Lecture 3: Monopoly price discrimination

ECON 7510
Cornell University
Adam Harris

Example

almohada holandesa
Dutch Flag Pillow
9,99 EUR

almohada francesa
French Flag Pillow
15,99 EUR

— What is strange about this screenshot?

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— Why might these two "different" products be priced so differently?

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Price discrimination requires market power. Otherwise $p_i = c_i$ for all i.

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→ FOC: $P_i(x_i^*) = c$

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Question: In reality, what other factors might limit ability to do this kind of price discrimination?

— Arbitrage (resale)

— Administrative costs

In some cases these factors can completely eliminate the ability to discriminate.

Factors limiting discrimination: Resale

— Go back to assuming constant marginal cost c.

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   — $b = $ monopoly price.

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— Student tickets for movies, theater, etc.

— Airline tickets: Price depends on how long in advance you book, as well as day of week, browser, operating system, location, etc.

— Retail: Different prices at stores in richer versus poorer neighborhoods.

Third-Degree Discrimination

Suppose monopolist can distinguish classes of consumers, but is limited to simple linear pricing (unit price p_i) within each group.

— Two groups $i = 1, 2$

— Demands $D_i(p_i)$

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With discrimination:

$$\max_{p_1, p_2} \sum_{i=1,2} (p_i - c) D_i(p_i)$$

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$\Rightarrow \frac{p_i^* - c}{p_i^*} = -\frac{1}{\epsilon_i(p_i^*)}$

Label the markets so that $p_1^* < p_2^*$. (WLOG)

Like monopoly pricing in two completely separate markets.

Backlash to Third-Degree Price Discrimination

UNIFORM PRICING IN U.S. RETAIL CHAINS*

STEFANO DELLAVIGNA AND MATTHEW GENTZKOW

We show that most U.S. food, drugstore, and mass-merchandise chains charge nearly uniform prices across stores, despite wide variation in consumer demographics and competition. Demand estimates reveal substantial within-chain variation in price elasticities and suggest that the median chain sacrifices $16 million of annual profit relative to a benchmark of optimal prices. In contrast, differ-

This could arise if consumers who observe different prices for the same item in multiple stores perceive this as unfair or a breach of an implicit contract. In a report on UK grocery pricing, the Competition Commission (2003) writes:

> Asda said that it would be commercial suicide for it to move away from its highly publicized national EDLP pricing strategy and a breach of its relationship of trust with its customers, and it would cause damage to its brand image, which was closely associated with a pricing policy that assured the lowest prices always.

Banning Third-Degree Price Discrimination?

FTC Issues Orders to Eight Companies Seeking Information on Surveillance Pricing

Agency seeks information about products and services that use personal data, including finances and browser history, to set individualized prices for the same goods or services

July 23, 2024

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Question: What's your intuition about the effects of a ban on third-degree price discrimination?

Banning Third-Degree Price Discrimination?

If we ban discrimination:

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Question: Which case does linear demand fall into? Isoelastic demand?

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— Deadweight loss

— Misallocation of goods sold, which might result from discriminatory and nonlinear pricing schemes

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Proposition: If 3rd degree discrimination does not increase total output relative to uniform pricing, then it reduces social welfare.

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Notation:

- $(\bar{p}, \bar{q}_i)$ are price and quantity for market i under uniform pricing
- (p_i, q_i) are price and quantity for market i under 3DPD

Proof: Welfare for market i is

$$W_i = \underbrace{S_i(p_i)}_{\text{CS}} + \underbrace{(p_i - c)\, q_i}_{\text{Profit}}$$

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Effect of price discrimination relative to uniform pricing:

$$\Delta W = \sum_i \left[S_i(p_i) - S_i(\bar{p}) \right] + \left(\sum_i (p_i - c) q_i - \sum_i (\bar{p} - c) \bar{q}_i \right)$$

Recall that $S_i'(p) = -D_i(p)$, so S_i is convex in p. Everywhere above its tangents, so

$$S_i(\bar{p}) > S_i(p_i) + S_i'(p_i)\, (\bar{p} - p_i)$$

Third-Degree Discrimination: Welfare Effects

$$\sum_i (p_i - c)(q_i - \bar{q}) < \Delta W < \sum_i (\bar{p} - c)(q_i - \bar{q})$$

$$= (\bar{p} - c) \sum_i (q_i - \bar{q}_i)$$

$$\leqslant 0 \text{ if } \sum_i q_i \leqslant \sum_i \bar{q}_i$$

So if discrimination doesn't increase output, it doesn't increase welfare.

