AALTO UNIV. SCHOOL OF ELECTRICAL ENGINEERING
COMNET Department
ELECTRICAL-E7460 Modeling and Simulation, fall 2018

Assignment 2
DL 25.11.2018
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Returning your solutions

Write a short report on the results. Return the report and your Mathematica files in a zip-file to the return folder on course web page in MyCourses (under Assignments section). The deadline for returning your solutions for this exercise is **Sunday 25.11.2018**.

Requirements for reporting:

- In the report, for each problem explain also briefly your solution.
- For each problem, provide the Mathematica file(s) that can be executed/evaluated, so that the assistant is able to easily verify that your solution works. It may be just one Mathematica file, where you have clearly documented each problem, or you can return each problem in its own Mathematica file.
- The assignment is graded 1, 3 or 5. Minimum requirement (for grade 1) is to complete problems 1 and 2.

All problems below are related to steady state properties. Remember then to implement some kind of initial transient control, e.g., by discarding a certain number of samples from the start. Note that we are not expecting a detailed analysis on the length of the transient in your results.

Problem 1

a) Consider the single-class M/G/1 PS queue. Assume first that the service times are exponentially distributed with mean 1. Simulate the system for $\lambda = \{0.2, 0.5, 0.8\}$ and estimate the mean delay of the jobs. Also, determine the 95% confidence intervals.

b) Assume now that the service times obey the hyperexponential distribution with two phases representing long and short service times. More precisely, consider a hyperexponential distribution with two phases γ_1 and γ_2 with weight parameter p. Then the pdf of the service times obeys

$$f(x) = p\gamma_1 e^{-\gamma_1 x} + (1-p)\gamma_2 e^{-\gamma_2 x}.$$

The distribution is clearly just a weighted sum of two exponentials and the interpretation is indeed that with probability p the service time S obeys the exponential distribution with rate γ_1 and otherwise the exp-distribution with rate γ_2. The mean is simply $E[S] = p/\gamma_1 + (1-p)/\gamma_2$. Now we fix $\gamma_1 = 0.1$ (mean is 10, long service times) and $\gamma_2 = 10$ (mean is 0.1, short service times). To have $E[S] = 1$ requires $p = \gamma_1(\gamma_2 E[S]-1)/(\gamma_2-\gamma_1) = 0.0909091$, but the variance is 17.2.

Repeat the same simulation as in a)-part and compare the results. Note that with these parameters the service times have 17 times greater variance than with exponential distribution and recall the discussion on the lectures on impact of service times and scheduling discipline. What do you observe? Are you able to explain your observations?

Problem 2

Modify your single-class M/G/1 PS simulator so that you can simulate both FIFO discipline and the SRPT discipline. Recall that the pseudo-code for FIFO is given in the lecture material, and SRPT is not a big change from PS.

Your task is to simulate the M/G/1 queue with two different service time distributions and compare the performance of the three disciplines (FIFO, PS, SRPT) to each other. Let us fix the mean service time as $1/\mu = 1$ and then the load $\rho = \lambda$.

a) Assume first that the service times obey the exponential distribution with mean $1/\mu = 1$. Simulate the system for $\lambda = \{0.4, 0.6, 0.8\}$ and estimate the mean delay together with its 95% confidence interval.

b) Repeat the same simulation experiment but assume now that the service times obey the hyperexponential distribution with two phases, as discussed in Problem 1 b).

c) Compare the results. What do you observe? To gain more insight you may also use the results of PS as a reference and evaluate the ratio of the delays for FIFO and SRPT relative to PS. If you want to visualize the results in a plot, you can of course simulate more load values. Note that with these parameters the service times have almost 17 times greater variance than with the exponential distribution and recall the discussion on the lectures on impact of service times and scheduling discipline.

d) (Additional task, not mandatory) By using the formulae given in the lectures, it is also possible to compare the simulation results against numerical results.

Hint: Note that the pseudo code for simulating the FIFO queue does not include estimating the delays. This in fact requires the use of a simple job list where the arrival time of each job is saved. Then at departure the job at the head of the list is deleted and the delay sample can be collected. So to implement the FIFO you need the job list as in the PS simulator but the logic changes according to the FIFO pseudo code. Changing the PS simulator to SRPT is more straight forward.

Problem 3

Your task is now to add one more class to the multiclass PS simulator, i.e., there will be class-3 flows arrive according to the Poisson process with rate λ_3, the flows are served at rate r_3 bit/s. In your implementation you may assume that the sizes L_3 are exponentially distributed (and hence the service times, as well), similarly as in the example code. Make the necessary modifications by following the example implementation of the 2-class simulator.

Your next task is to simulate the system and estimate the throughput of all classes as a function of the load. Recall the definition of the throughput from the lecture 9 slides. The sizes in all classes obey the same exponential distribution with mean 1 Mbit, i.e., $E[L] = 1$

Mbit. The service bit rates of all classes are $r_1 = 10$ Mbit/s, $r_2 = 5$ Mbit/s and $r_3 = 1$ Mbit/s. Also, we fix the arrival rate of classes 1 and 2, such that $\lambda_1 = 3$ and $\lambda_2 = 0.5$. Then the stability condition becomes

$$\frac{\lambda_1 E[L]}{r_1} + \frac{\lambda_2 E[L]}{r_2} + \frac{\lambda_3 E[L]}{r_3} < 1 \Rightarrow \lambda_3 < \frac{r_3}{E[L]}\left(1 - \frac{\lambda_1 E[L]}{r_1} + \frac{\lambda_2 E[L]}{r_2}\right) = 0.6.$$

Simulate the system as a function of $\lambda_3 = \{0.05, 0.1, 0.3, 0.5\}$ and estimate the throughput of of each class together with the 95% confidence intervals. Also, plot your results in a graph and compare to the exact result, as given in the lecture 9 slides.

Hint: Recall that the code in the lectures assumes that the service requirement is expressed in units of time. If your sample is originally in units of bits (size), say L, then the corresponding requirement in time is simply L/r_k, i.e., the size divided by the bit rate of that class. This is then used as the sample of the *service time*. Thus, the service time and the size obey the same distribution, but size is converted to time by dividing with the rate.

Problem 4

Consider the flow-level model of multiuser scheduling as discussed in lecture 9, slides 28-31. We study a system with 3 classes having similar rates as in Problem 3 but they use different amounts of codes:

- Class 1 corresponds to the near users that use $w_1 = 5$ codes and have service bit rate $r_1 = 10$ Mbit/s.
- Class 2 flows are mid range flows and can use $w_1 = 5$ codes and the rate is $r_2 = 5$ Mbit/s.
- Class 3 flows are far users and can use $w_3 = 10$ codes and their rate is $r_3 = 1$ Mbit/s.

Totally, the system has $W = 20$ codes. Modify the 3 class PS simulator you developed in problem 3 in order to take into account the unequal sharing as described in the model.

As in problem 3, we assume that the sizes of the flows are exponentially distributed with mean 1 Mbit. The arrival rate of classes 1 and 2, such that $\lambda_1 = 3$ and $\lambda_2 = 0.5$. Then the stability condition in fact becomes identical to the one in problem 3, i.e.,

$$\lambda_3 < 0.6.$$

Simulate the system as a function of $\lambda_3 = \{0.05, 0.1, 0.3, 0.5\}$ and estimate the throughput of each class together with the 95% confidence intervals. Can you make any observations by comparing with the results from problem 3.