Lecture 5: Dynamic Competition

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Slides draw upon lecture materials from Glenn & Sara Ellison (MIT).

Review: Bertrand competition

- 2 firms (could be N)
- $X(p)$ is market demand function. Assume $X(p)$ is weakly decreasing and $pX(p)$ is bounded.
- c is unit cost
- Firms simultaneously announce prices. All demand goes to lowest price firms.

Unique Nash equilibrium: $p_1^* = p_2^* = c$.
Bertrand is an "exemplifying theory." It illustrates forces using extreme assumptions that we would not see in practice.

- No product differentiation creates infinitely elastic firm-level demand
- Constant returns to scale with no capacity constraints
- One-shot interaction

Bertrand competition in a dynamic game

- N identical firms with constant marginal cost c. Discount factor δ.
- Bertrand competition at $t = 1, 2, \ldots$ with market demand $Q(p)$.
- Firms observe all prices at the end of each period.
- History: $h_{t-1} = \{p_1^\tau, \ldots, p_N^\tau\}_{\tau \leqslant t-1}$
- Firm i's strategy: $p_{it}(h_{t-1})$ for $t = 1, 2, \ldots$

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- **Question**: Can you think of another possible SPNE?

Bertrand competition in a dynamic game

A collusive SPE? That is, is it possible to sustain super-competitive prices?

- Proposed strategy:

$$p_{it}(h_{t-1}) = \begin{cases} p_m & \text{if } h_{t-1} = (p_m, \ldots, p_m) \\ c & \text{otherwise} \end{cases}$$

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- **Question**: What are current and future payoffs from collusion? From deviation?
  - Let $\pi_m = (p_m - c)\,Q(p_m)$
  - Current payoff from collusion: $\frac{1}{N}\pi_m$
  - Current payoff from deviation: π_m
  - Future payoff from collusion: $\frac{\delta}{1-\delta}\frac{1}{N}\pi_m$.
  - Future payoff from deviation: 0.
- Collusion sustainable if

$$\frac{1}{N}\frac{1}{1-\delta}\pi_m \geqslant \pi_m \;\Leftrightarrow\; \delta \geqslant \frac{N-1}{N}$$

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- This condition doesn't depend on the demand function.

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- **Question**: What features of a setting make it more likely that collusion can be sustained?
  - It is easier to sustain collusion when δ is larger.
    - Could explain how detection lags, smooth/lumpy demands, growth, and technological change affect markets.
  - It is easier to sustain collusion when N is smaller.

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  - It is easier to sustain collusion when N is smaller.
- Given the equilibrium multiplicity we must believe in some equilibrium selection or accept our inability to forecast.

Factors Limiting Collusive Pricing

In the real world, δ is often quite close to 1, which would mean that tacit collusion should be sustainable. But it's safe to say that tacit collusion is not a ubiquitous feature of real-world markets.

Question: What factors might make collusion difficult to sustain in practice?

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Question: What factors might make collusion difficult to sustain in practice?

1. Imperfect observation of actions
2. Variable demand
3. Cost shocks
4. Antitrust enforcement

Collusion with Imperfect Monitoring

Green-Porter (1984)

- Two firms with marginal cost c compete as in Bertrand at $t = 1, 2, \ldots$
- Market demand is noisy:

$$Q_t(p) = \begin{cases} Q(p) & \text{with probability } 1 - \alpha \\ 0 & \text{with probability } \alpha \end{cases}$$

- Demand goes to the lower-priced firm (or splits 50-50 if prices equal).
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Question: Do you think there is a SPNE in which both firms set $p_{it} = p^m$ in each period on the equilibrium path?

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Proof sketch: (By contradiction.) In such an SPNE firm 1 would need to keep charging p_m even if it got zero demand in the first million periods. Given this, firm 2 will want to cut its price to $p_m - \epsilon$.

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Proposition: For $\alpha < \frac{1}{2}$ and δ sufficiently close to 1 the model does have a partially collusive equilibrium in which firms initially set $p_{it} = p_m$, switch to $p_{it} = c$ for T periods every time some firm gets zero demand, and then go back to $p_{it} = p_m$ after the T periods are over (for some T).

