

# Tracking Object in Tundra Orbit For HIAD Applications

March 14, 2022

Anthony D'Amico

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## Introduction

The object of this project is to determine the orbital elements for an object in orbit at the time of loss of signal and after seven hours and twenty-five minutes of flight from loss of signal. The object is believed to be a SiriusXM satellite in a tundra orbit above the United States. NASA wants to target this satellite as a part of its new satellite de-orbiting system that uses a Hypersonic Inflatable Aerodynamic Decelerator (**HIAD**). The HIAD can attach to any satellite robotically.

The object is being tracked by the US Space Force from the NORAD tracking site at Peterson Air Force Base in Colorado Springs, Colorado. The coordinates of the tracking are **32.82366°N, 104.695°W**. The object is first detected and tracked near what is believed to be its apoapsis and is tracked until it is out of the line of sight of the tracking site. The last readings from the tracking site were given to NASA and were:

$$\begin{aligned}\vec{p} &= 4.37923\hat{X}_h + 2.37638\hat{Y}_h + 0.025397\hat{Z}_h DU \\ \dot{\vec{p}} &= 0.065168\hat{X}_h - 0.16369\hat{Y}_h - 0.380347\hat{Z}_h DU/TU\end{aligned}$$

These last readings were recorded on February 10, 2022 at 17:35 MST. NASA wants to intercept the object if it is the SiriusXM satellite, seven hours and twenty-five minutes from the last recorded observation.

In order for NASA to make an intercept, they need to know the following objectives:

### Objectives:

- I. ECI Position and Velocity Vectors
- II. Orbital Elements
- III. Kepler's Inverse Problem (New True Anomaly)
- IV. ECI Position and Velocity Vectors at New True Anomaly

SiriusXM Satellite Radio operates a constellation of three satellites, all in tundra orbits. A tundra orbit is a form of geosynchronous orbit that forms a figure-eight shape on a ground track map (**Figure 1**). These orbits tend to have an eccentricity between 0.2 and 0.3, an argument of periapsis of 270°, an inclination of 63.4°, and an orbital period of one sidereal day. Tundra orbits are mostly used for communication satellites due to a phenomenon called apogee dwell. This occurs when the satellite is in the smaller loop of the figure-eight and spends most of its time over this given area of Earth. The SiriusXM constellation is set up to always have at least one satellite in the smaller loop at all times in order to provide constant service to its customers.

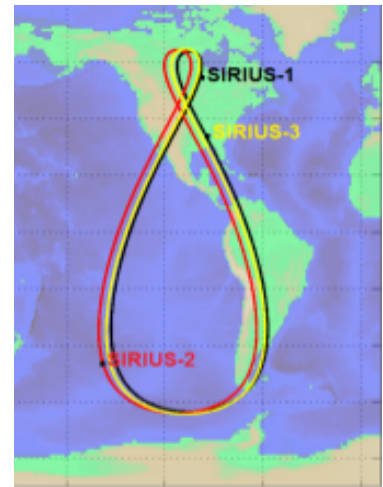


Figure 1

# Approach

## I. ECI Position and Velocity Vectors

The first goal of this project was to determine the position and velocity vectors in Earth-Centered inertial (ECI), equatorial coordinate system. In order to obtain this, the time of the last reading and position of the radar site must be converted to Sidereal time. The first step in converting to Sidereal time is calculating the Julian Day number ( $J_0$ ) with **Equation 1**.

$$J_0 = 367 * y - INT(\frac{7(y + INT(\frac{m+9}{12}))}{4}) + INT(\frac{275 * m}{9}) + d + 1721013.5 \text{ (Eq. 1)}$$

The Julian Century ( $T_0$ ) is then calculated using **Equation 2**.

$$T_0 = \frac{J_0 - 2451545}{36525} \text{ (Eq. 2)}$$

Using **Equation 3**, the reference Greenwich time is calculated using the Julian Century.

$$\theta_{g0} = 100.4606184 + 36000.77004T_0 + 0.000387933T_0^2 - 2.583 * 10^{-8}T_0^3 \text{ (Eq. 3)}$$

