Lecture 2: Firms, producer theory, and monopoly pricing

ECON 7510

Cornell University

Adam Harris

Theory of the firm: A brief overview

What is a firm? What determines its boundaries?

NB: This is question that is more often tackled by organizational economics rather than IO.

Tirole: “a cost-minimizing device" for production.

Three perspectives:

1. Loophole for the exercise of monopoly power
2. Static synergy
3. A long-term relationship

What is a firm? What determines its boundaries?

Loophole for the exercise of monopoly power

- Exercise of monopoly power can be disciplined both by regulators and by other firms.
- What happens in the market is often publicly observable; what happens within the firm is usually not.
- *Example*: Monopoly pricing.
    - Collusive price-setting is illegal; being a monopolist is not.
- *Example*: Price of intermediate good set by government

What is a firm? What determines its boundaries?

Static synergy

- If there are increasing returns to scale, having production concentrated in a smaller number of firms may be more efficient.
- “Economies of scale encourage the gathering of activities."

What is a firm? What determines its boundaries?

Long-term relationship

- Key idea: Idiosyncratic investment and asset specificity.

$\rightarrow$ Want *ex ante* assurance that future gains from trade will be exploited and shared.

- *Example*: Specific human capital. More efficient to work on the same task / with the same team every day.

- *Example*: Site specificity. Mine-mouth power plant.

What do firms do?

Profit-maximization hypothesis

- If shareholders ran the firm directly, it would be profit-maximizing.
- But, in practice, the firm is run by a manager, who may have different incentives:
  - Monetary incentives, e.g., distorting short-term vs long-term tradeoff
  - Growth for its own sake (prestige, ego, power, etc)
  - Mislead about technology to take pressure off
- To combat this, shareholders may try to monitor manager performance or put limits on managerial discretion. But all of these approaches are imperfect.
- These are important issues and are the focus of organizational economics.

What do firms do?

Is the assumption of profit-maximization “good enough”?

1. Yes, if internal organization issues and product-market/input-market choices are approximately “separable”.

   Example: Manager chooses q, e.

   $$\Pi = P(q)q - c(e, \epsilon)q - w$$

   where e, ϵ are not observed by shareholders.

What do firms do?

Is the assumption of profit-maximization "good enough"?

1. Yes, if internal organization issues and product-market/input-market choices are approximately "separable".

   Example: Manager chooses q, e.

   $$\Pi = P(q)q - c(e,\epsilon)q - w$$

   where e, ϵ are not observed by shareholders. Would expect inefficiently low effort and inefficiently high cost (moral hazard)

   $$e < e^* \;\Rightarrow\; \underbrace{c(e,\epsilon)}_{\tilde{c}} > \underbrace{c(e^*,\epsilon)}_{c^*}$$

What do firms do?

Is the assumption of profit-maximization "good enough"?

1. Yes, if internal organization issues and product-market/input-market choices are approximately "separable".

   Example: Manager chooses q, e.

   $$\Pi = P(q)q - c(e, \epsilon)q - w$$

   where e, ϵ are not observed by shareholders. Would expect inefficiently low effort and inefficiently high cost (moral hazard)

   $$e < e^* \;\Rightarrow\; \underbrace{c(e, \epsilon)}_{\tilde{c}} > \underbrace{c(e^*, \epsilon)}_{c^*}$$

   Then the quantity-choice problem is $\max_q P(q)q - \tilde{c}q$. The manager might as well choose the quantity that is profit-maximizing given marginal cost $\tilde{c}$.

What do firms do?

Is the assumption of profit-maximization “good enough”?

1. Yes, if internal organization issues and product-market/input-market choices are approximately “separable”.

   Example: Manager chooses q, e.

   $$\Pi = P(q)q - c(e, \epsilon)q - w$$

   where e, ϵ are not observed by shareholders. Would expect inefficiently low effort and inefficiently high cost (moral hazard)

   $$e < e^* \;\Rightarrow\; \underbrace{c(e, \epsilon)}_{\tilde{c}} > \underbrace{c(e^*, \epsilon)}_{c^*}$$

   Then the quantity-choice problem is $\max_q P(q)q - \tilde{c}q$. The manager might as well choose the quantity that is profit-maximizing given marginal cost $\tilde{c}$.

   So firm’s choice of quantity is *observational equivalent* to that of a profit-maximizing firm.

What do firms do?

Is the assumption of profit-maximization “good enough”?

1. Yes, if internal organization issues and product-market/input-market choices are approximately “separable”.

   Example: Manager chooses q, e.

   $$\Pi = P(q)q - c(e, \epsilon)q - w$$

   where e, ϵ are not observed by shareholders. Would expect inefficiently low effort and inefficiently high cost (moral hazard)

   $$e < e^* \;\Rightarrow\; \underbrace{c(e, \epsilon)}_{\tilde{c}} > \underbrace{c(e^*, \epsilon)}_{c^*}$$

   Then the quantity-choice problem is $\max_q P(q)q - \tilde{c}q$. The manager might as well choose the quantity that is profit-maximizing given marginal cost $\tilde{c}$.

   So firm’s choice of quantity is *observational equivalent* to that of a profit-maximizing firm. The fact that $\tilde{c} > c^*$ is sometimes referred to as *X-inefficiency*.

What do firms do?

Is the assumption of profit-maximization “good enough”?

2. Regardless, it is a necessary assumption.
   - As with any modeling choice, there’s a realism-versus-tractability tradeoff.
   - If we want to make progress/derive theoretical predictions about important IO questions—e.g., antitrust policy, innovation, regulation, etc.—we can’t also tackle the intra-firm incentives.

