Carleton University

SYSC 3600

Lab 3

Control of an Inverted Pendulum

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Contents

List of Figures

1 Introduction

This report is to discuss the unstable non-linear inverted pendulum system. The system was implanted in Simulink and tested using MATLAB.

2 PreLab Equation

Before proceeding with the experiment, the following equations were obtained

$$\omega_n = \sqrt{\frac{kp - (M+m)g}{Ml}} \tag{1}$$

Rearranging Equation 1 and solving for k_p

$$k_p = \omega_n^2 * Ml + Mg + mg \tag{2}$$

$$\zeta = \frac{\frac{k_d}{Ml}}{2\sqrt{\frac{kp-(M+m)g}{Ml}}} \tag{3}$$

Rearranging equation 3 and solving for k_d.

$$k_d = \zeta 2\sqrt{Ml(k_p - (M+m)g)} \tag{4}$$

$$k_d = 2\zeta\omega_n Ml \tag{5}$$

Then, the values of k_d and k_p were obtained from the above equations. Using $M = 1000kg$, $m = 200kg$, $l = 10m$, $g = 9.81m/s^2$, $\zeta = 0.7$ and $\omega_n = 0.5rad/s$.

$$k_d = 7000 \tag{6}$$

$$k_p = 14272 \tag{7}$$

3 System Simulation in Simulink

3.1 Inverted Pendulum Demo in Simulink

Simulink has a built in demo of the inverted pendulum. The Demo was observed by typing penddemo in MTLAB command prompt, an animation of the system was shown. When moving the cart in the demo the pendulum would over shoot before balancing, this over shoot is a result of an under damped system. The reason an under damped response is preferred to the over damped or critically damped repose is due to the fact that with overshooting the cart will shoot a little beyond where it is suppose to. This system involves feedback, therefore the damping will bring theta back to its desired state. An over damped system would never reach the desired state. Even though a critically damped system is mathematically possible, these systems are not achievable in the real world. We can come close, however all systems will always be slightly over damped or under damped.

3.2 Testing the Pendulum

To test the pendulum, the block diagram provided in the appendix along with MatLab code found in the lab manual were used.

The first plot below is a picture of the θ scope in Figure 3 of the lab manual.

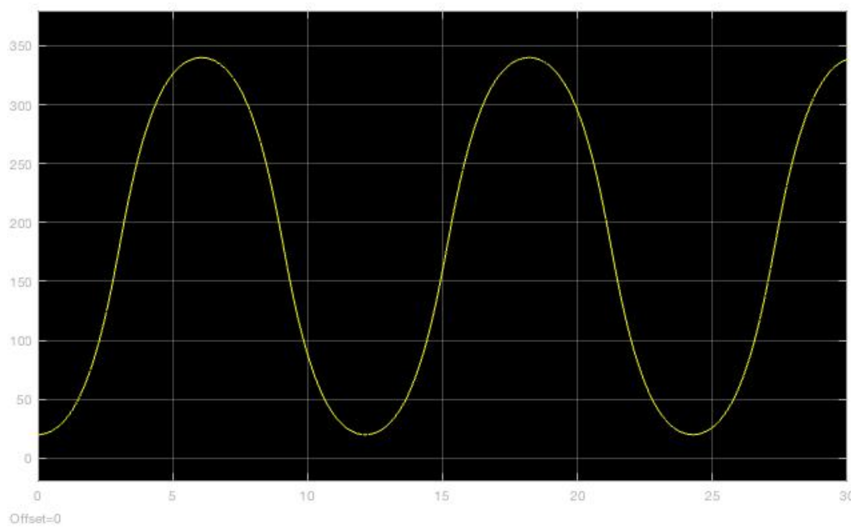

Figure 1: θ scope in Figure 3 of the Lab Manual

The plot below is figure of the x scope in Figure 3 of the lab manual.

