

# Lecture 6: Entry

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ECON 7510

Cornell University

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Slides draw upon lecture materials from Glenn & Sara Ellison (MIT).

## Entry with Homogeneous Products

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- **Stage 1:** A large number of potential entrants choose In/Out with entry cost  $K$ .
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**Question:** In general, what is an approach to solving a model like this one with finitely many stages?

We solve by backward induction, resulting in prices  $p^*(N)$  and variable profits  $\pi_v^*(N)$  if  $N$  firms enter. We then find  $N^*$  from the first stage.

## Entry with Homogeneous Products (cont.)

**Stage 2:** Firms that chose to enter play some game like Bertrand, Cournot, or price competition with search. Assume firms have variable costs  $c(Q)$ .

To analyze Stage 1, we need to know the relationship between  $N$  and each firm's variable profits (which is determined by equilibrium of Stage 2). A few examples:

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- **Cournot** (symmetric, with linear demand  $P(Q) = a - bQ$ ):  $\pi_v^*(N) = \frac{1}{(N+1)^2} \frac{(a-c)^2}{b}$



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- **Question:** Suppose there are three potential entrants. Suppose Stage 2 is symmetric Bertrand game and  $0 < K < \pi^m$ . How many PSNE are there?
- **Question:** Suppose there are  $M$  potential entrants. Suppose Stage 2 is symmetric Cournot game and  $K > 0$ . How many PSNE are there? What profits do entrants earn?

## Entry with Homogeneous Products (cont.)

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Suppose there are  $M$  potential entrants and each adopts a symmetric mixed strategy: Enter with probability  $\lambda$ .

**Question:** What is the equilibrium condition that pins down  $\lambda$ ?

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$$N \sim \text{Binomial}(M, \lambda)$$

Since  $\pi_v^*(N)$  is generally nonlinear in  $N$ , might need to compute the expectation numerically.

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If I enter, I earn zero profit if anyone else enters; if I'm the only one who enters, I earn  $\pi^m$ . So expected profit is  $(1 - \lambda)^{M-1} \pi^m$ .

So the unique symmetric mixed strategy equilibrium has

$$\lambda^* = 1 - \left( \frac{K}{\pi^m} \right)^{\frac{1}{M-1}}$$

## Entry with Homogeneous Products (cont.)

### Observations:

- With lower fixed costs, we get more entry.
- In many models,  $N^* \rightarrow \infty$  as  $K \rightarrow 0$ .
- With pure strategies, equilibrium profits are a rounding error, so profits are not directly related to factors that increase markups conditional on  $N$ .
- With Bertrand competition, positive (and perhaps large) probability of negative profits!

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**First best: Question:** What is first best here?

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**Second best:** Maximize welfare, taking second-period equilibrium strategies (e.g.,  $p^*(N)$  or  $q^*(N)$ ) as given. For the Cournot game,

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How does this compare to equilibrium number of firms  $N^*$ ?

## Efficiency of Entry with Homogeneous Products (cont.)

**Proposition:** Suppose  $W(N)$  is concave,  $\pi_v^*(N)$  is decreasing, and  $p^*(N)$  and  $q^*(N)$  can be extended to continuous functions with:

- $\frac{\partial}{\partial N}(N \cdot q^*(N)) > 0$
- $\frac{\partial}{\partial N}(q^*(N)) < 0$
- $p^*(N) - c'(q^*(N)) > 0$

Then:

$$N^* \geq N^{2B} - 1$$



## Efficiency of Entry with Homogeneous Products (cont.)

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**Proof:** Extend  $W(N)$  to a differentiable function using  $p^*(N)$  and  $q^*(N)$ . Let  $\hat{N}^{2B}$  be the solution to  $W'(\hat{N}^{2B}) = 0$ .