   $\rightarrow$ Let’s leave the internal principal-agent issues to the organizational economists.

What do firms do?

Is the assumption of profit-maximization "good enough"?

2. Regardless, it is a necessary assumption.
   - As with any modeling choice, there's a realism-versus-tractability tradeoff.
   - If we want to make progress/derive theoretical predictions about important IO questions—e.g., antitrust policy, innovation, regulation, etc.—we can't also tackle the intra-firm incentives.

   $\rightarrow$ Let's leave the internal principal-agent issues to the organizational economists.

MWG (p.127): "The firm is viewed merely as a "black box", able to transform inputs into outputs."

Producer theory: A review

Context

- Here, we'll briefly review some material from ECON 6090.
- From the ECON 6090 lecture notes:

  Assumptions 3.1:

  (i) L commodities
  (ii) Production plan $y \in \mathbb{R}^L$
     - Net input: good i such that $y_i < 0$
     - Net output: good j such that $y_j > 0$
  (iii) Production possibility set, $Y \subseteq \mathbb{R}^L$ of feasible production plans
  (iv) Prices, $p \geqslant 0$, are unaffected by the activity of the firm.

Context

- Here, we'll briefly review some material from ECON 6090.

- From the ECON 6090 lecture notes:

  Assumptions 3.1:

  (i) L commodities
  (ii) Production plan $y \in \mathbb{R}^L$
    - Net input: good i such that $y_i < 0$
    - Net output: good j such that $y_j > 0$
  (iii) Production possibility set, $Y \subseteq \mathbb{R}^L$ of feasible production plans
  (iv) Prices, $p \geqslant 0$, are unaffected by the activity of the firm.

- To some extent, "core IO" is the study of 3.1.(iv) violations.

Context

- Here, we'll briefly review some material from ECON 6090.
- From the ECON 6090 lecture notes:

  Assumptions 3.1:

  (i) L commodities
  (ii) Production plan $y \in \mathbb{R}^L$
    - Net input: good i such that $y_i < 0$
    - Net output: good j such that $y_j > 0$

  (iii) Production possibility set, $Y \subseteq \mathbb{R}^L$ of feasible production plans
  (iv) Prices, $p \geqslant 0$, are unaffected by the activity of the firm.
- To some extent, "core IO" is the study of 3.1.(iv) violations.

  $\rightarrow$ Today, we'll think about what happens when this assumption holds and when it does not.

Technological feasibility

***Assumptions 3.1*:**

(i) L commodities

(ii) Production plan $y \in \mathbb{R}^L$

- Net input: good i such that $y_i < 0$
- Net output: good j such that $y_j > 0$

(iii) Production possibility set, $Y \subseteq \mathbb{R}^L$ of feasible production plans

(iv) Prices, $p \geqslant 0$, are unaffected by the activity of the firm.

***Assumptions 3.2*:**

(i) Y is nonempty, closed and (strictly) convex.

(ii) Free disposal: If $y \in Y$ and $y' \leqslant y$, then $y' \in Y$.

Single-output case: $f(z) = \max_q q$ s.t. $(-z, q) \in Y$

Efficiency

Definition: A production plan $y \in Y$ is *efficient* if there does not exist a $y' \in Y$ such that $y' \geqslant y$ and $y_i' > y_i$ for some i.

Profit maximization

General case:

$$\pi(p) \equiv \max_{y} p \cdot y \text{ subject to } y \in Y$$

Single-output case:

$$\pi(p, w) \equiv \max_{z \in \mathbb{R}^{L-1}} pf(z) - w \cdot z$$

Profit maximization

General case:

$$\pi(p) \equiv \max_{y} p \cdot y \text{ subject to } y \in Y$$

Single-output case:

$$\pi(p, w) \equiv \max_{z \in \mathbb{R}^{L-1}} pf(z) - w \cdot z$$

FOC:

$$\underbrace{p \nabla f(z)}_{\text{MRP}} = w \;\Rightarrow\; \underbrace{\frac{f_i(z)}{f_{i'}(z)}}_{\text{MRTS}} = \frac{w_i}{w_{i'}}$$

Profit maximization implies cost minimization

$$\begin{aligned}
\pi(p, w) &\equiv \max_{z \in \mathbb{R}^{L-1}} pf(z) - w \cdot z \\
&= \max_q \left[\max_{z \in \mathbb{R}^{L-1}} pq - w \cdot z \text{ s.t. } f(z) = q \right] \\
&= \max_q pq - \left[\underbrace{\min_{z \in \mathbb{R}^{L-1}} w \cdot z \text{ s.t. } f(z) = q}_{\text{CMP}} \right] \\
&= \max_q pq - c(w, q)
\end{aligned}$$

Profit maximization (with product market power)

General case:

$$\pi(p) \equiv \max_{y} \mathbf{p}(\mathbf{y}) \cdot y \text{ subject to } y \in Y$$

Single-output case:

$$\pi(p, w) \equiv \max_{z \in \mathbb{R}^{L-1}} \mathbf{p}(\mathbf{f}(\mathbf{z})) f(z) - w \cdot z$$

FOC:

$$\underbrace{\left[p + \mathbf{p}'\left(\mathbf{f}(\mathbf{z})\right)\right] \nabla f(z)}_{\text{MRP}} = w \Rightarrow \underbrace{\frac{f_i(z)}{f_{i'}(z)}}_{\text{MRTS}} = \frac{w_i}{w_{i'}}$$

