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PART A

RULE OF INFERENCES

Introduction of Arguments

The arguments might be seen as a sequence of statements or propositions one after another. The premises and the conclusion are often included in it. While the conclusion is used to represent the end statement, the premises are used to demonstrate the remaining assertion in the argument. (*Arguments in Discrete Mathematics - Javatpoint*, n.d.)

Question 1 (a)

Question: Briefly explain about the rules of inferences and its applications.

- Definition rules of inferences:

A logical form or guide that consists of premises (or hypotheses) and forms a conclusion is referred to as the rules of inference, which is also known as inference rules (Calcworkshop LLC, 2021). The argument is valid if the conclusion (final statement) follows from the truth of the preceding statements (premises). Rules of inference are templates for building valid arguments. (Moura, 2010)

TABLE 1 Rules of Inference.		
<i>Rule of Inference</i>	<i>Tautology</i>	<i>Name</i>
$\begin{array}{l} p \\ p \rightarrow q \\ \hline \therefore q \end{array}$	$(p \wedge (p \rightarrow q)) \rightarrow q$	Modus ponens
$\begin{array}{l} \neg q \\ p \rightarrow q \\ \hline \therefore \neg p \end{array}$	$(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$	Modus tollens
$\begin{array}{l} p \rightarrow q \\ q \rightarrow r \\ \hline \therefore p \rightarrow r \end{array}$	$((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$	Hypothetical syllogism
$\begin{array}{l} p \vee q \\ \neg p \\ \hline \therefore q \end{array}$	$((p \vee q) \wedge \neg p) \rightarrow q$	Disjunctive syllogism

$\frac{p}{p \rightarrow (p \vee q)}$	$p \rightarrow (p \vee q)$	Addition
$\frac{p \wedge q}{(p \wedge q) \rightarrow p}$	$(p \wedge q) \rightarrow p$	Simplification
$\frac{\begin{array}{l} p \\ q \\ \hline \therefore p \wedge q \end{array}}{(p \wedge q) \rightarrow (p \wedge q)}$	$((p \wedge q) \rightarrow (p \wedge q))$	Conjunction
$\frac{\begin{array}{l} p \vee q \\ \neg p \vee r \\ \hline \therefore q \vee r \end{array}}{((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)}$	$((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$	Resolution

The most commonly used rule of inference as shown in the above table is Modus Ponens , Modus Tollens, Hypothetical Syllogism, Disjunctive Syllogism, Addition Simplification and Conjunction. This rule of inference can be used in the method of making conclusions from provided arguments as well as in the process of determining whether or not a particular argument is valid.

- Application Rules of Inferences

1. Artificial Intelligence

In artificial intelligence, inference rules are used to create proofs, and the proof itself is a sequence of conclusions that leads to the achievement of the intended objective. (*Rules of inference in Artificial Intelligence – Javatpoint*, n/a)

2. Philosophy

In philosophy, rule of inferences are used for the purpose of conducting argumentation analysis and drawing logical conclusions on a variety of ideas and theories. (Wikepedia, 2022)

3. Science

In scientific research, rule of inferences are used to make predictions and draw conclusions from experiments and data.

4. Law

In the course of legal procedures, rules of inference are utilized to assist with evidence interpretation and decision-making in the courtroom.

Question 1 (b)

Consider the arguments below:

“Amin doesn’t like mathematics or he attends the classes. If Amin attends the classes or study hard, then he will pass the examination. Amin failed the exam.”

Question i) : Using variables p, q, r and s, write down the primary statements

Answer:

Primary statement for the argument above:

p = Amin like mathematics.

q = Amin attends the classes.

r = Amin study hard.

s = Amin pass the examination.

Solution:

1- From the first statement “Amin doesn’t like mathematics or he attends the classes” can be divided by two statement in and put in two different variable. “Amin doesn’t like mathematics” can’t be put as variable ‘p’ because it has “doesn’t” that show the falsity statement. It need to change the structure sentence to be positive statement.

p = Amin like mathematics.

q = Amin attends the classes.

2- Next statement is “If Amin attends the classes or study hard, then he will pass the examination.” Can be separated by two statement for two different variables if the statement hasn’t declared in any variable.

q = Amin attends the classes.

r = Amin study hard.

3- The last statement “Amin failed the exam” can’t be declared in the variable ‘s’ because of word “failed” that show falsity statement. Change the structure of sentence to be positive statement.

s = Amin pass the examination.

Question ii) : Write the premises using variables in (i)

Answer:

Premises for variable in question (i) is :

$$1 - \sim p \vee q$$

$$2 - (q \vee r) \rightarrow s$$

$$3 - \sim s$$

Solution and Method:

1 - Amin doesn't like mathematics or he attends the classes.

$\sim p$ = Amin doesn't like mathematics

\vee = or (this symbol call “disjunction”, use to combine 2 statement by the word ‘or’)

q = he attends the classes.

$$\therefore \sim p \vee q$$

2 - If Amin attends the classes or study hard, then he will pass the examination.

q = Amin attends the classes

\vee = or (this symbol call “disjunction”, use to combine 2 statement by the word ‘or’)

r = study hard

s = he will pass the examination.

Use conditional statement (implication) when the form is ‘**if p then q**’, it will be denoted by ‘ $p \rightarrow q$ ’

$(q \vee r) \rightarrow s$ = If Amin attends the classes or study hard, then he will pass the examination.

$$\therefore (q \vee r) \rightarrow s$$

3 - Amin failed the exam.

