

Lecture 3: Monopoly price discrimination

ECON 7510

Cornell University

Adam Harris

Slides draw upon lecture materials from Glenn & Sara Ellison (MIT).

Example



almohada holandesa
Dutch Flag Pillow

9,99 EUR



almohada francesa
French Flag Pillow

15,99 EUR

- What is strange about this screenshot?

Example



- What is strange about this screenshot?
- Why might these two “different” products be priced so differently?

Price Discrimination

Questions: How would you define price discrimination?

Price Discrimination

Questions: How would you define price discrimination?

A classic definition is selling different units of the **same good** at **different prices**.

- Regular vs. student tickets at the theater
- This also applies to nonlinear prices: buy one get one 50% off, cell phone data

Price Discrimination

Questions: How would you define price discrimination?

A classic definition is selling different units of the **same good** at **different prices**.

- Regular vs. student tickets at the theater
- This also applies to nonlinear prices: buy one get one 50% off, cell phone data

The label is also used when firms sell **similar goods** at **different markups**:

- Stata/IC vs. Stata/SE, coach vs. business-class seat, smartphone 64GB version vs 256GB version

Price Discrimination

Questions: How would you define price discrimination?

A classic definition is selling different units of the **same good** at **different prices**.

- Regular vs. student tickets at the theater
- This also applies to nonlinear prices: buy one get one 50% off, cell phone data

The label is also used when firms sell **similar goods** at **different markups**:

- Stata/IC vs. Stata/SE, coach vs. business-class seat, smartphone 64GB version vs 256GB version
- How costs should matter is not clear, $\frac{p_i}{c_i} \neq \frac{p_j}{c_j}$ (?) or $p_i - c_i \neq p_j - c_j$ (?), but often differences are obvious.
- Can be considered discrimination to not discriminate when costs differ, e.g. free delivery.

Price Discrimination

Questions: How would you define price discrimination?

A classic definition is selling different units of the **same good** at **different prices**.

- Regular vs. student tickets at the theater
- This also applies to nonlinear prices: buy one get one 50% off, cell phone data

The label is also used when firms sell **similar goods** at **different markups**:

- Stata/IC vs. Stata/SE, coach vs. business-class seat, smartphone 64GB version vs 256GB version
- How costs should matter is not clear, $\frac{p_i}{c_i} \neq \frac{p_j}{c_j}$ (?) or $p_i - c_i \neq p_j - c_j$ (?), but often differences are obvious.
- Can be considered discrimination to not discriminate when costs differ, e.g. free delivery.

Question: Why are we talking about price discrimination in the “monopoly” part of the course?

Price Discrimination

Questions: How would you define price discrimination?

A classic definition is selling different units of the **same good** at **different prices**.

- Regular vs. student tickets at the theater
- This also applies to nonlinear prices: buy one get one 50% off, cell phone data

The label is also used when firms sell **similar goods** at **different markups**:

- Stata/IC vs. Stata/SE, coach vs. business-class seat, smartphone 64GB version vs 256GB version
- How costs should matter is not clear, $\frac{p_i}{c_i} \neq \frac{p_j}{c_j}$ (?) or $p_i - c_i \neq p_j - c_j$ (?), but often differences are obvious.
- Can be considered discrimination to not discriminate when costs differ, e.g. free delivery.

Question: Why are we talking about price discrimination in the “monopoly” part of the course?

Price discrimination requires market power. Otherwise $p_i = c_i$ for all i .

Pigou's Types of Price Discrimination

Pigou (1920) distinguished between three types of price discrimination:

- **First degree:**
- **Second degree:**
- **Third degree:**

The names are not helpful. And they aren't even sensibly ordered. But somehow they remain the standard.

Pigou's Types of Price Discrimination

Pigou (1920) distinguished between three types of price discrimination:

- **First degree: Perfect** price discrimination.
- **Second degree:**
- **Third degree:**

The names are not helpful. And they aren't even sensibly ordered. But somehow they remain the standard.

Pigou's Types of Price Discrimination

Pigou (1920) distinguished between three types of price discrimination:

- **First degree: Perfect** price discrimination. The firm knows consumer preferences completely and can price separately to each consumer.
- **Second degree:**
- **Third degree:**

The names are not helpful. And they aren't even sensibly ordered. But somehow they remain the standard.

Pigou's Types of Price Discrimination

Pigou (1920) distinguished between three types of price discrimination:

- **First degree: Perfect** price discrimination. The firm knows consumer preferences completely and can price separately to each consumer.
- **Second degree:** Pricing based on **self-selection**
- **Third degree:**

The names are not helpful. And they aren't even sensibly ordered. But somehow they remain the standard.