Profit maximization implies cost minimization (with product market power)

$$
\begin{aligned}
\pi(w) &\equiv \max_{z\in\mathbb{R}^{L-1}} \mathbf{p}\left(\mathbf{f}(\mathbf{z})\right) f(z) - w \cdot z \\
&= \max_q \left[\max_{z\in\mathbb{R}^{L-1}} \mathbf{p}\left(\mathbf{q}\right) q - w \cdot z \text{ s.t. } f(z) = q \right] \\
&= \max_q \mathbf{p}\left(\mathbf{q}\right) q - \left[\underbrace{\min_{z\in\mathbb{R}^{L-1}} w \cdot z \text{ s.t. } f(z) = q}_{\text{CMP}} \right] \\
&= \max_q \mathbf{p}\left(\mathbf{q}\right) q - c(w, q)
\end{aligned}
$$

Quantity choice under perfect competition

$$\pi(q) \equiv \max_q pq - c(q)$$

FOC:

$$p = c'(q)$$

Price equals marginal cost. Zero profit on the marginal unit.

Monopoly pricing

Quantity choice:

$$\pi(q) \equiv \max_q p(q)q - c(q)$$

FOC:

$$\left[p(q^m) + p'(q^m)q^m\right] = c'(q^m)$$

Monopoly pricing

Quantity choice:

$$\pi(q) \equiv \max_q p(q)q - c(q)$$

FOC:

$$\left[p(q^m) + p'(q^m)q^m\right] = c'(q^m)$$

Marginal revenue equals marginal cost.

Monopoly pricing

Quantity choice:

$$\pi(q) \equiv \max_q p(q)q - c(q)$$

FOC:

$$\left[p(q^m) + p'(q^m)q^m\right] = c'(q^m)$$

Marginal revenue equals marginal cost.

$$\Rightarrow \; p(q^m) = c'(q^m) - \underbrace{p'(q^m)}_{<0} q^m$$

$$> c'(q^m)$$

Positive profit on the marginal unit.

Monopoly pricing

Quantity choice:

$$\pi(q) \equiv \max_q p(q)q - c(q)$$

FOC:

$$\left[p(q^m) + p'(q^m)q^m\right] = c'(q^m)$$

Marginal revenue equals marginal cost.

$$\Rightarrow \; p(q^m) = c'(q^m) - \underbrace{p'(q^m)}_{<0} q^m$$

$$> c'(q^m)$$

Positive profit on the marginal unit. *How much profit?*

Monopoly pricing

Equivalently, price choice:

$$\pi(p) \equiv \max_p pD(p) - c\left(D(p)\right)$$

FOC:

$$\left[p^m D'(p^m) + D(p^m)\right] = c'\left(D(p^m)\right) D'(p^m)$$

Monopoly pricing

Equivalently, price choice:

$$\pi(p) \equiv \max_p pD(p) - c\left(D(p)\right)$$

FOC:

$$\left[p^m D'(p^m) + D(p^m)\right] = c'\left(D(p^m)\right) D'(p^m)$$

$$p^m - c'\left(D(p^m)\right) = -\frac{D(p^m)}{D'(p^m)}$$

$$\frac{p^m - c'\left(D(p^m)\right)}{p^m} = -\frac{D(p^m)}{D'(p^m)p^m}$$

“Lerner Index”: $\qquad L = -\frac{1}{\epsilon}$

Monopoly pricing

Equivalently, price choice:

$$\pi(p) \equiv \max_p pD(p) - c\left(D(p)\right)$$

FOC:

$$\left[p^m D'(p^m) + D(p^m)\right] = c'\left(D(p^m)\right) D'(p^m)$$

$$p^m - c'\left(D(p^m)\right) = -\frac{D(p^m)}{D'(p^m)}$$

$$\frac{p^m - c'\left(D(p^m)\right)}{p^m} = -\frac{D(p^m)}{D'(p^m)p^m}$$

"Lerner Index": $L = -\frac{1}{\epsilon}$

$$p^m = \left(\frac{1+\epsilon}{\epsilon}\right) c'\left(D(p^m)\right)$$

Monopoly pricing

Equivalently, price choice:

$$\pi(p) \equiv \max_{p} pD(p) - c\left(D(p)\right)$$

FOC:

$$\left[p^m D'(p^m) + D(p^m)\right] = c'\left(D(p^m)\right) D'(p^m)$$

$$p^m - c'\left(D(p^m)\right) = -\frac{D(p^m)}{D'(p^m)}$$

$$\frac{p^m - c'\left(D(p^m)\right)}{p^m} = -\frac{D(p^m)}{D'(p^m)p^m}$$

"Lerner Index": $\quad L = -\dfrac{1}{\epsilon}$

$$p^m = \left(\frac{1+\epsilon}{\epsilon}\right) c'\left(D(p^m)\right)$$

Question: What happens in the limiting cases (perfectly elastic and perfectly inelastic demand)?

Properties of monopoly pricing

- What point on the demand curve does monopolist choose?

Properties of monopoly pricing

- What point on the demand curve does monopolist choose?

$$p^m = \left(\frac{1+\epsilon}{\epsilon}\right) c'$$

$$p^m > 0 \Leftrightarrow \frac{1+\epsilon}{\epsilon} > 0$$

$$\Leftrightarrow \epsilon < -1$$

Elastic part of the demand curve.

$\rightarrow$ As long as demand is inelastic, $\frac{\partial \pi}{\partial p} > 0$, so increase price (i.e., decrease quantity) until you get to an elastic part of the demand curve.

