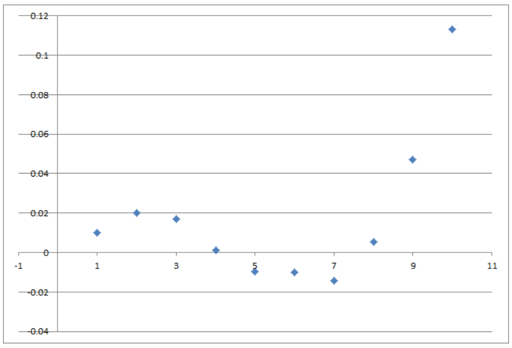
Statistics Practice Questions

Try to do these questions by hand or using a calculator (no computers).

1. (From 2014 Exam) After fitting a regression line to some data, the following residual plot is obtained.



   Write down an appropriate polynomial model for the data.

2. (a) Show that the linear square regression line passes through $(\bar{x}, \bar{y})$.

   (b) Does appending $(\bar{x}, \bar{y})$ to the data change the regression line? Does it change r^2?

3. (From 2015 Exam) In simple linear regression:

   (a) Express r^2 in terms of only SSR and SST. Does your formula also hold in the case of multiple linear regression?

   (b) Express F in terms of only SSE, SSR and n (where n is the number of data points).

   (c) How is SSE related to SSR and SST?

   (d) Using your answers to parts (a)–(c), express F in terms of only r^2 and n.

4. To derive the relation SST = SSE + SSR for simple linear regression, we need the following identity; prove it.

$$\sum_{i=1}^{n} (y_i - \hat{y}_i)(\hat{y}_i - \bar{y}) = 0.$$

   Hint: write $\hat{y}_i$ in terms of $\hat{\beta}_1$, but not of $\hat{\beta}_0$.

5. A statistician transforms the data points (x_i, y_i) into (x_i', y_i'), where $x_i' = ax_i + b$ and $y_i' = cy_i + d$.

   (a) If she computes the least square regression line for (x_i', y_i'), how will the correlation coefficient and the slope relate to those of the regression line for (x_i, y_i)?

   (b) Using (a), prove that the regression line for standardized data (see Week 10 Lecture 2) is $\hat{y}' = rx'$.

6. (From 2015 Exam) The weekly attendance numbers of an unpopular statistics course are recorded below.

Week	Attendance
1	69
2	63
3	55
4	57
5	60
6	44

(a) Test if the attendance numbers are uniformly distributed, using $\alpha = 0.05$ and $\chi^2_{5,0.95} = 11.071$.

(b) Find the least square regression line for Attendance vs Week.

Note: practice doing this question on a calculator.

7. The 95% confidence interval for β_1 in a simple linear regression is $[0.09366, 0.3274]$. Also, MSE = 3183 and $s_x = 67.50$. Estimate the sample size.

8. Is it possible for $r^2 < 0$? What about adjusted r^2?

9. (From 2015 Exam) A political party predicts that it will get 60% of all votes in the next election. Based on a poll of 100 voters, only 48 showed support for the party. We wish to test if the party's prediction is consistent with this, using $\alpha = 0.05$.

   (a) We can view this as a problem involving inference for a proportion. Test for $H_0 : p = 0.6$, by computing the two-sided p-value of $\hat{p} = 0.48$.

   (b) Alternatively, we can solve this problem using a chi-squared test. Namely, the expected number of supporters (respectively, non-supporters) is 60 (respectively, 40), while the observed number is 48 (respectively, 52). Compute the χ^2 statistic.

   (c) What is the p-value of the χ^2 statistic you found in part (b)?

10. (From 2016 Exam) For single factor ANOVA, if all the group sizes are the same $(n_1 = n_2 = \cdots n_k = n)$, then F may be expressed simply as

$$F = c(k,n)\,\frac{\sum_{i=1}^{k}(\bar{y}_i - \bar{\bar{y}})^2}{\sum_{i=1}^{k} s_i^2},$$

   where $c(k,n)$ is a function of k and n; find it.

11. (a) Is F in single factor ANOVA affected by the units of the data?

   (b) Is F_B in two-factor ANOVA always the same as F in the corresponding single factor ANOVA (obtained by ignoring factor A)? (You may experiment in *Excel* for this part.)

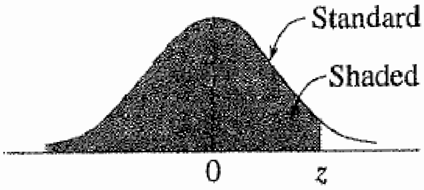
   (c) If $f_{2,5,0.95} = 5.786$, find $f_{5,2,0.05}$.

12. Other topics to revise: maximum likelihood, sign test, bootstrap/permutation test.

   Also try the *Excel* file; the first 4 problems can be done using a calculator; the remaining problems are more involved.

   Go through all problems from lectures, recitations and homework assignments.

Table A.3 Standard Normal Curve Areas $\Phi(z) = P(Z \le z)$ (*cont.*)



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0,4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9278	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1,7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2,0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998