Pigou's Types of Price Discrimination

Pigou (1920) distinguished between three types of price discrimination:

- **First degree: Perfect** price discrimination. The firm knows consumer preferences completely and can price separately to each consumer.
- **Second degree:** Pricing based on **self-selection**—consumers with different preferences choose to buy different goods at different prices, e.g., cell phone plans, health insurance contracts, types of gasoline.
- **Third degree:**

The names are not helpful. And they aren't even sensibly ordered. But somehow they remain the standard.

Pigou's Types of Price Discrimination

Pigou (1920) distinguished between three types of price discrimination:

- **First degree: Perfect** price discrimination. The firm knows consumer preferences completely and can price separately to each consumer.
- **Second degree:** Pricing based on **self-selection**—consumers with different preferences choose to buy different goods at different prices, e.g., cell phone plans, health insurance contracts, types of gasoline.
- **Third degree:** Pricing based on **limited observed characteristics**

The names are not helpful. And they aren't even sensibly ordered. But somehow they remain the standard.

Pigou's Types of Price Discrimination

Pigou (1920) distinguished between three types of price discrimination:

- **First degree: Perfect** price discrimination. The firm knows consumer preferences completely and can price separately to each consumer.
- **Second degree:** Pricing based on **self-selection**—consumers with different preferences choose to buy different goods at different prices, e.g., cell phone plans, health insurance contracts, types of gasoline.
- **Third degree:** Pricing based on **limited observed characteristics**, e.g. age, student status, gender, health status.

The names are not helpful. And they aren't even sensibly ordered. But somehow they remain the standard.

First-Degree Price Discrimination

First-Degree Discrimination

- Suppose each consumer i 's preferences are completely known by the firm, summarized by inverse demand $P_i(x_i)$. Suppose the seller can set customer-specific nonlinear prices and prevent resale. Constant marginal cost of production c .

First-Degree Discrimination

- Suppose each consumer i 's preferences are completely known by the firm, summarized by inverse demand $P_i(x_i)$. Suppose the seller can set customer-specific nonlinear prices and prevent resale. Constant marginal cost of production c .
- The maximum possible profit can be achieved by making each consumer a single take-it-or-leave-it offer (x_i, T_i) .

First-Degree Discrimination

- Suppose each consumer i 's preferences are completely known by the firm, summarized by inverse demand $P_i(x_i)$. Suppose the seller can set customer-specific nonlinear prices and prevent resale. Constant marginal cost of production c .
- The maximum possible profit can be achieved by making each consumer a single take-it-or-leave-it offer (x_i, T_i) .
- **Question:** Given quantity x_i , what price T_i will monopolist choose?

First-Degree Discrimination

- Suppose each consumer i 's preferences are completely known by the firm, summarized by inverse demand $P_i(x_i)$. Suppose the seller can set customer-specific nonlinear prices and prevent resale. Constant marginal cost of production c .
- The maximum possible profit can be achieved by making each consumer a single take-it-or-leave-it offer (x_i, T_i) .
- **Question:** Given quantity x_i , what price T_i will monopolist choose? Consumer i will accept this offer if and only if $T_i \leq \int_0^{x_i} P_i(s) ds$. Clearly setting $T_i = \int_0^{x_i} P_i(s) ds$ is optimal given x_i .

First-Degree Discrimination

- Suppose each consumer i 's preferences are completely known by the firm, summarized by inverse demand $P_i(x_i)$. Suppose the seller can set customer-specific nonlinear prices and prevent resale. Constant marginal cost of production c .
- The maximum possible profit can be achieved by making each consumer a single take-it-or-leave-it offer (x_i, T_i) .
- **Question:** Given quantity x_i , what price T_i will monopolist choose? Consumer i will accept this offer if and only if $T_i \leq \int_0^{x_i} P_i(s) ds$. Clearly setting $T_i = \int_0^{x_i} P_i(s) ds$ is optimal given x_i .
- **Question:** Quantity choice?

First-Degree Discrimination

- Suppose each consumer i 's preferences are completely known by the firm, summarized by inverse demand $P_i(x_i)$. Suppose the seller can set customer-specific nonlinear prices and prevent resale. Constant marginal cost of production c .
- The maximum possible profit can be achieved by making each consumer a single take-it-or-leave-it offer (x_i, T_i) .
- **Question:** Given quantity x_i , what price T_i will monopolist choose? Consumer i will accept this offer if and only if $T_i \leq \int_0^{x_i} P_i(s) ds$. Clearly setting $T_i = \int_0^{x_i} P_i(s) ds$ is optimal given x_i .
- **Question:** Quantity choice? The monopolist solves: $\max_{x_i} \int_0^{x_i} P_i(s) ds - cx_i$
- FOC:

First-Degree Discrimination

- Suppose each consumer i 's preferences are completely known by the firm, summarized by inverse demand $P_i(x_i)$. Suppose the seller can set customer-specific nonlinear prices and prevent resale. Constant marginal cost of production c .
- The maximum possible profit can be achieved by making each consumer a single take-it-or-leave-it offer (x_i, T_i) .
- **Question:** Given quantity x_i , what price T_i will monopolist choose? Consumer i will accept this offer if and only if $T_i \leq \int_0^{x_i} P_i(s) ds$. Clearly setting $T_i = \int_0^{x_i} P_i(s) ds$ is optimal given x_i .
- **Question:** Quantity choice? The monopolist solves: $\max_{x_i} \int_0^{x_i} P_i(s) ds - cx_i$
- FOC: $P_i(x_i^*) = c$

First-Degree Discrimination: Notes

Notes:

1. Quantities are the same as with perfect competition.
2. First-degree discrimination is socially optimal. (If we ignore distributional issues.)