Properties of monopoly pricing

- What point on the demand curve does monopolist choose?

$$p^m = \left(\frac{1+\epsilon}{\epsilon}\right) c'$$

$$p^m > 0 \Leftrightarrow \frac{1+\epsilon}{\epsilon} > 0$$

$$\Leftrightarrow \epsilon < -1$$

Elastic part of the demand curve.

$\rightarrow$ As long as demand is inelastic, $\frac{\partial \pi}{\partial p} > 0$, so increase price (i.e., decrease quantity) until you get to an elastic part of the demand curve.

- Inefficiency?

Properties of monopoly pricing

— What point on the demand curve does monopolist choose?

$$p^m = \left(\frac{1+\epsilon}{\epsilon}\right) c'$$

$$p^m > 0 \Leftrightarrow \frac{1+\epsilon}{\epsilon} > 0$$

$$\Leftrightarrow \epsilon < -1$$

Elastic part of the demand curve.

$\rightarrow$ As long as demand is inelastic, $\frac{\partial \pi}{\partial p} > 0$, so increase price (i.e., decrease quantity) until you get to an elastic part of the demand curve.

— Inefficiency? Yes, any deviation from $p = c'$ means quantity is inefficient.

Properties of monopoly pricing

- What point on the demand curve does monopolist choose?

$$p^m = \left(\frac{1+\epsilon}{\epsilon}\right) c'$$

$$p^m > 0 \Leftrightarrow \frac{1+\epsilon}{\epsilon} > 0$$

$$\Leftrightarrow \epsilon < -1$$

  Elastic part of the demand curve.

  $\rightarrow$ As long as demand is inelastic, $\frac{\partial \pi}{\partial p} > 0$, so increase price (i.e., decrease quantity) until you get to an elastic part of the demand curve.

- Inefficiency? Yes, any deviation from $p = c'$ means quantity is inefficient.

- p^m is weakly increasing in marginal cost.

Proof: p^m is weakly increasing in marginal cost.

- Suppose $c_2'(q) > c_1'(q)$ for all $q > 0$.
- Let (p_1, q_1) and (p_2, q_2) denote the corresponding monopoly prices and quantities.
  - **Key idea**: Both (p_1, q_1) and (p_2, q_2) are points on the demand curve, so both feasible for both monopolists.

Proof: p^m is weakly increasing in marginal cost.

- Suppose $c_2'(q) > c_1'(q)$ for all $q > 0$.
- Let (p_1, q_1) and (p_2, q_2) denote the corresponding monopoly prices and quantities.
  - **Key idea**: Both (p_1, q_1) and (p_2, q_2) are points on the demand curve, so both feasible for both monopolists.

$$p_1 q_1 - c_1(q_1) \geqslant p_2 q_2 - c_1(q_2)$$
$$p_2 q_2 - c_2(q_2) \geqslant p_1 q_1 - c_2(q_1)$$

Proof: p^m is weakly increasing in marginal cost.

- Suppose $c_2'(q) > c_1'(q)$ for all $q > 0$.
- Let (p_1, q_1) and (p_2, q_2) denote the corresponding monopoly prices and quantities.
    - **Key idea**: Both (p_1, q_1) and (p_2, q_2) are points on the demand curve, so both feasible for both monopolists.

$$p_1 q_1 - c_1(q_1) \geqslant p_2 q_2 - c_1(q_2)$$
$$p_2 q_2 - c_2(q_2) \geqslant p_1 q_1 - c_2(q_1)$$

- Combining the two:

$$[c_2(q_1) - c_1(q_1)] - [c_2(q_2) - c_1(q_2)] \geqslant 0$$

Proof: p^m is weakly increasing in marginal cost.

- Suppose $c_2'(q) > c_1'(q)$ for all $q > 0$.
- Let (p_1, q_1) and (p_2, q_2) denote the corresponding monopoly prices and quantities.
  - **Key idea**: Both (p_1, q_1) and (p_2, q_2) are points on the demand curve, so both feasible for both monopolists.

$$p_1 q_1 - c_1(q_1) \geqslant p_2 q_2 - c_1(q_2)$$
$$p_2 q_2 - c_2(q_2) \geqslant p_1 q_1 - c_2(q_1)$$

- Combining the two:

$$[c_2(q_1) - c_1(q_1)] - [c_2(q_2) - c_1(q_2)] \geqslant 0$$

which implies

$$\int_{q_2}^{q_1} \underbrace{[c_2'(x) - c_1'(x)]}_{>0\ \forall x} dx \geqslant 0$$

so $q_1 \geqslant q_2$, which means $p_1 \leqslant p_2$.

Questions for discussion

1. A retailer uses a simple heuristic, always setting prices 20% above marginal cost. Is that consistent with this model?

Questions for discussion

1. A retailer uses a simple heuristic, always setting prices 20% above marginal cost. Is that consistent with this model?

2. Could a corrective tax/subsidy be used to correct the distortion? If so, does that ensure efficiency?

Questions for discussion

1. A retailer uses a simple heuristic, always setting prices 20% above marginal cost. Is that consistent with this model?

2. Could a corrective tax/subsidy be used to correct the distortion? If so, does that ensure efficiency? **Yes**, a production subsidy could bring us back to the efficient quantity,

Questions for discussion

1. A retailer uses a simple heuristic, always setting prices 20% above marginal cost. Is that consistent with this model?

2. Could a corrective tax/subsidy be used to correct the distortion? If so, does that ensure efficiency? **Yes**, a production subsidy could bring us back to the efficient quantity, though still could be inefficiency if:

   - Distortion of product/quality choice.
   - "X-inefficiency"

Questions for discussion

1. A retailer uses a simple heuristic, always setting prices 20% above marginal cost. Is that consistent with this model?

