Assignment 2 ELEC 4700 Finite Difference Method

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Adam Heffernan 100977570 Completed February 22nd 2020

In this section we model two simple cases. a 1D and 2D case using Laplaces finite difference method in Matrix form as to make it easier to add boxes into the simulation. The dimmensions given in the problem states a 3/2 Length v. Width, as such I used 30 and 20 as my mesh sizes for the length and width of respectively for all problems solved in the scripts below. The initial Voltage at the boundary conditions is set to 1V for all cases modeled below.

Part 1A

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Assignment2_Part1(1)
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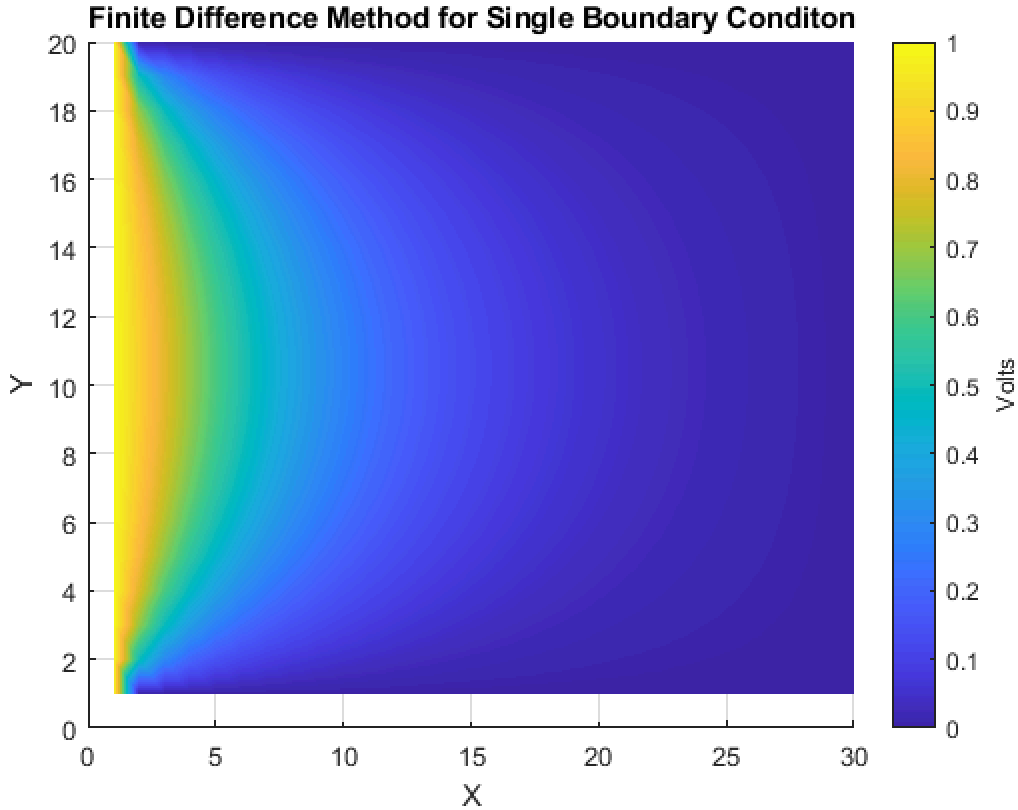

Figure 1 Above shows the 1D potential case where $X = V_0$ while $X = 0$ and the potential as we approach all 3 other sides goes to 0. That is $X = 0$ at $X = length$, $Y = 0$ while $Y = 0$ and $Y = Width$. In this case the voltage is constant in the Y direction at 0V and rises linearly to 1V at between $X = 0$ and $X = length$