The reference time is reduced by multiples of 360 in order to have an answer that is less than  $360^\circ$ . Using **Equation 4**, the sidereal time in Greenwich can be calculated. In order to do this, the time of the last radar reading must be converted to Universal Time (UT).

$$\theta_g = \theta_{g0} + 360.98564724(\frac{UT}{24}) \text{ (Eq. 4)}$$

To get the local sidereal time of the radar site, the longitude of the tracking station ( $\lambda_E$ ) is added to the Greenwich sidereal time (**Equation 5**). If the longitude is western, then it is made negative before adding it.

$$\theta = \theta_g + \lambda_E \text{ (Eq. 5)}$$

The range rate must have the radius of the Earth added into the  $\hat{Z}$  direction to get the proper relations.

$$\overline{r}_h = \overline{\rho} + R_E \hat{Z}$$

The ECI position vector can then be found using the sidereal time and **Equation 6**.

$$\begin{bmatrix} \hat{X} \\ \hat{Y} \\ \hat{Z} \end{bmatrix} = [D]^{-1} \begin{bmatrix} \hat{X}_h \\ \hat{Y}_h \\ \hat{Z}_h \end{bmatrix} \quad (\text{Eq. 6})$$

Where  $[D]^{-1}$  is equal to:

$$[D]^{-1} = \begin{bmatrix} \sin L \cos \theta & -\sin \theta & \cos L \cos \theta \\ \sin L \sin \theta & \cos \theta & \cos L \sin \theta \\ -\cos L & 0 & \sin L \end{bmatrix}$$

The L in the transformation matrix is the latitude of the radar site.

To get the velocity vector in ECI coordinates, the Coriolis acceleration ( $\bar{\omega}$ ) term is added to the ECI position vector and range rate in **Equation 7**.

$$\bar{v} = \bar{\rho} + (\bar{\omega} \times \bar{r}) \quad (\text{Eq. 7})$$

## II. Orbital Elements

The next step was to find the orbital elements of the orbit that was being tracked. This is important for the situation because then the elements can be checked against tundra orbit elements and confirm the orbit of the object. The specific angular momentum ( $\bar{h}$ ) was found by cross multiplying the ECI position and velocity vector (**Equation 8**).

$$\bar{h} = \bar{r} \times \bar{v} \quad (\text{Eq. 8})$$

The node vector can then be found by taking the cross-product of the unit vector  $\hat{Z}$  and  $\bar{h}$

| Orbit Shape | Eccentricity (e) |
|-------------|------------------|
| Circle      | $e = 0$          |
| Ellipse     | $0 < e < 1$      |
| Parabola    | $e = 1$          |
| Hyperbola   | $e > 1$          |

**Table 1**

(**Equation 9**). The eccentricity of an orbit defines its shape. **Table 1** shows the different eccentricity values for different orbital shapes. The eccentricity is found by taking the magnitude of the vector calculated using **Equation 10**. After finding these three important orbital elements, the rest can be found simply. The semi-latus rectum, which is the distance to the point where the true anomaly is  $90^\circ$ , is found using specific angular momentum and the gravitational parameter of Earth

(**Equation 11**). Using the semi-latus rectum and the eccentricity, the semi-major axis can be calculated (**Equation 12**).

$$\bar{n} = \hat{Z} \times \bar{h} \quad (\text{Eq. 9})$$

$$\bar{e} = \frac{1}{\mu} \left[ (v^2 - \frac{\mu}{r}) \bar{r} - (\bar{r} \cdot \bar{v}) \bar{v} \right] \quad (\text{Eq. 10})$$

$$p = \frac{h^2}{\mu} \quad (\text{Eq. 11})$$

$$a = \frac{p}{(1-e^2)} \quad (\text{Eq. 12})$$

The argument of periapsis is the angle between the longitude of the ascending node and periapsis. The argument of periapsis can be found using the eccentricity and node vectors in **Equation 13**. The longitude of the ascending node is the angle between the X-axis and where the spacecraft crosses the equatorial plane in a northerly direction. This angle is always measured eastward from the X-axis. The longitude of the ascending node can be calculated by using the node vector in **Equation 14**. The inclination of an orbit is the angle between the orbital plane and the XY plane. For an object orbiting Earth using an ECI coordinate system, the inclination is the angle between the orbital plane and the equator. The inclination is found using the specific angular momentum vector in **Equation 15**.