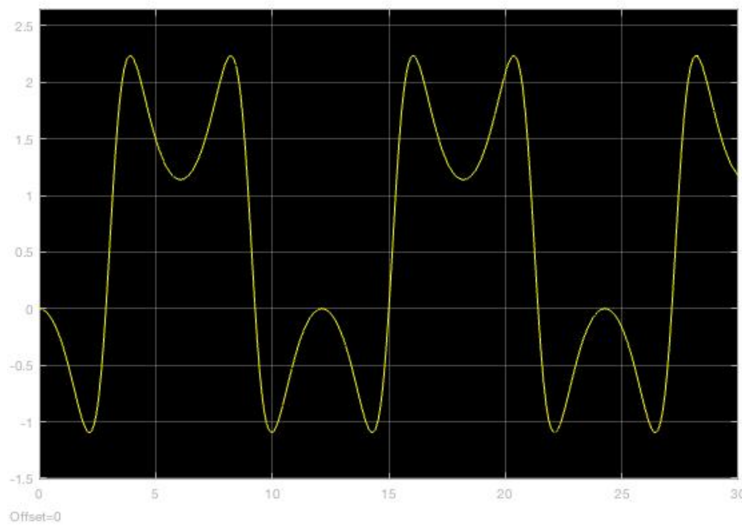

Figure 2: X-Scope in Figure 3 of the Lab Manual

The plot above is a plot of the inverted pendulum as the distance is varied.

This is an open loop system. Therefore the pendulum will have nothing stopping it's motion and can keep rotating. A zero input response to this system results in a sinusoidal output, this means that the pendulum will just keep rotating. There is no stopping and starting in this function, this is confirmed when running the simulation drawpendulum(time,theta) in MatLab. As the x position is varied the sinusoid begins to look weird as the cart is moving left and right at the same time that the pendulum is swinging. This slightly affects the rotational motion of the pendulum, however it does not stop the pendulum from rotating. This is confirmed when running the simulation drawpendulum(time,theta,x) in MatLab.

3.3 Simulation of PD Controlled Non-Linear Inverted Pendulum

A low pass with 3-dB cutoff at 100 rads/sec filter is place before the PD to attenuate high frequency components of error signal. the equation below is the transfer function of the filter is found below.

$$H(s) = \frac{1}{\frac{s}{\omega_{cf}} + 1} \tag{8}$$

$$H(s) = \frac{1}{\frac{s}{100} + 1} \tag{9}$$

$$H(s) = \frac{1}{0.01s + 1} \tag{10}$$

The plot below is the Magnitude plot in dB V. ω.

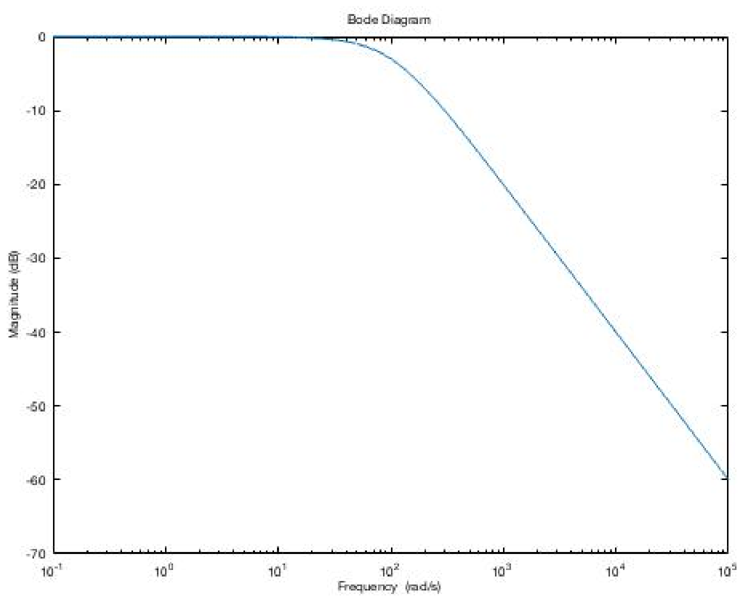

Figure 3: Magnitude Plot of Transfer Function for the Low-Pass Filter

The low pass filter has a midband gain of 0dB and drops to -60dB 3 decades from the cutoff frequency which corresponds to a slope of 20dB/decade. This is expected from a first order low pass filter.