Part 1B

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Assignment2_Part1(2)
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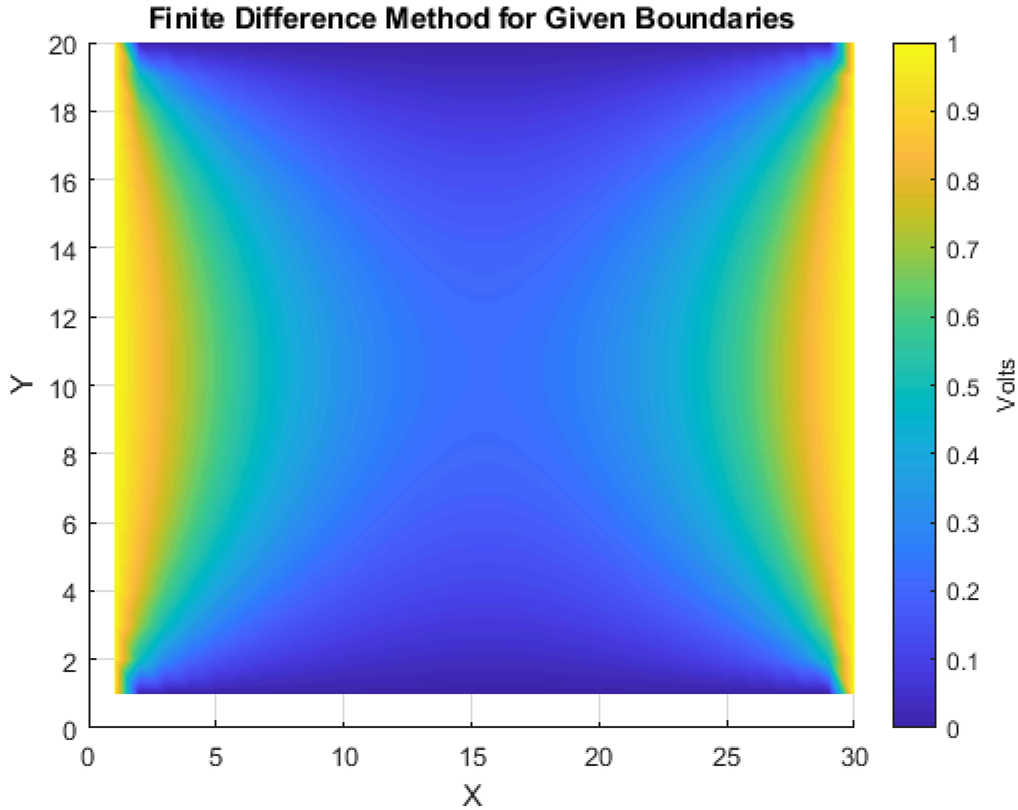
Finite Difference Method for Given Boundaries
Y
X
Volts
0
5
10
15
20
25
30
0
2
4
6
8
10
12
14
16
18
20
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

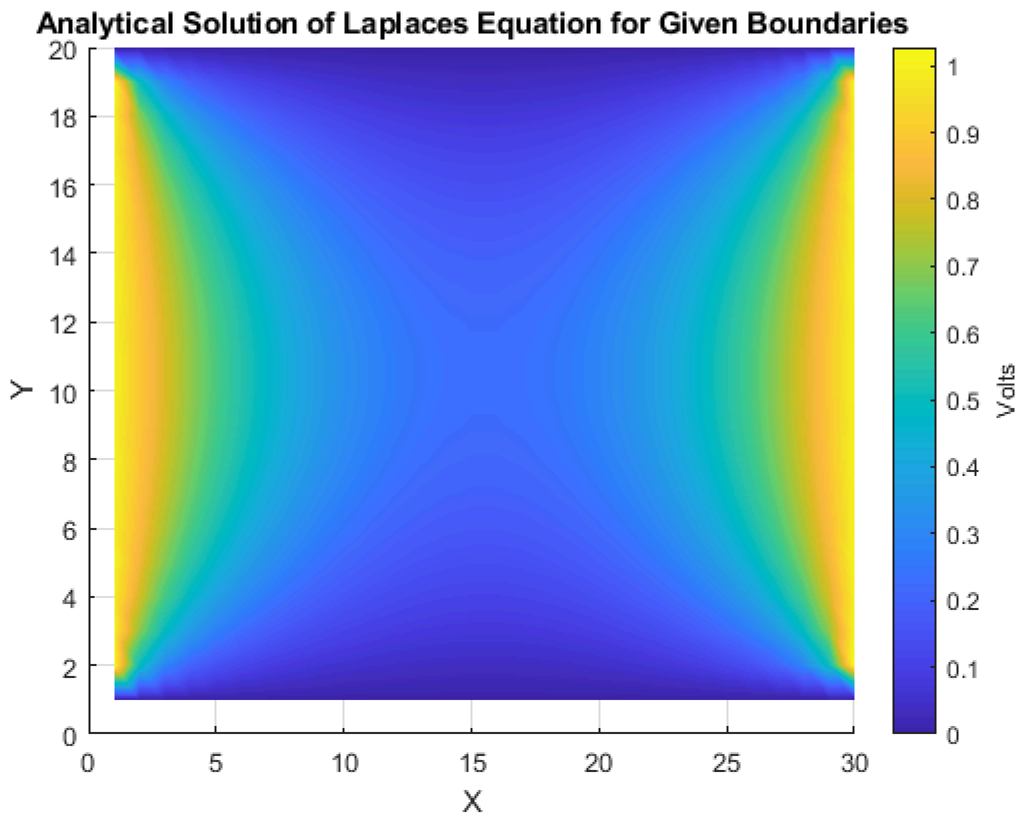
Analytical Solution of Laplaces Equation for Given Boundaries
Y
X
Volts
0
5
10
15
20
25
30
0
2
4
6
8
10
12
14
16
18
20
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