First-Degree Discrimination: Notes

Notes:

1. Quantities are the same as with perfect competition.
2. First-degree discrimination is socially optimal. (If we ignore distributional issues.)

Question: Suppose only pricing instrument available is two-part tariff: $T_i(x) = A_i + bx$. What is the best monopolist can do?

First-Degree Discrimination: Notes

Notes:

1. Quantities are the same as with perfect competition.
2. First-degree discrimination is socially optimal. (If we ignore distributional issues.)

Question: Suppose only pricing instrument available is two-part tariff: $T_i(x) = A_i + bx$. What is the best monopolist can do?

→ Can achieve 1DPD outcome by choosing $A_i = \int_0^{x_i} P_i(s) dx$ and $b = c$.

First-Degree Discrimination: Notes

Notes:

1. Quantities are the same as with perfect competition.
2. First-degree discrimination is socially optimal. (If we ignore distributional issues.)

Question: Suppose only pricing instrument available is two-part tariff: $T_i(x) = A_i + bx$. What is the best monopolist can do?

→ Can achieve 1DPD outcome by choosing $A_i = \int_0^{x_i} P_i(s) dx$ and $b = c$.

Of course, this is only possible if consumer preferences are perfectly observable.

Question: Does this result hinge on another specific assumption we made?

First-Degree Discrimination: Notes

Notes:

1. Quantities are the same as with perfect competition.
2. First-degree discrimination is socially optimal. (If we ignore distributional issues.)

Question: Suppose only pricing instrument available is two-part tariff: $T_i(x) = A_i + bx$. What is the best monopolist can do?

→ Can achieve 1DPD outcome by choosing $A_i = \int_0^{x_i} P_i(s) dx$ and $b = c$.

Of course, this is only possible if consumer preferences are perfectly observable.

Question: Does this result hinge on another specific assumption we made?

Question: In reality, what other factors might limit ability to do this kind of price discrimination?

First-Degree Discrimination: Notes

Notes:

1. Quantities are the same as with perfect competition.
2. First-degree discrimination is socially optimal. (If we ignore distributional issues.)

Question: Suppose only pricing instrument available is two-part tariff: $T_i(x) = A_i + bx$. What is the best monopolist can do?

→ Can achieve 1DPD outcome by choosing $A_i = \int_0^{x_i} P_i(s) dx$ and $b = c$.

Of course, this is only possible if consumer preferences are perfectly observable.

Question: Does this result hinge on another specific assumption we made?

Question: In reality, what other factors might limit ability to do this kind of price discrimination?

- Arbitrage (resale)
- Administrative costs

In some cases these factors can completely eliminate the ability to discriminate.

Factors limiting discrimination: Resale

- Go back to assuming constant marginal cost c .
- Suppose the monopolist cannot prevent resale.
- And the monopolist is limited to a two-part tariff: $T_i(x) = A_i + bx$.
- **Question:** What A_i , b does the monopolist chose?

Factors limiting discrimination: Resale

- Go back to assuming constant marginal cost c .
- Suppose the monopolist cannot prevent resale.
- And the monopolist is limited to a two-part tariff: $T_i(x) = A_i + bx$.
- **Question:** What A_i , b does the monopolist choose?
 - If $A_i > 0$, one consumer can buy the good for everyone and resell. So can't do better than $A_i = 0 \forall i$.

Factors limiting discrimination: Resale

- Go back to assuming constant marginal cost c .
- Suppose the monopolist cannot prevent resale.
- And the monopolist is limited to a two-part tariff: $T_i(x) = A_i + bx$.
- **Question:** What A_i , b does the monopolist chose?
 - If $A_i > 0$, one consumer can buy the good for everyone and resell. So can't do better than $A_i = 0 \forall i$.
 - $b = ?$

Factors limiting discrimination: Resale

- Go back to assuming constant marginal cost c .
- Suppose the monopolist cannot prevent resale.
- And the monopolist is limited to a two-part tariff: $T_i(x) = A_i + bx$.
- **Question:** What A_i , b does the monopolist chose?
 - If $A_i > 0$, one consumer can buy the good for everyone and resell. So can't do better than $A_i = 0 \forall i$.
 - $b =$ monopoly price.

Third-Degree Price Discrimination

Third-degree price discrimination

Third degree: Pricing based on **limited observed characteristics**, e.g. age, student status, gender, health status,...