$$\omega = \cos^{-1} \left( \frac{\bar{n} \cdot \bar{e}}{ne} \right) \quad (\text{Eq. 13})$$

$$\Omega = \cos^{-1} \left( \frac{n_x}{n} \right) \quad (\text{Eq. 14})$$

$$i = \cos^{-1} \left( \frac{h_z}{h} \right) \quad (\text{Eq. 15})$$

The true anomaly is the angle between periapsis and the current position of the spacecraft. True anomaly is the only orbital element that changes as the spacecraft moves in its orbit. It can be used to determine if the spacecraft is currently heading towards apoapsis or perigee. The true anomaly is found using the eccentricity and the position vector in **Equation 16**.

$$v = \cos^{-1} \left( \frac{\bar{e} \cdot \bar{r}}{er} \right) \quad (\text{Eq. 16})$$

The orbital period is defined as how long it takes for an object to complete one full rotation around its central body. For example, the orbital period of Earth is 365 days or one year. Finding the orbital period is vital to confirming this object is in a tundra orbit, especially since tundra orbits have orbital periods of one sidereal day. The period of an elliptical orbit can be found using the semi-major axis and gravitational parameter in **Equation 17**.

$$T = 2\pi \sqrt{\frac{a^3}{\mu}} \quad (\text{Eq. 17})$$

### III. Kepler's Inverse Problem (New True Anomaly)

The next objective is to find the new true anomaly of the spacecraft when the line of sight is reacquired, which occurs seven hours and twenty-five minutes after loss of signal. Since the true anomaly at this new position is unknown, this requires Kepler's Inverse Problem to solve it. The first step is to find the initial Eccentric Anomaly ( $E_0$ ), which is calculated with the eccentricity and the true anomaly from the last known position (**Equation 18**). Using the initial Eccentric Anomaly and the eccentricity, the initial Mean Anomaly ( $M_0$ ) can then be found (**Equation 19**). The mean motion ( $n$ ) is a constant value for the entire orbital path, and is calculated using the semimajor axis and gravitational parameter (**Equation 20**). The number of periapsis passes that occur during the time of flight can be calculated using the time from periapsis to the last known position ( $t_0$ ) and the time of flight (**Equation 21**).

$$E_0 = \cos^{-1}\left(\frac{e + \cos(v)}{1 + (e \cdot \cos(v))}\right) \text{ (Eq. 18)}$$

$$M_0 = E_0 - e \cdot \sin(E_0) \text{ (Eq. 19)}$$

$$n = \sqrt{\frac{\mu}{a^3}} \text{ (Eq. 20)}$$

$$t_0 = \frac{M_0}{n} \text{ (Eq. 21)}$$

The mean anomaly ( $M$ ) at the current position of the spacecraft can be calculated using the initial mean anomaly ( $M_0$ ), mean motion, and the time of flight (**Equation 22**).

$$M = (tof * n) - 2\pi k + M_0 \text{ (Eq. 22)}$$

Kepler's Inverse Problem requires iteration to find the eccentric anomaly at the current position. In order to start the problem, an initial eccentric anomaly ( $E_1$ ) must be chosen. A general rule to choosing the initial value is pick 0.5 for low eccentricity values and pick  $\pi$  for larger values. The eccentric anomaly should converge to a value within five iterations, but could be more or less depending on how close the value for the initial eccentric anomaly is to the actual value.