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Proof sketch:

- Write V_m for the PDV of payoffs at $t = 0$ and V_p for the PDV of payoffs at the start of the punishment phase. We need to show that there are no profitable single period deviations in any states of this process.

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- **Question**: Does firm want to deviate during the *punishment phase*? No, a firm that deviates can't earn positive profits in the period in which it deviates and it does not affect pricing in any other period.
- It remains just to show that firms don't want to deviate during the *collusive phase*.

Collusion with Imperfect Monitoring

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Proof sketch (cont'd):

- **Question**: In punishment phase, payoff from cooperation is V_m. What is payoff from deviation in terms of V_m, π_m, and V_p?

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- So, to show that firms don't want to deviate in the cooperate phase we want to show that $V_m \geqslant \pi_m(1-\alpha)+\delta\,V_p$.

- **Question**: What are the equations defining V_m, V_p?

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- **Question**: What are the equations defining V_m, V_p?

$$V_m = (1-\alpha)\left(\frac{\pi_m}{2} + \delta V_m\right) + \alpha\delta V_p$$

$$V_p = \delta^T V_m$$

- Substituting the second expression into the first the solution is

$$V_m = \frac{(1-\alpha)\pi_m/2}{1-[(1-\alpha)\delta + \alpha\delta^{T+1}]}$$

$$V_p = \delta^T V_m$$

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Proof sketch (cont'd):

- The required inequality is then $(1-\delta^{T+1})V_m \geqslant \pi_m(1-\alpha)$. Cancelling the common $\pi_m(1-\alpha)$ terms and multiplying by the denominators gives

$$(1-\delta^{T+1}) \geqslant 2(1-[(1-\alpha)\delta+\alpha\delta^{T+1}])$$

$$\iff 2(1-\alpha)\delta+(2\alpha-1)\delta^{T+1} \geqslant 1$$

- For $\alpha < \frac{1}{2}$ this will hold if δ is sufficiently close to 1 and T is sufficiently large.

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Remarks:

1. We see price wars in equilibrium. They can be part of a well-functioning cartel.
2. Optimal collusion may have T finite.
3. Price wars are triggered by random demand shocks that resemble firms cheating. In this model, low demand triggers price wars.
4. In a model with continuous demand shocks players will use cutoffs in the observed variable and would sometimes get away with cheating.
5. Sustaining collusion is more difficult when demand is noisier. (Here, when α is larger.)

Collusion with Imperfect Monitoring

More general analyses

- Green and Porter (Econometrica 1984) also consider a Cournot-like model with continuous demand shocks.
  - Firms choose quantities $q_1, q_2, \ldots, q_N$
  - $p(Q) = \theta P(Q)$ where $\theta \sim F(.)$ with unshifting support and $E(\theta) = 1$
  - G-P focus on symmetric "trigger price" equilibria – produce q^* but revert to static Cournot equilibrium for T periods if realized price is below $\hat{p}$.

- Abreu, Pearce, and Stacchetti (Econometrica 1990) provide a more general result on optimal strongly symmetric equilibria in models of this type. They focus on the equilibrium payoff set and use dynamic programming arguments to characterize extreme points. In the GP model, it can involve two-sided triggers to enter and exit price wars.

- Fudenberg, Levine, and Maskin (Econometrica 1994) show that in many models we can avoid the inefficiency in the GP and APS equilibria by using asymmetric strategies in which we punish firms that appear to have cheated by transferring market share from firms that appear to have cheated to other firms, avoiding the inefficiency of price wars.

- Recent private monitoring papers include Awaya-Krishna (AER 2016) and Sugaya-Wolitzky (JPE 2018).

Collusion with Cyclical Demand

Rotemberg-Saloner (1986)

- Two firms with marginal cost c compete as in Bertrand at $t = 1, 2, \dots$
- Market demand is noisy: $Q_t(p)$ is $Q_L(p)$ or $Q_H(p)$, each with prob. $\frac{1}{2}$
- Key difference from G-P: Firms observe period t demand state before choosing p_{it}

Consider possible collusive equilibria in which firms charge p_H if demand is high and no one has cheated, p_L if demand is low and no one has cheated, and c if anyone has ever deviated.

