AALTO UNIV. SCHOOL OF ELECTRICAL ENGINEERING	**Assignment 1**
COMNET Department	DL 11.11.2018
ELEC-E7460 Modeling and Simulation, fall 2018	Lassila

Returning your solutions

Write a short report on the results. Return the report and your Mathematica files in a zip-file in the assignment folder in MyCourses or to the lecturer (Pasi.Lassila@aalto.fi). The deadline for returning your solutions for this exercise is **Sunday 11.11.2018**.

Requirements for reporting:

- In the report, for each problem explain also briefly your solution.
- For each problem, provide the Mathematica file(s) that can be executed/evaluated, so that the assistant is able to easily verify that your solution works. It may be just one Mathematica file, where you have clearly documented each problem, or you can return each problem in its own Mathematica file.
- The assignment is graded 1, 3 or 5. Minimum requirement (for grade 1) is to complete problems 1 and 2.

Problems

1. Consider a 5-state Markov process $X(t)$ with state space $S = \{1, 2, 3, 4, 5\}$ and the following transition rate matrix

$$Q = \begin{pmatrix} -3 & 1 & 0 & 0 & 2 \\ 0 & -2 & 2 & 0 & 0 \\ 1 & 1 & -3 & 1 & 0 \\ 1 & 0 & 0 & -3 & 2 \\ 0 & 0 & 0 & 1 & -1 \end{pmatrix}.$$

   Assume that the process starts at time 0 from state 1. Let the random variable T_1 represent the time it takes for the process from time 0 to return for the first time back to state 1, i.e., it may visit several other states in between but eventually it return back to state 1 and then the cycle immediately ends. Estimate $\mathrm{E}[T_1]$. Additionally, determine the 95% confidence interval of your estimates. *Hint*: Recall that in Lecture 8 you have code for generating sample paths of a Markov process. For comparison, the exact answer is $E[T_1] = 47/12$.

2. Consider still the same process $X(t)$ from the previous problem. Let π_i denote the fraction of time that the process spends in state i in the steady state. Your task is now to estimate the fractions π_i by simulation, also show the 95% confidence intervals of the estimates from your simulations. *Hint*: This is clearly now related to steady state simulation so you need to implement initial transient control, as well!

Also recall that you can check the correctness of your solution by solving the steady state distribution from the matrix equation

$$\pi = e \cdot (Q + E)^{-1},$$

where E is a matrix with 1 in all elements and e is a vector with 1 in all elements. For more information, see Lecture 7.

3. In the above problems 1 and 2, the simulation of the transitions (if you used the code from the lectures) was based on selecting in each state the minimum of a set of exponentially distributed random variables. An alternative method is characterized in pseudo-code in Lecture 7 on slide 33 (so-called Method 2), where in each state the jump to the next state is sampled from the discrete distribution of the jump probabilities. Implement this approach and repeat the same estimation of the steady state fraction of time the process spends in each state, i.e., the same as in problem 2.

4. Consider the flow-level model for streaming CBR traffic as given in the Lecture 7. Assume that the streaming video flows arrive according to a Poisson process with rate λ and their durations are exponentially distributed with mean 1 minutes and the transmission rate of each flow is $r = 1$ Mbit/s. The link capacity is $C = 10$ Mbps.

   a) Estimate the loss ratio p_{loss} for an increasing value of the arrival rate of flows $\lambda = [1, 3, 5, 6, 8, 9]$ 1/min.

   b) Plot the results as a function of λ. Additionally, you can in the same figure plot the numerical solution of p_{loss} following from the formula on slide 29.

   c) Determine the 95% confidence intervals of the estimates, as well. You can give them in a table. Are you able to visually represent them in the plot? Google information for ErrorBar.

   Hint: Recall that in Lecture 8, you have code for simulating the M/M/1 queue!