Lecture 4: Static Competition

ECON 7510
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Slides draw upon lecture materials from Glenn & Sara Ellison (MIT).

Introduction

- Strategic interaction among firms involves many decision variables. They differ in the longevity of their effects.

- Today, we'll discuss the most short-run of these, focusing on the determination of prices and markups holding technologies and market structure fixed.

- We focus on markups because deadweight loss is an important welfare concern, but will also highlight other welfare considerations.

- We'll start with a quick review of classic models and then spend more time on the differentiated product demand models that are now most commonly used.

Competition with undifferentiated goods

Two models of competition for undifferentiated goods

1. Cournot competition: Quantity choice

2. Bertrand competition: Price choice

Cournot Competition (1838)

- N firms
- Inverse demand $P(X)$ for homogeneous good
- Cost functions $c_i(x_i)$
- Firms simultaneously choose outputs $x_1, x_2, \ldots, x_N$. Price is $P(\sum_i x_i)$

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- Literal interpretation might be appropriate for commodities like wheat, natural gas, iron ore.
- A more widely applicable interpretation: reduced form for a situation where firms choose capacities of factories that will always run at full capacity.
- Not a go-to empirical model these days, but still potentially useful for thinking about the effects of competition/market power.

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Nash equilibrium FOC: If $(x_1^*, \ldots, x_N^*)$ is a NE then

$$\left[P\left(\sum_i x_i^*\right) - c_i'(x_i^*)\right] + P'\left(\sum_i x_i^*\right) x_i^* = 0 \text{ for all } i \text{ with } x_i^* > 0$$

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A useful interpretation: i is a monopolist, facing the *residual demand curve*:

- Let $X_{-i} = \sum_{j \neq i} x_j$.
- Then, let $\tilde{P}(x_i) = P(x_i + X_{-i})$.
- Firm i's problem:

$$\max_{x_i} \left[\tilde{P}(x_i) - c_i(x_i)\right] x_i$$

- FOC:

$$\frac{\tilde{P}(x_i) - c'(x_i)}{\tilde{P}(x_i)} = -\tilde{P}'(x_i)\frac{x_i}{\tilde{P}(x_i)} = -\frac{1}{\tilde{\epsilon}}$$

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3. The industry-wide Lerner index is $\frac{P - \sum_i \frac{x_i^*}{X^*} c_i'(x_i^*)}{P} = -\frac{H}{\epsilon}$, where $H = \sum_i \left(\frac{x_i^*}{X^*}\right)^2$ is the industry "Herfindahl Index".

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6. Firm outputs are *usually* "strategic substitutes": $\frac{\partial BR_i}{\partial x_{-i}} < 0$ where $BR_i = x_i^*(X_{-i})$. **Question**: How would we prove this? What conditions on P'' and C'' are required?

Bertrand Competition (1883)

- 2 firms (could be N)
- $X(p)$ is market demand function. Assume $X(p)$ is weakly decreasing and $pX(p)$ is bounded.
- Symmetric, constant marginal costs c
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Bertrand is an "exemplifying theory." It illustrates forces using extreme assumptions that we would not see in practice.

- No product differentiation creates infinitely elastic firm-level demand
- Constant returns to scale with no capacity constraints
- One-shot interaction

With asymmetric costs, $c_1 < c_2$, an equilibrium is $p_1^* = p_2^* = c_2$ with all consumers purchasing from firm 1.

Competition with differentiated goods

Hotelling Competition (1929)

- Continuum of consumers with types $\theta \sim U[0, 1]$ have unit demands
- Utility is $v - t\theta - p_1$ if buy from firm 1, $v - t(1 - \theta) - p_2$ if buy from firm 2, and 0 if they don't buy.
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If v is sufficiently large relative to p_1, p_2, then all consumers will purchase from one firm or the other. The indifferent type has

$$v - t\theta - p_1 = v - t(1-\theta) - p_2 \Rightarrow \theta = \frac{1}{2} + \frac{p_2 - p_1}{2t}$$

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Assuming that equilibrium prices are sufficiently low relative to v so that this case applies:

$$BR_i(p_j) = \arg\max_p \, (p - c) \left(\frac{1}{2} + \frac{p_j - p}{2t} \right)$$

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$$\Rightarrow BR_i(p_j) = \frac{1}{2}(c + t + p_j)$$

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- **Question**: Markups are proportional to t. How do we interpret that?
- **Question**: What if both firms observe each consumer's θ?
- In an N-firm circular version markups decline like $1/N$ as in Cournot.
- Actions are “strategic complements”: firms increase prices when rivals increase prices.

