Section 4.1 Antiderivatives and Indefinite Integration

1. $\frac{d}{dx}\left(\frac{2}{x^3} + C\right) = \frac{d}{dx}\left(2x^{-3} + C\right) = -6x^{-4} = \frac{-6}{x^4}$

2. $\frac{d}{dx}\left(2x^4 - \frac{1}{2x} + C\right) = \frac{d}{dx}\left(2x^4 - \frac{1}{2}x^{-1} + C\right)$

$= 8x^3 + \frac{1}{2}x^{-2} = 8x^3 + \frac{1}{2x^2}$

3. $\frac{dy}{dt} = 9t^2$

$y = 3t^3 + C$

Check: $\frac{d}{dt}\left[3t^3 + C\right] = 9t^2$

4. $\frac{dy}{dt} = 5$

$y = 5t + C$

Check: $\frac{d}{dt}[5t + C] = 5$

5. $\frac{dy}{dx} = x^{3/2}$

$y = \frac{2}{5}x^{5/2} + C$

Check: $\frac{d}{dx}\left[\frac{2}{5}x^{5/2} + C\right] = x^{3/2}$

6. $\frac{dy}{dx} = 2x^{-3}$

$y = \frac{2x^{-2}}{-2} + C = -\frac{1}{x^2} + C$

Check: $\frac{d}{dx}\left[-\frac{1}{x^2} + C\right] = 2x^{-3}$

	Given	*Rewrite*	*Integrate*	*Simplify*
7.	$\int \sqrt[3]{x}\, dx$	$\int x^{1/3}\, dx$	$\frac{x^{4/3}}{4/3} + C$	$\frac{3}{4}x^{4/3} + C$
8.	$\int \frac{1}{4x^2}\, dx$	$\frac{1}{4}\int x^{-2}\, dx$	$\frac{1}{4}\frac{x^{-1}}{-1} + C$	$-\frac{1}{4x} + C$
9.	$\int \frac{1}{x\sqrt{x}}\, dx$	$\int x^{-3/2}\, dx$	$\frac{x^{-1/2}}{-1/2} + C$	$-\frac{2}{\sqrt{x}} + C$
10.	$\int \frac{1}{(3x)^2}\, dx$	$\frac{1}{9}\int x^{-2}\, dx$	$\frac{1}{9}\left(\frac{x^{-1}}{-1}\right) + C$	$-\frac{1}{9x} + C$

11. $\int \left(3x^3 - 6x^2 + 2\right) dx = \frac{3}{4}x^4 - 2x^3 + 2x + C$

Check: $\frac{d}{dx}\left[\frac{3}{4}x^4 - 2x^3 + 2x + C\right] = 3x^3 - 6x^2 + 2$

12. $\int \left(x^2 + 7\right) dx = \frac{1}{3}x^3 + 7x + C$

Check: $\frac{d}{dx}\left(\frac{1}{3}x^3 + 7x + C\right) = x^2 + 7$

13. $\int \left(x^{3/2} + 2x + 1\right) dx = \frac{2}{5}x^{5/2} + x^2 + 2x + C$

Check: $\frac{d}{dx}\left(\frac{2}{5}x^{5/2} + x^2 + x + C\right) = x^{3/2} + 2x + 1$

14. $\int \left(\sqrt{x} + \frac{1}{2\sqrt{x}}\right) dx = \int \left(x^{1/2} + \frac{1}{2}x^{-1/2}\right) dx$

$= \frac{x^{3/2}}{3/2} + \frac{1}{2}\left(\frac{x^{1/2}}{1/2}\right) + C$

$= \frac{2}{3}x^{3/2} + x^{1/2} + C$

Check: $\frac{d}{dx}\left(\frac{2}{3}x^{3/2} + x^{1/2} + C\right) = x^{1/2} + \frac{1}{2}x^{-1/2}$