The plot below is the Phase plot in Phase Angle V. ω. The phase shift is 90 degrees in this first order low pass filter. The corner frequency of this low pass filter corresponds to a 45 degree phase shift in the output of the filter when compared to the input.

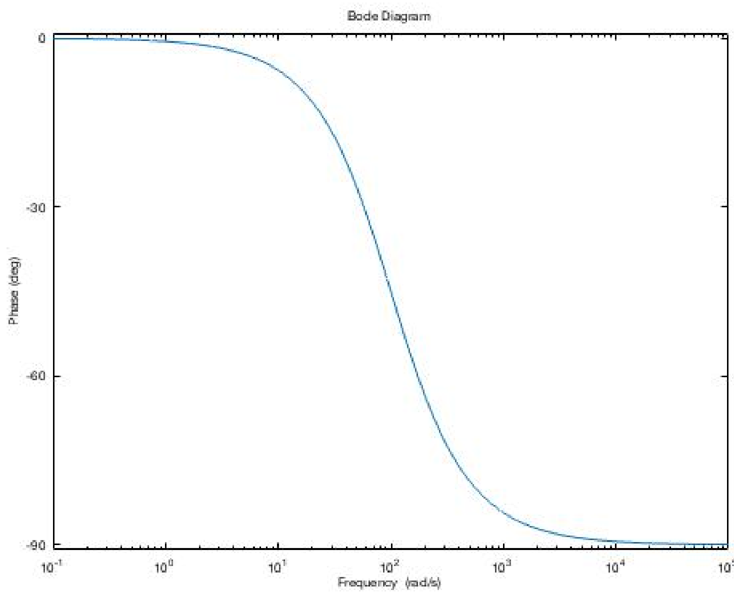

Figure 4: Phase Plot of Transfer Function for the Low-Pass Filter

3.4 Simulating the PD controlled inverted pendulum

The low–pass filter was implemented in Simulink and the pendulum was tested with different angles.

Pendulum tested at 5 degrees. The PD controller was able to balance the pendulum position.

