Lecture 2: Firms, producer theory, and monopoly pricing

ECON 7510
Cornell University
Adam Harris

Producer theory: A review

Context

- Here, we'll briefly review some material from ECON 6090.

- From the ECON 6090 lecture notes:

  Assumptions 3.1:

  (i) L commodities
  (ii) Production plan $y \in \mathbb{R}^L$
     - Net input: good i such that $y_i < 0$
     - Net output: good j such that $y_j > 0$
  (iii) Production possibility set, $Y \subseteq \mathbb{R}^L$ of feasible production plans
  (iv) Prices, $p \geqslant 0$, are unaffected by the activity of the firm.

Context

- Here, we'll briefly review some material from ECON 6090.
- From the ECON 6090 lecture notes:

  Assumptions 3.1:

  (i) L commodities
  (ii) Production plan $y \in \mathbb{R}^L$
    - Net input: good i such that $y_i < 0$
    - Net output: good j such that $y_j > 0$

  (iii) Production possibility set, $Y \subseteq \mathbb{R}^L$ of feasible production plans
  (iv) Prices, $p \geqslant 0$, are unaffected by the activity of the firm.

- To some extent, "core IO" is the study of 3.1.(iv) violations.

Context

- Here, we'll briefly review some material from ECON 6090.
- From the ECON 6090 lecture notes:

  Assumptions 3.1:

  (i) L commodities
  (ii) Production plan $y \in \mathbb{R}^L$
     - Net input: good i such that $y_i < 0$
     - Net output: good j such that $y_j > 0$
  (iii) Production possibility set, $Y \subseteq \mathbb{R}^L$ of feasible production plans
  (iv) Prices, $p \geqslant 0$, are unaffected by the activity of the firm.

- To some extent, "core IO" is the study of 3.1.(iv) violations.

  $\rightarrow$ Today, we'll think about what happens when this assumption holds and when it does not.

Technological feasibility

***Assumptions 3.1*:**

(i) L commodities

(ii) Production plan $y \in \mathbb{R}^L$

- Net input: good i such that $y_i < 0$
- Net output: good j such that $y_j > 0$

(iii) Production possibility set, $Y \subseteq \mathbb{R}^L$ of feasible production plans

(iv) Prices, $p \geqslant 0$, are unaffected by the activity of the firm.

***Assumptions 3.2*:**

(i) Y is nonempty, closed and (strictly) convex.

(ii) Free disposal: If $y \in Y$ and $y' \leqslant y$, then $y' \in Y$.

Single-output case: $f(z) = \max_q q$ s.t. $(-z, q) \in Y$

Efficiency

Definition: A production plan $y \in Y$ is *efficient* if there does not exist a $y' \in Y$ such that $y' \geqslant y$ and $y_i' > y_i$ for some i.

Profit maximization

General case:

$$\pi(p) \equiv \max_{y} p \cdot y \text{ subject to } y \in Y$$

Single-output case:

$$\pi(p, w) \equiv \max_{z \in \mathbb{R}^{L-1}} pf(z) - w \cdot z$$

Profit maximization

General case:

$$\pi(p) \equiv \max_{y} p \cdot y \text{ subject to } y \in Y$$

Single-output case:

$$\pi(p, w) \equiv \max_{z \in \mathbb{R}^{L-1}} pf(z) - w \cdot z$$

FOC:

Profit maximization

General case:

$$\pi(p) \equiv \max_y p \cdot y \text{ subject to } y \in Y$$

Single-output case:

$$\pi(p, w) \equiv \max_{z \in \mathbb{R}^{L-1}} pf(z) - w \cdot z$$

FOC:

$$\underbrace{p \nabla f(z)}_{\text{MRP}} = w \;\Rightarrow\; \underbrace{\frac{f_i(z)}{f_{i'}(z)}}_{\text{MRTS}} = \frac{w_i}{w_{i'}}$$

Profit maximization implies cost minimization

$$\pi(p, w) \equiv \max_{z \in \mathbb{R}^{L-1}} pf(z) - w \cdot z$$

Profit maximization implies cost minimization

$$
\begin{aligned}
\pi(p, w) &\equiv \max_{z \in \mathbb{R}^{L-1}} pf(z) - w \cdot z \\
&= \max_q \left[\max_{z \in \mathbb{R}^{L-1}} pq - w \cdot z \text{ s.t. } f(z) = q \right] \\
&= \max_q pq - \left[\underbrace{\min_{z \in \mathbb{R}^{L-1}} w \cdot z \text{ s.t. } f(z) = q}_{\text{CMP}} \right] \\
&= \max_q pq - C(w, q)
\end{aligned}
$$

Profit maximization (with product market power)

General case:

$$\pi(p) \equiv \max_{y} p(y) \cdot y \text{ subject to } y \in Y$$

Single-output case:

$$\pi(p, w) \equiv \max_{z \in \mathbb{R}^{L-1}} p(f(z))f(z) - w \cdot z$$

Profit maximization (with product market power)

General case:

$$\pi(p) \equiv \max_{y} p(y) \cdot y \text{ subject to } y \in Y$$

Single-output case:

$$\pi(p, w) \equiv \max_{z \in \mathbb{R}^{L-1}} p(f(z))f(z) - w \cdot z$$

FOC:

Profit maximization (with product market power)

General case:

$$\pi(p) \equiv \max_y p(y) \cdot y \text{ subject to } y \in Y$$

Single-output case:

$$\pi(p, w) \equiv \max_{z \in \mathbb{R}^{L-1}} p(f(z))f(z) - w \cdot z$$

FOC:

$$\underbrace{\left[p + p'(f(z))\right] \nabla f(z)}_{\text{MRP}} = w \Rightarrow \underbrace{\frac{f_i(z)}{f_{i'}(z)}}_{\text{MRTS}} = \frac{w_i}{w_{i'}}$$

