Multi Arm Bandit Problem

April 26, 2019

1 Statistical and Machine Learning (01.113) - HW5 Question 3

In this problem, we will study the multi-armed ($k = 10$) bandit problem.

```
In [1]: import matplotlib.pyplot as plt
        import numpy as np

        T = 1000;
```

1.0.1 ϵ-greedy policy

Behave greedily most of the time, but explore once in a while:

$$A_t = \begin{cases} i^* = \arg\max_i Q_t(i) & \text{with probability } 1 - \epsilon \\ j, j \neq i^* & \text{each with probability } \frac{\epsilon}{k-1} \end{cases}$$

Balances exploitation vs exploration, but does not select intelligently between the k1 non-greedy actions.

```
In [2]: # greed and UCB function
        def e_greedy(Q,N,t,e):
            """
            Q: 1-d array of action-values
            N: 1-d array of action counts (X)
            t: time (total number of actions taken thus far) (X)
            e: exploration probability

            Returns
                next action to be made
            """
            current_action = np.argmax(Q)
            if np.random.rand()<e: # explore other action with equal probability
                other_actions = [i for i in range(len(Q)) if i != current_action]
                next_action = np.random.choice(other_actions)
            else:
                next_action = current_action
            return next_action
```

1.0.2 Upper confidence bound (UCB)

Select action at time t according to

$$A_t = \arg\max_i \left(Q_t(i) + c\sqrt{\frac{\log t}{N_t(i)}} \right)$$

where $N_t(i)$ denotes the number of times action i has been selected prior to time t, and c is the exploration constant; increasing it favours exploration and decreasing it favours exploitation.

```
In [3]: # the denominator means that:
        ## number of times of A_i increases, we will choose A_i lesser
        ## recall that exploration is choosing other decisions
        ## exploitation is choose current best

        def UCB(Q,N,t,c):
            """
            Q: 1-d array of action-values
            N: 1-d array of action counts
            t: time (total number of actions taken thus far)
            c: exploration constant : increasing it favours exploration \\
                                      and decreasing it favours exploitation.
            Returns
                next action to be made
            """
            assert t == sum(N);
            return np.argmax(Q+c*np.sqrt((np.log(t))/(N)))
```

Write a function that performs one run (1000 time steps), updates Q incrementally and records the reward received at each time step:

$$Q_{n+1} = Q_n + \frac{1}{n+1}\left(R_{n+1} - Q_n\right)$$

At each time step, when action a is taken, the reward r is sampled from a normal distribution with mean $true\_means[a]$ and standard deviation 1.

```
In [4]: def test_run(policy,param):
            true_means=np.random.normal(0,1,10) # true rewards

            # START, initialize
            rewards=np.zeros(T+1) # reward tracker
            Q=np.zeros(10) #action values
            N = np.zeros(10) # action counts

            for t in range(T):
                a=policy(Q,N,t,param) # next action (a)
                r=np.random.normal(true_means[a],1) # observed reward at (t) when (a)

                #NewEstimate = OldEstimate + StepSize[Target - OldEstimate]
```

```
            step_size = (1 / (N[a]+1))
            Q[a] += step_size * (r - Q[a])
            N[a] += 1 # update counter
            rewards[t+1]=r # store reward
        return rewards
```

1.0.3 (c) Use the following function to average over 2000 runs and plot the results:

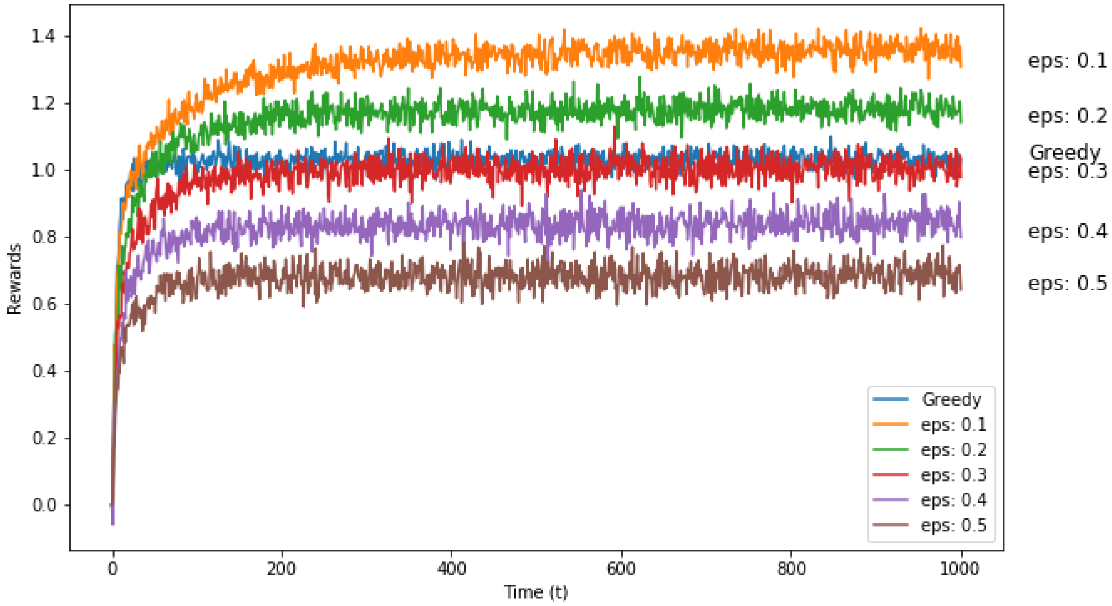
At approximately what value of ϵ does the ϵ-greedy method switch from being better than the greedy policy to being worse than it?

```
In [5]: def simulate_greedy(constants):
            data = []
            for eps in epsilons:
                print("eps: ", eps)
                ave_eg=np.zeros(T+1)
                for i in range(2000):
                    eg=test_run(e_greedy, eps) #choose parameter
                    ave_eg+=(eg-ave_eg)/(i+1)
                data.append(ave_eg)
            return data
```

```
In [10]: time = np.arange(T+1)

         epsilons = [0, 0.1, 0.2, 0.3, 0.4 ,0.5]
         data_greedy = simulate_greedy(epsilons)

         plt.figure(figsize=(10, 6))
         for i in range(len(data_greedy)):
             if i == 0:
                 label = "Greedy"
             else:
                 label = f"eps: {round(epsilons[i], 2)}"
             plt.plot(time, data_greedy[i], label=label)
             plt.text(time[-1] + 80, data_greedy[i][-1], label, fontsize=12)
         plt.xlabel("Time (t)")
         plt.ylabel("Rewards")
         plt.legend()
         plt.show()
```



We notice that at $\epsilon = 0.3$ has approx. the same performance as greedy, $\epsilon = 0.4$ performs worst, $\epsilon = 0.2$ performs better

```
In [28]: epsilons = [0, 0.27, 0.275, 0.28, 0.285, 0.29, 0.295, 0.3]
         data_greedy_new = simulate_greedy(epsilons)

         # set 400 as warm up time required to reach steady state
         reward_means = [np.mean(rewards[400:]) for rewards in data_greedy_new]
         for i in range(len(reward_means)):
             eps = epsilons[i]
             base_mean = reward_means[0]
             r_mean = reward_means[i]
             perform = "Better" if r_mean > base_mean else \
                         "Worst" if r_mean < base_mean else ""
             print(f"eps: {eps} Average Rewards: {reward_means[i]} ({perform})")
```

```
eps: 0 Average Rewards: 1.0134459686714616 ()
eps: 0.27 Average Rewards: 1.0695985950382605 (Better)
eps: 0.275 Average Rewards: 1.0648032754828465 (Better)
eps: 0.28 Average Rewards: 1.062069782616915 (Better)
eps: 0.285 Average Rewards: 1.033637678572049 (Better)
eps: 0.29 Average Rewards: 1.0366515661034263 (Better)
eps: 0.295 Average Rewards: 1.020058512818882 (Better)
eps: 0.3 Average Rewards: 0.9980903970582163 (Worst)
```