Figure 2 above shows the 1D potential case where $X = V_0$ while $X = 0$ and $X = length$ and the potential as we approach the Y boundaries goes to 0. That is $Y = 0V$ while $Y = 0$ and $Y = Width$. In this case the voltage is constant in the Y direction at 0V and rises linearly to 1V at between $X = 0$ and $X = length$. Figure 2 above shows the analytical solution to the differential equation where the boundary conditions in the X direction approach V_0 in this case our $V_0 = 1V$. By looking at the corresponding matrix and figures we can see that these two solutions are closely approaching eachother. This makes sense as the analytical solution will require infinite iterations in order to truly represent the numerical solution to this equation.

Part 2A

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Assignment2_Part2(1)
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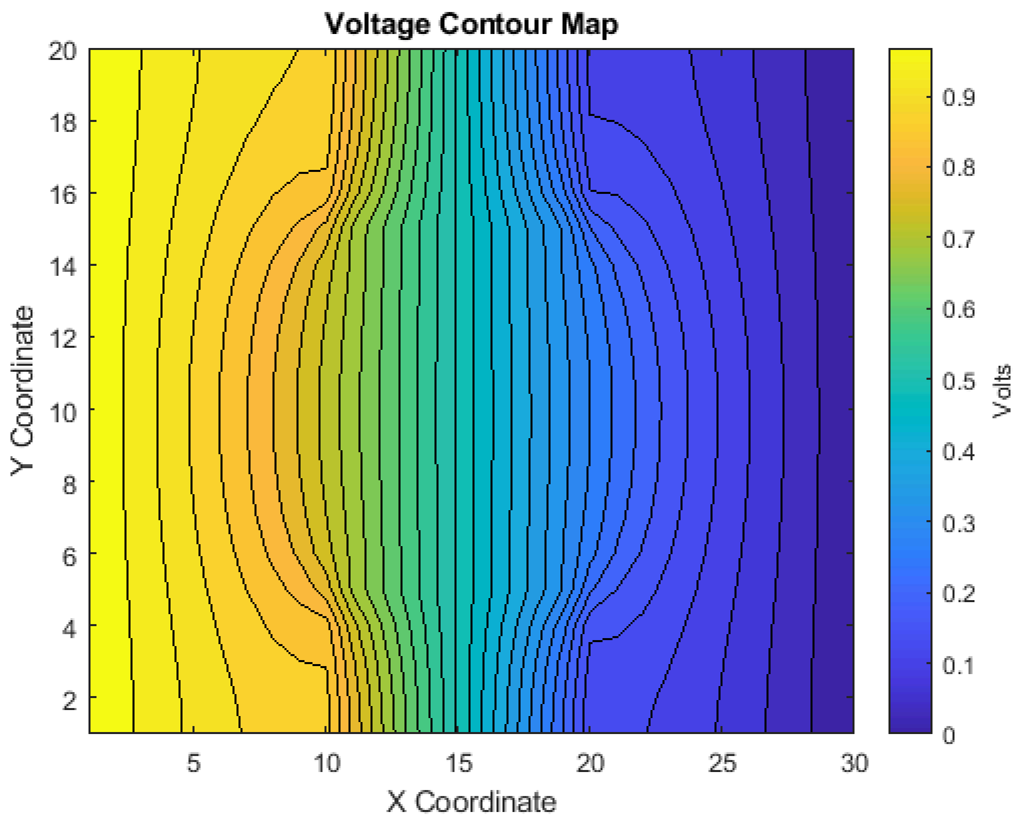
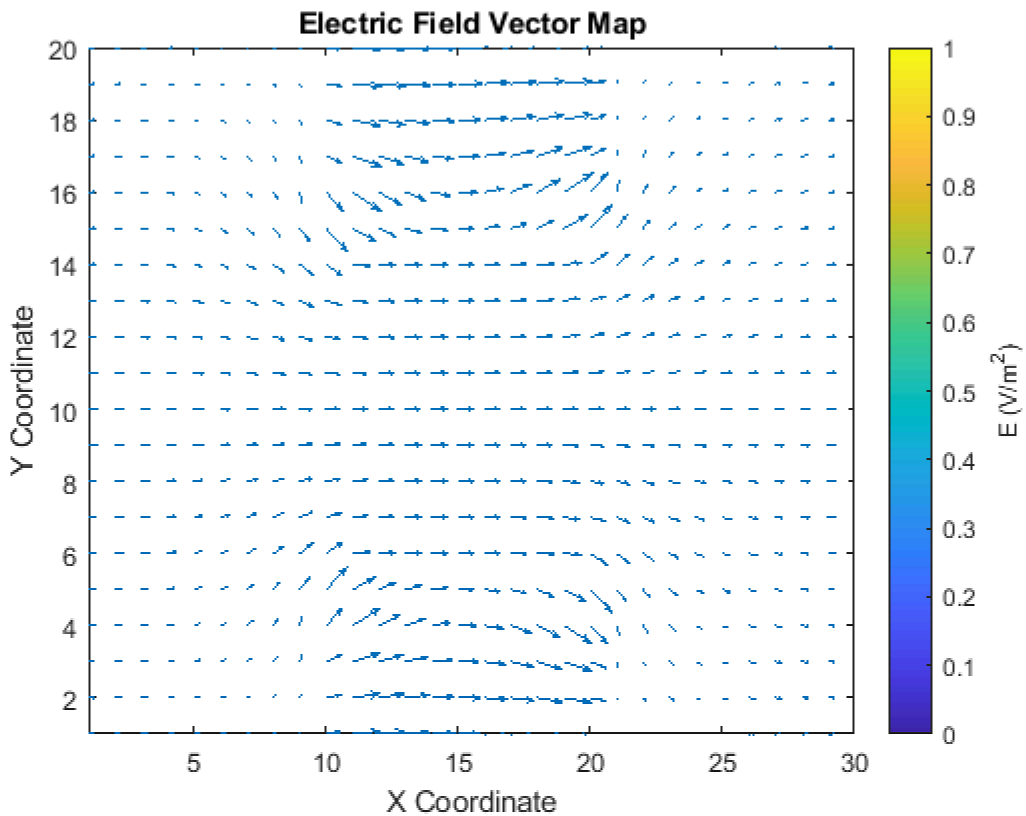
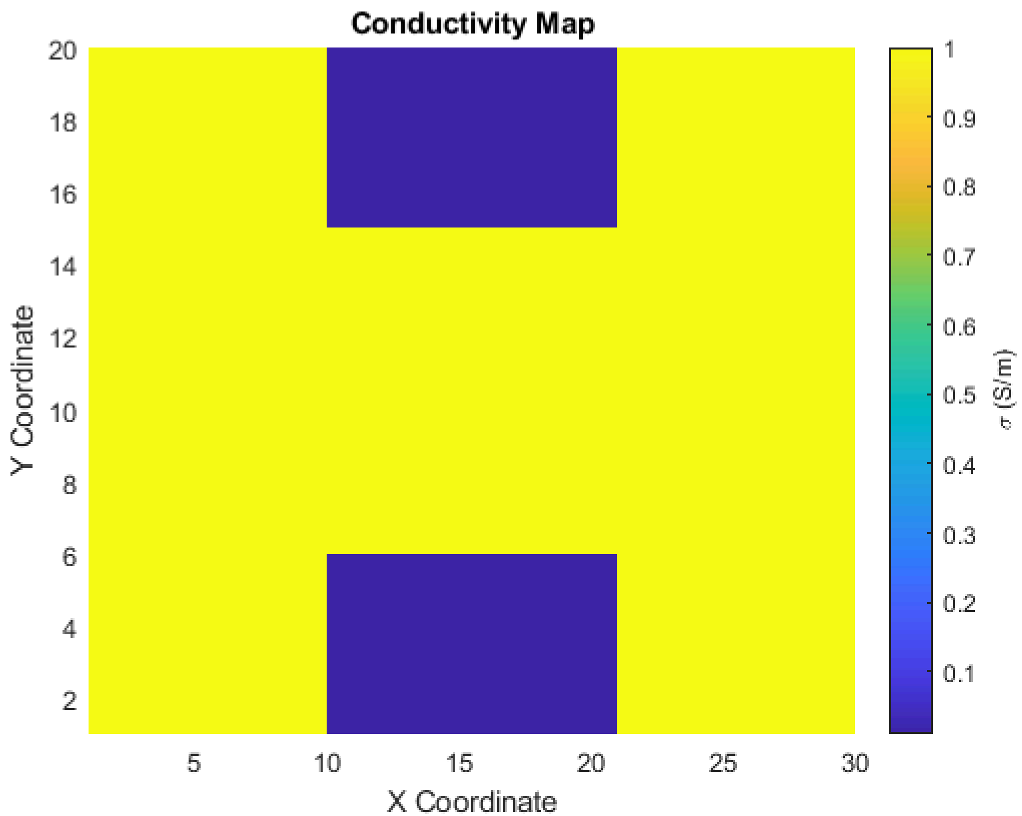
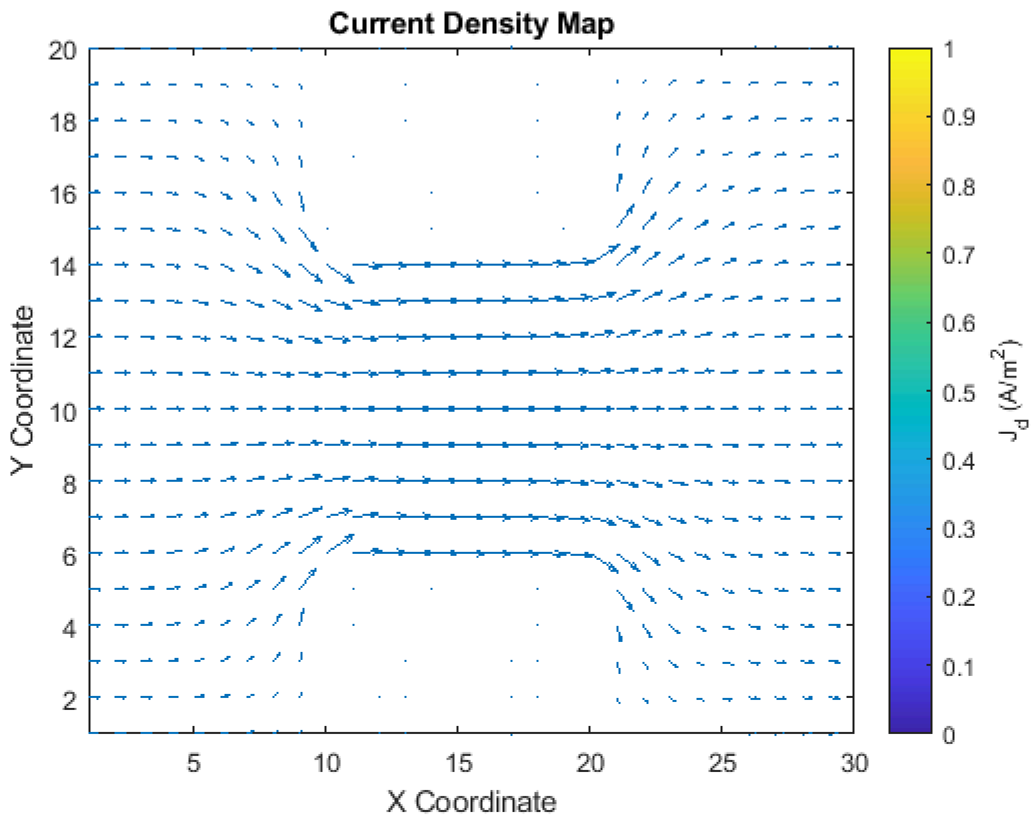



