Statistics

Week 5: Inference for Single Samples (Chapters 6 & 7)

ESD, SUTD

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SINGAPORE UNIVERSITY OF TECHNOLOGY AND DESIGN

Established in collaboration with MIT

HW and exam

Homework 2 will be available next week. Homework 1 solutions are on *eDimension*.

Exam 1:

- Friday 3 March, 2:30-4:30pm, in CC14 (2.507).
- Everything taught up to next Tuesday's class will be examinable.
- Allowed: non-programmable, scientific calculator; one A4 sheet with *handwritten* notes on both sides.
- Provided: a normal distribution table, and all required values from other distributions.
- Let me know ASAP if you cannot sit the exam. Note that you are not automatically granted a makeup exam.

Outline

Inequalities in H_0

For a one-sided alternative hypothesis, such as $H_1 : \mu > \mu_0$, it does not matter if we use

$$H_0 : \mu = \mu_0 \quad \text{or} \quad H_0 : \mu \leq \mu_0.$$

If we use the latter, then the maximum p-value is still obtained at the boundary, when $\mu = \mu_0$.

One-sided tests are used when the deviation is expected to be in a particular direction. They should *not* be used as a device to make a statistically non-significant result significant.

Hypothesis test: a summary

Here are some equivalent ways to perform a hypothesis test for the mean μ, assuming that σ is known and the sample size is large:

- Calculate the appropriate $(1-\alpha)$-level confidence interval around $\bar{x}$, and check if μ_0 falls outside it.
- Calculate the p-value and compare it with α.
- Calculate the z-statistic, $(\bar{x}-\mu_0)/(\sigma/\sqrt{n})$, and compare it with the appropriate critical value ($z_{1-\alpha}$ or $z_{1-\alpha/2}$).

Hypothesis test: exercise

The procedure is similar if σ is unknown but the population is normal: we use the t-distribution instead.

Exercise: suppose that you selected a random sample of 36 SUTD students, and found that on average, they spend 20.0 hours on homework per week, with a sample standard deviation of 3.0 hours.

For the hypotheses $H_0 : \mu = 19$ vs $H_1 : \mu > 19$,

(1) Find the p-value.

(2) Can H_0 be rejected if $\alpha = 5\%$? $\alpha = 1\%$?

Outline

Power: exercise 1

Exercise: previous research showed that the amount of time children spend watching TV per week has $\mu = 22.6$h and $\sigma = 6.1$h. A market research firm believes that the stated mean is now too low. A random sample of 60 children are taken to measure the number of hours they watch TV. A hypothesis test at the $\alpha = 0.01$ level is carried out.

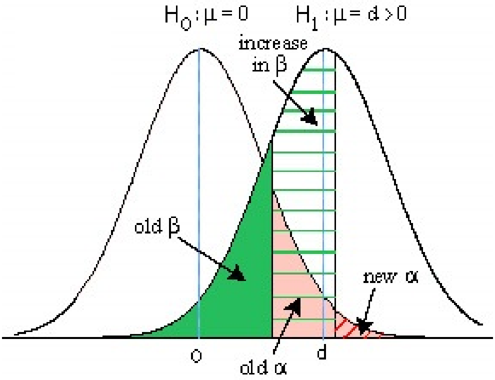
(1) State H_0 and H_1.

(2) Can we use the CLT?

(3) Suppose the *true* mean for this population is 25 hours. What is β, and what is the power in this case? (Draw a picture!)

α and β

α and β *cannot* be reduced simultaneously, unless we increase the sample size.



Power calculation – formula

Assume that σ is known, and that n is large so we may use the z-distribution.

Consider the problem of testing $H_0 : \mu = \mu_0$ vs $H_1 : \mu > \mu_0$. Then the power $(1 - \beta)$, as a function of μ, is given by

$$1 - \beta = \Phi\left(\frac{(\mu - \mu_0)\sqrt{n}}{\sigma} - z_{1-\alpha}\right).$$

Proof: generalize from Exercise 1. You should also figure out the corresponding formulas for $H_1 : \mu < \mu_0$ and $H_1 : \mu \neq \mu_0$

Note: in situations where we need to use the t-distribution, the power calculation is less straightforward.

Sample size determination – part 1

We can now relate the required sample size to α and β.

With the assumptions on the previous slides, the minimum sample size required for an α-level hypothesis test with power of $(1-\beta)$ is

$$n = \left(\frac{(z_{1-\alpha} + z_{1-\beta})\sigma}{\mu - \mu_0}\right)^2,$$

rounded to the next integer.

Sample size determination – part 2

Consider a $(1-\alpha)$ two-sided confidence interval for μ using the z-distribution. What is the relationship between the width of the interval and the sample size?

If the width of the CI is $2E$, then we require the minimal sample size to be

$$n = \left(\frac{z_{1-\alpha/2}\,\sigma}{E}\right)^2,$$

rounded to the next integer.

Exercise: Find the required sample size for a 95% CI, whose width is $\sigma/4$.

Power: exercise 2

Changes in test scores for students retaking the SAT without coaching has $\mu = 15$ and $\sigma = 40$. The changes in the scores are roughly normally distributed. A coaching program claims that on average it can improve the mean score by at least 35 points. A 0.01-level test of $H_0 : \mu = 15$ vs $H_1 : \mu > 15$ is to be conducted. Find the number of students that must be tested in order to have at least 90% power for detecting an increase of 35 points or more.