Complex Networks W3: Growing Network Models - Scale-Free Networks

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Contents

1 General

1.1 Terminology

Term	Definition
Hub	Nodes of very high degree
Clique	A subgraph which is well-connected (every node has a link to every other node)

2 Scale Free Networks

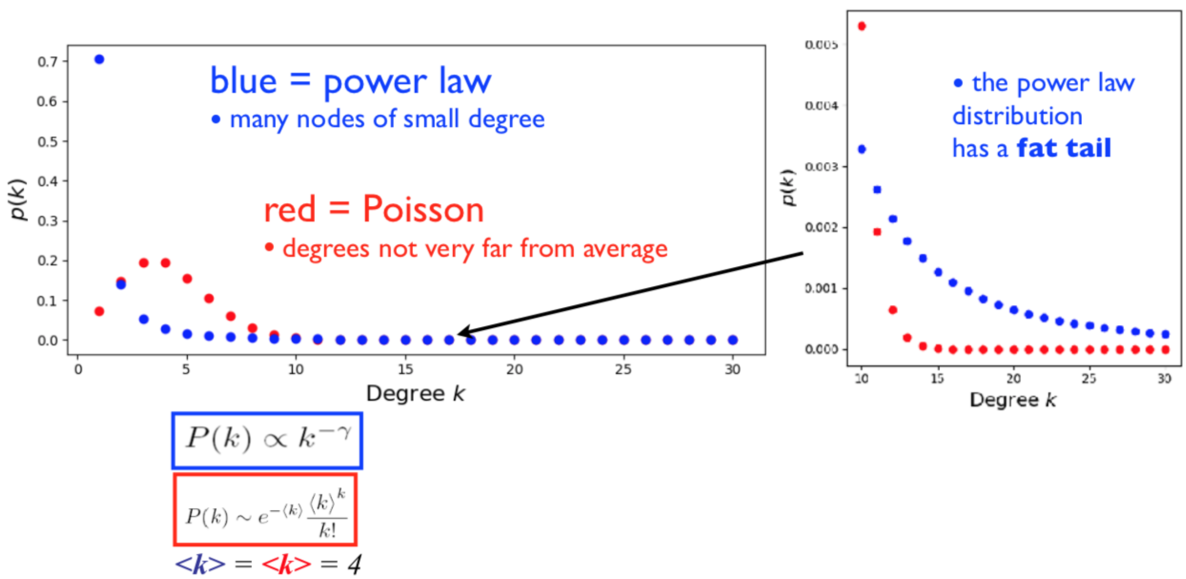
2.1 Description

Networks with power-law degree distribution

2.2 Degree Distribution

2.2.1 Power Law vs Poisson

- Linear Scale



- Log Scale

2.2.2 Form of Distribution

$$P(\alpha k) \propto (\alpha k)^{-\gamma} = \alpha^{-\gamma} P(k)$$

2.3 Identifying Power Law Distribution

- Fat tail (e.g. for degree distro, high probability of low degree)

2.4 Possible Explanation or Power Law Distribution

The rich get richer analogy (note that this result is the Pareto Law)

3 Scale-Free Generation Models

3.1 Barabasi-Albert Model

3.1.1 Starting Condition

Small seed network of a few connected nodes

3.1.2 Algorithm Loop

1. Add a new node with m stubs
2. Connect each stub to existing node i, chosen with probability $p_i = \frac{k_i}{\sum k_i}$

3.1.3 Properties

Property	Notes
Power law distribution	$P(k) = \frac{2m^2}{k^3}$
Average degree $< k > \approx 2m$	m new edges added with each node
Ultra-small World	$l \propto \frac{\ln N}{\ln(\ln N)}$

3.2 Random Walks

3.2.1 Starting Condition

Small seed network of a few connected nodes

3.2.2 Algorithm Loop

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3.3 Preferential Attachment

3.3.1 What It Is

- Step 2 of the loop in Barabasi-Albert loop part of algorithm
- Connecting a node to another node chosen with probability $p_i = \frac{k_i}{\sum k_i}$

3.3.2 Why Hubs Exist

- Most new links are achieved by following existing links
- The probability that a random link leads to a hub is higher
  - Rich get richer $\rightarrow$ probability gets continually higher

3.3.3 Friendship Paradox Reasoning

- Basically, if Person A chooses a friend Person B at random, then Person B probably has more friends than Person A
- Direct consequence of the existence of hubs + Power Law
  - Person A likely has few friends (low degree)
  - Person B likely has many friends (random edge more likely connects to a hub)

3.3.4 Friendship Paradox Examples

Network	$\langle k \rangle$	$\langle k_{nn} \rangle$	$p(k_{nn} > k)$
Short messages	2.2	146	0.62
Airports & flights	11	65	0.93
Protein interaction	3.0	20	0.85
Internet AS	13	1445	0.96

k_{nn} is the nearest neighbor

4 NetworkX Learnings

4.1 Cliques

4.1.1 Notes

The clique problem is NP-Complete (so it could take quite a while to run)
Further reading

4.1.2 Useful Functions

More functions here

Function	**Output/Description**	Notes
`nx.enumerate_all_cliques(G)`	Returns **all** cliques in an **undirected** graph	Compare to `nx.find_cliques`. Double check this
`nx.find_cliques(G)`	Returns all maximal cliques in a graph	Compare to `nx.enumerate_all_cliques`
`nx.graph_clique_number(G[, cliques])`	Returns size of max clique in graph	Pass in cliques, get max size of those passed in cliques
`nx.cliques_containing_node(G[, nodes, cliques])`	Returns a list of cliques containing cliques (if 1 node passed) or a list of lists containing cliques (if list of nodes passed)	