$= \sqrt{x} + \frac{1}{2\sqrt{x}}$

15. $\int \sqrt[3]{x^2}\, dx = \int x^{2/3}\, dx = \dfrac{x^{5/3}}{5/3} + C = \dfrac{3}{5}x^{5/3} + C$

Check: $\dfrac{d}{dx}\left(\dfrac{3}{5}x^{5/3} + C\right) = x^{2/3} = \sqrt[3]{x^2}$

16. $\int \left(\sqrt[4]{x^3} + 1\right) dx = \int \left(x^{3/4} + 1\right) dx = \dfrac{4}{7}x^{7/4} + x + C$

Check: $\dfrac{d}{dx}\left(\dfrac{4}{7}x^{7/4} + x + C\right) = x^{3/4} + 1 = \sqrt[4]{x^3} + 1$

17. $\int \dfrac{1}{x^5}\, dx = \int x^{-5}\, dx = \dfrac{x^{-4}}{-4} + C = -\dfrac{1}{4x^4} + C$

Check: $\dfrac{d}{dx}\left(\dfrac{-1}{4x^4} + C\right) = \dfrac{d}{dx}\left(-\dfrac{1}{4}x^{-4} + C\right)$

$$= -\frac{1}{4}\left(-4x^{-5}\right) = \frac{1}{x^5}$$

18. $\int \dfrac{3}{x^7}\, dx = \int 3x^{-7}\, dx = \dfrac{3x^{-6}}{-6} + C = -\dfrac{1}{2x^6} + C$

Check: $\dfrac{d}{dx}\left(-\dfrac{1}{2x^6} + C\right) = \dfrac{d}{dx}\left(-\dfrac{1}{2}x^{-6} + C\right)$

$$= \left(-\frac{1}{2}\right)(-6)x^{-7} = \frac{3}{x^7}$$

19. $\int \dfrac{x + 6}{\sqrt{x}}\, dx = \int \left(x^{1/2} + 6x^{-1/2}\right) dx$

$$= \frac{x^{3/2}}{3/2} + 6\frac{x^{1/2}}{1/2} + C$$

$$= \frac{2}{3}x^{3/2} + 12x^{1/2} + C$$

$$= \frac{2}{3}x^{1/2}(x + 18) + C$$

Check: $\dfrac{d}{dx}\left(\dfrac{2}{3}x^{3/2} + 12x^{1/2} + C\right)$

$$= \frac{2}{3}\left(\frac{3}{2}x^{1/2}\right) + 12\left(\frac{1}{2}x^{-1/2}\right)$$

$$= x^{1/2} + 6x^{-1/2} = \frac{x + 6}{\sqrt{x}}$$

20. $\int \dfrac{x^4 - 3x^2 + 5}{x^4}\, dx = \int \left(1 - 3x^{-2} + 5x^{-4}\right) dx$

$$= x - \frac{3x^{-1}}{-1} + \frac{5x^{-3}}{-3} + C$$

$$= x + \frac{3}{x} - \frac{5}{3x^3} + C$$

Check:

$\dfrac{d}{dx}\left[x + \dfrac{3}{x} - \dfrac{5}{3x^3} + C\right] = \dfrac{d}{dx}\left[x + 3x^{-1} - \dfrac{5}{3}x^{-3} + C\right.$

$$= 1 - 3x^{-2} + 5x^{-4}$$

$$= 1 - \frac{3}{x^2} + \frac{5}{x^4}$$

$$= \frac{x^4 - 3x^2 + 5}{x^4}$$

21. $\int (x + 1)(3x - 2)\, dx = \int \left(3x^2 + x - 2\right) dx$

$$= x^3 + \frac{1}{2}x^2 - 2x + C$$

Check: $\dfrac{d}{dx}\left(x^3 + \dfrac{1}{2}x^2 - 2x + C\right) = 3x^2 + x - 2$

$$= (x + 1)(3x - 2)$$

22. $\int \left(4t^2 + 3\right)^2 dt = \int \left(16t^4 + 24t^2 + 9\right) dt$

$$= \frac{16t^5}{5} + 8t^3 + 9t + C$$

Check: $\dfrac{d}{dt}\left(\dfrac{16t^5}{5} + 8t^3 + 9t + C\right) = 16t^4 + 24t^2 + 9$

$$= \left(4t^2 + 3\right)^2$$

23. $\int (5\cos x + 4\sin x)\, dx = 5\sin x - 4\cos x + C$

Check:

