Lecture 3: Monopoly price discrimination

ECON 7510
Cornell University
Adam Harris

Slides draw upon lecture materials from Glenn & Sara Ellison (MIT).

Example


almohada holandesa
Dutch Flag Pillow
9,99 EUR
almohada francesa
French Flag Pillow
15,99 EUR

Price Discrimination

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Question: Why are we talking about price discrimination in the "monopoly" part of the course? Price discrimination requires market power. Otherwise $p_i = c_i$ for all i.

Pigou's Types of Price Discrimination

Pigou (1920) distinguished between three types of price discrimination:

- **First degree**: **Perfect** price discrimination.
- **Second degree**:
- **Third degree**:

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→ FOC: $P_i(x_i^*) = c$

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Question: In reality, what factors might limit firms' ability to do this kind of price discrimination?

- Non-observability of consumer preferences
- Arbitrage (resale)
- Administrative costs

In some cases these factors can completely eliminate the ability to discriminate.

Factors limiting discrimination: Resale

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- Student tickets for movies, theater, etc.
- Airline tickets: Price depends on how long in advance you book, as well as day of week, browser, operating system, etc.
- Retail: Different prices at stores in richer versus poorer neighborhoods.

Third-Degree Discrimination

Now suppose monopolist can distinguish classes of consumers, but is limited to simple linear pricing (unit price p_i) within each group.

- Two groups $i = 1, 2$
- Demands $X_i(p_i)$
- Constant marginal cost c

With discrimination:

$$\max_{p_1, p_2} \sum_{i=1,2} (p_i - c) X_i(p_i)$$

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$\Rightarrow \frac{p_i^* - c}{p_i^*} = -\frac{1}{\epsilon_i(p_i^*)}$

Label the markets so that $p_1^* < p_2^*$.

Like monopoly pricing in two completely separate markets.

Backlash to Third-Degree Price Discrimination

UNIFORM PRICING IN U.S. RETAIL CHAINS*

STEFANO DELLAVIGNA AND MATTHEW GENTZKOW

We show that most U.S. food, drugstore, and mass-merchandise chains charge nearly uniform prices across stores, despite wide variation in consumer demographics and competition. Demand estimates reveal substantial within-chain variation in price elasticities and suggest that the median chain sacrifices $16 million of annual profit relative to a benchmark of optimal prices. In contrast, differ-

This could arise if consumers who observe different prices for the same item in multiple stores perceive this as unfair or a breach of an implicit contract. In a report on UK grocery pricing, the Competition Commission (2003) writes:

> Asda said that it would be commercial suicide for it to move away from its highly publicized national EDLP pricing strategy and a breach of its relationship of trust with its customers, and it would cause damage to its brand image, which was closely associated with a pricing policy that assured the lowest prices always.

Banning Third-Degree Price Discrimination?

FTC Issues Orders to Eight Companies Seeking Information on Surveillance Pricing

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July 23, 2024

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Question: What's your intuition about the effects of a ban on third-degree price discrimination?

Banning Third-Degree Price Discrimination?

If we ban discrimination:

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Two cases:

1. If $\pi_i(p) \equiv (p-c)X_i(p)$ is single-peaked and concave in p for $p \leqslant p_2^*$, then $p^* \in (p_1^*, p_2^*)$.
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   In this case, no one is better off when discrimination is banned.

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- Deadweight loss
- Misallocation of goods sold, which might result from discriminatory and nonlinear pricing schemes

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Notation:

- $(\bar{p}, \bar{q}_i)$ are price and quantity for market i under uniform pricing
- (p_i, q_i) are price and quantity for market i under 3DPD

Proof: Welfare for market i is

$$W_i = \underbrace{S_i(p_i)}_{\text{CS}} + \underbrace{(p_i - c)\, q_i}_{\text{Profit}}$$

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Effect of price discrimination relative to uniform pricing:

$$\Delta W = \sum_i [S_i(p_i) - S_i(\bar{p})] + \left(\sum_i (p_i - c) q_i - \sum_i (\bar{p} - c) \bar{q}_i \right)$$