Electric Field Vector Map
Y Coordinate
X Coordinate
E (V/m^2)


Conductivity Map
Y Coordinate
X Coordinate
σ (S/m)



In Part 2 we added the bottlenecks created in Assignment 1. The length and width of the mesh grid was kept the same as in Part 1. Two boxes with low conductivity were created with a conductivity value of $\sigma = 1e^-2$. The rest of the mesh grid was populated with a conductivity value of $\sigma = 1$ The first figure shows the potential voltage lines for this mesh grid. The low conducivity zone is inversely proportional to a high resistance area. Therefore the greatest drop in voltage is across the boxes which makes sense. In figure 5 we see the electric field. The elctric field is completely in the X-direction as the potential at the boundaries in the Y-direction is 0. In figure 6 we are shown a conductivity map, the map corresponds exactly to the matrix we created to represent the conductivity in the mesh grid. Figure 7 shows the current for the specified mesh size above. The current density is equal to the conductivity multiplied by the electric field at each point in space. We map this result, the current density vectors are all in the same direction as the Electric Field. As seen from the figure the boxes have little to no current.

Part 2B Varrying mesh sizes

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Assignment2_Part2(2)
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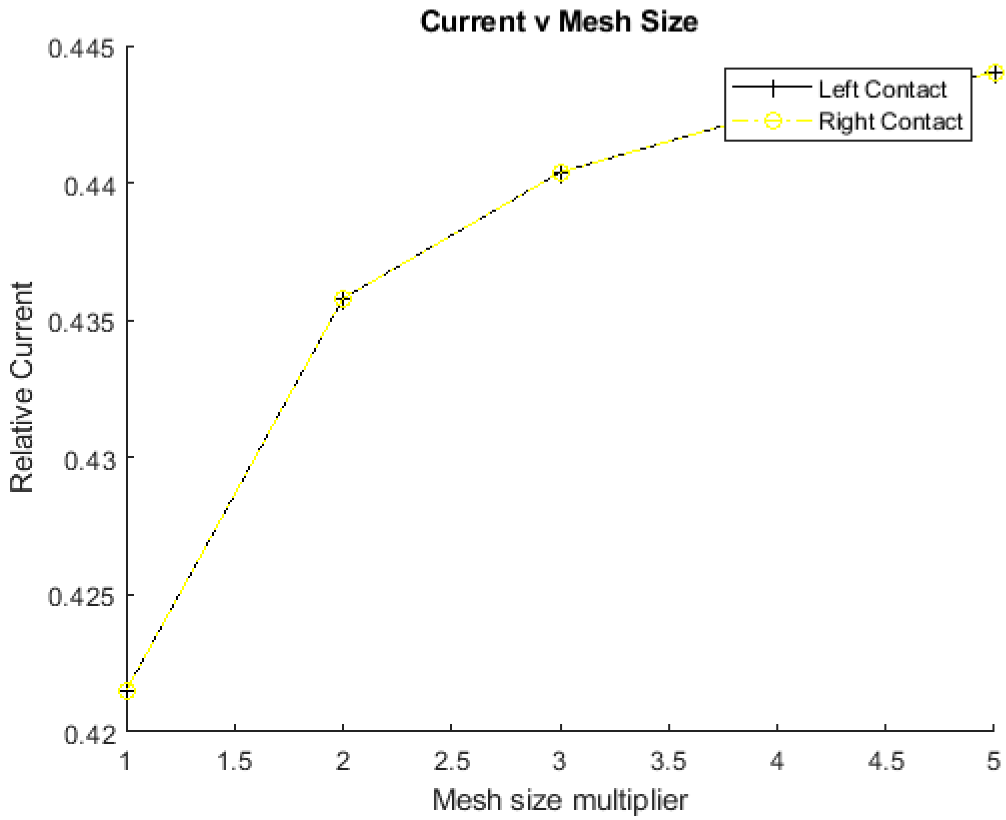

Figure 8 shown above is plotting the relative current of the mesh grid versus a varrying mesh size. As we can see from the plot as the mesh size increases so to does relative current. This makes sense as when the mesh size is increased the electric field is also correspondingly increased. Since Current density is directly proportional to the Electric field multiplied by the conductivity.