$\sim s$ = Amin failed the exam. (put negation symbol because its show the falsity of the statement)

Question iii) :Find the conclusion of the arguments by using the rules of inference.

Answer:

- 1- $\sim p \vee q$
- 2- $(q \vee r) \rightarrow s$
- 3- $\sim s$
- 4- $\sim (q \vee r)$ (3),(2) Modus Tollens.
- 5- $\sim q \wedge \sim r$ (4) De Morgan Law.
- 6- $\sim q$ (5) Simplification
- 7- $\sim p$ (1),(6) Disjunctive Syllogism

\therefore Hence, Amin doesn't like Mathematics.

Solution:

- 1- $\sim p \vee q$
 - 2- $(q \vee r) \rightarrow s$
 - 3- $\sim s$
- } take and write back the premises from the question ii).

- 4- $\sim (q \vee r)$ (3),(2) Modus Tollens.

How to solve:

Take the third premises and second premises to simplify the “ $\sim s$ ” and “ $(q \vee r) \rightarrow s$ ” using rules of inferences “**Modus Tollens**”.

$$\begin{array}{c} (3) \quad \sim s \\ (2) \quad (q \vee r) \rightarrow s \\ \hline \end{array}$$

$$\therefore \sim (q \vee r)$$

- 5- $\sim q \wedge \sim r$ (4) De Morgan Law.

How to solve:

Take the fourth premises to simplify the “ $\sim (q \vee r)$ ” by using laws of logic which is “**De Morgan Law**”

$$\begin{array}{c} (4) \quad \sim (q \vee r) \\ \hline \\ \therefore \sim q \wedge \sim r \end{array}$$

6- $\sim q$ (5) Simplification

How to solve:

Take the five premises to simplify the “ $\sim q \wedge \sim r$ ” by using the rule of inferences “**Simplification**”.

(5) $\sim q \wedge \sim r$

$\therefore \sim q$

7- p (1),(6) Disjunctive Syllogism

How to solve:

Take the first premises and sixth premises to simplify the “ $\sim p \vee q$ ” and “ $\sim q$ ” by using the rule of inferences “**Disjunctive Syllogism**”

(1) $\sim p \vee q$

(6) $\sim q$

$\therefore \sim p$

Summary

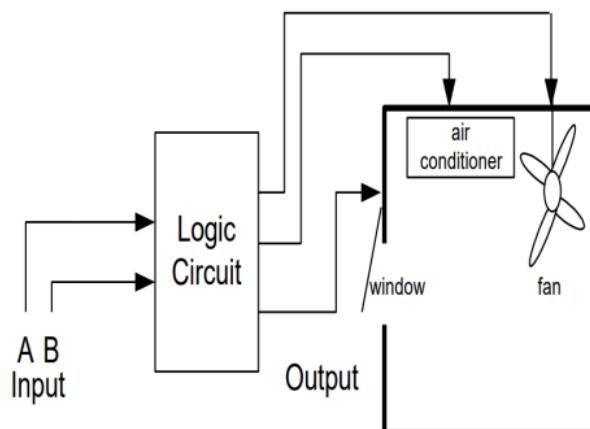
Based on the solution above, rules of inferences that we use is Modus Tollens, Simplification, and Disjunctive Syllogism to simplify the premises. However, we also use Law of Logic to expand the expression “ $\sim (q \vee r)$ ” by using De Morgan Law. From the rules of inferences and law of logic that we use, we can get the last conclusion which is “ $\sim p$ ” that means Amin doesn’t like Mathematics.

PART B

LAW OF LOGIC

Question of Air-Cooling System

Air cooling system created to control the temperature of the room. This system is consisting of three output variables, which are air conditioner, fan and window. Two switches A and B use to control these three output variables. Below is a logic circuit to control an AIR-COOLING



The logic circuit will function as follows:

COOLNESS	INPUT AB	OUTPUT
OFF	00	Close fan, air conditioner, and window
FRESH AIR	01	Open fan, window and close air conditioner
COOL	10	Open air conditioner and close fan, window
COOLER	11	Open fan, air conditioner and close window

Assume 1 = open / true and 0 = close / false.

INPUT		FAN (F)	AIR CONDITIONER (C)	WINDOW (W)
A	B			
F	F	F	F	F
F	T	T	F	T
T	F	F	T	F
T	T	T	T	F

Solve the proposition and state the conclusion.

$$(\neg A \wedge B) \Leftrightarrow W$$

Answer and Solution :

Step 1: Create a truth table

- Develop the truth table to show the truth value of compound propositions given truth values of propositions from which they constructed. For this proposition, it has 2 variables which is (A,B) then the truth table must be constructed with 4 rows.

Answer:

A	B	$\neg A$	$\neg A \wedge B$	W	$(\neg A \wedge B) \Leftrightarrow W$
T	T	F	F	F	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	F	F	T

Step 2: Conclusion

- Give a conclusion based on the value of truth table and refer the question information.

Conclusion Answer:

∴ Based on the truth table above, we can see the value from the expression is valid and true when we compare it with the given output. So, we can conclude that to get the fresh air of coolness when we control the temperature of the room, it needs to be off switch A and open the switch B. Not only that, when we off the switch A and open the switch B it will open the fan and window but it will off the air-conditioner.

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