2. Could a corrective tax/subsidy be used to correct the distortion? If so, does that ensure efficiency? **Yes**, a production subsidy could bring us back to the efficient quantity, though still could be inefficiency if:

   - Distortion of product/quality choice.
   - "X-inefficiency"

3. Suppose you know that firm A is a monopolist and you know p, $\epsilon(p)$, $c'(D(p))$. However, you observe that $p \neq \left(\frac{1+\epsilon}{\epsilon}\right) c'(D(p))$. What could explain this?

Questions for discussion

1. A retailer uses a simple heuristic, always setting prices 20% above marginal cost. Is that consistent with this model?

2. Could a corrective tax/subsidy be used to correct the distortion? If so, does that ensure efficiency? **Yes**, a production subsidy could bring us back to the efficient quantity, though still could be inefficiency if:

   - Distortion of product/quality choice.
   - "X-inefficiency"

3. Suppose you know that firm A is a monopolist and you know p, $\epsilon(p)$, $c'(D(p))$. However, you observe that $p \neq \left(\frac{1+\epsilon}{\epsilon}\right) c'(D(p))$. What could explain this?

   - Dynamic considerations: Reputation, resale market for durable goods, learning by doing, etc.
   - Multiproduct firm.
     - Supply side: Products' costs are non-separable.
     - Demand side: Cross-elasticities of demand.

Multi-product monopoly

Setup

- Cost function: $C(q_1, \dots, q_n)$
- Demand: $D_1(\mathbf{p}), \dots, D_n(\mathbf{p})$

$$\pi(\mathbf{p}) = \sum_i p_i D_i(\mathbf{p}) - C\left(D_1(\mathbf{p}), \dots, D_n(\mathbf{p})\right)$$

First order condition

$$\pi(\mathbf{p}) = \sum_i p_i D_i(\mathbf{p}) - C\left(D_1(\mathbf{p}), \ldots, D_n(\mathbf{p})\right)$$

FOC: For each i,

$$\left(D_i + p_i \frac{\partial D_i}{\partial p_i}\right) + \sum_{j \neq i} p_j \frac{\partial D_j}{\partial p_i} = \frac{\partial C}{\partial q_i} \frac{\partial D_i}{\partial p_i} + \sum_{j \neq i} \frac{\partial C}{\partial q_j} \frac{\partial D_j}{\partial p_i}$$

Questions:

1. What's new here relative to single-product monopoly pricing?

First order condition

$$\pi(\mathbf{p}) = \sum_i p_i D_i(\mathbf{p}) - C\left(D_1(\mathbf{p}), \ldots, D_n(\mathbf{p})\right)$$

FOC: For each i,

$$\left(D_i + p_i \frac{\partial D_i}{\partial p_i}\right) + \sum_{j \neq i} p_j \frac{\partial D_j}{\partial p_i} = \frac{\partial C}{\partial q_i} \frac{\partial D_i}{\partial p_i} + \sum_{j \neq i} \frac{\partial C}{\partial q_j} \frac{\partial D_j}{\partial p_i}$$

Questions:

1. What's new here relative to single-product monopoly pricing?
2. What's the interpretation of each term?

Case 1: Separable costs, inseparable demands

Suppose $C(q_1, \ldots, q_n) = \sum_i C_i(q_i)$. Can we derive an expression for L_i for each i?

A definition that will prove useful:

$$\epsilon_{ij} = \frac{\partial D_j}{\partial p_i} \times \frac{p_i}{D_j}$$

Case 1: Separable costs, inseparable demands

Suppose $C(q_1, \ldots, q_n) = \sum_i C_i(q_i)$. Can we derive an expression for L_i for each i?

A definition that will prove useful:

$$\epsilon_{ij} = \frac{\partial D_j}{\partial p_i} \times \frac{p_i}{D_j}$$

The FOC becomes

$$\frac{p_i - C_i'}{p_i} = -\frac{1}{\epsilon_{ii}} - \sum_{j \neq i} \frac{(p_j - C_j')D_j \epsilon_{ij}}{p_i D_i \epsilon_{ii}}$$

- **Case 1a: Goods are substitutes**: Then, $\epsilon_{ij} > 0 \ \forall i \neq j$.

Case 1: Separable costs, inseparable demands

Suppose $C(q_1, \dots, q_n) = \sum_i C_i(q_i)$. Can we derive an expression for L_i for each i?

A definition that will prove useful:

$$\epsilon_{ij} = \frac{\partial D_j}{\partial p_i} \times \frac{p_i}{D_j}$$

The FOC becomes

$$\frac{p_i - C_i'}{p_i} = -\frac{1}{\epsilon_{ii}} - \sum_{j \neq i} \frac{(p_j - C_j')D_j \epsilon_{ij}}{p_i D_i \epsilon_{ii}}$$

- **Case 1a: Goods are substitutes**: Then, $\epsilon_{ij} > 0 \; \forall i \neq j$. In this case, $L_i > -\frac{1}{\epsilon_{ii}}$.

Case 1: Separable costs, inseparable demands

Suppose $C(q_1, \ldots, q_n) = \sum_i C_i(q_i)$. Can we derive an expression for L_i for each i?