$\dfrac{d}{dx}(5\sin x - 4\cos x + C) = 5\cos x + 4\sin x$

24. $\int \left(\theta^2 + \sec^2\theta\right) d\theta = \dfrac{1}{3}\theta^3 + \tan\theta + C$

Check: $\dfrac{d}{d\theta}\left(\dfrac{1}{3}\theta^3 + \tan\theta + C\right) = \theta^2 + \sec^2\theta$

25. $\int \left(2 \sin x - 5e^x\right) dx = -2 \cos x - 5e^x + C$

Check: $\frac{d}{dx}\left(-2 \cos x - 5e^x + C\right) = 2 \sin x - 5e^x$

26. $\int (\sec y)(\tan y - \sec y)\, dy = \int \left(\sec y \tan y - \sec^2 y\right) dy$

$= \sec y - \tan y + C$

Check: $\frac{d}{dy}(\sec y - \tan y + C) = \sec y \tan y - \sec^2 y$

$= (\sec y)(\tan y - \sec y)$

40. $f''(x) = x^2$

$f'(0) = 8$

$f(0) = 4$

$f'(x) = \int x^2\,dx = \frac{1}{3}x^3 + C_1$

$f'(0) = 0 + C_1 = 8 \Rightarrow C_1 = 8$

$f'(x) = \frac{1}{3}x^3 + 8$

$f(x) = \int \left(\frac{1}{3}x^3 + 8\right)dx = \frac{1}{12}x^4 + 8x + C_2$

$f(0) = 0 + 0 + C_2 = 4 \Rightarrow C_2 = 4$

$f(x) = \frac{1}{12}x^4 + 8x + 4$

41. $f''(x) = x^{-3/2}$

$f'(4) = 2$

$f(0) = 0$

$f'(x) = \int x^{-3/2}\,dx = -2x^{-1/2} + C_1 = -\frac{2}{\sqrt{x}} + C_1$

$f'(4) = -\frac{2}{2} + C_1 = 2 \Rightarrow C_1 = 3$

$f'(x) = -\frac{2}{\sqrt{x}} + 3$

$f(x) = \int \left(-2x^{-1/2} + 3\right)dx = -4x^{1/2} + 3x + C_2$

$f(0) = 0 + 0 + C_2 = 0 \Rightarrow C_2 = 0$

$f(x) = -4x^{1/2} + 3x = -4\sqrt{x} + 3x$

42. $f''(x) = \sin x$

$f'(0) = 1$

$f(0) = 6$

$f'(x) = \int \sin x\,dx = -\cos x + C_1$

$f'(0) = -1 + C_1 = 1 \Rightarrow C_1 = 2$

$f'(x) = -\cos x + 2$

$f(x) = \int (-\cos x + 2)\,dx = -\sin x + 2x + C_2$

$f(0) = 0 + 0 + C_2 = 6 \Rightarrow C_2 = 6$

$f(x) = -\sin x + 2x + 6$

43. $f''(x) = e^x$

$f'(0) = 2$

$f(0) = 5$

$f'(x) = \int e^x\,dx = e^x + C_1$

$f'(0) = 2 = e^0 + C_1 \Rightarrow C_1 = 1$

$f'(x) = e^x + 1$

$f(x) = \int \left(e^x + 1\right)dx = e^x + x + C_2$

$f(0) = 5 = e^0 + 0 + C_2 \Rightarrow C_2 = 4$

$f(x) = e^x + x + 4$

44. $f''(x) = \frac{2}{x^2}$

$f'(1) = 4$

$f(1) = 3$

$f'(x) = \int \frac{2}{x^2}\,dx = \int 2x^{-2}\,dx = -\frac{2}{x} + C_1$

$f'(1) = 4 = -2 + C_1 \Rightarrow C_1 = 6$

$f'(x) = -\frac{2}{x} + 6$

$f(x) = \int \left(-\frac{2}{x} + 6\right)dx = -2\ln|x| + 6x + C_2$