Profit maximization implies cost minimization (with product market power)

$$
\begin{aligned}
\pi(w) &\equiv \max_{z \in \mathbb{R}^{L-1}} p(f(z))f(z) - w \cdot z \\
&= \max_{q} \left[\max_{z \in \mathbb{R}^{L-1}} p(q)q - w \cdot z \text{ s.t. } f(z) = q \right] \\
&= \max_{q} p(q)q - \left[\underbrace{\min_{z \in \mathbb{R}^{L-1}} w \cdot z \text{ s.t. } f(z) = q}_{\text{CMP}} \right] \\
&= \max_{q} p(q)q - C(w, q)
\end{aligned}
$$

Quantity choice under perfect competition

$$\pi(q) \equiv \max_{q} pq - C(q)$$

Quantity choice under perfect competition

$$\pi(q) \equiv \max_q pq - C(q)$$

FOC:

$$p = C'(q)$$

Price equals marginal cost. Zero profit on the marginal unit.

Monopoly pricing

Quantity choice:

$$\pi(q) \equiv \max_q p(q)q - C(q)$$

Monopoly pricing

Quantity choice:

$$\pi(q) \equiv \max_q p(q)q - C(q)$$

FOC:

$$\left[p(q^m) + p'(q^m)q^m\right] = C'(q^m)$$

Monopoly pricing

Quantity choice:

$$\pi(q) \equiv \max_{q} p(q)q - C(q)$$

FOC:

$$\left[p(q^m) + p'(q^m)q^m\right] = C'(q^m)$$

Marginal revenue equals marginal cost.

Monopoly pricing

Quantity choice:

$$\pi(q) \equiv \max_q p(q)q - C(q)$$

FOC:

$$\left[p(q^m) + p'(q^m)q^m\right] = C'(q^m)$$

Marginal revenue equals marginal cost.

$$\Rightarrow \; p(q^m) = C'(q^m) - \underbrace{p'(q^m)}_{<0} q^m$$

$$> c'(q^m)$$

Positive profit on the marginal unit.

Monopoly pricing

Quantity choice:

$$\pi(q) \equiv \max_q p(q)q - C(q)$$

FOC:

$$\left[p(q^m) + p'(q^m)q^m\right] = C'(q^m)$$

Marginal revenue equals marginal cost.

$$\Rightarrow \ p(q^m) = C'(q^m) - \underbrace{p'(q^m)}_{<0} q^m$$

$$> c'(q^m)$$

Positive profit on the marginal unit. *How much profit?*

Monopoly pricing

Equivalently, price choice: (Notation: $D = p^{-1}$.)

$$\pi(p) \equiv \max_p pD(p) - C\left(D(p)\right)$$

Monopoly pricing

Equivalently, price choice: (Notation: $D = p^{-1}$.)

$$\pi(p) \equiv \max_p pD(p) - C\left(D(p)\right)$$

FOC:

$$\left[p^m D'(p^m) + D(p^m)\right] = C'\left(D(p^m)\right) D'(p^m)$$

Monopoly pricing

Equivalently, price choice: (Notation: $D = p^{-1}$.)

$$\pi(p) \equiv \max_p pD(p) - C\left(D(p)\right)$$

FOC:

$$\left[p^m D'(p^m) + D(p^m)\right] = C'\left(D(p^m)\right) D'(p^m)$$

$$p^m - C'\left(D(p^m)\right) = -\frac{D(p^m)}{D'(p^m)}$$

$$\frac{p^m - C'\left(D(p^m)\right)}{p^m} = -\frac{D(p^m)}{D'(p^m)p^m}$$

"Lerner Index": $\quad L = -\frac{1}{\epsilon}$

Monopoly pricing

Equivalently, price choice: (Notation: $D = p^{-1}$.)

$$\pi(p) \equiv \max_{p} pD(p) - C\left(D(p)\right)$$

FOC:

$$\left[p^m D'(p^m) + D(p^m)\right] = C'\left(D(p^m)\right) D'(p^m)$$

$$p^m - C'\left(D(p^m)\right) = -\frac{D(p^m)}{D'(p^m)}$$

$$\frac{p^m - C'\left(D(p^m)\right)}{p^m} = -\frac{D(p^m)}{D'(p^m)p^m}$$

"Lerner Index": $\quad L = -\frac{1}{\epsilon}$

Question: What does this imply about perfectly elastic demand? Unit elastic demand?

Properties of monopoly pricing

- What point on the demand curve does monopolist choose?

Properties of monopoly pricing

— What point on the demand curve does monopolist choose?

$$p^m = \left(\frac{1+\epsilon}{\epsilon}\right) C'$$

$$p^m > 0 \Leftrightarrow \frac{1+\epsilon}{\epsilon} > 0$$

$$\Leftrightarrow \epsilon < -1$$

Elastic part of the demand curve.

$\rightarrow$ As long as demand is inelastic, $\frac{\partial \pi}{\partial p} > 0$, so increase price (i.e., decrease quantity) until you get to an elastic part of the demand curve.

Properties of monopoly pricing

- What point on the demand curve does monopolist choose?

$$p^m = \left(\frac{1+\epsilon}{\epsilon}\right) C'$$

$$p^m > 0 \Leftrightarrow \frac{1+\epsilon}{\epsilon} > 0$$

$$\Leftrightarrow \epsilon < -1$$

Elastic part of the demand curve.

$\rightarrow$ As long as demand is inelastic, $\frac{\partial \pi}{\partial p} > 0$, so increase price (i.e., decrease quantity) until you get to an elastic part of the demand curve.

- Inefficiency?