A definition that will prove useful:

$$\epsilon_{ij} = \frac{\partial D_j}{\partial p_i} \times \frac{p_i}{D_j}$$

The FOC becomes

$$\frac{p_i - C_i'}{p_i} = -\frac{1}{\epsilon_{ii}} - \sum_{j \neq i} \frac{(p_j - C_j')D_j \epsilon_{ij}}{p_i D_i \epsilon_{ii}}$$

- **Case 1a: Goods are substitutes**: Then, $\epsilon_{ij} > 0 \ \forall i \neq j$. In this case, $L_i > -\frac{1}{\epsilon_{ii}}$.
- **Case 1b: Goods are complements**: Then, $\epsilon_{ij} < 0 \ \forall i \neq j$.

Case 1: Separable costs, inseparable demands

Suppose $C(q_1, \ldots, q_n) = \sum_i C_i(q_i)$. Can we derive an expression for L_i for each i?

A definition that will prove useful:

$$\epsilon_{ij} = \frac{\partial D_j}{\partial p_i} \times \frac{p_i}{D_j}$$

The FOC becomes

$$\frac{p_i - C_i'}{p_i} = -\frac{1}{\epsilon_{ii}} - \sum_{j \neq i} \frac{(p_j - C_j')D_j \epsilon_{ij}}{p_i D_i \epsilon_{ii}}$$

- **Case 1a: Goods are substitutes**: Then, $\epsilon_{ij} > 0 \; \forall i \neq j$. In this case, $L_i > -\frac{1}{\epsilon_{ii}}$.
- **Case 1b: Goods are complements**: Then, $\epsilon_{ij} < 0 \; \forall i \neq j$. In this case, $L_i < -\frac{1}{\epsilon}$.

Case 1: Separable costs, inseparable demands

Suppose $C(q_1, \ldots, q_n) = \sum_i C_i(q_i)$. Can we derive an expression for L_i for each i?

A definition that will prove useful:

$$\epsilon_{ij} = \frac{\partial D_j}{\partial p_i} \times \frac{p_i}{D_j}$$

The FOC becomes

$$\frac{p_i - C_i'}{p_i} = -\frac{1}{\epsilon_{ii}} - \sum_{j \neq i} \frac{(p_j - C_j')D_j \epsilon_{ij}}{p_i D_i \epsilon_{ii}}$$

- **Case 1a: Goods are substitutes**: Then, $\epsilon_{ij} > 0 \; \forall i \neq j$. In this case, $L_i > -\frac{1}{\epsilon_{ii}}$.
- **Case 1b: Goods are complements**: Then, $\epsilon_{ij} < 0 \; \forall i \neq j$. In this case, $L_i < -\frac{1}{\epsilon}$. *Example*: Loss leader pricing

Case 2: Separable demands, inseparable costs

Suppose $q_i = D_i(p_i)$ for each i.

Example: Learning by doing in a two-period game.

- At $t = 1$, cost is $C_1(q_1)$.
- At $t = 2$, cost is $C_2(q_2, q_1)$, where $\partial C_2 / \partial q_1 < 0$.

Case 2: Separable demands, inseparable costs

Suppose $q_i = D_i(p_i)$ for each i.

Example: Learning by doing in a two-period game.

- At $t = 1$, cost is $C_1\left(q_1\right)$.
- At $t = 2$, cost is $C_2\left(q_2, q_1\right)$, where $\partial C_2 / \partial q_1 < 0$.
- **Question**: What do you predict happens here?

Case 2: Separable demands, inseparable costs

Suppose $q_i = D_i(p_i)$ for each i.

Example: Learning by doing in a two-period game.

- At $t = 1$, cost is $C_1\left(q_1\right)$.
- At $t = 2$, cost is $C_2\left(q_2, q_1\right)$, where $\partial C_2 / \partial q_1 < 0$.
- **Question**: What do you predict happens here?
- Profit is $p_1 D_1(p_1) - C_1\left(D_1(p_1)\right) + \delta\left[p_2 D_2(p_2) - C_2\left(D_2(p_2), D_1(p_1)\right)\right]$.

Case 2: Separable demands, inseparable costs

Suppose $q_i = D_i(p_i)$ for each i.

Example: Learning by doing in a two-period game.

- At $t = 1$, cost is $C_1(q_1)$.
- At $t = 2$, cost is $C_2(q_2, q_1)$, where $\partial C_2 / \partial q_1 < 0$.
- **Question**: What do you predict happens here?
- Profit is $p_1 D_1(p_1) - C_1(D_1(p_1)) + \delta [p_2 D_2(p_2) - C_2(D_2(p_2), D_1(p_1))]$.
- FOC wrt p_1:

$$
\begin{aligned}
D_1 + p_1 D_1' &= C_1' D_1' + \delta \left(\partial C_2 / \partial q_1 \right) D_1' \\
L_1 &= -\frac{1}{\epsilon_1} + \delta \frac{1}{p_1} \frac{\partial C_2}{\partial q_1} < -\frac{1}{\epsilon_1}
\end{aligned}
$$

Durable goods monopolist

Setup

- Many goods last more than one “period”. **Question**: What does this mean for the monopolist?

Setup

- Many goods last more than one "period". **Question**: What does this mean for the monopolist?
- The monopolist tomorrow is "in competition with" the monopolist today.
- Suppose there are measure 1 consumers, whose lifetime discounted utility from a good is $v \sim U[0, 1]$.
  - $\rightarrow$ **Question**: What is the demand function?

Setup

- Many goods last more than one “period”. **Question**: What does this mean for the monopolist?
- The monopolist tomorrow is “in competition with” the monopolist today.
- Suppose there are measure 1 consumers, whose lifetime discounted utility from a good is $v \sim U[0,1]$.
  - $\rightarrow$ **Question**: What is the demand function? $D(p) = 1 - p$.

