Lectures 9: Introduction to dynamic choices

ECON 4620
Cornell University
Adam Harris

Part 2: Dynamic choices

Overview of the next month

- **Today**: Motivating examples: Solving dynamic problems (using analytical methods).
- **3/4, 3/6, 3/11**: Dynamic programming—a systematic computational approach to solving dynamic models.
- **3/13, 3/18**: Applications to firm choices: investment, advertising, etc.
- **3/20**: Review session.
- **3/25**: Prelim #2.

PS4 (which I'll post later today) is due on 3/11; PS5 is due on 3/18.

Dynamic choices: A motivating example

What do we mean by **dynamic**? Choices being made at different points in time.

Dynamic choices: A motivating example

What do we mean by **dynamic**? Choices being made at different points in time.

Example:

- Consider a monopolist who produces maple syrup in two periods $t = 1, 2$.
- In each period, monopolist faces inverse demand curve $P(q) = 160 - q$ and has constant marginal cost $c_t = 120$
- What is the monopolist's optimal choice of q_1, q_2?

Dynamic choices: A motivating example

What do we mean by **dynamic**? **Interrelated** choices being made at different points in time. Current choice affects future payoffs.

Example:

- Consider a monopolist who produces maple syrup in two periods $t = 1, 2$.
- In each period, monopolist faces inverse demand curve $P(q) = 160 - q$ and has constant marginal cost $c_t = 120$
- What is the monopolist's optimal choice of q_1, q_2?

Dynamic choices: A motivating example

What do we mean by **dynamic**? **Interrelated** choices being made at different points in time. Current choice affects future payoffs.

Example:

- Consider a monopolist who produces maple syrup in two periods $t = 1, 2$.
- In each period, monopolist faces inverse demand curve $P(q) = 160 - q$ and has constant marginal cost $c_t = 120 - Q_t^{\text{past}}$ where Q_t^{past} is the total quantity produced in all past periods. (*Learning by doing*.)
- What is the monopolist's optimal choice of q_1, q_2?

Anatomy of a dynamic problem

Elements common to static and dynamic problem:

- **Payoff/loss function**: The objective is to maximize payoff or minimize loss.

Anatomy of a dynamic problem

Elements common to static and dynamic problem:

- **Payoff/loss function**: The objective is to maximize payoff or minimize loss.

Elements specific to dynamic problem:

- **Time**: Discrete versus continuous, finite versus infinite.

Anatomy of a dynamic problem

Elements common to static and dynamic problem:

- **Payoff/loss function**: The objective is to maximize payoff or minimize loss.

Elements specific to dynamic problem:

- **Time**: Discrete versus continuous, finite versus infinite.
- **State variable(s)**: What gets "carried forward" from one period to another? In other words, what aspect of history affects payoffs today? (E.g., capital stock, brand reputation, production technology, inventory).

Anatomy of a dynamic problem

Elements common to static and dynamic problem:

- **Payoff/loss function**: The objective is to maximize payoff or minimize loss.

Elements specific to dynamic problem:

- **Time**: Discrete versus continuous, finite versus infinite.
- **State variable(s)**: What gets "carried forward" from one period to another? In other words, what aspect of history affects payoffs today? (E.g., capital stock, brand reputation, production technology, inventory).
- **Transition process**: How does current state and action affect next period's state?

Dynamic choices: Another example

- Again, consider a monopolist who produces maple syrup in two periods $t = 1, 2$.
- Again, in each period, the monopolist faces inverse demand curve $P(q) = 160 - q$.
- Now, rather than learning by doing, marginal cost of production in each period is determined by *capital stock*: $c_t = 120 - K_t$, where K_t is the capital stock in period t.
- The monopolist can improve next period's capital stock through investment I_t: $K_{t+1} = K_t + I_t$. But investment is costly: ${I_t}^2$.
- What are the monopolist's optimal choices?

Dynamic choices: Another example

- Again, consider a monopolist who produces maple syrup in two periods $t = 1, 2$.
- Again, in each period, the monopolist faces inverse demand curve $P(q) = 160 - q$.
- Now, rather than learning by doing, marginal cost of production in each period is determined by *capital stock*: $c_t = 120 - K_t$, where K_t is the capital stock in period t.
- The monopolist can improve next period's capital stock through investment I_t: $K_{t+1} = K_t + I_t$. But investment is costly: ${I_t}^2$.
- What are the monopolist's optimal choices of q_1, q_2, q_3, I_1, I_2?

Dynamic choices: A stochastic example

- Again, consider a monopolist who produces maple syrup in two periods $t = 1, 2$.
- Again, in each period, the monopolist faces inverse demand curve $P(q) = 160 - q$.
- In period 1, marginal cost is 80. In period 2, marginal cost is 80 if the trees are healthy, but 140 if the trees are unhealthy.
- The monopolist can improve the chances of healthy trees by investing in fertilizer. The cost of each ton of fertilizer is 375, and the probability of healthy trees in period 2 is $\Lambda(I - 3)$ where I is the number of tons of fertilizer used and Λ is the logistic function.
- The monopolist is risk neutral. That is, the monopolist's objective is maximizing *expected profit*.
- What are the monopolist's optimal choices of q_1, q_2^H, q_2^U, I?

Dynamic choices: A stochastic example

- Again, consider a monopolist who produces maple syrup in two periods $t = 1, 2$.
- Again, in each period, the monopolist faces inverse demand curve $P(q) = 160 - q$.
- In period 1, marginal cost is 80. In period 2, marginal cost is 80 if the trees are healthy, but 140 if the trees are unhealthy.
- The monopolist can improve the chances of healthy trees by investing in fertilizer. The cost of each ton of fertilizer is 375, and the probability of healthy trees in period 2 is $\Lambda(I - 3)$ where I is the number of tons of fertilizer used and Λ is the logistic function.
- The monopolist is risk neutral. That is, the monopolist's objective is maximizing *expected profit*.
- What are the monopolist's optimal choices of q_1, q_2^H, q_2^U, I?
- What if the cost of fertilizer were 376 per ton instead of 375?