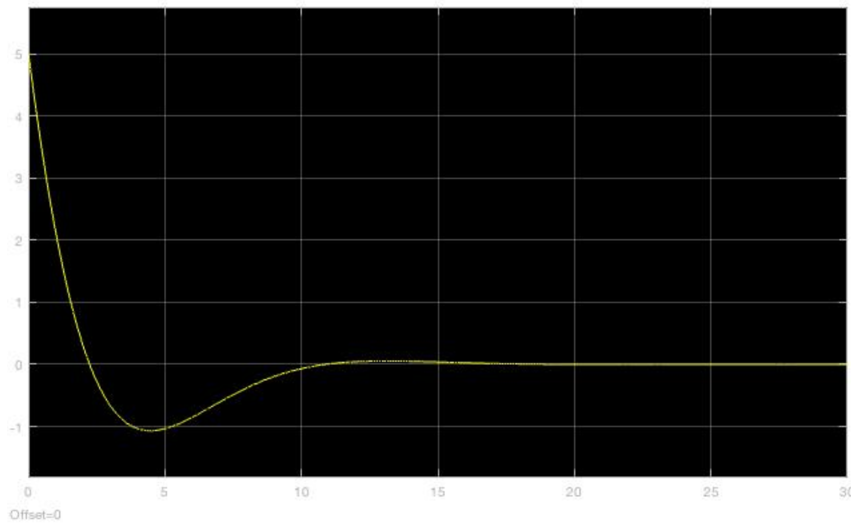

Figure 5: 5° with k_p =14272, k_d = 7000

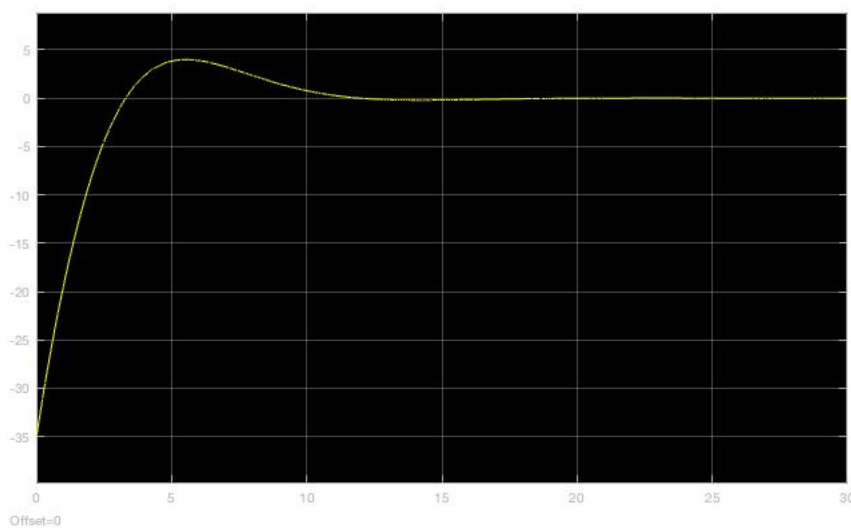

Figure 6: -35° with k_p =14272, k_d = 7000

Pendulum tested at -35 degrees. The PD controller was able to balance the pendulums position.

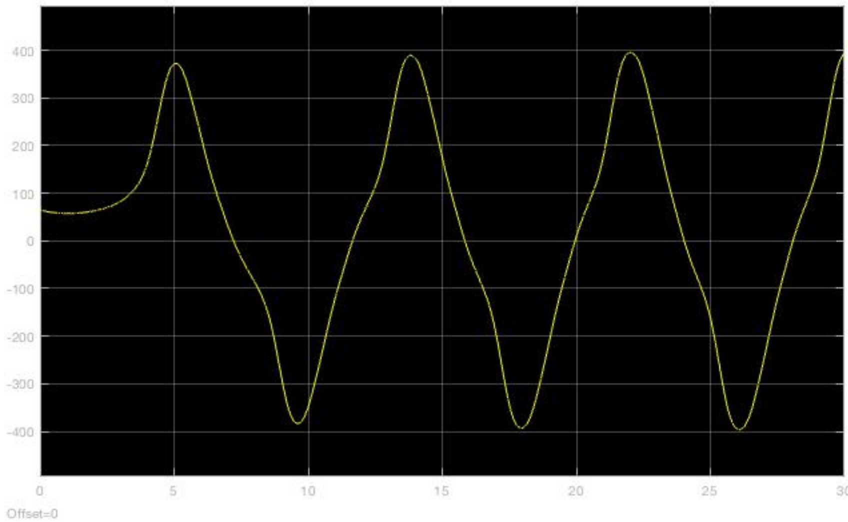

Figure 7: 65° with k_p =14272, k_d = 7000

Pendulum tested at 65 degrees. The PD controller in this case was unable to balance the pendulum. The pendulum simply keeps rotating. The system failed to keep the pendulum inverted. This may be due to an over damped system.

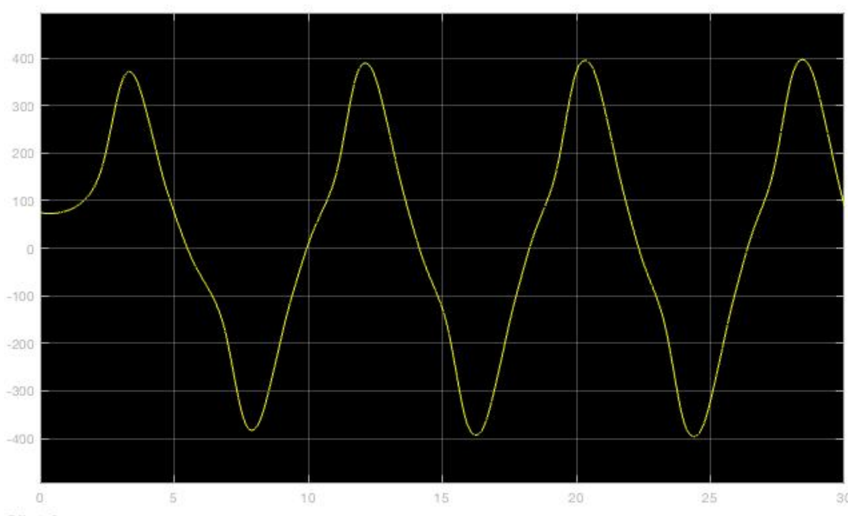

Figure 8: 75° with k_p =14272, k_d = 7000

Pendulum tested at 75 degrees. The PD controller in this case was unable to balance the pendulum. The pendulum simply will keep rotating. The system failed to keep the pendulum inverted. This may be due to an over damped system.