Question: Examples?

Third-degree price discrimination

Third degree: Pricing based on **limited observed characteristics**, e.g. age, student status, gender, health status,...

Question: Examples?

- Student tickets for movies, theater, etc.
- Airline tickets: Price depends on how long in advance you book, as well as day of week, browser, operating system, location, etc.
- Retail: Different prices at stores in richer versus poorer neighborhoods.

Third-Degree Discrimination

Suppose monopolist can distinguish classes of consumers, but is limited to simple linear pricing (unit price p_i) within each group.

- Two groups $i = 1, 2$
- Demands $D_i(p_i)$
- Constant marginal cost c

With discrimination:

$$\max_{p_1, p_2} \sum_{i=1,2} (p_i - c) D_i(p_i)$$

Third-Degree Discrimination

Suppose monopolist can distinguish classes of consumers, but is limited to simple linear pricing (unit price p_i) within each group.

- Two groups $i = 1, 2$
- Demands $D_i(p_i)$
- Constant marginal cost c

With discrimination:

$$\max_{p_1, p_2} \sum_{i=1,2} (p_i - c) D_i(p_i) \Rightarrow D_i(p_i^*) + (p_i^* - c) D_i'(p_i^*) = 0 \text{ for all } i$$

Third-Degree Discrimination

Suppose monopolist can distinguish classes of consumers, but is limited to simple linear pricing (unit price p_i) within each group.

- Two groups $i = 1, 2$
- Demands $D_i(p_i)$
- Constant marginal cost c

With discrimination:

$$\max_{p_1, p_2} \sum_{i=1,2} (p_i - c) D_i(p_i) \Rightarrow D_i(p_i^*) + (p_i^* - c) D_i'(p_i^*) = 0 \text{ for all } i$$

$$\Rightarrow \frac{p_i^* - c}{p_i^*} = -\frac{1}{\epsilon_i(p_i^*)}$$

Label the markets so that $p_1^* < p_2^*$. (WLOG)

Like monopoly pricing in two completely separate markets.

Backlash to Third-Degree Price Discrimination

UNIFORM PRICING IN U.S. RETAIL CHAINS*

STEFANO DELLA VIGNA AND MATTHEW GENTZKOW

We show that most U.S. food, drugstore, and mass-merchandise chains charge nearly uniform prices across stores, despite wide variation in consumer demographics and competition. Demand estimates reveal substantial within-chain variation in price elasticities and suggest that the median chain sacrifices \$16 million of annual profit relative to a benchmark of optimal prices. In contrast, differ-


This could arise if consumers who observe different prices for the same item in multiple stores perceive this as unfair or a breach of an implicit contract. In a report on UK grocery pricing, the [Competition Commission \(2003\)](#) writes:

Asda said that it would be commercial suicide for it to move away from its highly publicized national EDLP pricing strategy and a breach of its relationship of trust with its customers, and it would cause damage to its brand image, which was closely associated with a pricing policy that assured the lowest prices always.

Banning Third-Degree Price Discrimination?

FTC Issues Orders to Eight Companies Seeking Information on Surveillance Pricing

Agency seeks information about products and services that use personal data, including finances and browser history, to set individualized prices for the same goods or services

July 23, 2024 |   

Banning Third-Degree Price Discrimination?

FTC Issues Orders to Eight Companies Seeking Information on Surveillance Pricing

Agency seeks information about products and services that use personal data, including finances and browser history, to set individualized prices for the same goods or services

July 23, 2024 |   

Question: What's your intuition about the effects of a ban on third-degree price discrimination?

Banning Third-Degree Price Discrimination?

If we ban discrimination:

$$\max_p (p - c) (D_1(p) + D_2(p)) \Rightarrow \sum_i [D_i(p^*) + (p^* - c) D_i'(p^*)] = 0$$

Banning Third-Degree Price Discrimination?

If we ban discrimination:

$$\max_p (p - c) (D_1(p) + D_2(p)) \Rightarrow \sum_i [D_i(p^*) + (p^* - c) D_i'(p^*)] = 0$$

Two cases:

1. If $\pi_i(p) \equiv (p - c) D_i(p)$ is concave in p for $0 \leq p \leq p_2^*$, then $p^* \in (p_1^*, p_2^*)$.

Question: Welfare effects?

Banning Third-Degree Price Discrimination?

If we ban discrimination:

$$\max_p (p - c) (D_1(p) + D_2(p)) \Rightarrow \sum_i [D_i(p^*) + (p^* - c) D_i'(p^*)] = 0$$

Two cases:

1. If $\pi_i(p) \equiv (p - c) D_i(p)$ is concave in p for $0 \leq p \leq p_2^*$, then $p^* \in (p_1^*, p_2^*)$.

Question: Welfare effects? Here, banning discrimination helps one group and hurts the other.

