Statistics 2018

Homework Problem Set 4

Due: 11:59pm, Monday 23 April.

Submit this problem set on *eDimension*.

Refer to the *Excel* spreadsheet for the data. Show working.

Question 1. Refer to the spreadsheet for the corneal thickness data of glaucoma patients.

(a) Do the *sign test* to determine if the corneal thickness differs between an eye affected with glaucoma and an unaffected eye. Use $\alpha = 0.05$.

(b) Repeat part (a) using the *Wilcoxon signed rank test.* Do the two tests always give the same result when applied to the same data?

Question 2. In ANOVA for single factor experiments, define $\text{MST} = \text{SST}/(N-1)$. Is it possible that MST = MSA + MSE? Fully justify your answer.

Question 3. Water salinity measurements at three sites are given in the spreadsheet, and an ANOVA table is produced. However, one entry (marked by X) has been accidentally deleted.

(a) What can you conclude from the ANOVA F?

(b) Find X with help from the ANOVA table. (Hint: you can do this using any method you like, but check your answer.)

Question 4. Refer to the sugar content example given in the spreadsheet. Use the Bonferroni method to determine which shelves have significantly different mean sugar content. Use $\alpha = 0.05$.

Question 5. Refer to the spreadsheet 'IQ', which records some adopted children's IQ with the socioeconomic status of their biological parents as well as adoptive parents.

(a) Sketch a line chart for the cell means, and from it comment on whether there is any significant interaction.

(b) Construct an ANOVA table for this two-factor experiment. What conclusions can you draw from it?

Question 6. Suppose x is the number of iid Bernoulli trials with success probability p required to achieve the first success. Show that the MLE of p equals $\hat{p} = 1/x$.

Question 7. Show that the gamma distribution is a conjugate prior for the Poisson distribution. In particular, suppose that $x_1, \ldots, x_n$ is a random sample from a Poisson distribution

$$f(x|\theta) = \frac{e^{-\theta}\theta^x}{x!}, \quad \text{for } x = 0, 1, 2, \ldots$$

and the prior on θ is Gamma(α, β) distribution

$$\pi(\theta) = \frac{1}{\Gamma(\alpha)} \beta^{\alpha} \theta^{\alpha-1} e^{-\beta\theta}.$$

(a) Show that the posterior distribution of θ is Gamma$(\alpha + \sum_i x_i, \beta + n)$.

(b) Interpret how the posterior mean of θ,

$$E(\theta | x_1, \ldots, x_n) = \frac{\alpha + \sum_i x_i}{\beta + n},$$

depends on n. What happens to this estimate as $n \to \infty$?