Statistics

Week 12: Two-Factor Experiments (Chapters 13)

ESD, SUTD

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Randomized block design

ANOVA can also be used in randomized block design (not in this course, see textbook Section 12.4).

For example, to compare the flight distances of 3 types of golf balls, we can get each golfer to hit one ball of each type in random order.

The golfer is the blocking (noise) factor. F values can then be used to test for treatment effects and blocking effects.

Outline

Introduction

We now consider experiments with two factors, which we call A and B. Factor A has $a \geq 2$ levels, and factor B has $b \geq 2$ levels.

Example: measure the compression during a crash, vs car size (factor A) and car speed (factor B).

We consider all treatment combinations (in total, ab of them). For each treatment combination, we make n observations (called replicates).

See *Excel* sheet *anova2* for an example.

We omit the mathematical details in this course (but see textbook Section 13.1 if you are interested).

Layout

	Factor B levels			
Factor A levels	1	2	$\cdots$	b
1	y_{111} $\vdots$ y_{11n}	y_{121} $\vdots$ y_{12n}	$\cdots$	y_{1b1} $\vdots$ y_{1bn}
2	y_{211} $\vdots$ y_{21n}	y_{221} $\vdots$ y_{22n}	$\cdots$	y_{2b1} $\vdots$ y_{2bn}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
a	y_{a11} $\vdots$ y_{a1n}	y_{a21} $\vdots$ y_{a2n}	$\cdots$	y_{ab1} $\vdots$ y_{abn}

Interactions

We wish to test if the treatments have the same effect. Do we really need a new method to analyze this experimental set-up, or can we just test for each factor separately?

It turns out that we cannot just test each factor separately, because there might exist **interactions**: when the effects of one factor depend on the levels of the other factor.

To visualize interactions, we can create a *line chart* for each level of Factor A. In each line chart, plot the cell means against the levels of Factor B.

Interactions – example

(From Recitation 1) suppose we have the following cell means:

	High stress before exam	Low stress before exam
Study hard during semester	90	100
Not study hard during semester	60	50

Exercise

If the lines are almost parallel, then what does that say about the level of interaction?

ANOVA

There are three natural null hypotheses to look at:

H_{0A} : the levels of Factor A all have the same mean

H_{0B} : the levels of Factor B all have the same mean

H_{0AB} : there is no interaction between Factors A and B

These null hypotheses are tested by looking at the values of F_A, F_B, and F_{AB} (computable in *Excel*) respectively.

If each null hypothesis is true, then it can be shown that the corresponding F follows an F distribution.

Excel

One should look at F_{AB} first. If H_{0AB} is rejected, then the presence of interaction means that the other two hypothesis tests are no longer very meaningful.

In *Excel*, go to `Data Analysis` $\rightarrow$ `Anova: Two-Factor With Replication`.

Include the column and row headings in the input range, and enter the correct 'Rows per sample'.

Exercise

Do the analysis for *anova2*.