→ **Question:** How should we think of this in terms of distributional concerns? Suppose we have a preference for progressive redistribution.

Banning Third-Degree Price Discrimination?

If we ban discrimination:

$$\max_p (p - c) (D_1(p) + D_2(p)) \Rightarrow \sum_i [D_i(p^*) + (p^* - c) D_i'(p^*)] = 0$$

Two cases:

1. If $\pi_i(p) \equiv (p - c) D_i(p)$ is concave in p for $0 \leq p \leq p_2^*$, then $p^* \in (p_1^*, p_2^*)$.

Question: Welfare effects? Here, banning discrimination helps one group and hurts the other.

→ **Question:** How should we think of this in terms of distributional concerns? Suppose we have a preference for progressive redistribution.

2. Otherwise, the monopolist may choose to serve only one market and set $p^* = p_2^*$.

→ **Question:** Distributional analysis in this case?

Banning Third-Degree Price Discrimination?

If we ban discrimination:

$$\max_p (p - c) (D_1(p) + D_2(p)) \Rightarrow \sum_i [D_i(p^*) + (p^* - c) D_i'(p^*)] = 0$$

Two cases:

1. If $\pi_i(p) \equiv (p - c) D_i(p)$ is concave in p for $0 \leq p \leq p_2^*$, then $p^* \in (p_1^*, p_2^*)$.

Question: Welfare effects? Here, banning discrimination helps one group and hurts the other.

→ **Question:** How should we think of this in terms of distributional concerns? Suppose we have a preference for progressive redistribution.

2. Otherwise, the monopolist may choose to serve only one market and set $p^* = p_2^*$.

→ **Question:** Distributional analysis in this case?

In this case, no one is better off when discrimination is banned.

Banning Third-Degree Price Discrimination?

If we ban discrimination:

$$\max_p (p - c) (D_1(p) + D_2(p)) \Rightarrow \sum_i [D_i(p^*) + (p^* - c) D_i'(p^*)] = 0$$

Two cases:

1. If $\pi_i(p) \equiv (p - c) D_i(p)$ is concave in p for $0 \leq p \leq p_2^*$, then $p^* \in (p_1^*, p_2^*)$.

Question: Welfare effects? Here, banning discrimination helps one group and hurts the other.

→ **Question:** How should we think of this in terms of distributional concerns? Suppose we have a preference for progressive redistribution.

2. Otherwise, the monopolist may choose to serve only one market and set $p^* = p_2^*$.

→ **Question:** Distributional analysis in this case?

In this case, no one is better off when discrimination is banned.

Question: Which case does linear demand fall into? Isoelastic demand?

Third-Degree Discrimination: Welfare Effects

Question: Does being allowed to 3DPD (weakly) increase *producer* surplus?

Third-Degree Discrimination: Welfare Effects

Question: Does being allowed to 3DPD (weakly) increase *producer* surplus?

- Yes. At worst, the monopolist can just set the uniform price p^* .

Third-Degree Discrimination: Welfare Effects

Question: Does being allowed to 3DPD (weakly) increase *producer* surplus?

- Yes. At worst, the monopolist can just set the uniform price p^* .

When analyzing welfare effects of 3DPD, there are two sources of potential inefficiency we need to consider. **Question:** What are they?

Third-Degree Discrimination: Welfare Effects

Question: Does being allowed to 3DPD (weakly) increase *producer* surplus?

- Yes. At worst, the monopolist can just set the uniform price p^* .

When analyzing welfare effects of 3DPD, there are two sources of potential inefficiency we need to consider. **Question:** What are they?

- Deadweight loss
- Misallocation of goods sold, which might result from discriminatory and nonlinear pricing schemes

Third-Degree Discrimination: Welfare Effects

Proposition: If 3rd degree discrimination does not increase total output relative to uniform pricing, then it reduces social welfare.

Third-Degree Discrimination: Welfare Effects

Proposition: If 3rd degree discrimination does not increase total output relative to uniform pricing, then it reduces social welfare.

Notation:

- (\bar{p}, \bar{q}_i) are price and quantity for market i under uniform pricing
- (p_i, q_i) are price and quantity for market i under 3DPD

Proof: Welfare for market i is

$$W_i = \underbrace{S_i(p_i)}_{\text{CS}} + \underbrace{(p_i - c) q_i}_{\text{Profit}}$$

Third-Degree Discrimination: Welfare Effects

Proposition: If 3rd degree discrimination does not increase total output relative to uniform pricing, then it reduces social welfare.