Part 2C-1 Varrying Box Width

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Assignment2_Part2(3)
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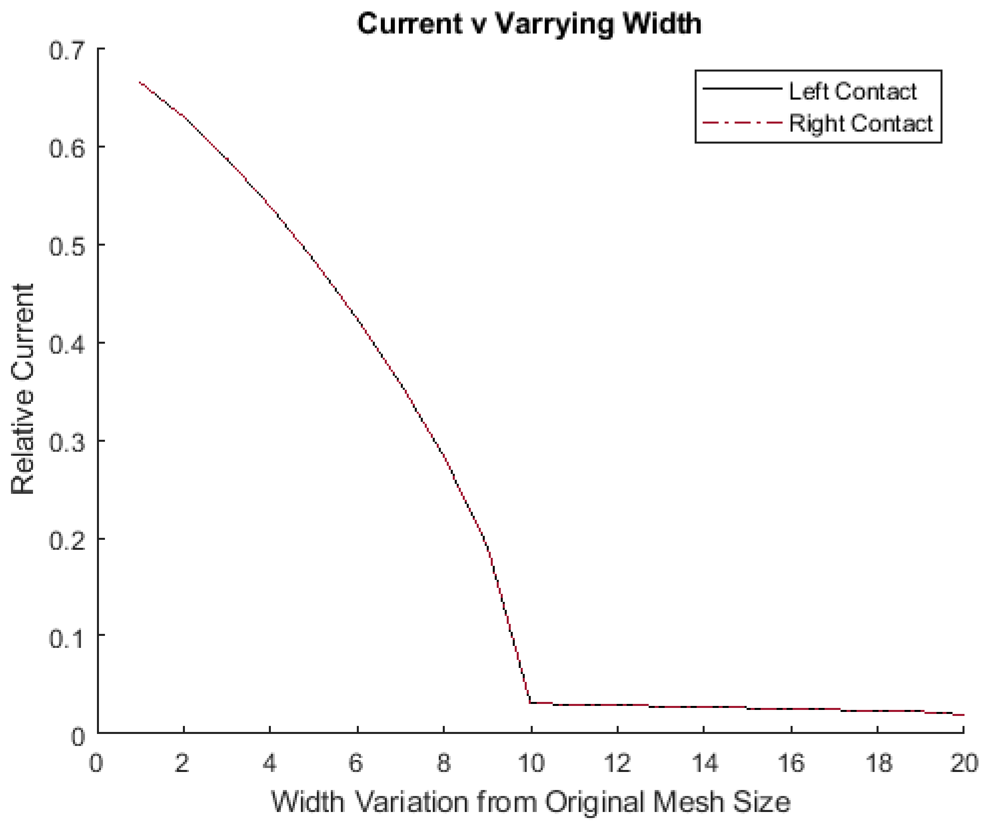

Figure 9 shown above is plotting the relative current of the mesh grid versus varrying bottle neck widths. As the width is varried the path of low resitance (or high conductivity) is made smaller and smaller. With an increase in resistance and having fixed voltages at the boundaries it makes sense the relative current in the mesh grid would decrease as the width of the box increases to compensate for $V = I * R$.

Part 2C-2 Varrying Box Length

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Assignment2_Part2(4)
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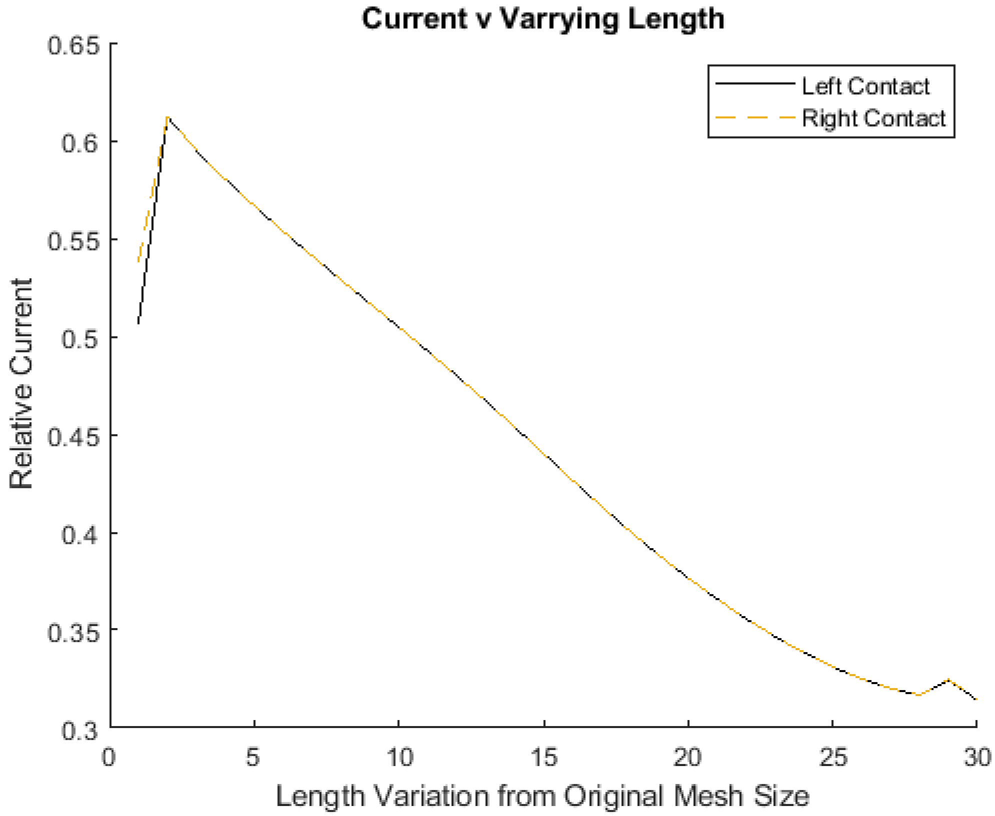