Properties of monopoly pricing

- What point on the demand curve does monopolist choose?

$$p^m = \left(\frac{1+\epsilon}{\epsilon}\right) C'$$
$$p^m > 0 \Leftrightarrow \frac{1+\epsilon}{\epsilon} > 0$$
$$\Leftrightarrow \epsilon < -1$$

  Elastic part of the demand curve.

  $\rightarrow$ As long as demand is inelastic, $\frac{\partial \pi}{\partial p} > 0$, so increase price (i.e., decrease quantity) until you get to an elastic part of the demand curve.

- Inefficiency? Yes, any deviation from $p = C'$ means quantity is inefficient.

Properties of monopoly pricing

- What point on the demand curve does monopolist choose?

$$p^m = \left(\frac{1+\epsilon}{\epsilon}\right) C'$$
$$p^m > 0 \Leftrightarrow \frac{1+\epsilon}{\epsilon} > 0$$
$$\Leftrightarrow \epsilon < -1$$

  Elastic part of the demand curve.

  $\rightarrow$ As long as demand is inelastic, $\frac{\partial \pi}{\partial p} > 0$, so increase price (i.e., decrease quantity) until you get to an elastic part of the demand curve.

- Inefficiency? Yes, any deviation from $p = C'$ means quantity is inefficient.

- p^m is weakly increasing in marginal cost.

Proof: p^m is weakly increasing in marginal cost.

- Suppose $C_2'(q) > C_1'(q)$ for all $q > 0$.
- Let (p_1, q_1) and (p_2, q_2) denote the corresponding monopoly prices and quantities.
  - **Key idea**: Both (p_1, q_1) and (p_2, q_2) are points on the demand curve, so both feasible for both monopolists.

Proof: p^m is weakly increasing in marginal cost.

- Suppose $C_2'(q) > C_1'(q)$ for all $q > 0$.
- Let (p_1, q_1) and (p_2, q_2) denote the corresponding monopoly prices and quantities.
  - **Key idea**: Both (p_1, q_1) and (p_2, q_2) are points on the demand curve, so both feasible for both monopolists.

$$p_1 q_1 - C_1(q_1) \geqslant p_2 q_2 - C_1(q_2)$$
$$p_2 q_2 - C_2(q_2) \geqslant p_1 q_1 - C_2(q_1)$$

Proof: p^m is weakly increasing in marginal cost.

- Suppose $C_2'(q) > C_1'(q)$ for all $q > 0$.
- Let (p_1, q_1) and (p_2, q_2) denote the corresponding monopoly prices and quantities.
  - **Key idea**: Both (p_1, q_1) and (p_2, q_2) are points on the demand curve, so both feasible for both monopolists.

$$p_1 q_1 - C_1(q_1) \geqslant p_2 q_2 - C_1(q_2)$$
$$p_2 q_2 - C_2(q_2) \geqslant p_1 q_1 - C_2(q_1)$$

- Combining the two:

$$[C_2(q_1) - C_1(q_1)] - [C_2(q_2) - C_1(q_2)] \geqslant 0$$

Proof: p^m is weakly increasing in marginal cost.

- Suppose $C_2'(q) > C_1'(q)$ for all $q > 0$.
- Let (p_1, q_1) and (p_2, q_2) denote the corresponding monopoly prices and quantities.
  - **Key idea**: Both (p_1, q_1) and (p_2, q_2) are points on the demand curve, so both feasible for both monopolists.

$$p_1 q_1 - C_1(q_1) \geqslant p_2 q_2 - C_1(q_2)$$
$$p_2 q_2 - C_2(q_2) \geqslant p_1 q_1 - C_2(q_1)$$

- Combining the two:

$$[C_2(q_1) - C_1(q_1)] - [C_2(q_2) - C_1(q_2)] \geqslant 0$$

which implies

$$\int_{q_2}^{q_1} \underbrace{[C_2'(x) - C_1'(x)]}_{>0 \ \forall x} dx \geqslant 0$$

so $q_1 \geqslant q_2$, which means $p_1 \leqslant p_2$.

Questions for discussion

1. What does the previous result imply about the recent discussion about whether firms “eat tariffs” or pass them on to consumers?

Questions for discussion

1. What does the previous result imply about the recent discussion about whether firms “eat tariffs” or pass them on to consumers?
2. Suppose a retailer uses a simple heuristic, always setting prices 20% above marginal cost. Is that consistent with the retailer following the monopoly pricing rule described above?

Questions for discussion

1. What does the previous result imply about the recent discussion about whether firms “eat tariffs” or pass them on to consumers?
2. Suppose a retailer uses a simple heuristic, always setting prices 20% above marginal cost. Is that consistent with the retailer following the monopoly pricing rule described above?
3. Could a corrective tax/subsidy be used to correct the distortion? If so, does that ensure efficiency?

Questions for discussion

1. What does the previous result imply about the recent discussion about whether firms “eat tariffs” or pass them on to consumers?
2. Suppose a retailer uses a simple heuristic, always setting prices 20% above marginal cost. Is that consistent with the retailer following the monopoly pricing rule described above?
3. Could a corrective tax/subsidy be used to correct the distortion? If so, does that ensure efficiency? **Yes**, a production subsidy could bring us back to the efficient quantity,

Questions for discussion

1. What does the previous result imply about the recent discussion about whether firms "eat tariffs" or pass them on to consumers?
2. Suppose a retailer uses a simple heuristic, always setting prices 20% above marginal cost. Is that consistent with the retailer following the monopoly pricing rule described above?
3. Could a corrective tax/subsidy be used to correct the distortion? If so, does that ensure efficiency? **Yes**, a production subsidy could bring us back to the efficient quantity, though still could be inefficiency if:
   - Distortion of product/quality choice.
   - "X-inefficiency."