Setup

- Many goods last more than one “period”. **Question**: What does this mean for the monopolist?
- The monopolist tomorrow is “in competition with” the monopolist today.
- Suppose there are measure 1 consumers, whose lifetime discounted utility from a good is $v \sim U[0, 1]$.
  - → **Question**: What is the demand function? $D(p) = 1 - p$.
- Time is discrete, $t = 1, 2, \ldots$. Discount rate is δ.

Setup

- Many goods last more than one "period". **Question**: What does this mean for the monopolist?
- The monopolist tomorrow is "in competition with" the monopolist today.
- Suppose there are measure 1 consumers, whose lifetime discounted utility from a good is $v \sim U[0, 1]$.
  - → **Question**: What is the demand function? $D(p) = 1 - p$.
- Time is discrete, $t = 1, 2, \ldots$. Discount rate is δ.
- If the firm disappeared after $t = 1$, then at $t = 1$, the firm charges $p^m =$ **?**

Setup

- Many goods last more than one “period”. **Question**: What does this mean for the monopolist?
- The monopolist tomorrow is “in competition with” the monopolist today.
- Suppose there are measure 1 consumers, whose lifetime discounted utility from a good is $v \sim U[0, 1]$.
  - $\rightarrow$ **Question**: What is the demand function? $D(p) = 1 - p$.
- Time is discrete, $t = 1, 2, \ldots$. Discount rate is δ.
- If the firm disappeared after $t = 1$, then at $t = 1$, the firm charges $p^m = \frac{1}{2}$ and sells $q^m = \frac{1}{2}$.

Setup

- Many goods last more than one "period". **Question**: What does this mean for the monopolist?
- The monopolist tomorrow is "in competition with" the monopolist today.
- Suppose there are measure 1 consumers, whose lifetime discounted utility from a good is $v \sim U[0, 1]$.
  - $\rightarrow$ **Question**: What is the demand function? $D(p) = 1 - p$.
- Time is discrete, $t = 1, 2, \ldots$. Discount rate is δ.
- If the firm disappeared after $t = 1$, then at $t = 1$, the firm charges $p^m = \frac{1}{2}$ and sells $q^m = \frac{1}{2}$.
- If the monopolist (is surprised to find that she) continues to exist at $t = 2$, faces *residual demand* $D(p) =$ **?**

Setup

- Many goods last more than one "period". **Question**: What does this mean for the monopolist?
- The monopolist tomorrow is "in competition with" the monopolist today.
- Suppose there are measure 1 consumers, whose lifetime discounted utility from a good is $v \sim U[0,1]$.
  - $\rightarrow$ **Question**: What is the demand function? $D(p) = 1 - p$.
- Time is discrete, $t = 1, 2, \ldots$. Discount rate is δ.
- If the firm disappeared after $t = 1$, then at $t = 1$, the firm charges $p^m = \frac{1}{2}$ and sells $q^m = \frac{1}{2}$.
- If the monopolist (is surprised to find that she) continues to exist at $t = 2$, faces *residual demand* $D(p) = \frac{1}{2} - p$ and charges $p^m =$ **?**

Setup

- Many goods last more than one "period". **Question**: What does this mean for the monopolist?
- The monopolist tomorrow is "in competition with" the monopolist today.
- Suppose there are measure 1 consumers, whose lifetime discounted utility from a good is $v \sim U[0,1]$.
  - $\rightarrow$ **Question**: What is the demand function? $D(p) = 1 - p$.
- Time is discrete, $t = 1, 2, \ldots$. Discount rate is δ.
- If the firm disappeared after $t = 1$, then at $t = 1$, the firm charges $p^m = \frac{1}{2}$ and sells $q^m = \frac{1}{2}$.
- If the monopolist (is surprised to find that she) continues to exist at $t = 2$, faces *residual demand* $D(p) = \frac{1}{2} - p$ and charges $p^m = \frac{1}{4}$ and sells $q^m = \frac{1}{4}$.

Setup

- Many goods last more than one “period”. **Question**: What does this mean for the monopolist?
- The monopolist tomorrow is “in competition with” the monopolist today.
- Suppose there are measure 1 consumers, whose lifetime discounted utility from a good is $v \sim U[0,1]$.
  - $\rightarrow$ **Question**: What is the demand function? $D(p) = 1 - p$.
- Time is discrete, $t = 1, 2, \ldots$. Discount rate is δ.
- If the firm disappeared after $t = 1$, then at $t = 1$, the firm charges $p^m = \frac{1}{2}$ and sells $q^m = \frac{1}{2}$.
- If the monopolist (is surprised to find that she) continues to exist at $t = 2$, faces *residual demand* $D(p) = \frac{1}{2} - p$ and charges $p^m = \frac{1}{4}$ and sells $q^m = \frac{1}{4}$.
- **Question**: What might consumers do?

Setup

- Many goods last more than one "period". **Question**: What does this mean for the monopolist?
- The monopolist tomorrow is "in competition with" the monopolist today.
- Suppose there are measure 1 consumers, whose lifetime discounted utility from a good is $v \sim U[0, 1]$.
  - $\rightarrow$ **Question**: What is the demand function? $D(p) = 1 - p$.
- Time is discrete, $t = 1, 2, \ldots$. Discount rate is δ.
- If the firm disappeared after $t = 1$, then at $t = 1$, the firm charges $p^m = \frac{1}{2}$ and sells $q^m = \frac{1}{2}$.
- If the monopolist (is surprised to find that she) continues to exist at $t = 2$, faces *residual demand* $D(p) = \frac{1}{2} - p$ and charges $p^m = \frac{1}{4}$ and sells $q^m = \frac{1}{4}$.
- **Question**: What might consumers do? If consumers anticipate that $p_2 = \frac{1}{4}$, they may decide to wait. And if the monopolist were not myopic, she'd anticipate this waiting....

