# Assignment 1
# ELEC-E7460 Modeling and Simulation, fall 2018
# Adam Ilyas 725819

## Problem 1: Simulation of a simple general Markov process

In[2]:=
```
Needs["HypothesisTesting`"]
```

In[3]:=
```
Q = {
   { -3, 1, 0, 0, 2 },
   {0, -2, 2, 0, 0},
   {1, 1, -3, 1, 0},
   {1, 0, 0, -3, 2},
   {0, 0, 0, 1, -1}
  }
```

Out[3]= {{-3, 1, 0, 0, 2}, {0, -2, 2, 0, 0}, {1, 1, -3, 1, 0}, {1, 0, 0, -3, 2}, {0, 0, 0, 1, -1}}

In[4]:=
```
MatrixForm[Q]
```

Out[4]//MatrixForm=

$$\begin{pmatrix} -3 & 1 & 0 & 0 & 2 \\ 0 & -2 & 2 & 0 & 0 \\ 1 & 1 & -3 & 1 & 0 \\ 1 & 0 & 0 & -3 & 2 \\ 0 & 0 & 0 & 1 & -1 \end{pmatrix}$$

In[ ]:=
```
generateEventTimes[Q_, state_] :=
   Table[
    If[Q[[state, nextState]] > 0,
     RandomVariate[ExponentialDistribution[Q[[state, nextState]]]],
     Infinity],
    {nextState, 1, Length[Q]}
   ];
```

In[]:=
```
state = 1;
simLength = 30;
simTime = 0;
step = 0;

While[simTime ≤ simLength,
  eventTimes = generateEventTimes[Q, state];
  timeInState = Min[eventTimes];
  simTime += timeInState;
  state = Position[eventTimes, timeInState][[1, 1]];
  #Print[{timeInState, state}];
 ];
```

Here we build a simulator that returns the simulation time if a cycle from to 1 back to 1 happened.

In[]:=
```
cycleSimulator[Q_, st_] := Module[
   {simTime, eventTimes, timeInState, res, state},
   state = st;
   simTime = 0;
   res = {};
   While[True,
    eventTimes = generateEventTimes[Q, state];
    timeInState = Min[eventTimes];
    simTime += timeInState;
    state = Position[eventTimes, timeInState][[1, 1]];
    res = Append[res, {simTime, state}];
    If[state == st, Break[], True];
   ];
   simTime
  ];
res = simulator[Q, 1]
```

Out[]=
```
2.26813
```

We repeat the simulation a large enough number of times, each time storing the simulation time in a table
and we find the mean and the confidence interval of this table.

In[]:=
```
simResult = Table[simulator[Q, 1], 1000]
Mean[simResult]
MeanCI[simResult]
```

Out[]=
```
3.91614
```

General: $0.00383984^{499.5}$ is too small to represent as a normalized machine number; precision may be lost.

Out[]=
```
{3.69906, 4.13322}
```

Our expected time that a cycle from an initial state 1 back, back to state 1, after a 1000 simulation is 3.91. Our confidence interval is {3.69906,4.13322}

# Problem 2: estimate the fractions $\pi$_i by simulation

```
In[•]:= piSimulator[Q_, simLength_] := Module[
          {simTime, eventTimes, timeInState, state, pi},
          state = 1;
          simTime = 0;
          pi = ConstantArray[0, 5];
          While[True,
           eventTimes = generateEventTimes[Q, state];
           timeInState = Min[eventTimes];
           pi[[state]] += timeInState / simLength;
           simTime += timeInState;
           state = Position[eventTimes, timeInState][[1, 1]];
           If[simTime > simLength, Break[], True];
          ];
          pi
         ];

        piSimulator[Q, 1000]
Out[•]= {0.0810011, 0.0749499, 0.0477375, 0.211173, 0.585492}

In[•]:= piResult = Table[piSimulator[Q, 1000], 1000]


In[•]:= meanPi = Mean[piResult]
        meanCI = MeanCI[piResult]
Out[•]= {0.0854831, 0.0641268, 0.0427434, 0.212814, 0.595549}

        General: 0.00383984^499.5 is too small to represent as a normalized machine number; precision may be lost.
Out[•]= {{0.0850497, 0.063532, 0.0423492, 0.212237, 0.594459},
         {0.0859166, 0.0647216, 0.0431376, 0.213391, 0.596638}}
```

The mean fraction of time for each state, and its confidence interval is as follows:

```
In[•]:= For[i = 1, i ≤ 5, i++,
         Print[StringForm["State `` mean: ``, Confidence Interval: ``",
           i, meanPi[[i]], meanCI[[1, i]]], meanCI[[2, i]]]
        ]
        State 1 mean: 0.08548314615260438`, Confidence Interval: 0.08504973874723055`0.0859166
        State 2 mean: 0.06412677261454684`, Confidence Interval: 0.0635319881298627`0.0647216
        State 3 mean: 0.04274341189428288`, Confidence Interval: 0.04234924214065339`0.0431376
        State 4 mean: 0.2128141923355341`, Confidence Interval: 0.21223726265939785`0.213391
        State 5 mean: 0.595548635607118`, Confidence Interval: 0.5944592307478451`0.596638
```