$f(1) = 3 = 6 + C_2 \Rightarrow C_2 = -3$

$f(x) = -2\ln|x| + 6x - 3$

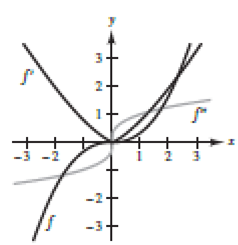
45. They are the same. In both cases you are finding a function $F(x)$ such that $F'(x) = f(x)$.

46. $f(x) = \tan^2 x \Rightarrow f'(x) = 2\tan x \cdot \sec^2 x$

$g(x) = \sec^2 x \Rightarrow g'(x) = 2\sec x \cdot \sec x \tan x = f'(x)$

The derivatives are the same, so f and g differ by a constant. In fact, $\tan^2 x + 1 = \sec^2 x$.

47. Because f'' is negative on $(-\infty, 0)$, f' is decreasing on $(-\infty, 0)$. Because f'' is positive on $(0, \infty)$, f' is increasing on $(0, \infty)$. f' has a relative minimum at $(0, 0)$. Because f' is positive on $(-\infty, \infty)$, f is increasing on $(-\infty, \infty)$.



48. $f(0) = -4$. Graph of f' is given.

(a) $f'(4) \approx -1$

The graph of f' is given, so at $x = 4, f'(4)$ is about -1.

(b) No. The slopes of the tangent lines are greater than 2 on $[0, 2]$. Therefore, f must increase more than 4 units on $[0, 4]$.

(c) No, $f(5) < f(4)$ because f is decreasing on $[4, 5]$.

(d) f is a maximum at $x = 3.5$ because $f'(3.5) \approx 0$ and the First Derivative Test.

(e) f is concave upward when f' is increasing on $(-\infty, 1)$ and $(5, \infty)$. f is concave downward on $(1, 5)$. Points of inflection at $x = 1, 5$.

49. (a) $h(t) = \int (1.5t + 5)\,dt = 0.75t^2 + 5t + C$

$h(0) = 0 + 0 + C = 12 \Rightarrow C = 12$

$h(t) = 0.75t^2 + 5t + 12$

(b) $h(6) = 0.75(6)^2 + 5(6) + 12 = 69$ cm

50. $\dfrac{dP}{dt} = k\sqrt{t},\ 0 \le t \le 10$

$P(t) = \int kt^{1/2}\,dt = \dfrac{2}{3}kt^{3/2} + C$

$P(0) = 0 + C = 500 \Rightarrow C = 500$

$P(1) = \dfrac{2}{3}k + 500 = 600 \Rightarrow k = 150$

$P(t) = \dfrac{2}{3}(150)t^{3/2} + 500 = 100t^{3/2} + 500$

$P(7) = 100(7)^{3/2} + 500 \approx 2352$ bacteria

51. $a(t) = -32$ ft/sec^2

$v(t) = \int -32\,dt = -32t + C_1$

$v(0) = 60 = C_1$

$s(t) = \int (-32t + 60)\,dt = -16t^2 + 60t + C_2$

$s(0) = 6 = C_2$

$s(t) = -16t^2 + 60t + 6$, Position function

The ball reaches its maximum height when

$v(t) = -32t + 60 = 0$

$32t = 60$

$t = \frac{15}{8}$ seconds.