The derivative for the mean anomaly with respect to eccentric anomaly is found in each iteration using **Equation 23**. The difference between the mean anomaly and the mean anomaly for that iteration ( $M - M_i$ ) represents how close the eccentric anomaly for that iteration ( $E_i$ ) is to the actual mean anomaly. This difference is calculated using **Equation 24**. The eccentric anomaly for the next iteration is calculating using **Equation 25**. Once the difference between the mean anomaly and the iterative mean anomaly reaches zero, the iterative eccentric anomaly is the eccentric anomaly for the current position.

$$\frac{dM}{dE} = 1 - e \cos(E) \text{ (Eq. 23)}$$

$$(M - M_i) = M - (E_i - e \sin(E_i)) \text{ (Eq. 24)}$$

$$E_{i+1} = E_i + \left( \frac{M - M_i}{\frac{dM}{dE}} \right) \text{ (Eq. 25)}$$

The true anomaly of this position can then be calculated using the eccentricity and the eccentric anomaly found in Kepler's Inverse Problem (**Equation 26**).

$$v = \cos^{-1} \left( \frac{\cos(E) - e}{1 - e \cos(E)} \right) \text{ (Eq. 26)}$$

#### IV. ECI Position and Velocity Vectors at New True Anomaly

The final objective is to find the ECI position and velocity vectors at the current position of the spacecraft. In order to do this, the position of the spacecraft needs to be found. This can be calculated using the semi-latus rectum, eccentricity, and the new true anomaly and using **Equation 27**. After the position is found, the perifocal position and velocity vectors can be found using **Equations 28 & 29**.

$$r = \frac{p}{1 + e \cos(v)} \text{ (Eq. 27)}$$

$$\overline{r}_w = r \cdot \cos(v) \widehat{X}_w + r \cdot \sin(v) \widehat{Y}_w \text{ (Eq. 28)}$$

$$\overline{v}_w = \sqrt{\frac{\mu}{p}} (-\sin(v) \widehat{X}_w + (e + \cos(v)) \widehat{Y}_w) \text{ (Eq. 29)}$$

The perifocal position and velocity vectors can be transformed to ECI vectors using **Equation 30**, where  $[R]$  is a transformation matrix.

$$\begin{bmatrix} \widehat{X} \\ \widehat{Y} \\ \widehat{Z} \end{bmatrix} = [R] \begin{bmatrix} \widehat{X}_w \\ \widehat{Y}_w \\ \widehat{Z}_w \end{bmatrix} \text{ (Eq. 30)}$$

The transformation matrix can be shown as:

$$[R] = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix}$$



$$\begin{aligned}
R_{11} &= \cos(\Omega)\cos(\omega) - \sin(\Omega)\sin(\omega)\cos(i) \\
R_{12} &= -\cos(\Omega)\sin(\omega) - \sin(\Omega)\cos(\omega)\cos(i) \\
R_{13} &= \sin(\Omega)\sin(i) \\
R_{21} &= \sin(\Omega)\cos(\omega) + \cos(\Omega)\sin(\omega)\cos(i) \\
R_{22} &= -\sin(\Omega)\sin(\omega) + \cos(\Omega)\cos(\omega)\cos(i) \\
R_{23} &= -\cos(\Omega)\sin(i) \\
R_{31} &= \sin(\omega)\sin(i) \\
R_{32} &= \cos(\omega)\sin(i) \\
R_{33} &= \cos(i)
\end{aligned}$$

## Results

### I. ECI Position and Velocity Vectors

The object that is believed to be the SiriusXM satellite is tracked through most of its orbit from the NORAD tracking site at Peterson Air Force Base near Colorado Springs, Colorado. The last reading before the targeted NASA intercept was taken on February 10, 2022 at 17:35 MST. The local sidereal time of the radar site at the time of this last reading was calculated to be 44.6277°.

Using the local sidereal time and the latitude of the radar site, the range and range rate were converted into ECI position and range rate vectors. These vectors are:

$$\begin{aligned}
\bar{r} &= 0.853038\hat{X} + 4.181108\hat{Y} - 2.768923\hat{Z} \text{ DU} \\
\bar{\dot{r}} &= -0.066819\hat{X} - 0.29596\hat{Y} - 0.28922\hat{Z} \text{ DU/TU}
\end{aligned}$$

Since the Z component of the ECI position vector is negative, this means the spacecraft is currently over the Southern Hemisphere. The magnitude of the position vector is 5.0869 DU, which means the spacecraft is currently 32,444.2 kilometers away from the center of the Earth.