Notation:

- (\bar{p}, \bar{q}_i) are price and quantity for market i under uniform pricing
- (p_i, q_i) are price and quantity for market i under 3DPD

Proof: Welfare for market i is

$$W_i = \underbrace{S_i(p_i)}_{\text{CS}} + \underbrace{(p_i - c) q_i}_{\text{Profit}}$$

Effect of price discrimination relative to uniform pricing:

$$\Delta W = \sum_i [S_i(p_i) - S_i(\bar{p})] + \left(\sum_i (p_i - c) q_i - \sum_i (\bar{p} - c) \bar{q}_i \right)$$

Recall that $S'_i(p) = -D_i(p)$, so S_i is convex in p . Everywhere above its tangents, so

$$S_i(\bar{p}) > S_i(p_i) + S'_i(p_i) (\bar{p} - p_i)$$

Third-Degree Discrimination: Welfare Effects

$$\begin{aligned}\sum_i (p_i - c) (q_i - \bar{q}) &< \Delta W < \sum_i (\bar{p} - c) (q_i - \bar{q}) \\ &= (\bar{p} - c) \sum_i (q_i - \bar{q}_i) \\ &\leq 0 \text{ if } \sum_i q_i \leq \sum_i \bar{q}_i\end{aligned}$$

So if discrimination doesn't increase output, it doesn't increase welfare.

Questions:

1. What do we think in general? Would we expect 3DPD to increase or decrease welfare? Examples?

Third-Degree Discrimination: Welfare Effects

$$\begin{aligned}\sum_i (p_i - c) (q_i - \bar{q}) &< \Delta W < \sum_i (\bar{p} - c) (q_i - \bar{q}) \\ &= (\bar{p} - c) \sum_i (q_i - \bar{q}_i) \\ &\leq 0 \text{ if } \sum_i q_i \leq \sum_i \bar{q}_i\end{aligned}$$

So if discrimination doesn't increase output, it doesn't increase welfare.

Questions:

1. What do we think in general? Would we expect 3DPD to increase or decrease welfare? Examples? Can go either way. A clear example of increasing welfare is if the monopolist will only serve high types under uniform pricing.

Third-Degree Discrimination: Welfare Effects

$$\begin{aligned}\sum_i (p_i - c) (q_i - \bar{q}) &< \Delta W < \sum_i (\bar{p} - c) (q_i - \bar{q}) \\ &= (\bar{p} - c) \sum_i (q_i - \bar{q}_i) \\ &\leq 0 \text{ if } \sum_i q_i \leq \sum_i \bar{q}_i\end{aligned}$$

So if discrimination doesn't increase output, it doesn't increase welfare.

Questions:

1. What do we think in general? Would we expect 3DPD to increase or decrease welfare? Examples? Can go either way. A clear example of increasing welfare is if the monopolist will only serve high types under uniform pricing.
2. What about equity concerns?

Third-Degree Discrimination: Welfare Effects

$$\begin{aligned}\sum_i (p_i - c) (q_i - \bar{q}) &< \Delta W < \sum_i (\bar{p} - c) (q_i - \bar{q}) \\ &= (\bar{p} - c) \sum_i (q_i - \bar{q}_i) \\ &\leq 0 \text{ if } \sum_i q_i \leq \sum_i \bar{q}_i\end{aligned}$$

So if discrimination doesn't increase output, it doesn't increase welfare.

Questions:

1. What do we think in general? Would we expect 3DPD to increase or decrease welfare? Examples? Can go either way. A clear example of increasing welfare is if the monopolist will only serve high types under uniform pricing.
2. What about equity concerns? Could go either way: financial aid vs health or life insurance

Third-Degree Discrimination: Welfare Effects

Corollary:

If demands are linear and all markets are served under uniform pricing, then 3DPD

You'll consider this case in Problem Set 1.

Second-degree price discrimination

Second-Degree Discrimination: Two Type Model

Suppose the monopolist cannot observe consumer preferences but sells a good of variable quality/quantity and can prevent resale between consumers.

- Consumers of type θ get utility $v(q, \theta) - T$ if they buy quality/quantity q at total price T , and utility 0 if they do not purchase.

Question: Why is it okay to think of q as either quality or quantity?

Second-Degree Discrimination: Two Type Model

Suppose the monopolist cannot observe consumer preferences but sells a good of variable quality/quantity and can prevent resale between consumers.

- Consumers of type θ get utility $v(q, \theta) - T$ if they buy quality/quantity q at total price T , and utility 0 if they do not purchase.

Question: Why is it okay to think of q as either quality or quantity?

Utilities satisfy:

$$\begin{aligned}\frac{\partial v}{\partial q} &> 0, & \frac{\partial v}{\partial \theta} &> 0 \\ \frac{\partial^2 v}{\partial \theta \partial q} &> 0, & \frac{\partial^2 v}{\partial q^2} &< 0\end{aligned}$$

Second-Degree Discrimination: Two Type Model (Cont.)

- Assume the cost of producing a quality q good/producing quantity q is cq .
- Assume there are just two types with $\theta_2 > \theta_1$, and write $v_i(q)$ for $v(q, \theta_i)$.

If θ were observable, this would be a first-degree discrimination model.

Second-Degree Discrimination: Two Type Model (Cont.)