Questions for discussion

1. What does the previous result imply about the recent discussion about whether firms "eat tariffs" or pass them on to consumers?
2. Suppose a retailer uses a simple heuristic, always setting prices 20% above marginal cost. Is that consistent with the retailer following the monopoly pricing rule described above?
3. Could a corrective tax/subsidy be used to correct the distortion? If so, does that ensure efficiency? **Yes**, a production subsidy could bring us back to the efficient quantity, though still could be inefficiency if:
   - Distortion of product/quality choice.
   - "X-inefficiency."
4. Suppose you know that firm is a monopolist and you know p, $\epsilon(p)$, $c'(D(p))$. However, you observe that $p \neq \left(\frac{1+\epsilon}{\epsilon}\right) c'(D(p))$. What could explain this?

Questions for discussion

1. What does the previous result imply about the recent discussion about whether firms "eat tariffs" or pass them on to consumers?

2. Suppose a retailer uses a simple heuristic, always setting prices 20% above marginal cost. Is that consistent with the retailer following the monopoly pricing rule described above?

3. Could a corrective tax/subsidy be used to correct the distortion? If so, does that ensure efficiency? **Yes**, a production subsidy could bring us back to the efficient quantity, though still could be inefficiency if:
   - Distortion of product/quality choice.
   - "X-inefficiency."

4. Suppose you know that firm is a monopolist and you know p, $\epsilon(p)$, $c'(D(p))$. However, you observe that $p \neq \left(\frac{1+\epsilon}{\epsilon}\right) c'(D(p))$. What could explain this?
   - Dynamic considerations: Reputation, resale market for durable goods, learning by doing, etc.
   - Multiproduct firm.

Multi-product monopoly

Setup

- Cost function: $C(q_1, \dots, q_n)$
- Demand: $D_1(\mathbf{p}), \dots, D_n(\mathbf{p})$

$$\pi(\mathbf{p}) = \sum_i p_i D_i(\mathbf{p}) - C\left(D_1(\mathbf{p}), \dots, D_n(\mathbf{p})\right)$$

Setup

- Cost function: $C(q_1, \ldots, q_n)$
- Demand: $D_1(\mathbf{p}), \ldots, D_n(\mathbf{p})$

$$\pi(\mathbf{p}) = \sum_i p_i D_i(\mathbf{p}) - C\left(D_1(\mathbf{p}), \ldots, D_n(\mathbf{p})\right)$$

Question: In what ways might the choice of price for one product involve considerations about other products?

First order condition

$$\pi(\mathbf{p}) = \sum_i p_i D_i(\mathbf{p}) - C\left(D_1(\mathbf{p}), \ldots, D_n(\mathbf{p})\right)$$

FOC: For each i,

First order condition

$$\pi(\mathbf{p}) = \sum_i p_i D_i(\mathbf{p}) - C\left(D_1(\mathbf{p}), \ldots, D_n(\mathbf{p})\right)$$

FOC: For each i,

$$\left(D_i + p_i \frac{\partial D_i}{\partial p_i}\right) + \sum_{j \neq i} p_j \frac{\partial D_j}{\partial p_i} = \frac{\partial C}{\partial q_i} \frac{\partial D_i}{\partial p_i} + \sum_{j \neq i} \frac{\partial C}{\partial q_j} \frac{\partial D_j}{\partial p_i}$$

Questions:

1. What's new here relative to single-product monopoly pricing?

First order condition

$$\pi(\mathbf{p}) = \sum_i p_i D_i(\mathbf{p}) - C\left(D_1(\mathbf{p}), \ldots, D_n(\mathbf{p})\right)$$

FOC: For each i,

$$\left(D_i + p_i \frac{\partial D_i}{\partial p_i}\right) + \sum_{j \neq i} p_j \frac{\partial D_j}{\partial p_i} = \frac{\partial C}{\partial q_i}\frac{\partial D_i}{\partial p_i} + \sum_{j \neq i} \frac{\partial C}{\partial q_j}\frac{\partial D_j}{\partial p_i}$$

Questions:

1. What's new here relative to single-product monopoly pricing?
2. What's the interpretation of each term?

Case 1: Separable costs, inseparable demands

Suppose $C(q_1, \ldots, q_n) = \sum_i C_i(q_i)$. Can we derive an expression for L_i for each i?

A definition that will prove useful:

$$\epsilon_{ij} = \frac{\partial D_j}{\partial p_i} \times \frac{p_i}{D_j}$$

Case 1: Separable costs, inseparable demands

Suppose $C(q_1, \ldots, q_n) = \sum_i C_i(q_i)$. Can we derive an expression for L_i for each i?

A definition that will prove useful:

$$\epsilon_{ij} = \frac{\partial D_j}{\partial p_i} \times \frac{p_i}{D_j}$$

The FOC becomes

Case 1: Separable costs, inseparable demands

Suppose $C(q_1, \ldots, q_n) = \sum_i C_i(q_i)$. Can we derive an expression for L_i for each i?

A definition that will prove useful:

$$\epsilon_{ij} = \frac{\partial D_j}{\partial p_i} \times \frac{p_i}{D_j}$$

The FOC becomes

$$\frac{p_i - C_i'}{p_i} = -\frac{1}{\epsilon_{ii}} - \sum_{j \neq i} \frac{(p_j - C_j')D_j\epsilon_{ij}}{p_i D_i \epsilon_{ii}}$$

Case 1: Separable costs, inseparable demands

Suppose $C(q_1, \ldots, q_n) = \sum_i C_i(q_i)$. Can we derive an expression for L_i for each i?