Figure 10 shown above is plotting the relative current of the mesh grid versus varrying bottle neck lengths. As the length is varried, the path of low resitance (or high conductivity) is made smaller and smaller. With an increase in resistance, and having fixed voltages at the boundaries it makes sense the relative current in the mesh grid would decrease as the length of the box increases to compensate for $V = I * R$.

Part 2D Varrying the conductivity density map

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Assignment2_Part2(5)
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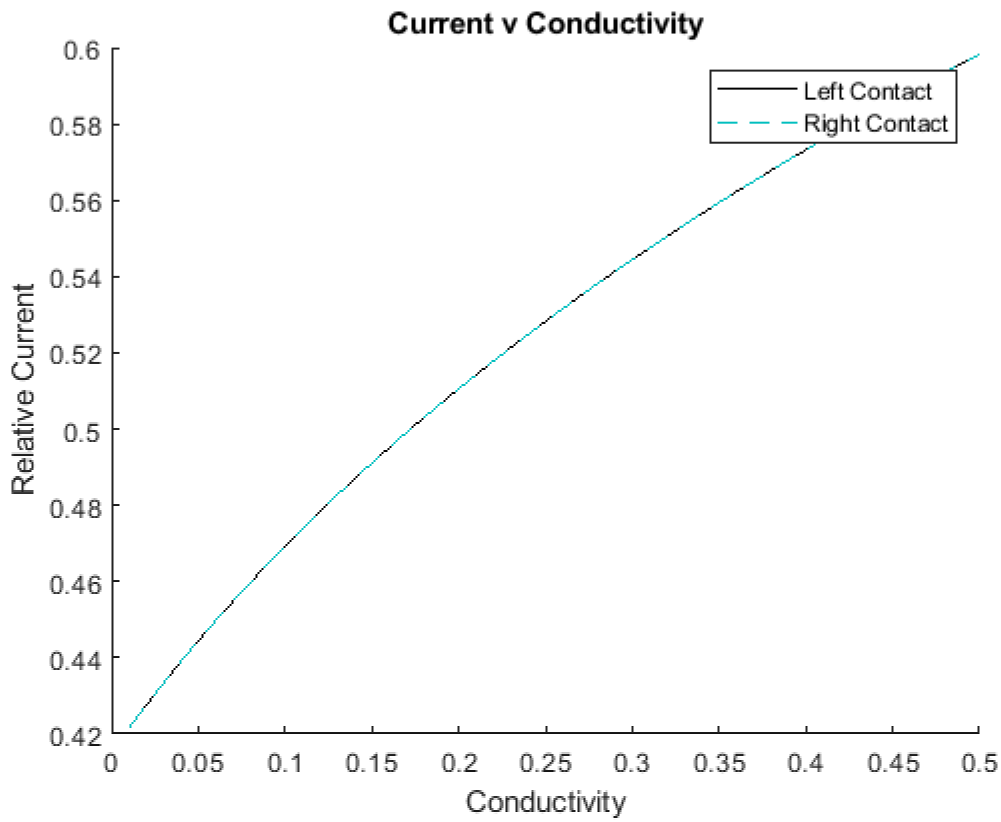

Figure 11 shown above is plotting the relative current of the mesh grid versus varrying conductivity in the grid. As the conductvity of the boxes is increased the overall resistance in the mesh grid decreases. The current should increase to comensate for $V = I * R$. As shown above in figure 11, the current increases with increasing conductivity.

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