Recall that $S'(p) = -D(p)$, so S is convex in p. Everywhere above its tangents, so

$$S_i(p_i) > S_i(\bar{p}) + S_i'(\bar{p})\,(p_i - \bar{p})$$

Third-Degree Discrimination: Welfare Effects

$$\begin{aligned}\sum_i (p_i - c)(q_i - \bar{q}) < \Delta W < \sum_i (\bar{p} - c)(q_i - \bar{q}) \\ = (\bar{p} - c) \sum_i (q_i - \bar{q}_i) \\ \lesseqgtr 0 \text{ if } \sum_i q_i \lesseqgtr \sum_i \bar{q}_i\end{aligned}$$

So if discrimination doesn't increase output, it doesn't increase welfare.

Questions:

1. What do we think in general? Would we expect 3DPD to increase or decrease welfare? Examples?

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1. What do we think in general? Would we expect 3DPD to increase or decrease welfare? Examples? Can go either way. A clear example of increasing welfare is if the monopolist will only serve high types under uniform pricing.
2. What about equity concerns? Could go either way: financial aid vs health or life insurance

Third-Degree Discrimination: Welfare Effects

Corollary:

If demands are linear and all markets are served under uniform pricing, then 3DPD

You'll consider this case in Problem Set 1.

Second-degree price discrimination

Second-Degree Discrimination: Two Type Model

Suppose the monopolist cannot observe consumer preferences but sells a good of variable quality/quantity and can prevent resale between consumers.

- Consumers of type θ get utility $v(q, \theta) - T$ if they buy quality/quantity q at total price T, and utility 0 if they do not purchase.

Question: Why is it okay to think of q as either quality or quantity?

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Utilities satisfy:

$$\frac{\partial v}{\partial q} > 0, \quad \frac{\partial v}{\partial \theta} > 0$$
$$\frac{\partial^2 v}{\partial \theta \partial q} > 0, \quad \frac{\partial^2 v}{\partial q^2} < 0$$

Second-Degree Discrimination: Two Type Model (Cont.)

- Assume the cost of producing a quality q good/producing quantity q is cq.
- Assume there are just two types with $\theta_2 > \theta_1$, and write $v_i(q)$ for $v(q, \theta_i)$.

If θ were observable, this would be a first-degree discrimination model.

Second-Degree Discrimination: Two Type Model (Cont.)

With θ unobservable, the monopolist will want to allow consumers to choose (q, T) from a menu of offers.

- **Question**: In the two-type case, do we need to think of the monopolist as offering a full menu (continuum of options)?

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The monopolist's profit-maximization problem is:

$$\max_{q_1, T_1, q_2, T_2} T_1 + T_2 - c(q_1 + q_2)$$

subject to: **What constraints?** In other words, what prevents $T_1, T_2 = \infty$ and $q_1, q_2 = 0$?

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subject to:

$$\begin{aligned}
(IR1) \quad & v_1(q_1) - T_1 \geqslant 0 \\
(IR2) \quad & v_2(q_2) - T_2 \geqslant 0 \\
(IC1) \quad & v_1(q_1) - T_1 \geqslant v_1(q_2) - T_2 \\
(IC2) \quad & v_2(q_2) - T_2 \geqslant v_2(q_1) - T_1
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Second-Degree Discrimination: Two Type Model (Cont.)

$$\max_{q_1, T_1, q_2, T_2} T_1 + T_2 - c(q_1 + q_2)$$

subject to:

$$\begin{array}{ll} (IR1) & v_1(q_1) - T_1 \geqslant 0 \\ (IR2) & v_2(q_2) - T_2 \geqslant 0 \\ (IC1) & v_1(q_1) - T_1 \geqslant v_1(q_2) - T_2 \\ (IC2) & v_2(q_2) - T_2 \geqslant v_2(q_1) - T_1 \end{array}$$

The first step in solving problems like this is to figure out which constraints are binding.
Question: Which two do you think will not bind?