A definition that will prove useful:

$$\epsilon_{ij} = \frac{\partial D_j}{\partial p_i} \times \frac{p_i}{D_j}$$

The FOC becomes

$$\frac{p_i - C_i'}{p_i} = -\frac{1}{\epsilon_{ii}} - \sum_{j \neq i} \frac{(p_j - C_j') D_j \epsilon_{ij}}{p_i D_i \epsilon_{ii}}$$

Question: Is the markup of the multi-product firm larger or smaller than that of the single-product firm?

Case 1: Separable costs, inseparable demands

Suppose $C(q_1, \ldots, q_n) = \sum_i C_i(q_i)$. Can we derive an expression for L_i for each i?

A definition that will prove useful:

$$\epsilon_{ij} = \frac{\partial D_j}{\partial p_i} \times \frac{p_i}{D_j}$$

The FOC becomes

$$\frac{p_i - C_i'}{p_i} = -\frac{1}{\epsilon_{ii}} - \sum_{j \neq i} \frac{(p_j - C_j')D_j \epsilon_{ij}}{p_i D_i \epsilon_{ii}}$$

Question: Is the markup of the multi-product firm larger or smaller than that of the single-product firm?

- **Case 1a: Goods are substitutes**: Then, $\epsilon_{ij} > 0 \; \forall i \neq j$.

Case 1: Separable costs, inseparable demands

Suppose $C(q_1, \ldots, q_n) = \sum_i C_i(q_i)$. Can we derive an expression for L_i for each i?

A definition that will prove useful:

$$\epsilon_{ij} = \frac{\partial D_j}{\partial p_i} \times \frac{p_i}{D_j}$$

The FOC becomes

$$\frac{p_i - C_i'}{p_i} = -\frac{1}{\epsilon_{ii}} - \sum_{j \neq i} \frac{(p_j - C_j')D_j \epsilon_{ij}}{p_i D_i \epsilon_{ii}}$$

Question: Is the markup of the multi-product firm larger or smaller than that of the single-product firm?

- **Case 1a: Goods are substitutes**: Then, $\epsilon_{ij} > 0 \; \forall i \neq j$. In this case, $L_i > -\frac{1}{\epsilon_{ii}}$.

Case 1: Separable costs, inseparable demands

Suppose $C(q_1, \ldots, q_n) = \sum_i C_i(q_i)$. Can we derive an expression for L_i for each i?

A definition that will prove useful:

$$\epsilon_{ij} = \frac{\partial D_j}{\partial p_i} \times \frac{p_i}{D_j}$$

The FOC becomes

$$\frac{p_i - C_i'}{p_i} = -\frac{1}{\epsilon_{ii}} - \sum_{j \neq i} \frac{(p_j - C_j')D_j \epsilon_{ij}}{p_i D_i \epsilon_{ii}}$$

Question: Is the markup of the multi-product firm larger or smaller than that of the single-product firm?

- **Case 1a: Goods are substitutes**: Then, $\epsilon_{ij} > 0 \ \forall i \neq j$. In this case, $L_i > -\frac{1}{\epsilon_{ii}}$.
- **Case 1b: Goods are complements**: Then, $\epsilon_{ij} < 0 \ \forall i \neq j$.

Case 1: Separable costs, inseparable demands

Suppose $C(q_1, \ldots, q_n) = \sum_i C_i(q_i)$. Can we derive an expression for L_i for each i?

A definition that will prove useful:

$$\epsilon_{ij} = \frac{\partial D_j}{\partial p_i} \times \frac{p_i}{D_j}$$

The FOC becomes

$$\frac{p_i - C_i'}{p_i} = -\frac{1}{\epsilon_{ii}} - \sum_{j \neq i} \frac{(p_j - C_j')D_j \epsilon_{ij}}{p_i D_i \epsilon_{ii}}$$

Question: Is the markup of the multi-product firm larger or smaller than that of the single-product firm?

- **Case 1a: Goods are substitutes**: Then, $\epsilon_{ij} > 0 \; \forall i \neq j$. In this case, $L_i > -\frac{1}{\epsilon_{ii}}$.
- **Case 1b: Goods are complements**: Then, $\epsilon_{ij} < 0 \; \forall i \neq j$. In this case, $L_i < -\frac{1}{\epsilon}$.

Case 2: Separable demands, inseparable costs

Suppose $q_i = D_i(p_i)$ for each i.

Example: Learning by doing in a two-period game.

- At $t = 1$, cost is $C_1(q_1)$.
- At $t = 2$, cost is $C_2(q_2, q_1)$, where $\partial C_2 / \partial q_1 \overset{?}{\gtrless} 0$.
- **Question**: What do you predict happens here?

Case 2: Separable demands, inseparable costs

Suppose $q_i = D_i(p_i)$ for each i.

Example: Learning by doing in a two-period game.

- At $t = 1$, cost is $C_1(q_1)$.
- At $t = 2$, cost is $C_2(q_2, q_1)$, where $\partial C_2 / \partial q_1 < 0$.
- **Question**: What do you predict happens here?
- Profit is $p_1 D_1(p_1) - C_1(D_1(p_1)) + \delta\,[p_2 D_2(p_2) - C_2(D_2(p_2), D_1(p_1))]$.

Case 2: Separable demands, inseparable costs

Suppose $q_i = D_i(p_i)$ for each i.

Example: Learning by doing in a two-period game.

- At $t = 1$, cost is $C_1(q_1)$.
- At $t = 2$, cost is $C_2(q_2, q_1)$, where $\partial C_2 / \partial q_1 0$.
- **Question**: What do you predict happens here?
- Profit is $p_1 D_1(p_1) - C_1(D_1(p_1)) + \delta [p_2 D_2(p_2) - C_2(D_2(p_2), D_1(p_1))]$.
- FOC wrt p_1:

$$D_1 + p_1 D_1' = C_1' D_1' + \delta (\partial C_2 / \partial q_1) D_1'$$
$$L_1 = -\frac{1}{\epsilon_1} + \delta \frac{1}{p_1} \frac{\partial C_2}{\partial q_1} < -\frac{1}{\epsilon_1}$$

Durable goods monopolist

Setup

- Many goods last more than one “period”. **Question**: What does this mean for the monopolist?

