ELEC-E7460 Modeling and Simulation, fall 2018
Assignment 2

Adam Ilyas 725819

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1 Single-class M/G/1 PS queue.

a) Consider the single-class M/G/1 PS queue. Assume first that the service times are expo- nentially distributed with mean 1. Simulate the system for $\lambda = \{0.2, 0.5, 0.8\}$ and estimate the mean delay of the jobs. Also, determine the 95% confidence intervals.
Ans.

$$\text{When } \lambda = 0.2, Mean : 1.2577,$$

$$95\% = \text{Confidence Interval} = \{1.24539, 1.26992\}$$

$$\text{When } \lambda = 0.5, Mean : 1.9946,$$

$$95\% = \text{Confidence Interval} = \{1.94877, 2.04039\}$$

$$\text{When } \lambda = 0.8, Mean : 4.9683,$$

$$95\% = \text{Confidence Interval} = \{4.76486, 5.17172\}$$

b) Assume now that the service times obey the hyperexponential distribution with two phases representing long and short service times. More precisely, consider a hyperexponential distribution with two phases γ_1 and γ_2 with weight parameter p. Then the pdf of the service times obeys

$$f(x) = p\gamma_1 e^{\gamma_1 x} + p\gamma_2 e^{\gamma_2 x}$$

Repeat the same simulation as in a)-part and compare the results. Note that with these parameters the service times have 17 times greater variance than with exponential distribution and recall the discussion on the lectures

on impact of service times and scheduling discipline. What do you observe? Are you able to explain your observations?
Ans.

When $\lambda = 0.2, Mean : 1.2519,$

$95\% = \text{Confidence Interval} = \{1.2366, 1.26729\}$

When $\lambda = 0.5, Mean : 2.009,$

$95\% = \text{Confidence Interval} = \{1.96898, 2.04908\}$

When $\lambda = 0.8, Mean : 4.9208,$

$95\% = \text{Confidence Interval} = \{4.78287, 5.05881\}$

We observe that for hyperexponential Service times, the confidence interval for the mean delay is larger as compared to the queue simulation with exponential service times. This is due to the larger variance in the arrival times and as such, we are less certain of delay times

2 Single-class M/G/1 PS simulator (PS FIFO SRPT)

a) Your task is to simulate the M/G/1 queue with two different service time distributions and compare the performance of the three disciplines (FIFO, PS, SRPT) to each other.
Let us fix the mean service time as $1/\mu = 1$ and then the load $\rho = \lambda$.
Assume first that the service times obey the exponential distribution with mean $1/\mu = 1$. Simulate the system for $\lambda = \{0.4, 0.6, 0.8\}$ and estimate the mean delay together with its 95% confidence interval.
Ans.

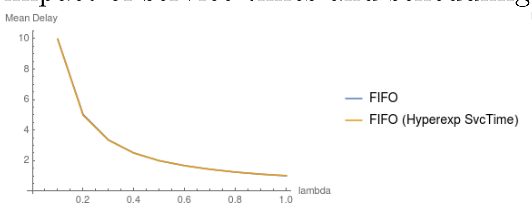
Discipline	Lambda	Mean Delay	Confidence Interval
FIFO	0.4	2.5039	2.48541, 2.52243
	0.6	1.6762	1.66237, 1.69009
	0.8	1.2534	1.24877, 1.25803
PS	0.4	1.6698	1.6388, 1.70088
	0.6	2.4347	2.37107,2.49829
	0.8	4.9521	4.74702,5.15716
SRPT	0.4	1.3894	1.36088,1.41799
	0.6	1.2858	1.26539,1.30612
	0.8	0.90992	0.882442,0.937395

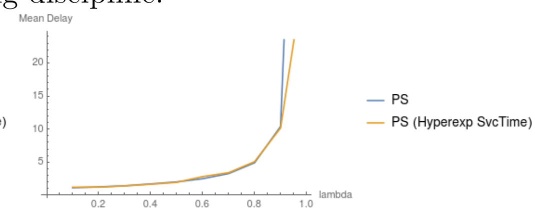
b) Repeat the same simulation experiment but assume now that the service times obey the hyperexponential distribution with two phases, as discussed in Problem 1 b).

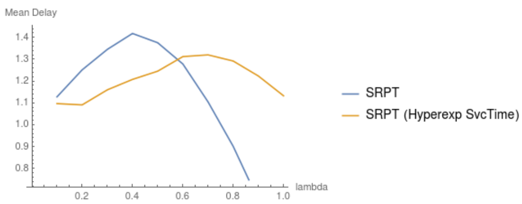
Ans.

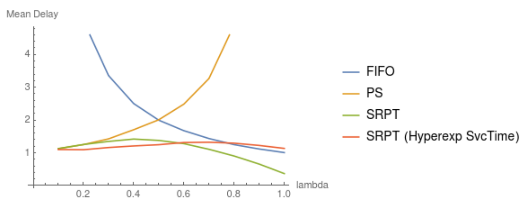
Discipline	Lambda	Mean Delay	Confidence Interval
FIFO	0.4	2.4924	2.47442,2.5104
	0.6	1.6733	1.6636,1.68298
	0.8	1.2599	1.24886,1.271
PS	0.4	1.6311	1.53437,1.72792
	0.6	2.5578	2.40994,2.70569
	0.8	4.4229	3.81458,5.03125
SRPT	0.4	1.19764	1.14057,1.25471
	0.6	1.2599	1.18239,1.33745
	0.8	0.90992	0.882442,0.937395

c) Compare the results. What do you observe? To gain more insight you may also use the results of PS as a reference and evaluate the ratio of the delays for FIFO and SRPT relative to PS. If you want to visualize the results in a plot, you can of course simulate more load values. Note that with these parameters the service times have almost 17 times greater variance than with the exponential distribution and recall the discussion on the lectures on impact of service times and scheduling discipline.









We can see the affect of increase/decreasing arrival rate has for throughput. 1) FIFO works well with higher arrival rate 2) PS works well with lower arrival rate 3) SRPT remain relatively constant as compared to FIFO and PS.

3 Multiclass PS Queue

Your task is now to add one more class to the multiclass PS simulator, i.e., there will be class-3 flows arrive according to the Poisson process with rate λ_3 , the flows are served at rate r_3 bit/s. In your implementation you may assume that the sizes L_3 are exponentially distributed (and hence the service times, as well), similarly as in the example code. Make the necessary modifications by following the example implementation of the 2-class simulator. Your next task is to simulate the system and estimate the throughput of all classes as a function of the load.

The service bit rates of all classes are r_1 = 10 Mbit/s, r_2 = 5 Mbit/s and r_3 = 1 Mbit/s. Also, we fix the arrival rate of classes 1 and 2, such that $\lambda_1 = 3$ and $\lambda_2 = 0.5$

Simulate the system as a function of $\lambda 3 = \{0.05, 0.1, 0.3, 0.5\}$ and estimate the throughput of of each class together with the 95% confidence intervals. Also, plot your results in a graph and compare to the exact result, as given in the lecture 9 slides.