Durable goods with commitment

- Suppose the monopolist can commit the a path of future prices.
- Can't do better than $p_t = p^m$ for all t. [Bulow (1982)]

Durable goods with commitment

- Suppose the monopolist can commit the a path of future prices.
- Can't do better than $p_t = p^m$ for all t. [Bulow (1982)]
- **Question**: How likely is it that the monopolist actually has commitment power?

Coase Conjecture

Consider an extreme commitment problem. Suppose the monopolist has no commitment power, can set prices at $t = 0, \Delta T, 2\Delta T, 3\Delta T, \ldots$, and consumers are fully rational and get gross utility θe^{-rt} if they purchase at t.

Coase Conjecture: Under some conditions the monopolist's profits go to zero as $\Delta T \to 0$.

Intuition:

- Suppose that $\theta \sim U(0, 1)$ and $c \in [0, 1]$
- Suppose that in the limiting time path of prices is not an immediate drop to zero.
- Hence, prices must be dropping linearly in T at some point and quantities must also be proportional to ΔT.
- Suppose the monopolist jumps ahead in its price sequence and charges $p_{r+\Delta T}$ instead of p_r.
- The gain from having all sales occur ΔT earlier is first order in ΔT.
- The loss from earning less on the sales at time t is second order in ΔT the price difference and quantity on which you get the lower price are both of order ΔT. Hence, it is better to jump ahead and cut prices faster.

If Coase is right, then monopolies are fine I guess?

If Coase is right, then monopolies are fine I guess?

- The required conditions are actually somewhat narrow: Whether the result is true depends on whether the lower-bound of the value distribution is above or below c.

If Coase is right, then monopolies are fine I guess?

- The required conditions are actually somewhat narrow: Whether the result is true depends on whether the lower-bound of the value distribution is above or below c.
- Durable goods producers don't seem to earn zero. **Why? What was Coase missing?**

If Coase is right, then monopolies are fine I guess?

- The required conditions are actually somewhat narrow: Whether the result is true depends on whether the lower-bound of the value distribution is above or below c.
- Durable goods producers don't seem to earn zero. **Why? What was Coase missing?**
  - Delays/costs of changing prices (unlikely to be explanation)
  - Reputation
  - Inflows of new high willingness-to-pay customers
  - Per-period fixed costs
  - Strategic actions:
    - Rent rather than sell (but can run into moral hazard or antitrust problems)
    - Most favored customer contracts or money back guarantee
    - Destroy/limit ability to produce
    - Convince consumers that marginal cost is higher than it really is

Product quality

Monopoly and Product Quality

Suppose the monopolist also chooses the quality $s \in \mathbb{R}$ of its good. Suppose it has a constant marginal cost $c(s)$ with $c'(s) > 0$, i.e. $C(q, s) = qc(s)$. Suppose a unit mass of consumers with types $\theta \sim U(0, 1)$ have unit demands:
Utility from one unit is $v(s; \theta) - p$ where $v_s(s; \theta) > 0$ and $v_\theta(s; \theta) > 0$. If the firm sells q units, it will sell to consumers with $\theta \in [1 - q, 1]$. Hence, the price at which it can sell q units of quality s is $v(s; 1 - q)$.
The monopolist solves:

$$\max_{q,s} q[v(s; 1 - q) - c(s)]$$

FOC:

$$\frac{\partial v}{\partial s}(s_m; 1 - q_m) = \frac{\partial c}{\partial s}(s_m)$$

The marginal cost of increasing s equals the marginal benefit to the marginal consumer.

Product Quality: Monopoly versus First-Best

Monopolist FOC: (Chooses optimal quality for the *marginal* consumer)

$$\frac{\partial v}{\partial s}(s_m; 1 - q_m) = \frac{\partial c}{\partial s}(s_m)$$

First-Best FOC: (Chooses optimal quality for the *average* consumer)

$$\int_{v^{-1}(s^*, c(s^*))}^{1} \left[\frac{\partial v}{\partial s}(s^*; \theta) - \frac{\partial c}{\partial s}(s^*) \right] d\theta = 0$$

Product Quality: Monopoly versus First-Best

Monopolist FOC: (Chooses optimal quality for the *marginal* consumer)

$$\frac{\partial v}{\partial s}(s_m; 1 - q_m) = \frac{\partial c}{\partial s}(s_m)$$

First-Best FOC: (Chooses optimal quality for the *average* consumer)

$$\int_{v^{-1}(s^*, c(s^*))}^{1} \left[\frac{\partial v}{\partial s}(s^*; \theta) - \frac{\partial c}{\partial s}(s^*)\right] d\theta = 0$$

Question: Does the monopolist choose higher or lower s than the social planner?

Product Quality: Monopoly versus First-Best

Monopolist FOC: (Chooses optimal quality for the *marginal* consumer)

$$\frac{\partial v}{\partial s}(s_m; 1 - q_m) = \frac{\partial c}{\partial s}(s_m)$$

First-Best FOC: (Chooses optimal quality for the *average* consumer)

$$\int_{v^{-1}(s^*, c(s^*))}^{1} \left[\frac{\partial v}{\partial s}(s^*; \theta) - \frac{\partial c}{\partial s}(s^*) \right] d\theta = 0$$

Question: Does the monopolist choose higher or lower s than the social planner?

Takeaway: Quality will almost certainly be distorted, but we have no general prediction on the direction of the distortion.