Setup

- Many goods last more than one “period”. **Question**: What does this mean for the monopolist?
- The monopolist tomorrow is “in competition with” the monopolist today.
- Suppose there are measure 1 consumers, whose lifetime discounted utility from a good is $v \sim U[0, 1]$.
  - $\rightarrow$ **Question**: What is the demand function?

Setup

- Many goods last more than one “period”. **Question**: What does this mean for the monopolist?
- The monopolist tomorrow is “in competition with” the monopolist today.
- Suppose there are measure 1 consumers, whose lifetime discounted utility from a good is $v \sim U[0,1]$.
  - $\rightarrow$ **Question**: What is the demand function? $D(p) = 1 - p$.

Setup

- Many goods last more than one “period”. **Question**: What does this mean for the monopolist?
- The monopolist tomorrow is “in competition with” the monopolist today.
- Suppose there are measure 1 consumers, whose lifetime discounted utility from a good is $v \sim U[0, 1]$.
  - → **Question**: What is the demand function? $D(p) = 1 - p$.
- Suppose zero marginal cost, and suppose time is discrete, $t = 1, 2, \ldots$. Discount rate is δ.

Setup

- Many goods last more than one “period”. **Question**: What does this mean for the monopolist?
- The monopolist tomorrow is “in competition with” the monopolist today.
- Suppose there are measure 1 consumers, whose lifetime discounted utility from a good is $v \sim U[0, 1]$.
  - → **Question**: What is the demand function? $D(p) = 1 - p$.
- Suppose zero marginal cost, and suppose time is discrete, $t = 1, 2, \ldots$. Discount rate is δ.
- If the firm disappeared after $t = 1$, then at $t = 1$, the firm charges $p^m =$ **?**

Setup

- Many goods last more than one "period". **Question**: What does this mean for the monopolist?
- The monopolist tomorrow is "in competition with" the monopolist today.
- Suppose there are measure 1 consumers, whose lifetime discounted utility from a good is $v \sim U[0, 1]$.
  - → **Question**: What is the demand function? $D(p) = 1 - p$.
- Suppose zero marginal cost, and suppose time is discrete, $t = 1, 2, \ldots$. Discount rate is δ.
- If the firm disappeared after $t = 1$, then at $t = 1$, the firm charges $p^m = \frac{1}{2}$ and sells $q^m = \frac{1}{2}$.

Setup

- Many goods last more than one "period". **Question**: What does this mean for the monopolist?
- The monopolist tomorrow is "in competition with" the monopolist today.
- Suppose there are measure 1 consumers, whose lifetime discounted utility from a good is $v \sim U[0,1]$.
  - → **Question**: What is the demand function? $D(p) = 1 - p$.
- Suppose zero marginal cost, and suppose time is discrete, $t = 1, 2, \ldots$. Discount rate is δ.
- If the firm disappeared after $t = 1$, then at $t = 1$, the firm charges $p^m = \frac{1}{2}$ and sells $q^m = \frac{1}{2}$.
- If the monopolist (is surprised to find that she) continues to exist at $t = 2$, faces *residual demand* $D(p) =$ **?**

Setup

- Many goods last more than one "period". **Question**: What does this mean for the monopolist?
- The monopolist tomorrow is "in competition with" the monopolist today.
- Suppose there are measure 1 consumers, whose lifetime discounted utility from a good is $v \sim U[0,1]$.
  - → **Question**: What is the demand function? $D(p) = 1 - p$.
- Suppose zero marginal cost, and suppose time is discrete, $t = 1, 2, \ldots$. Discount rate is δ.
- If the firm disappeared after $t = 1$, then at $t = 1$, the firm charges $p^m = \frac{1}{2}$ and sells $q^m = \frac{1}{2}$.
- If the monopolist (is surprised to find that she) continues to exist at $t = 2$, faces *residual demand* $D(p) = \frac{1}{2} - p$ and charges $p^m =$ **?**

Setup

- Many goods last more than one "period". **Question**: What does this mean for the monopolist?
- The monopolist tomorrow is "in competition with" the monopolist today.
- Suppose there are measure 1 consumers, whose lifetime discounted utility from a good is $v \sim U[0,1]$.
  - → **Question**: What is the demand function? $D(p) = 1 - p$.
- Suppose zero marginal cost, and suppose time is discrete, $t = 1, 2, \ldots$. Discount rate is δ.
- If the firm disappeared after $t = 1$, then at $t = 1$, the firm charges $p^m = \frac{1}{2}$ and sells $q^m = \frac{1}{2}$.
- If the monopolist (is surprised to find that she) continues to exist at $t = 2$, faces *residual demand* $D(p) = \frac{1}{2} - p$ and charges $p^m = \frac{1}{4}$ and sells $q^m = \frac{1}{4}$.

Setup

- Many goods last more than one "period". **Question**: What does this mean for the monopolist?
- The monopolist tomorrow is "in competition with" the monopolist today.
- Suppose there are measure 1 consumers, whose lifetime discounted utility from a good is $v \sim U[0, 1]$.
  - → **Question**: What is the demand function? $D(p) = 1 - p$.
- Suppose zero marginal cost, and suppose time is discrete, $t = 1, 2, \ldots$. Discount rate is δ.
- If the firm disappeared after $t = 1$, then at $t = 1$, the firm charges $p^m = \frac{1}{2}$ and sells $q^m = \frac{1}{2}$.
- If the monopolist (is surprised to find that she) continues to exist at $t = 2$, faces *residual demand* $D(p) = \frac{1}{2} - p$ and charges $p^m = \frac{1}{4}$ and sells $q^m = \frac{1}{4}$.
- **Question**: What might consumers do?

