3. When  $\underline{\theta}$  and  $\bar{\theta}$  are too close together firm L is shut out of the market.

# Empirical relevance

Suppose we wanted to take a model of competition to the data.

- What issues might we encounter using the Hotelling model empirically?
- What issues might we encounter using the vertical differentiation model empirically?

## More flexible class of models

Variants of Hotelling's model (sometimes with some vertical differentiation as well) have become the dominant approach in empirical IO.

The standard  $N$ -firm implementation assumes consumers have an  $N + 1$  dimensional type  $(\epsilon_{i0}, \epsilon_{i1}, \dots, \epsilon_{iN})$  with joint CDF  $F$  and utility is

$$u_{ij} = \begin{cases} v_j - \alpha p_j + \epsilon_{ij} & \text{if } i \text{ purchases good } j \\ \epsilon_{i0} & \text{if } i \text{ consumes "outside good"} \end{cases}$$



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Demand in this model in the general case is given by an  $N$ -dimensional integral:

$$x_j(p_1, \dots, p_N) = \int_{\{\epsilon_{i0}, \epsilon_{i1}, \dots, \epsilon_{iN} | u_{ij} > u_{ik} \forall k \neq j\}} dF(\epsilon_{i0}, \epsilon_{i1}, \dots, \epsilon_{iN})$$

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Empirical papers sometimes approximate this by simulating draws of the  $\epsilon_{ik}$ . A more tractable special case is when there is no outside good, the  $v_j$  are all equal, and the  $\epsilon_{ik}$  are iid with density  $f$ .

Demand when others all charge  $p$  is a one-dimensional integral:

$$x_i(p_i, p, \dots, p) = \int \left[ 1 - F \left( \theta + \frac{p_i - p}{(N-1)F(\theta)^{N-2}f(\theta)} \right) \right] d\theta$$

# Horizontal Differentiation

Perloff and Salop (REStud 1985) analyze this symmetric model and show:

**Proposition:** In this model the symmetric NE prices are

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3. In the “logit” model,  $F(\epsilon) = e^{-e^{-(\epsilon+\gamma)}}$ ,  $p_N^* = c + k \left(1 + \frac{1}{N-1}\right) \frac{1}{\alpha}$  for some constant  $k$ .

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**Question:** What does UPP look like in the case of logit demand? For simplicity, suppose there are  $N$  symmetric firms pre-merger.

# Next time

Dynamic competition