Questions:

1. What do we think in general? Would we expect 3DPD to increase or decrease welfare? Examples?

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2. What about equity concerns? Could go either way: financial aid vs health or life insurance

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So if discrimination doesn't increase output, it doesn't increase welfare.

Questions:

1. What do we think in general? Would we expect 3DPD to increase or decrease welfare? Examples? Can go either way. A clear example of increasing welfare is if the monopolist will only serve high types under uniform pricing.

2. What about equity concerns? Could go either way: financial aid vs health or life insurance

(Another one of the key ambiguous theoretical results I referenced in Lecture 1.)

Third-Degree Discrimination: Welfare Effects

Corollary:

If demands are linear and all markets are served under uniform pricing, then 3DPD

You'll consider this case in Problem Set 1.

Second-degree price discrimination

Second-Degree Discrimination: Two Type Model

Suppose the monopolist cannot observe consumer preferences but sells a good of variable quality/quantity and can prevent resale between consumers.

— Consumers of type θ get utility $v(q, \theta) - T$ if they buy quality/quantity q at total price T, and utility 0 if they do not purchase.

Question: Why is it okay to think of q as either quality or quantity?

Second-Degree Discrimination: Two Type Model

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— Consumers of type θ get utility $v(q, \theta) - T$ if they buy quality/quantity q at total price T, and utility 0 if they do not purchase.

Question: Why is it okay to think of q as either quality or quantity?

Utilities satisfy:

$$\frac{\partial v}{\partial q} > 0, \quad \frac{\partial v}{\partial \theta} > 0$$

$$\frac{\partial^2 v}{\partial \theta \partial q} > 0, \quad \frac{\partial^2 v}{\partial q^2} < 0$$

— Assume the cost of producing a quality q good/producing quantity q is cq.

— Assume there are just two types with $\theta_2 > \theta_1$, and write $v_i(q)$ for $v(q, \theta_i)$.

If θ were observable, this would be a first-degree discrimination model.

Second-Degree Discrimination: Two Type Model (Cont.)

With θ unobservable, the monopolist will want to allow consumers to choose (q, T) from a menu of offers.

— **Question**: In the two-type case, do we need to think of the monopolist as offering a full menu (continuum of options)?

Second-Degree Discrimination: Two Type Model (Cont.)

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— **Question**: In the two-type case, do we need to think of the monopolist as offering a full menu (continuum of options)? No, it suffices to offer a two-item menu $(q_1, T_1), (q_2, T_2)$. WLOG, suppose $\theta_2 > \theta_1$.

The monopolist's profit-maximization problem is:

Second-Degree Discrimination: Two Type Model (Cont.)

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The monopolist's profit-maximization problem is:

$$\max_{q_1, T_1, q_2, T_2} T_1 + T_2 - c(q_1 + q_2)$$

subject to: **What constraints?** In other words, what prevents $T_1, T_2 = \infty$ and $q_1, q_2 = 0$?

Second-Degree Discrimination: Two Type Model (Cont.)

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subject to:

$$
\begin{aligned}
(IR1) \quad & v_1(q_1) - T_1 \geqslant 0 \\
(IR2) \quad & v_2(q_2) - T_2 \geqslant 0 \\
(IC1) \quad & v_1(q_1) - T_1 \geqslant v_1(q_2) - T_2 \\
(IC2) \quad & v_2(q_2) - T_2 \geqslant v_2(q_1) - T_1
\end{aligned}
$$

Second-Degree Discrimination: Two Type Model (Cont.)

$$\max_{q_1, T_1, q_2, T_2} T_1 + T_2 - c(q_1 + q_2)$$

subject to:

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The first step in solving problems like this is to figure out which constraints are binding.
Question: Which two do you think will not bind?

