Tutorial 1 Solutions

1. The first problem is a straightforward application of Amdahl's law discussed in the lecture, which is an idealized notion of how much a program execution can benefit from parallelization. Note that Amdahl's law follows intuitively if one compares the time T_1 to execute a program sequentially to the time T_N required to execute it on N processors in parallel. Recall that P is the fraction of code that can be parallelized. We have that

$$\begin{aligned} T_N &= \frac{P}{N} T_1 + (1 - P)\, T_1 \\ \Leftrightarrow \frac{T_1}{T_N} &= \frac{1}{(1 - P) + \frac{P}{N}}, \end{aligned} \tag{1}$$

   where Equation (1) is also referred to as *speedup*. We obtain the following values.

P	N	Speedup
0.9	10	5.2632
	100	9.1743
	1000	9.9108
0.95	10	6.8966
	100	16.8067
	1000	19.6271
0.99	10	9.1743
	100	50.2513
	1000	90.9918

2. a) Since for the disk seek time $t_s[s]$ measured in seconds we have $t_s = 0.01\,s$ and the transfer rate $r[b/s]$ measured in bytes per second we have $r = 150 * 10^6\ b/s$, the amount of data that can be read in the same time needed to perform a seek is

$$t_s * r = 10^{-2}\,s\ *\ 150 * 10^6\,b/s = 1.5\,\text{MB}.$$

   b) We divide the disk capacity $C[b] = 6 * 10^{12} b$ by the transfer rate r and obtain

$$\frac{C}{r} = \frac{6 * 10^{12} b}{150 * 10^6\ b/s} \approx 11.12\,\text{hours}.$$

c) Now we assume the data is not read sequentially but in blocks of fixed size. Further, reading a single block also requires a disk seek, which will slow down the *effective* transfer rate of the disk. Let $B[b]$ denote the block size in bytes and $r_e[b/s]$ denote the effective transfer rate, which can be computed as

$$r_e = \frac{B}{\frac{B}{r} + t_s} = \frac{4 * 10^3\, b}{\frac{4*10^3\, b}{150*10^6\, b/s} + 10^{-2}\, s} = \frac{6 * 10^8}{1504}\, \frac{b}{s}.$$

As before, by dividing the capacity to be read by the transfer rate we determine the time required for reading as

$$\frac{C}{r_e} = \frac{6 * 10^{12} * (1504)}{6 * 10^8}\, s = 10^4 * (1504)\, s \approx 174.074\,\text{days}.$$

d) The only difference to the previous question is the change in block size (the new block size is $B' = 64 * 10^6\, b$). By computing the new effective transfer rate r'_e we obtain

$$r'_e = \frac{B'}{\frac{B'}{r} + t_s} = \frac{64 * 10^6\, b}{\frac{64*10^6\, b}{150*10^6\, b/s} + 10^{-2}\, s} = \frac{150 * 64 * 10^6}{64 + 1.5}\, \frac{b}{s} = \frac{96 * 10^8}{65.5}\, \frac{b}{s}$$

and thus for the time required for reading follows

$$\frac{C}{r'_e} = \frac{6 * 10^{12} * 65.5}{96 * 10^8}\, s \approx 11.37\,\text{hours}.$$

3. Let N be the (unknown) number of transistors in a common CPU in the year 2015 and correspondingly N_F the number of future transistors in the year $F \geq 2015$. Due to the doubling rate of two years we have

$$N_F = 2^{\frac{F-2015}{2}} * N.$$

Since the number of transistors required per core is assumed stay the same, we can calculate the number of cores in the year F as

$$\frac{N_F}{N/4} = 4 * 2^{\frac{F-2015}{2}} = 2^{2+\frac{F-2015}{2}}.$$

So for $F = 2025$ we obtain 128 cores and for $F = 2035$ we have 4096 cores in a hypothetical future CPU, assuming the current trend continues.

4. The problem states that currently available hard-disk bandwidth (i.e., reading and writing speed) doubles every 2.8 years. In the same time period, however, hard disk capacity grows by a factor of 2.4, which means that capacity will eventually outgrow bandwidth to an extent at which the hypothetical hard disk will become unusable. More precisely, the question asks for the time at which it will take longer to fill the hard disk with data than its planned lifetime of 5 years.

Consider the evolution of the two variables bandwidth (transfer rate) $r_i[b/s]$ and capacity $C_i[b]$ after i doublings of disk bandwidth. We initially have $r_0 = 150 * 10^6\, b/s$ and $C_0 = 6 * 10^{12}\, b$. From the description follows

$$r_i = 2 * r_{i-1} = 2^i * r_0, \quad C_i = 2.4 * C_{i-1} = 2.4^i * C_0, \quad \text{for } i > 0.$$

We are interested in the smallest integer k, such that at the time of the k-th doubling we have

$$\frac{C_k}{r_k} = \left(\frac{2.4}{2}\right)^k * \frac{C_0}{r_0} = 1.2^k * \frac{10^6}{25} > L, \tag{2}$$

where $L[s]$ is the planned lifetime of the disk in seconds, which means $L = 157784630\,s$ (conversion courtesy of google). Solving numerically for k yields $k = 46$.[1] From this we obtain

$$r_k = 2^{46} * 150 * 10^6\, b/s \approx 1.0555 * 10^{10}\, \mathrm{TB}/s$$

and

$$C_k = 2.4^{46} * 6 * 10^{12}\, b \approx 1.8529 * 10^{18}\, \mathrm{TB},$$

which will hypothetically be achieved in the year

$$2015 + k * 2.8 = 2144.$$

[1]One method is to use a program that starts from $k = 0$, increments k by one in each iteration and terminates when the condition (2) is satisfied.