CS-E5740 Complex Networks, Answers to exercise set 1

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1 Basic network properties

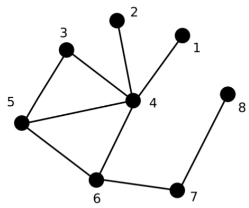

Figure 1: The graph for exercise 1.

a. Adjacency matrix

A network data structure in the form of a matrix used to represent a graph

$$a_{ij} = \begin{cases} 1 & if \quad (j,i) \in E \\ 0 & if \quad (j,i) \notin E \end{cases}$$

Hence, the adjacency matrix for the graph in figure 1 is as follows:

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

b. Edge density

The edge density of a network is the fraction of edges out of possible edges:

$$\rho = \frac{m}{\binom{N}{2}} = \frac{2m}{N(N-1)}$$

where m is the number of edges and N is the number of vertices.
Total possible edges is $\frac{N(N-1)}{2}$ which is $1 + 2 + 3... + (N-2) + (N-1)$

The edge density ρ of graph 1 is $\frac{18}{56} = 0.321$

c. Degree and Degree Distribution

The degree k_i of vertex v_i is the number of edges it is incident to.

Vertices	Degree
1	1
2	1
3	2
4	5
5	3
6	3
7	2
8	1

The degree distribution $P(k)$ is the probability that the degree of a randomly chosen node is k.

$$P(k) = \frac{N_k}{N}$$

where N_k is the number of nodes of degree k.

The degree distribution $P(k)$ of graph 1:

k	P(k)
1	0.375
2	0.25
3	0.25
4	0
5	0.125

d. The mean degree $\langle k \rangle$ of the graph

The average degree $\langle k \rangle$ of a network is:

$$\langle k \rangle = \sum_i \frac{k_i}{N} = \frac{2m}{N}$$

where m is the number of edges, N is the number of vertices, k_i is the degree for vertice i

The mean degree $\langle k \rangle$ for graph 1 is 2.25

e. The diameter d of the graph.

Diameter d is the largest distance in the network (It is the shortest distance between the two most distant nodes in the network.): $\max_{i,j \in V} d_{ij}$

Diameter of graph 1: 4

f. The clustering coefficient C_i

The clustering coefficient C_i for each node $i \in V$ that has degree $k_i > 1$. For nodes with $k_i = 0{,}1$, we define $C_i = 0$.

Clustering coefficient defined for node i as the fraction of edges between its neighbours out of possible edges between its neighbours:

$$C_i = \frac{E_i}{\binom{k_i}{2}} = \frac{2E_i}{k_i(k_i - 1)}$$

Vertices	C_i
1	0
2	0
3	1
4	0.2
5	0.66
6	0.33
7	0
8	0

Average clustering coefficient (averaged over all nodes) is $c = \frac{1}{N}\sum_i c_i = \frac{2.2}{8} = 0.275$