The velocity vector in ECI coordinates was then found using the Coriolis acceleration constant and the range rate. The velocity vector at the time of this reading was:

$$\bar{v} = -0.31279\hat{X} - 0.24578\hat{Y} - 0.28922\hat{Z} \text{ DU/TU}$$

The spacecraft's current velocity is 0.49183 DU/TU, or 3.891 kilometers per second. Due to the nature of tundra orbits, the periapsis is always at the southernmost point in the orbital plane. Since the Z-components in the position and velocity vectors are both negative, this means the spacecraft is currently approaching its periapsis.

## II. Orbital Elements

The last range and range rate readings taken before the targeted intercept of the satellite by NASA with its HIAD prototype were essential in defining the satellite's orbit. The orbit of the satellite needs to be defined in order to have a successful intercept because NASA has to calculate where the satellite is going to be when they want to intercept it.

The eccentricity of the orbit was calculated using the position and velocity vectors, as well as the gravitational parameter of Earth. The eccentricity came out to equal 0.30. This is within the criteria of a tundra orbit, which typically has an eccentricity value between 0.2 and 0.3. With an eccentricity of 0.3, the orbit is more circular and shorter than most elliptical orbits. The semi-major axis of the orbit was determined to be 6.610 DU, or approximately 42,158.6 kilometers. This is close to the expected value of 42,164 kilometers needed to have an orbital period of one sidereal day. The difference between the calculated value and the expected value can be attributed to rounding cutoffs from earlier calculations.

The argument of periapsis was measured to be  $270^\circ$ . This means the periapsis occurs at the southernmost point in the orbit. An argument of periapsis of  $270^\circ$  places the apoapsis over the Northern Hemisphere. The majority of tundra orbits have an argument of periapsis of  $270^\circ$ . This is because most objects in tundra orbits are communications satellites, which serve high latitude regions in the Northern Hemisphere. The apogee dwell effect allows the satellite to spend almost eight hours in the region it covers.

The longitude of the ascending node was  $239.51^\circ$  W. This represents the point where the satellite crosses the equator moving north. This point is over the Amazon Rainforest in South America. Since tundra orbits are geosynchronous, this means most of the ground track of the orbit is over North and South America. The orbital inclination of the satellite was calculated to be  $63.40^\circ$ . This inclination matches perfectly with tundra orbits' inclinations of  $63.40^\circ$ .

The orbital period of the satellite was calculated to be 86,156 seconds which is very close to the expected orbital period of 86,164 seconds or one sidereal day. This orbital period is what allows the orbit to be geosynchronous since the one revolution takes exactly one sidereal day. Since the true anomaly requires the current position of the satellite in order to calculate it, it is the only orbital element that changes throughout the course of the orbit. The true anomaly at the current position of the satellite was calculated to be  $307.49^\circ$ . This means the satellite is currently heading towards its periapsis.

## III. Kepler's Inverse Problem (New True Anomaly)

In order for NASA to make the targeted intercept, they need to know the true anomaly after seven hours and twenty-five minutes of flight from the current position of the satellite. Finding the true anomaly will allow NASA to determine the position and velocity vectors at their planned intercept between the SiriusXM satellite and the HIAD prototype.

The initial eccentric and mean anomalies, calculated at the current position of the satellite, are  $320.21^\circ$  and  $331.21^\circ$  respectively. Both of these values lead the true anomaly, which

is expected in the orbital plane from apoapsis to periapsis. These values, as well as the mean motion, were used to find the mean anomaly for the intercept position,  $M = 82.77^\circ$ .