$s\left(\frac{15}{8}\right) = -16\left(\frac{15}{8}\right)^2 + 60\left(\frac{15}{8}\right) + 6 = 62.25$ feet

52. $a(t) = -32$ ft/sec^2

$v(t) = \int -32\,dt = -32t + C_1$

$v(0) = 0 + C_1 = V_0 \Rightarrow C_1 = V_0$

$s'(t) = -32t + V_0$

$s(t) = \int (-32t + V_0)dt = -16t^2 + V_0t + C_2$

$s(0) = 0 + 0 + C_2 = S_0 \Rightarrow C_2 = S_0$

$s(t) = -16t^2 + V_0t + S_0$

$s'(t) = -32t + v_0 = 0$ when $t = \frac{v_0}{32}$ = time to reach maximum height.

$$s\left(\frac{v_0}{32}\right) = -16\left(\frac{v_0}{32}\right)^2 + v_0\left(\frac{v_0}{32}\right) = 555$$

$$-\frac{v_0^2}{64} + \frac{v_0^2}{32} = 555$$

$$v_0^2 = 35{,}520$$

$$v_0 \approx 188.468 \text{ ft/sec}$$

53. $v_0 = 16$ ft/sec

$s_0 = 64$ ft

(a) $s(t) = -16t^2 + 16t + 64 = 0$

$-16(t^2 - t - 4) = 0$

$t = \dfrac{1 \pm \sqrt{17}}{2}$

Choosing the positive value,

$t = \dfrac{1 + \sqrt{17}}{2} \approx 2.562$ seconds.

(b) $v(t) = s'(t) = -32t + 16$

$v\left(\dfrac{1 + \sqrt{17}}{2}\right) = -32\left(\dfrac{1 + \sqrt{17}}{2}\right) + 16$

$= -16\sqrt{17} \approx -65.970$ ft/sec

54. $a(t) = -9.8$

$v(t) = \int -9.8\,dt = -9.8t + C_1$

$v(0) = v_0 = C_1 \Rightarrow v(t) = -9.8t + v_0$

$f(t) = \int (-9.8t + v_0)\,dt = -4.9t^2 + v_0t + C_2$

$f(0) = s_0 = C_2 \Rightarrow f(t) = -4.9t^2 + v_0t + s_0$

So, $f(t) = -4.9t^2 + 10t + 2$.

$v(t) = -9.8t + 10 = 0$ (Maximum height when $v = 0$.)

$9.8t = 10$

$t = \dfrac{10}{9.8}$

$f\left(\dfrac{10}{9.8}\right) \approx 7.1$ m

55. From Exercise 54, $f(t) = -4.9t^2 + v_0t + 2$. If

$f(t) = 200 = -4.9t^2 + v_0t + 2$,

Then $v(t) = -9.8t + v_0 = 0$

for this t value. So, $t = v_0/9.8$ and you solve

$$\begin{aligned}
-4.9\left(\frac{v_0}{9.8}\right)^2 + v_0\left(\frac{v_0}{9.8}\right) + 2 &= 200\\
\frac{-4.9v_0^2}{(9.8)^2} + \left(\frac{v_0^2}{9.8}\right) &= 198\\
-4.9v_0^2 + 9.8v_0^2 &= (9.8)^2 198\\
4.9v_0^2 &= (9.8)^2 198\\
v_0^2 &= 3880.8\\
v_0 &\approx 62.3 \text{ m/sec.}
\end{aligned}$$

56. From Exercise 54, $f(t) = -4.9t^2 + 1800$. (Using the canyon floor as position 0.)

$$\begin{aligned}
f(t) = 0 &= -4.9t^2 + 1800\\
4.9t^2 &= 1800\\
t^2 &= \frac{1800}{4.9} \Rightarrow t \approx 9.2 \text{ sec}
\end{aligned}$$

57. $a = -1.6$

$v(t) = \int -1.6\,dt = -1.6t + v_0 = -1.6t$, because the stone was dropped, $v_0 = 0$.