Setup

- Many goods last more than one "period". **Question**: What does this mean for the monopolist?
- The monopolist tomorrow is "in competition with" the monopolist today.
- Suppose there are measure 1 consumers, whose lifetime discounted utility from a good is $v \sim U[0,1]$.
  - $\rightarrow$ **Question**: What is the demand function? $D(p) = 1 - p$.
- Suppose zero marginal cost, and suppose time is discrete, $t = 1, 2, \ldots$. Discount rate is δ.
- If the firm disappeared after $t = 1$, then at $t = 1$, the firm charges $p^m = \frac{1}{2}$ and sells $q^m = \frac{1}{2}$.
- If the monopolist (is surprised to find that she) continues to exist at $t = 2$, faces *residual demand* $D(p) = \frac{1}{2} - p$ and charges $p^m = \frac{1}{4}$ and sells $q^m = \frac{1}{4}$.
- **Question**: What might consumers do? If consumers anticipate that $p_2 = \frac{1}{4}$, they may decide to wait. And if the monopolist were not myopic, she'd anticipate this waiting....

Durable goods with commitment

- Suppose the monopolist can commit to a path of future prices.
- Can't do better than $p_t = p^m$ for all t. [Bulow (1982)]

Durable goods with commitment

- Suppose the monopolist can commit to a path of future prices.
- Can't do better than $p_t = p^m$ for all t. [Bulow (1982)]
- **Question**: How likely is it that the monopolist actually has commitment power?

Coase Conjecture

Consider an extreme commitment problem. Suppose the monopolist has no commitment power, can set prices at $t = 0, \Delta T, 2\Delta T, 3\Delta T, \ldots$, and consumers are fully rational and get gross utility θe^{-rt} if they purchase at t.

Coase Conjecture: Under some conditions the monopolist's profits go to zero as $\Delta T \to 0$.

Intuition:

- Suppose that $\theta \sim U(0,1)$ and $c \in [0,1]$
- Suppose that in the limiting time path of prices is not an immediate drop to zero.
- Hence, prices must be dropping linearly in T at some point and quantities must also be proportional to ΔT.
- Suppose the monopolist jumps ahead in its price sequence and charges $p_{r+\Delta T}$ instead of p_r.
- The gain from having all sales occur ΔT earlier is first order in ΔT.
- The loss from earning less on the sales at time t is second order in ΔT the price difference and quantity on which you get the lower price are both of order ΔT. Hence, it is better to jump ahead and cut prices faster.

If Coase is right, then monopolies are fine I guess?

If Coase is right, then monopolies are fine I guess?

- The required conditions are actually somewhat narrow: Whether the result is true depends on whether the lower-bound of the value distribution is above or below c.

If Coase is right, then monopolies are fine I guess?

- The required conditions are actually somewhat narrow: Whether the result is true depends on whether the lower-bound of the value distribution is above or below c.
- Durable goods producers don't seem to earn zero profit. **Why? What was Coase missing?**

If Coase is right, then monopolies are fine I guess?

- The required conditions are actually somewhat narrow: Whether the result is true depends on whether the lower-bound of the value distribution is above or below c.
- Durable goods producers don't seem to earn zero profit. **Why? What was Coase missing?**
  - Delays/costs of changing prices (unlikely to be explanation)
  - Reputation
  - Inflows of new high willingness-to-pay customers
  - Per-period fixed costs
  - Strategic actions:
    - Rent rather than sell (but can run into moral hazard or antitrust problems)
    - Most favored customer contracts or money back guarantee
    - Destroy/limit ability to produce
    - Convince consumers that marginal cost is higher than it really is

Product quality

Monopoly and Product Quality

- Suppose the monopolist also chooses the quality $s \in \mathbb{R}$ of its good.
- Suppose it has a constant marginal cost $c(s)$ with $c'(s) > 0$, i.e. $C(q,s) = qc(s)$.
- Suppose a unit mass of consumers with types $\theta \sim U(0,1)$ have unit demands: Utility from one unit is $v(s;\theta) - p$ where $v_s(s;\theta) > 0$ and $v_\theta(s;\theta) > 0$.

Monopoly and Product Quality

- Suppose the monopolist also chooses the quality $s \in \mathbb{R}$ of its good.
- Suppose it has a constant marginal cost $c(s)$ with $c'(s) > 0$, i.e. $C(q, s) = qc(s)$.
- Suppose a unit mass of consumers with types $\theta \sim U(0, 1)$ have unit demands: Utility from one unit is $v(s; \theta) - p$ where $v_s(s; \theta) > 0$ and $v_\theta(s; \theta) > 0$.
- **Question**: At what price can the firm sell q units?

Monopoly and Product Quality

- Suppose the monopolist also chooses the quality $s \in \mathbb{R}$ of its good.
- Suppose it has a constant marginal cost $c(s)$ with $c'(s) > 0$, i.e. $C(q, s) = qc(s)$.
- Suppose a unit mass of consumers with types $\theta \sim U(0, 1)$ have unit demands: Utility from one unit is $v(s; \theta) - p$ where $v_s(s; \theta) > 0$ and $v_\theta(s; \theta) > 0$.
- **Question**: At what price can the firm sell q units? It will sell to consumers with $\theta \in [1 - q, 1]$. Hence, the price at which it can sell q units of quality s is $v(s; 1 - q)$.