Using the mean anomaly at the intercept position, as well as the eccentricity of the orbit and an initial guess for eccentric anomaly, Kepler's Inverse Problem can be applied to find the eccentric anomaly. Kepler's Inverse Problem was derived from Kepler's Problem to find a new position after a specified time of flight. Kepler's Inverse Problem uses an iterative method to find the eccentric anomaly, and ultimately the true anomaly, of the new position. **Table 2** shows the iterative history of Kepler's Inverse Problem used for this scenario. Since the eccentricity value was lower, an initial value of 0.5 was chosen for the eccentric anomaly. The eccentric anomaly converged after four iterations to a value of 1.740303 radians, or  $99.71^\circ$ .

| i | $E_i$    | $M - M_i$                | dM/dE    | $E_{i+1}$ |
|---|----------|--------------------------|----------|-----------|
| 1 | 0.5      | 1.088446                 | 0.736753 | 1.977355  |
| 2 | 1.977355 | -0.257204                | 1.118623 | 1.747426  |
| 3 | 1.747426 | -0.007491                | 1.052708 | 1.740310  |
| 4 | 1.740310 | $-7.491 \times 10^{-12}$ | 1.050603 | 1.740303  |
| 5 | 1.740303 | 0                        | 1.050603 | 1.740303  |

**Table 2**

The true anomaly at the time of the intercept can be calculated using the eccentric anomaly found in Kepler's Inverse Problem and the orbit's eccentricity. The true anomaly of the satellite seven hours and twenty-five minutes from the current position is  $116.49^\circ$ . This means the satellite will be heading towards its apoapsis at the time of the intercept. This true anomaly passes the check since both the mean and eccentric anomalies lag the true anomaly, which is expected as the satellite moves from periapsis to apoapsis. Also, this means the satellite passes through periapsis on its way to the intercept position.

#### IV. ECI Position and Velocity Vectors at New True Anomaly

The last thing NASA needs to conduct the intercept between the SiriusXM satellite and HIAD prototype is the ECI position and velocity vectors at the time of the intercept. The intercept will occur seven hours and twenty-five minutes from the last reading taken from the NORAD tracking site.

The semi-latus rectum, eccentricity, and true anomaly of the intercept were used to find the perifocal position vector:

$$\bar{r}_w = -3.09809\hat{X} + 6.21568\hat{Y} \text{ DU}$$

In the perifocal coordinate system, there is no  $\hat{Z}$  component since it is in the direction of the specific angular momentum, which is orthogonal to the orbital plane. The perifocal velocity vector is then found using Earth's gravitational parameter, semi-latus rectum, eccentricity, and the new true anomaly:

$$\bar{v}_w = -0.36490\hat{X} - 0.05958\hat{Y} \text{ DU/TU}$$

The perifocal position and velocity vectors are then converted to ECI coordinate system using a transformation matrix. The transformation matrix orients the vectors to the equatorial plane using the inclination, argument of periapsis, and longitude of the ascending node. The final position and velocity vectors in the ECI system are:

$$\begin{aligned}\bar{r} &= -1.958269\hat{X} - 6.059794\hat{Y} + 2.770723\hat{Z} \text{ DU} \\ \bar{v} &= 0.171032\hat{X} - 0.031535\hat{Y} + 0.326276\hat{Z} \text{ DU/TU}\end{aligned}$$

Since the  $\hat{Z}$  component of ECI position vector is positive, the satellite will be over the Northern Hemisphere at the time of the intercept. The magnitude of the position is 6.9449 DU or approximately 44,294.6 kilometers from the center of the Earth. At the time of the intercept, the satellite will be heading towards it apopsis. This was determined because the  $\hat{Z}$  components of the velocity and position vectors are positive. When the HIAD prototype and SiriusXM satellite intercept, the satellite will be traveling at a velocity of 0.36973 DU/TU or 2.92 kilometers per second.

## Conclusion

The NORAD tracking site at Peterson Air Force Base in Colorado Springs, Colorado was tracking an object in orbit which was believed to be a SiriusXM Satellite. They were tracking this satellite as it is the target for the first HIAD prototype made by NASA. The last reading was made on February 10, 2022 at 17:35 MST, which was seven hours and twenty-five minutes before the targeted intercept with the prototype.