Vertical Differentiation

- Firms L and H produce goods of quality s_L and s_H, respectively, with $s_L < s_H$.
- Consumers with types $\theta \sim U[\underline{\theta}, \bar{\theta}]$ have unit demands with utility $u_i(\theta) = \theta s_i - p_i$ if buy from i and 0 from outside good. For simplicity assume mass $\bar{\theta} - \underline{\theta}$ of consumers.
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Given prices p_L, p_H, let $\hat{\theta}_{LH}$ be the solution to $u_L(\hat{\theta}_{LH}) = u_H(\hat{\theta}_{LH})$, and let θ_{0L} be the solution to $u_L(\theta_{0L}) = 0$.
When $\theta_{0L} < \underline{\theta} < \hat{\theta}_{LH}$, demands are given by

$$D_H(p_L, p_H) = \bar{\theta} - \hat{\theta}_{LH} = \bar{\theta} - \frac{p_H - p_L}{s_H - s_L}$$

$$D_L(p_L, p_H) = \hat{\theta}_{LH} - \underline{\theta} = \frac{p_H - p_L}{s_H - s_L} - \underline{\theta}$$

Vertical Differentiation

Again, finding BRs and NE is easy with linear demand curves:

$$BR_H(p_L) = \arg\max_p \, (p - c)\left(\bar{\theta} - \frac{p - p_L}{s_H - s_L}\right) = \frac{1}{2}\left(p_L + c + \bar{\theta}(s_H - s_L)\right)$$

$$BR_L(p_H) = \frac{1}{2}\left(p_H + c - \underline{\theta}(s_H - s_L)\right)$$

The solution to these is the NE provided $\bar{\theta} \geqslant 2\underline{\theta}$ and $\frac{c+\bar{\theta}-2\underline{\theta}}{3(s_H - s_L)} < \underline{\theta} s_L$:

$$p_L^* = c + \frac{\bar{\theta} - 2\underline{\theta}}{3(s_H - s_L)} \quad p_H^* = c + \frac{2\bar{\theta} - \underline{\theta}}{3(s_H - s_L)}$$

Notes:

1. Vertical differentiation also creates finite elasticities and positive markups.
2. Firm H sets a higher price and earns higher profits.
3. When $\underline{\theta}$ and $\bar{\theta}$ are too close together firm L is shut out of the market.

Empirical relevance

Suppose we wanted to take a model of competition to the data.

- What issues might we encounter using the Hotelling model empirically?
- What issues might we encounter using the vertical differentiation model empirically?

More flexible class of models

Variants of Hotelling's model (sometimes with some vertical differentiation as well) have become the dominant approach in empirical IO.

The standard N-firm implementation assumes consumers have an $N+1$ dimensional type $(\epsilon_{i0}, \epsilon_{i1}, \ldots, \epsilon_{iN})$ with joint CDF F and utility is

$$u_{ij} = \begin{cases} v_j - \alpha p_j + \epsilon_{ij} & \text{if } i \text{ purchases good } j \\ \epsilon_{i0} & \text{if } i \text{ consumes "outside good"} \end{cases}$$

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Demand in this model in the general case is given by an N-dimensional integral:

$$x_j(p_1, \ldots, p_N) = \int_{\{\epsilon_{i0}, \epsilon_{i1}, \ldots, \epsilon_{iN} | u_{ij} > u_{ik} \forall k \neq j\}} dF(\epsilon_{i0}, \epsilon_{i1}, \ldots, \epsilon_{iN})$$

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Empirical papers sometimes approximate this by simulating draws of the ϵ_{ik}. A more tractable special case is when there is no outside good, the v_j are all equal, and the ϵ_{ik} are iid with density f. Demand when others all charge p is a one-dimensional integral:

$$x_i(p_i, p, \ldots, p) = \int \left[1 - F\left(\theta + \frac{p_i - p}{(N-1)F(\theta)^{N-2} f(\theta)}\right)\right] d\theta$$

Horizontal Differentiation

Perloff and Salop (REStud 1985) analyze this symmetric model and show:
Proposition: In this model the symmetric NE prices are

$$p_N^* = c + \frac{1}{M(N)} \frac{1}{\alpha}$$

with $M(N) = N(N-1) \int_{-\infty}^{\infty} F(\epsilon)^{N-2} f(\epsilon)^2 d\epsilon$

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2. If ϵ is bounded above or $\lim_{\epsilon\to\infty} \frac{f'(\epsilon)}{f(\epsilon)} = -\infty$, then $\lim_{N\to\infty} p_N^* = c$.

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2. If ϵ is bounded above or $\lim_{\epsilon \to \infty} \frac{f'(\epsilon)}{f(\epsilon)} = -\infty$, then $\lim_{N \to \infty} p_N^* = c$.
3. In the "logit" model, $F(\epsilon) = e^{-e^{-(\epsilon+\gamma)}}$, $p_N^* = c + k \left(1 + \frac{1}{N-1}\right) \frac{1}{\alpha}$ for some constant k.

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The merger creates "upward pricing pressure" for product i if

$$(p_j - c_j - \Delta c_j)\underbrace{\left(-\frac{\frac{\partial x_j}{\partial p_i}}{\frac{\partial x_i}{\partial p_i}}\right)}_{\text{Diversion ratio}} + \Delta c_i > 0$$

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Question: What does diversion ratio look like in the case of logit demand? For simplicity, suppose there are N symmetric firms pre-merger.

Next time

Dynamic competition