3.5 Additional Questions

1. If the reference angle is set to some other value than 0, no the pendulum will not be kept at this angle as gravities forces will work to bring the pendulum down to the ground. The system is in a equilibrium steady-state at 0 degrees. All forces on the pendulum are 0 at this position hence why it is able to remain standing. Therefore by changing the geometry of the system we would also need to change the input forces necessary to keep the pendulum at rest if the reference angle was anything other than 0 degrees. The fact that the pendulum is coupled to the cart means that if the pendulum starts on an angle it will start to move the cart in an opposite direction to the motion of the pendulum. Therefore a feedback control system must be used in order to keep the pendulum in an upright position.
2. There should be two initial conditions for Eq.13 of the lab manual. They are θ_o and $\dot{\theta}_o$. The purpose of θ_o is to provide an initial condition for the starting angle of the pendulum it was not assumed to be 0 in the implementation used in this lab. The purpose of $\dot{\theta}_o$ was the initial angular velocity of the pendulum before the PD controller begins to balance the system. This value was assumed to be zero in Figure. 10 of the lab manual. $\dot{\theta}_o$ is therefore omitted in Figure. 10 of the lab manual.

4 Conclusion

After completion of this report, a better understanding of the inverted pendulum and its controlling properties was obtained. Also a better understanding between open and closed loop systems was achieved. A low pass filter was designed and used with a PD controller which leads to more experience with manipulating transfer functions. Finally a better understanding of the simulink toolbox and its components was obtained from the complexity of the blocks in the simulation model.