With θ unobservable, the monopolist will want to allow consumers to choose (q, T) from a menu of offers.

- **Question:** In the two-type case, do we need to think of the monopolist as offering a full menu (continuum of options)?

Second-Degree Discrimination: Two Type Model (Cont.)

With θ unobservable, the monopolist will want to allow consumers to choose (q, T) from a menu of offers.

- **Question:** In the two-type case, do we need to think of the monopolist as offering a full menu (continuum of options)? No, it suffices to offer a two-item menu $(q_1, T_1), (q_2, T_2)$. WLOG, suppose $\theta_2 > \theta_1$.

The monopolist's profit-maximization problem is:

Second-Degree Discrimination: Two Type Model (Cont.)

With θ unobservable, the monopolist will want to allow consumers to choose (q, T) from a menu of offers.

- **Question:** In the two-type case, do we need to think of the monopolist as offering a full menu (continuum of options)? No, it suffices to offer a two-item menu $(q_1, T_1), (q_2, T_2)$. WLOG, suppose $\theta_2 > \theta_1$.

The monopolist's profit-maximization problem is:

$$\max_{q_1, T_1, q_2, T_2} T_1 + T_2 - c(q_1 + q_2)$$

subject to: **What constraints?** In other words, what prevents $T_1, T_2 = \infty$ and $q_1, q_2 = 0$?

Second-Degree Discrimination: Two Type Model (Cont.)

With θ unobservable, the monopolist will want to allow consumers to choose (q, T) from a menu of offers.

- **Question:** In the two-type case, do we need to think of the monopolist as offering a full menu (continuum of options)? No, it suffices to offer a two-item menu $(q_1, T_1), (q_2, T_2)$. WLOG, suppose $\theta_2 > \theta_1$.

The monopolist's profit-maximization problem is:

$$\max_{q_1, T_1, q_2, T_2} T_1 + T_2 - c(q_1 + q_2)$$

subject to:

$$(IR1) \quad v_1(q_1) - T_1 \geq 0$$

$$(IR2) \quad v_2(q_2) - T_2 \geq 0$$

$$(IC1) \quad v_1(q_1) - T_1 \geq v_1(q_2) - T_2$$

$$(IC2) \quad v_2(q_2) - T_2 \geq v_2(q_1) - T_1$$

Second-Degree Discrimination: Two Type Model (Cont.)

$$\max_{q_1, T_1, q_2, T_2} T_1 + T_2 - c(q_1 + q_2)$$

subject to:

$$(IR1) \quad v_1(q_1) - T_1 \geq 0$$

$$(IR2) \quad v_2(q_2) - T_2 \geq 0$$

$$(IC1) \quad v_1(q_1) - T_1 \geq v_1(q_2) - T_2$$

$$(IC2) \quad v_2(q_2) - T_2 \geq v_2(q_1) - T_1$$

The first step in solving problems like this is to figure out which constraints are binding.

Question: Which two do you think will not bind?

Second-Degree Discrimination: Two Type Model (Cont.)

$$\max_{q_1, T_1, q_2, T_2} T_1 + T_2 - c(q_1 + q_2)$$

subject to:

$$(IR1) \quad v_1(q_1) - T_1 \geq 0$$

$$(IR2) \quad v_2(q_2) - T_2 \geq 0$$

$$(IC1) \quad v_1(q_1) - T_1 \geq v_1(q_2) - T_2$$

$$(IC2) \quad v_2(q_2) - T_2 \geq v_2(q_1) - T_1$$

The first step in solving problems like this is to figure out which constraints are binding.

Question: Which two do you think will not bind?

- (IC2) + (IR1) imply (IR2).
- (IC1) seems unlikely to bind. Students rarely consider buying first-class tickets.

Second-Degree Discrimination: Two Type Model (Cont.)

The simplified problem is:

$$\max_{q_1, T_1, q_2, T_2} T_1 + T_2 - c(q_1 + q_2)$$

$$(IR1) \quad v_1(q_1) - T_1 \geq 0$$

$$(IC2) \quad v_2(q_2) - T_2 \geq v_2(q_1) - T_1$$

Question: What else can we say about the two remaining constraints?

Second-Degree Discrimination: Two Type Model (Cont.)

The simplified problem is:

$$\max_{q_1, T_1, q_2, T_2} T_1 + T_2 - c(q_1 + q_2)$$

$$(IR1) \quad v_1(q_1) - T_1 \geq 0$$

$$(IC2) \quad v_2(q_2) - T_2 \geq v_2(q_1) - T_1$$

Question: What else can we say about the two remaining constraints?

- Clearly, one wants to increase T_1 if (IR1) doesn't bind. This implies $T_1 = v_1(q_1)$.
- One will also want to increase T_2 if (IC2) doesn't bind. This implies:

$$T_2 = T_1 + (v_2(q_2) - v_2(q_1)) = v_1(q_1) + v_2(q_2) - v_2(q_1)$$

Second-Degree Discrimination: Two Type Model (Cont.)