Monopoly and Product Quality

- Suppose the monopolist also chooses the quality $s \in \mathbb{R}$ of its good.
- Suppose it has a constant marginal cost $c(s)$ with $c'(s) > 0$, i.e. $C(q, s) = qc(s)$.
- Suppose a unit mass of consumers with types $\theta \sim U(0, 1)$ have unit demands: Utility from one unit is $v(s; \theta) - p$ where $v_s(s; \theta) > 0$ and $v_\theta(s; \theta) > 0$.
- **Question**: At what price can the firm sell q units? It will sell to consumers with $\theta \in [1 - q, 1]$. Hence, the price at which it can sell q units of quality s is $v(s; 1 - q)$.

The monopolist solves:

$$\max_{q,s} q[v(s; 1 - q) - c(s)]$$

Monopoly and Product Quality

- Suppose the monopolist also chooses the quality $s \in \mathbb{R}$ of its good.
- Suppose it has a constant marginal cost $c(s)$ with $c'(s) > 0$, i.e. $C(q,s) = qc(s)$.
- Suppose a unit mass of consumers with types $\theta \sim U(0,1)$ have unit demands: Utility from one unit is $v(s;\theta) - p$ where $v_s(s;\theta) > 0$ and $v_\theta(s;\theta) > 0$.
- **Question**: At what price can the firm sell q units? It will sell to consumers with $\theta \in [1-q, 1]$. Hence, the price at which it can sell q units of quality s is $v(s; 1-q)$.

The monopolist solves:

$$\max_{q,s} q[v(s;1-q) - c(s)]$$

FOC:

$$\frac{\partial v}{\partial s}(s_m; 1-q_m) = \frac{\partial c}{\partial s}(s_m)$$

Monopoly and Product Quality

- Suppose the monopolist also chooses the quality $s \in \mathbb{R}$ of its good.
- Suppose it has a constant marginal cost $c(s)$ with $c'(s) > 0$, i.e. $C(q,s) = qc(s)$.
- Suppose a unit mass of consumers with types $\theta \sim U(0,1)$ have unit demands: Utility from one unit is $v(s;\theta) - p$ where $v_s(s;\theta) > 0$ and $v_\theta(s;\theta) > 0$.
- **Question**: At what price can the firm sell q units? It will sell to consumers with $\theta \in [1-q, 1]$. Hence, the price at which it can sell q units of quality s is $v(s; 1-q)$.

The monopolist solves:

$$\max_{q,s} q[v(s;1-q) - c(s)]$$

FOC:

$$\frac{\partial v}{\partial s}(s_m; 1-q_m) = \frac{\partial c}{\partial s}(s_m)$$

The marginal cost of increasing s equals the marginal benefit to the marginal consumer.

Product Quality: Monopoly versus First-Best

Monopolist FOC: (Chooses optimal quality for the *marginal* consumer)

$$\frac{\partial v}{\partial s}(s_m; 1 - q_m) = \frac{\partial c}{\partial s}(s_m)$$

First-Best FOC:

Question: Does the monopolist choose higher or lower s than the social planner?

Product Quality: Monopoly versus First-Best

Monopolist FOC: (Chooses optimal quality for the *marginal* consumer)

$$\frac{\partial v}{\partial s}(s_m; 1 - q_m) = \frac{\partial c}{\partial s}(s_m)$$

First-Best FOC:

A fact that might be useful to recall (Leibniz's Rule):

$$\frac{d}{dx}\int_{a(x)}^{b(x)} f(x,u)\,du = f\big(x, b(x)\big)\cdot b'(x) - f\big(x, a(x)\big)\cdot a'(x) + \int_{a(x)}^{b(x)} \frac{\partial}{\partial x} f(x,u)\,du.$$

Question: Does the monopolist choose higher or lower s than the social planner?

Product Quality: Monopoly versus First-Best

Monopolist FOC: (Chooses optimal quality for the *marginal* consumer)

$$\frac{\partial v}{\partial s}(s_m; 1 - q_m) = \frac{\partial c}{\partial s}(s_m)$$

First-Best FOC: (Chooses optimal quality for the *average* consumer)

$$\int_{v^{-1}(s^*, c(s^*))}^{1} \left[\frac{\partial v}{\partial s}(s^*; \theta) - \frac{\partial c}{\partial s}(s^*)\right] d\theta = 0$$

A fact that might be useful to recall (Leibniz's Rule):

$$\frac{d}{dx}\int_{a(x)}^{b(x)} f(x,u)\,du = f\big(x, b(x)\big) \cdot b'(x) - f\big(x, a(x)\big) \cdot a'(x) + \int_{a(x)}^{b(x)} \frac{\partial}{\partial x} f(x,u)\,du.$$

Question: Does the monopolist choose higher or lower s than the social planner?

Product Quality: Monopoly versus First-Best

Monopolist FOC: (Chooses optimal quality for the *marginal* consumer)

$$\frac{\partial v}{\partial s}(s_m; 1 - q_m) = \frac{\partial c}{\partial s}(s_m)$$

First-Best FOC: (Chooses optimal quality for the *average* consumer)

$$\int_{v^{-1}(s^*, c(s^*))}^{1} \left[\frac{\partial v}{\partial s}(s^*; \theta) - \frac{\partial c}{\partial s}(s^*)\right] d\theta = 0$$

A fact that might be useful to recall (Leibniz's Rule):

$$\frac{d}{dx}\int_{a(x)}^{b(x)} f(x,u)\,du = f\big(x, b(x)\big) \cdot b'(x) - f\big(x, a(x)\big) \cdot a'(x) + \int_{a(x)}^{b(x)} \frac{\partial}{\partial x} f(x,u)\,du.$$

Question: Does the monopolist choose higher or lower s than the social planner?

Takeaway: Quality will almost certainly be distorted, but we have no general prediction on the direction of the distortion.