AALTO UNIV. SCHOOL OF ELECTRICAL ENGINEERING **Exercise 4**
COMNET Department 19.10.2018
ELEC-E7460 Modeling and Simulation, fall 2018 Lassila / Gebrehiwot

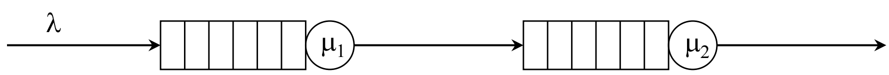
1. Assume that we want to generate uniformly distributed points within the unit circle $x^2 + y^2 = 1$. Give an algorithm based on the inverse transformation technique to directly generate samples of the x- and y-coordinates. Hint: consider x and y in the polar coordinates (r, ϕ), justify that r and ϕ are independent, derive the marginal distribution of ϕ-component and consider the probability of points falling inside a circle with radius r.

2. Let us model a cell being served by a base station in a wireless network as a circle with radius $R_{\max} = 100$ m. Inside the cell a user, if he is at a distance r from the base station, can achieve a transmission rate $c(r)$ that decays due to path loss effects as a function of the distance according to

$$c(r) = \begin{cases} C_{\max}, & r \leq R_{\min}, \\ C_{\max}\left(\frac{R_{\min}}{r}\right)^{\alpha}, & R_{\min} < r \leq R_{\max}, \end{cases}$$

where $C_{\max} = 100$ Mbit/s, $R_{\min} = 10$ m and $\alpha = 3$. Consider that the base station is located at point $(0, 0)$. At point $(70\text{ m}, 0)$ there is another circular cell with radius $R_2 = 20$ m, which represents the service area of a low power base station inside the coverage of the high power base station. Assume that the location of a user is uniformly distributed inside the larger circle (high power base station's coverage area). Let X denote the random variable for the location of the user. What is the mean achievable transmission rate of the user $E[c(X)]$ inside the large cell given that the user is not in the area of the small cell area? *Hint*: Apply the rejection method to generate uniformly distributed points inside the large cell and not inside the small cell.

3. Consider a simple tandem queuing network with two queues as shown in the figure below. Jobs arrive according to a Poisson process with rate λ. The service times of the jobs in the two queues are i.i.d. and obey the log-normal distribution with mean μ_1 and μ_2, respectively. Hence, the system is a tandem network of M/G/1 queues. We also assume the system is stable, i.e., $\lambda < \min(\mu_1, \mu_2)$. (i) Identify the regeneration points of this system so that one could apply the regenerative method for estimating, e.g., the mean delay of the jobs. (ii) How does the situation change if the service times are i.i.d. and exponentially distributed?



4. Devise an algorithm by using the composite method to generate rv:s obeying the density
$$f(x) = \frac{1}{2} + x^3 + 2x^7, \qquad 0 \le x \le 1.$$
Hint: compute the cumulative distribution.

5. The random variable Y is defined as follows: $Y = X_1/(1 + X_2/10)$, where $X_1 \sim X_2 \sim \mathrm{Exp}(1)$, and X_1 and X_2 are independent. Devise a method based on the use of a control variable for estimating the expectation of Y, $\mathrm{E}[Y]$.

   Implement in Mathematica both a direct method and the control variable method. Generate 10000 samples using both methods and compare your point estimates and their 95 % confidence intervals.