5 Appendix A

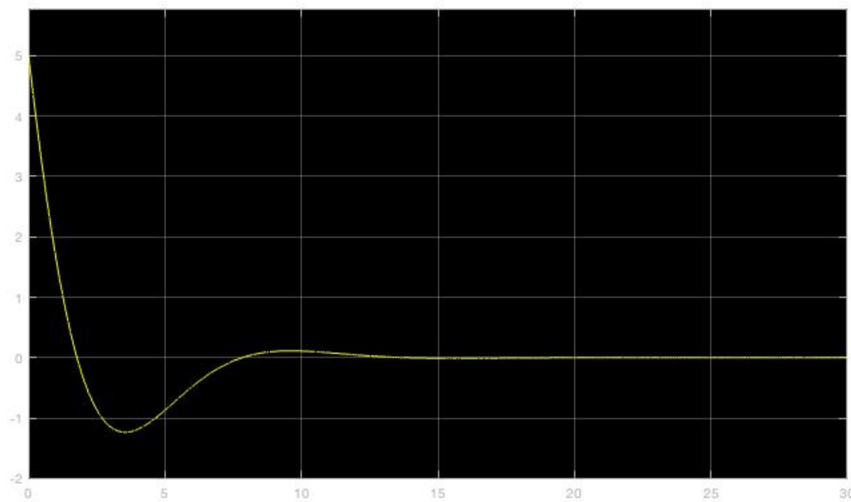

Figure 9: 5° with k_p =16000, k_d = 8000

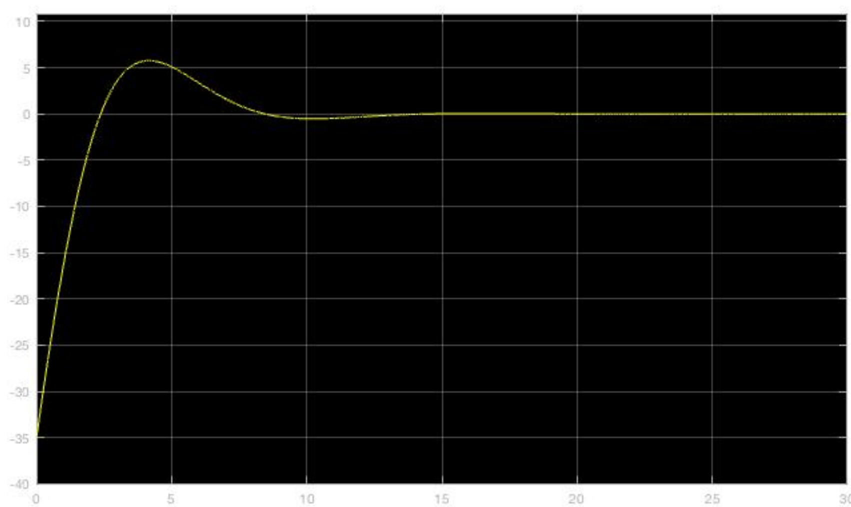

Figure 10: -35° with k_p =16000, k_d = 8000

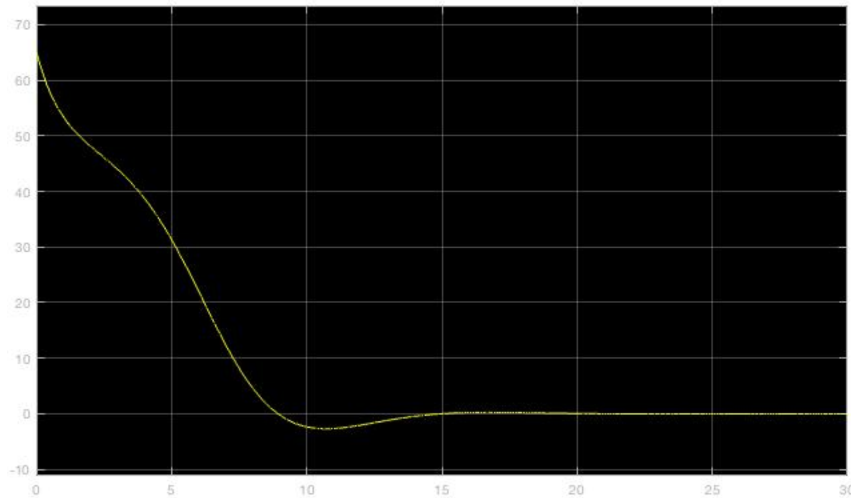

Figure 11: 65° with k_p =16000, k_d = 8000

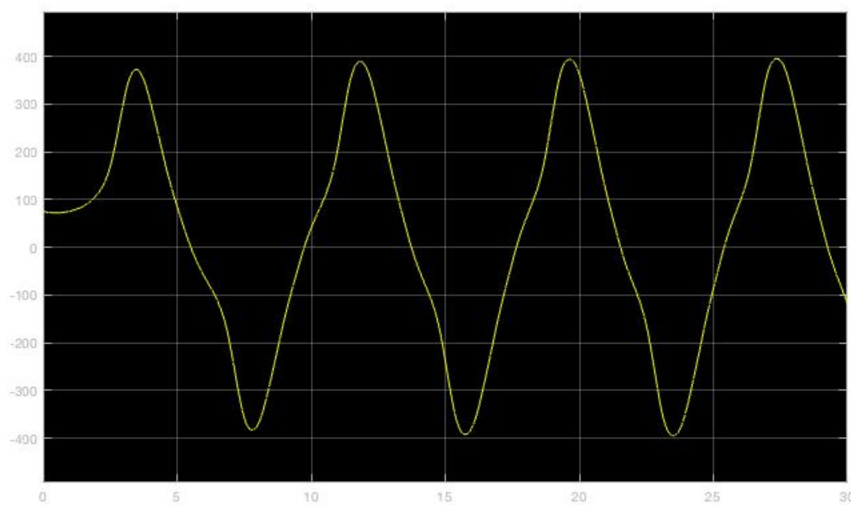

Figure 12: 75° with k_p =16000, k_d = 8000