Second-Degree Discrimination: Two Type Model (Cont.)

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The first step in solving problems like this is to figure out which constraints are binding.
Question: Which two do you think will not bind?

— (IC2) + (IR1) imply (IR2).

— (IC1) seems unlikely to bind. Students rarely consider buying first-class tickets.

Second-Degree Discrimination: Two Type Model (Cont.)

The simplified problem is:

$$\max_{q_1, T_1, q_2, T_2} T_1 + T_2 - c(q_1 + q_2)$$

$$(IR1) \quad v_1(q_1) - T_1 \geqslant 0$$
$$(IC2) \quad v_2(q_2) - T_2 \geqslant v_2(q_1) - T_1$$

Question: What else can we say about the two remaining constraints?

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The simplified problem is:

$$\max_{q_1, T_1, q_2, T_2} T_1 + T_2 - c(q_1 + q_2)$$

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$$(IC2) \quad v_2(q_2) - T_2 \geqslant v_2(q_1) - T_1$$

Question: What else can we say about the two remaining constraints?

— Clearly, one wants to increase T_1 if (IR1) doesn't bind. This implies $T_1 = v_1(q_1)$.

— One will also want to increase T_2 if (IC2) doesn't bind. This implies:

$$T_2 = T_1 + (v_2(q_2) - v_2(q_1)) = v_1(q_1) + v_2(q_2) - v_2(q_1)$$

Second-Degree Discrimination: Two Type Model (Cont.)

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Question: What now?

Second-Degree Discrimination: Two Type Model (Cont.)

The simplified problem is:

$$\max_{q_1, T_1, q_2, T_2} T_1 + T_2 - c(q_1 + q_2)$$

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Question: What now? With these T_1 and T_2, both constraints hold with equality. Substitute and then we have an unconstrained problem.

Question: Before we substitute, what welfare conclusions can we draw from these equalities?

Second-Degree Discrimination: Two Type Model (Cont.)

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Question: What now? With these T_1 and T_2, both constraints hold with equality. Substitute and then we have an unconstrained problem.

Question: Before we substitute, what welfare conclusions can we draw from these equalities? Low type gets zero surplus. High type gets *information rents*.

Second-Degree Discrimination: Two Type Model (Cont.)

Substitution gives an unconstrained optimization problem:

$$\max_{q_1, q_2} v_1(q_1) + v_2(q_2) + [v_1(q_1) - v_2(q_1)] - c(q_1 + q_2)$$

First order conditions give:

$v_2'(q_2) = c.$

$v_1'(q_1) + [v_1'(q_1) - v_2'(q_1)] = c.$

Second-Degree Discrimination: Two Type Model (Cont.)

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Question: What economic conclusions can we draw from these FOCs?

Second-Degree Discrimination: Two Type Model (Cont.)

Substitution gives an unconstrained optimization problem:

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First order conditions give:

$v_2'(q_2) = c.$

$v_1'(q_1) + \underbrace{[v_1'(q_1) - v_2'(q_1)]}_{<0} = c.$

Question: What economic conclusions can we draw from these FOCs?

— High type's quality is undistorted.

— $v_1'(q_1) > v_1'(q_1^*)$, so $q_1 < q_1^*$. So quality to the low type is inefficiently low.

Second-Degree Discrimination: Continuum of Types

Modern IO theory papers will work in a continuum-of-types model.

— Continuum of consumers with types θ with density $f(\theta)$ on $[\underline{\theta}, \bar{\theta}]$.

— Type θ consumer's gross utility from quality/quantity x is $v(x, \theta)$.

The model generalizes insights from the two-type model.

Looking ahead

— **Problem Set 1**: Due Th 9/11. You now have everything you need to complete this.

— **Next time**: Static competition