The last reading was taken in range and range rate vectors and were converted into Earth-Centered, Inertial position and velocity vectors. These vectors allowed NASA to assess the position and velocity of the satellite with respect to the Earth. These vectors revealed that the satellite was over the Southern Hemisphere and heading towards it periapsis at the time of the reading. It was at a distance of 5.0869 DU, or 32,444.2 kilometers, and orbiting at a velocity of 0.49183 DU/TU, or 3.89 kilometers per second.

The orbital elements of the satellite's orbits were then calculated from the ECI position and velocity vectors. These elements are critical in defining the orbital plane of the satellite and confirming that it is the SiriusXM satellite in a tundra orbit. The calculated orbital elements confirmed that the spacecraft being tracked was the SiriusXM satellite. The eccentricity, semi-major axis, argument of periapsis, inclination, and orbital period all matched the criteria of a tundra orbit.

NASA was targeting to intercept the satellite seven hours and twenty-five minutes after the last reading was taken. In order to determine the position of the intercept, the true anomaly at this point needed to be calculated. The new true anomaly was calculated using the iterative method of Kepler's Inverse Problem. The true anomaly at the point of intercept was determined to be  $116.49^\circ$ . This true anomaly meant that the satellite will pass through its periapsis on its way to the intercept point, as well as shows the satellite is heading towards its apoapsis.

The last piece of data that NASA needed to conduct the intercept between the SiriusXM and HIAD prototype was the ECI position and velocity vectors at the time of intercept. These vectors showed the intercept will occur over the Northern Hemisphere, and confirmed that the satellite will be heading towards its apoapsis. The intercept will occur at a distance of 6.9449 DU, or 44,294.6 kilometers, and at a velocity of 0.36973 DU/TU or 2.92 kilometers per second.

NASA can proceed with its planned intercept of the SiriusXM satellite and its HIAD prototype. The object being tracked by the NORAD radar site was confirmed to be the SiriusXM satellite that is the target of this demonstration. The location of the intercept, as well as the position and velocity vectors with respect to Earth, were provided to NASA. These vectors should satisfy all the needs of the NASA team.

# Calculations

Julian Day Number ( $J_0$ ) = 2459620

Julian Century ( $T_0$ ) = 0.2211

Reference Sidereal Time ( $\theta_{G0}$ ) = 139.56°

Greenwich Sidereal Time ( $\theta_G$ ) = 149.32°

Local Sidereal Time ( $\theta$ ) = 44.63°

$R_1$  = 5.0869 DU

$R_1$  = 32,444.2 km

$V_1$  = 0.49183 DU/TU

$V_1$  = 3.89 km/s

Specific Angular Momentum ( $h$ ) = 2.453 DU<sup>2</sup>/TU

Eccentricity ( $e$ ) = 0.30

Semi-Latus Rectum ( $p$ ) = 6.016 DU

Semi-Latus Rectum ( $p$ ) = 38,367.9 km

Semi-Major Axis ( $a$ ) = 6.610 DU

Semi-Major Axis ( $a$ ) = 42,161.6 km

Argument of Periapsis ( $\omega$ ) = 270°

Longitude of the Ascending Node ( $\Omega$ ) = 239.51°

Inclination ( $i$ ) = 63.40°

Initial True Anomaly ( $v_1$ ) = 307.49°

Orbital Period ( $T$ ) = 86,156 seconds

Initial Eccentric Anomaly ( $E_0$ ) = 5.589 radians

Initial Eccentric Anomaly ( $E_0$ ) = 320.21°

Initial Mean Anomaly ( $M_0$ ) = 5.781 radians

Initial Mean Anomaly ( $M_0$ ) = 331.21°

Mean Motion ( $n$ ) = 7.293 \* 10<sup>-5</sup>

New Mean Anomaly ( $M$ ) = 1.445 radians

New Mean Anomaly ( $M$ ) = 82.77°

New Eccentric Anomaly ( $E$ ) = 1.740 radians

New Eccentric Anomaly ( $E$ ) = 99.71°

New True Anomaly ( $v_2$ ) = 116.49°

$R_2$  = 6.945 DU

$R_2$  = 44,294.6 km

$V_2$  = 0.36973 DU/TU

$V_2$  = 2.92 km/s