The second-best number of firms will be  $N^{2B} \leq \lceil \hat{N}^{2B} \rceil$ , assuming  $W(N)$  is single-peaked. The first-order condition defining  $\hat{N}^{2B}$  gives:

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$$\pi_v^*(N) + \underbrace{N(p^*(N) - c'(q^*(N))) q^{*'}(N)}_{\equiv \delta < 0} - K|_{N=\hat{N}^{2B}} = 0$$

$$\Rightarrow \pi_v^*(\hat{N}^{2B}) = K - \delta > K$$

$$\pi_v^*(\hat{N}^*) = K \text{ and } N^* = \lfloor \hat{N}^* \rfloor$$

$$\Rightarrow N^* \geq \lfloor \hat{N}^{2B} \rfloor$$

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1. In numerical examples one often finds that many more firms enter than is socially optimal because most of the marginal entrant's demand is business stealing, e.g. in  $N$  firm Cournot with  $D(p) = 1 - p$ , we have  $q^*(N) = \frac{1}{N+1}$ . The increase in total output from the  $N$ th firm is just  $\frac{N}{N+1} - \frac{N-1}{N} = \frac{1}{N(N+1)}$ .

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2. We can get one firm too few in Bertrand-like environments, and the welfare loss from the slightly insufficient entry can be large. **Why?** In a pure SPNE of a Bertrand model only one firm enters, so free entry leads to monopoly. If  $K$  is not too large, the social planner would prefer to have two firms enter, leading to  $p^* = c$ .



## Welfare Effects of Entry

Beyond the homogeneous goods environment, entry can be too high or too low.

$$W_N = \sum_{i=1}^N \pi_v^*(N) + CS(N) - N \cdot K$$

$$W_{N+1} = \sum_{i=1}^{N+1} \pi_v^*(N+1) + CS(N+1) - (N+1) \cdot K$$

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1. Business stealing can lead to excessive entry.
2. Firms do not internalize gains in consumer surplus. This can lead to insufficient entry.

Our previous theorem implied that with homogeneous goods the second is outweighed by first (except perhaps for leading to one firm too few). On the margin, a reduction in price is just a transfer that increases CS by the same amount by which it reduces profit.

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Consider a Hotelling-like model with mass 1 of consumers who get utility  $v - p_j - t \cdot d_{ij}$  from buying at distance  $d_{ij}$  arranged around a circle of circumference one.

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- We can show that  $\pi_v^*(N) = t/N^2$ , so  $N^* = \sqrt{t/K}$
- $N^{2B} = \sqrt{t/4K} = \frac{1}{2} N^*$
- Intuitively, the extra CS-improving effect from entry exists, but is fairly weak with these preferences; most of the firm's marginal demand is still due to business stealing.
- Things can work out differently with other distributions of idiosyncratic tastes.

# Firm Dynamics with Learning

Jovanovic (1982) discusses entry, growth, and exit in a model with learning.

## Setup:

- Continuum of small potential entrants with a fixed cost  $K$  of entry and liquidation value  $w$ .
- Firms have unknown types  $\theta_i \sim N(\theta_0, \sigma_\theta^2)$ .
- Firm  $i$ 's period  $t$  cost is  $c(q_{it})f(\theta_i + \epsilon_{it})$ , with  $N$  convex and  $f$  increasing and bounded.
- Firms don't know their types but get signals every time they produce and update their beliefs.

Firms act as price takers with optimal behavior depending on the mean and variance of their posterior beliefs.

$$q_{it} \in \operatorname{argmax}_q E_{\theta_i} [p_t q - c(q)f(\theta_i + \epsilon_{it})]$$

# Firm Dynamics with Learning (cont.)

## Optimal Behavior:

- Low  $E(\theta_i)$  implies high  $q_{it}$ .
- Medium  $E(\theta_i)$  implies low  $q_{it}$ .
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## Empirical Predictions:

1. Small firms grow faster and fail more often.
2. Bigger firms have higher profits.
3. Larger firms' profits are more serially correlated.

## Other topics related to entry

- Theory: Entry with vertical differentiation
- Theory: Strategic actions (e.g., investment) to deter or accommodate entry.
- Empirical work on excessive entry
- Empirical methods: Use observed entry decisions to infer entry costs (moment inequality methods)

# Next time

Start Part 2 of the course