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Question: If one of these inequalities were violated, which would it be?

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Question: If one of these inequalities were violated, which would it be? (0.1). Note that the continuation payoff is state-independent, whereas the short-run gain from deviating is not.

Collusion with Cyclical Demand

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The constraints required for SPE are:

$$\frac{1}{2}\pi_H(p_H) \leqslant \delta\left(\frac{1}{1-\delta}\frac{1}{2}\left(\frac{1}{2}\pi_H(p_H)+\frac{1}{2}\pi_L(p_L)\right)\right)$$

$$\frac{1}{2}\pi_L(p_L) \leqslant \delta\left(\frac{1}{1-\delta}\frac{1}{2}\left(\frac{1}{2}\pi_H(p_H)+\frac{1}{2}\pi_L(p_L)\right)\right)$$

- When $\delta \approx 1$ neither constraint is binding and firms can collude on (p_L^m, p_H^m).
- When δ is smaller, the first binds. The best SPE has $p_L = p_L^m$, $p_H < p_H^m$.
- When $\delta = \frac{1}{2}$ both bind. $\pi_H(p_H) = \pi_L(p_L) \Rightarrow p_L < p_H$.
- For smaller δ no collusion is possible and both firms set $p_{it} = c$.

Remarks:

1. For intermediate δ the model predicts that optimal markups are countercyclical.
2. If we added imperfect observation, the intuition that collusion is more difficult when demand is high should carry over. Whether this results in lower markups, price wars being more likely, or both will be model dependent.

Private Cost Shocks

Athey-Bagwell (RAND 2001)

Suppose that the members of a cartel have private cost shocks in each period. Efficiency requires that the low-cost firm produce in each period. With no limits on contracting, side payments to losing bidders could be used to induce firms to reveal costs.

Without side payments we can consider dynamic equilibria.

Suppose $N = 2$ and costs $c_{it} \in \{c_L, c_H\}$ are iid across firms and time.

1. When δ is close to one firms can achieve full collusion via strategies in which announcing low costs today gives the other firm priority to produce in the next period when both firms' costs are low.
2. When δ is not as close to one, firms will price below the monopoly price and not produce efficiently. If a firm has lost too much future priority, it will be tempted to deviate from the collusive price. To make deviating less attractive, prices are reduced and production can be assigned to the high priority firm regardless of the cost realization.

Antitrust Authorities

Harrington (RAND 2004): Cartel pricing dynamics in the presence of an antitrust authority

Another constraint on tacit collusion is that firms must coordinate on one of many equilibria and discussing pricing can violate antitrust laws. Antitrust rules, however, can also have unintended consequences.

Suppose that the probability that collusion will be detected is

$$\phi(\vec{p}_1, \vec{p}_2, \ldots, \vec{p}_t) = \sum_{js} g(p_{js} - p_{js-1})$$

with convex g minimized at 0.

Suppose that damages accrue according to

$$x_{jt} = \beta x_{jt-1} + \gamma\pi(p_{jt})$$

1. Long run prices can be higher with antitrust penalties than without because penalties provide an additional reason not to deviate.
2. Collusion may feature prices that build up over time.
3. More patient firms raise prices more slowly and reach higher price levels.

Multimarket Contact

Bernheim-Whinston (RAND 1990)

When firms compete in many different markets does this make collusion easier?

- Two firms choose prices in N markets at $t = 0, 1, 2, \ldots$
- Prices are chosen simultaneously for all markets. All period t outcomes observed before choosing period $t + 1$ prices.

Firms can be punished in all markets in response to a deviation. But they can also gain by deviating simultaneously in all markets.

Observations:

1. When markets are identical (or with Bertrand competition in all markets) multimarket contact does not make collusion easier or harder.
2. When markets are asymmetric, firms can use spare punishment capacity in the easy-to-collude markets to enable collusion in other markets.

Multimarket Contact

Empirical question: Can/do firms actually leverage potential for multi-market punishment in practice?

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Question: What factors could lead to firms failing to exploit MMC?

Next time

Models of entry