The simplified problem is:

$$\max_{q_1, T_1, q_2, T_2} T_1 + T_2 - c(q_1 + q_2)$$

$$(IR1) \quad v_1(q_1) - T_1 \geq 0$$

$$(IC2) \quad v_2(q_2) - T_2 \geq v_2(q_1) - T_1$$

Question: What else can we say about the two remaining constraints?

- Clearly, one wants to increase T_1 if (IR1) doesn't bind. This implies $T_1 = v_1(q_1)$.
- One will also want to increase T_2 if (IC2) doesn't bind. This implies:

$$T_2 = T_1 + (v_2(q_2) - v_2(q_1)) = v_1(q_1) + v_2(q_2) - v_2(q_1)$$

Question: What now?

Second-Degree Discrimination: Two Type Model (Cont.)

The simplified problem is:

$$\max_{q_1, T_1, q_2, T_2} T_1 + T_2 - c(q_1 + q_2)$$

$$(IR1) \quad v_1(q_1) - T_1 \geq 0$$

$$(IC2) \quad v_2(q_2) - T_2 \geq v_2(q_1) - T_1$$

Question: What else can we say about the two remaining constraints?

- Clearly, one wants to increase T_1 if (IR1) doesn't bind. This implies $T_1 = v_1(q_1)$.
- One will also want to increase T_2 if (IC2) doesn't bind. This implies:

$$T_2 = T_1 + (v_2(q_2) - v_2(q_1)) = v_1(q_1) + v_2(q_2) - v_2(q_1)$$

Question: What now? With these T_1 and T_2 , both constraints hold with equality. Substitute and then we have an unconstrained problem.

Question: Before we substitute, what welfare conclusions can we draw from these equalities?

Second-Degree Discrimination: Two Type Model (Cont.)

The simplified problem is:

$$\max_{q_1, T_1, q_2, T_2} T_1 + T_2 - c(q_1 + q_2)$$

$$(IR1) \quad v_1(q_1) - T_1 \geq 0$$

$$(IC2) \quad v_2(q_2) - T_2 \geq v_2(q_1) - T_1$$

Question: What else can we say about the two remaining constraints?

- Clearly, one wants to increase T_1 if (IR1) doesn't bind. This implies $T_1 = v_1(q_1)$.
- One will also want to increase T_2 if (IC2) doesn't bind. This implies:

$$T_2 = T_1 + (v_2(q_2) - v_2(q_1)) = v_1(q_1) + v_2(q_2) - v_2(q_1)$$

Question: What now? With these T_1 and T_2 , both constraints hold with equality. Substitute and then we have an unconstrained problem.

Question: Before we substitute, what welfare conclusions can we draw from these equalities? Low type gets zero surplus. High type gets *information rents*.

Second-Degree Discrimination: Two Type Model (Cont.)

Substitution gives an unconstrained optimization problem:

$$\max_{q_1, q_2} v_1(q_1) + v_2(q_2) + [v_1(q_1) - v_2(q_1)] - c(q_1 + q_2)$$

First order conditions give:

$$v_2'(q_2) = c.$$

$$v_1'(q_1) + [v_1'(q_1) - v_2'(q_1)] = c.$$

Second-Degree Discrimination: Two Type Model (Cont.)

Substitution gives an unconstrained optimization problem:

$$\max_{q_1, q_2} v_1(q_1) + v_2(q_2) + [v_1(q_1) - v_2(q_1)] - c(q_1 + q_2)$$

First order conditions give:

$$v_2'(q_2) = c.$$

$$v_1'(q_1) + [v_1'(q_1) - v_2'(q_1)] = c.$$

Question: What economic conclusions can we draw from these FOCs?

Second-Degree Discrimination: Two Type Model (Cont.)

Substitution gives an unconstrained optimization problem:

$$\max_{q_1, q_2} v_1(q_1) + v_2(q_2) + [v_1(q_1) - v_2(q_1)] - c(q_1 + q_2)$$

First order conditions give:

$$v_2'(q_2) = c.$$

$$v_1'(q_1) + \underbrace{[v_1'(q_1) - v_2'(q_1)]}_{<0} = c.$$

Question: What economic conclusions can we draw from these FOCs?

- High type's quality is undistorted.
- $v_1'(q_1) > v_1'(q_1^*)$, so $q_1 < q_1^*$. So quality to the low type is inefficiently low.

Second-Degree Discrimination: Continuum of Types

Modern IO theory papers will work in a continuum-of-types model.

- Continuum of consumers with types θ with density $f(\theta)$ on $[\underline{\theta}, \bar{\theta}]$.
- Type θ consumer's gross utility from quality/quantity x is $v(x, \theta)$.

The model generalizes insights from the two-type model.

Next time

- **Problem Set 1:** Due Th 9/11. You now have everything you need to complete this.
- **Next time:** Static competition