SUTD ESD

Statistics 2017

Homework Assignment 2

Due: 1pm, Tuesday 14 March.

Question 1. In each of the following cases, state the null hypothesis and the alternative hypothesis about the population mean μ.

(a) A commuter's average commuting time from home to work is 25 minutes. He wants to try a different route for a month to see if it would cut down the average commuting time.

(b) A consumer watchdog group suspects that a yogurt advertised to be 98% fat free has actually a higher fat content. The group plans to measure the fat contents of 25 yogurt cups (each containing 170 grams) to verify its suspicion.

Question 2. A random sample of size 100, drawn from a normal distribution, has $\bar{x} = 16.3$.

(a) Calculate the 90% two-sided confidence interval for μ, if $\sigma = 6$.

(b) Calculate the 90% two-sided confidence interval for μ, if $s = 6$ and σ is unknown.

(c) Calculate the upper and lower 90% one-sided confidence intervals for μ, if $s = 6$ and σ is unknown.

(d) Why does the CI in (b) have to be wider than the CI in (a)?

Question 3. A random sample $X_1, X_2, \ldots, X_{150}$ is drawn from a population with $\mu = 40$ and $\sigma = 15$ but with an unknown distribution. Let U represent the sample mean of the first 50 observations, and let V represent the sample mean of the last 100 observations.

(a) What are the approximate distributions of U and V?

(b) Find $P(38 \le U \le 42)$ and $P(38 \le V \le 42)$.

Question 4. Consider testing $H_0 : \mu = 0$ vs $H_1 : \mu \ne 0$ based on a random sample of size n from an $N(\mu, 1)$ distribution.

(a) Calculate the p-values for the following three cases: (i) $\bar{x} = 0.1$, $n = 100$; (ii) $\bar{x} = 0.1$, $n = 400$; (iii) $\bar{x} = 0.1$, $n = 900$.

(b) Given the significance level $\alpha = 1\%$, conduct the hypothesis tests for the three cases in (a).

Question 5. Assume that σ is known and n is large.
(a) For the hypothesis test $H_0 : \mu = \mu_0$ vs $H_1 : \mu \ne \mu_0$, show that the power is given by

$$1 - \beta = \Phi\Big(\frac{(\mu - \mu_0)\sqrt{n}}{\sigma} - z_{1-\alpha/2}\Big) + \Phi\Big(\frac{(\mu_0 - \mu)\sqrt{n}}{\sigma} - z_{1-\alpha/2}\Big).$$

(b) Suppose that $|\mu - \mu_0|$ is large. Argue that one of the terms on the right hand side of part (a) is negligible, and hence find an approximate formula for n in terms of the other quantities.

Question 6. The manufacturer of a type of fabric claims that its durability has a standard deviation σ of 3500 units. The company's quality control department evaluated the fabric by testing 25 samples, and obtained a sample standard deviation of 4569 units. Assume the durability measurements are normally distributed.

Set up the hypotheses to check if the actual standard deviation is greater than the claimed value, and perform a hypothesis test at the 5% significance level.

Question 7. To shorten the time of an assembly process, an engineer has designed a new method. With the old method, the average time for assembly is 10 minutes. To test her new method, an experiment is conducted, involving 15 workers trained in the method.

(a) Set up the appropriate hypotheses for the average assembly time.

(b) Suppose that the sample mean for the 15 workers is 8.7 minutes. If $\sigma = 2$ minutes and assuming normality, is there statistically significant evidence to reject H_0? Use $\alpha = 0.05$.

(c) The engineer claims that the new method will reduce the average time by 1.5 minutes. What is the probability that the experiment can detect the claimed improvement?

Question 8. A restaurant purchased a new oven, which is hoped to have more even heating than the old oven. By testing 9 locations inside each oven on the same temperature setting, it is found that the sample standard deviation for the temperature in the old oven is $s_1 = 2.3$, while that for the new oven is $s_2 = 1.1$. Set up a hypothesis test with $\alpha = 0.05$ to check whether the new oven indeed provides more even heating.

Question 9. A person claims to be able to taste whether tea or milk was added first to a cup of English tea. To test her claim, 12 cups of visually indistinguishable tea are prepared, of which 6 of the cups are prepared tea-first, the other 6 milk-first. Being aware of this experimental setup, she would always try to pick 6 of the cups as tea-first, and the other 6 as milk-first. After tasting each cup of tea, she correctly identifies 5 of the tea-first cups (making 1 mistake), and 5 of the milk-first cups (also making 1 mistake).

Compute the p-value, that is, the probability that one can do at least as well as her by guessing, and hence perform a hypothesis test at the $\alpha = 0.05$ level.