Second-Degree Discrimination: Two Type Model (Cont.)

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\end{aligned}$$

The first step in solving problems like this is to figure out which constraints are binding.
Question: Which two do you think will not bind?

- (IC2) + (IR1) imply (IR2).
- (IC1) seems unlikely to bind. Students rarely consider buying first-class tickets.

Second-Degree Discrimination: Two Type Model (Cont.)

The simplified problem is:

$$\max_{q_1, T_1, q_2, T_2} T_1 + T_2 - c(q_1 + q_2)$$

$$\begin{aligned} (IR1) \quad & v_1(q_1) - T_1 \geqslant 0 \\ (IC2) \quad & v_2(q_2) - T_2 \geqslant v_2(q_1) - T_1 \end{aligned}$$

Question: What else can we say about the two remaining constraints?

Second-Degree Discrimination: Two Type Model (Cont.)

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$$(IC2) \quad v_2(q_2) - T_2 \geqslant v_2(q_1) - T_1$$

Question: What else can we say about the two remaining constraints?

- Clearly, one wants to increase T_1 if (IR1) doesn't bind. This implies $T_1 = v_1(q_1)$.
- One will also want to increase T_2 if (IC2) doesn't bind. This implies:

$$T_2 = T_1 + (v_2(q_2) - v_2(q_1)) = v_1(q_1) + v_2(q_2) - v_2(q_1)$$

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Question: What now?

Second-Degree Discrimination: Two Type Model (Cont.)

The simplified problem is:

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Question: What now? With these T_1 and T_2, both constraints hold with equality. Substitute and then we have an unconstrained problem.

Question: Before we substitute, what welfare conclusions can we draw from these equalities?

Second-Degree Discrimination: Two Type Model (Cont.)

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Question: What now? With these T_1 and T_2, both constraints hold with equality. Substitute and then we have an unconstrained problem.

Question: Before we substitute, what welfare conclusions can we draw from these equalities? Low type gets zero surplus. High type gets *information rents*.

Second-Degree Discrimination: Two Type Model (Cont.)

Substitution gives an unconstrained optimization problem:

$$\max_{q_1,q_2} v_1(q_1) + v_2(q_2) + [v_1(q_1) - v_2(q_1)] - c(q_1 + q_2)$$

First order conditions give:

$$v_2'(q_2) = c.$$

$$v_1'(q_1) + [v_1'(q_1) - v_2'(q_1)] = c.$$

Second-Degree Discrimination: Two Type Model (Cont.)

Substitution gives an unconstrained optimization problem:

$$\max_{q_1,q_2} v_1(q_1) + v_2(q_2) + [v_1(q_1) - v_2(q_1)] - c(q_1 + q_2)$$

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$$v_1'(q_1) + [v_1'(q_1) - v_2'(q_1)] = c.$$

Question: What economic conclusions can we draw from these FOCs?

Second-Degree Discrimination: Two Type Model (Cont.)

Substitution gives an unconstrained optimization problem:

$$\max_{q_1,q_2} v_1(q_1) + v_2(q_2) + [v_1(q_1) - v_2(q_1)] - c(q_1 + q_2)$$

First order conditions give:

$$v_2'(q_2) = c.$$

$$v_1'(q_1) + \underbrace{[v_1'(q_1) - v_2'(q_1)]}_{<0} = c.$$

Question: What economic conclusions can we draw from these FOCs?

- High type's quality is undistorted.
- $v_1'(q_1) > v_1'(q_1*)$, so $q_1 < q_1^*$. So quality to the low type is inefficiently low.

Second-Degree Discrimination: Continuum of Types

Modern IO theory papers will work in a continuum-of-types model.

- Continuum of consumers with types θ with density $f(\theta)$ on $[\theta, \theta]$.
- Type θ consumer's gross utility from quality/quantity x is $v(x, \theta)$.

The model generalizes insights from the two-type model.

Next time

- **Problem Set 1**: Due W 9/18, but you now have everything you need to complete this.
- **No class on Monday (9/9)**
- **Next time**: Static competition