$s(t) = \int(-1.6t)\,dt = -0.8t^2 + s_0$

$$\begin{aligned}
s(20) = 0 \Rightarrow -0.8(20)^2 + s_0 &= 0\\
s_0 &= 320
\end{aligned}$$

So, the height of the cliff is 320 meters.

$v(t) = -1.6t$

$v(20) = -32$ m/sec

58. $\int v\,dv = -GM\int \frac{1}{y^2}\,dy$

$$\frac{1}{2}v^2 = \frac{GM}{y} + C$$

When $y = R$, $v = v_0$.

$$\begin{aligned}
\frac{1}{2}v_0^2 &= \frac{GM}{R} + C\\
C &= \frac{1}{2}v_0^2 - \frac{GM}{R}\\
\frac{1}{2}v^2 &= \frac{GM}{y} + \frac{1}{2}v_0^2 - \frac{GM}{R}\\
v^2 &= \frac{2GM}{y} + v_0^2 - \frac{2GM}{R}\\
v^2 &= v_0^2 + 2GM\left(\frac{1}{y} - \frac{1}{R}\right)
\end{aligned}$$

59. $x(t) = t^3 - 6t^2 + 9t - 2, \quad 0 \le t \le 5$

(a) $$\begin{aligned}
v(t) = x'(t) &= 3t^2 - 12t + 9\\
&= 3(t^2 - 4t + 3) = 3(t-1)(t-3)\\
a(t) = v'(t) &= 6t - 12 = 6(t-2)
\end{aligned}$$

(b) $v(t) > 0$ when $0 < t < 1$ or $3 < t < 5$.

(c) $a(t) = 6(t-2) = 0$ when $t = 2$.

$v(2) = 3(1)(-1) = -3$

60. $x(t) = (t-1)(t-3)^2 \quad 0 \le t \le 5$

$= t^3 - 7t^2 + 15t - 9$

(a) $v(t) = x'(t) = 3t^2 - 14t + 15 = (3t-5)(t-3)$

$a(t) = v'(t) = 6t - 14$

(b) $v(t) > 0$ when $0 < t < \frac{5}{3}$ and $3 < t < 5$.

(c) $a(t) = 6t - 14 = 0$ when $t = \frac{7}{3}$.

$v\left(\frac{7}{3}\right) = \left(3\left(\frac{7}{3}\right) - 5\right)\left(\frac{7}{3} - 3\right) = 2\left(-\frac{2}{3}\right) = -\frac{4}{3}$

61. $v(t) = \frac{1}{\sqrt{t}} = t^{-1/2} \quad t > 0$

$x(t) = \int v(t)dt = 2t^{1/2} + C$

$x(1) = 4 = 2(1) + C \Rightarrow C = 2$

Position function: $x(t) = 2t^{1/2} + 2 = 2\sqrt{t} + 2$

Acceleration function:

$a(t) = v'(t) = -\frac{1}{2}t^{-3/2} = -\frac{1}{2t^{3/2}}$

62. (a) $a(t) = \cos t$

$$\begin{aligned}
v(t) &= \int a(t)\,dt\\
&= \int \cos t\,dt\\
&= \sin t + C_1 = \sin t \quad (\text{because } v_0 = 0)
\end{aligned}$$

$f(t) = \int v(t)\,dt = \int \sin t\,dt = -\cos t + C_2$

$f(0) = 3 = -\cos(0) + C_2 = -1 + C_2 \Rightarrow C_2 = 4$

$f(t) = -\cos t + 4$

(b) $v(t) = 0 = \sin t$ for $t = k\pi$, $k = 0, 1, 2, \ldots$

63. (a) $v(0) = 25 \text{ km/h} = 25 \cdot \frac{1000}{3600} = \frac{250}{36}$ m/sec

$v(13) = 80 \text{ km/h} = 80 \cdot \frac{1000}{3600} = \frac{800}{36}$ m/sec

$a(t) = a$ (constant acceleration)

$v(t) = at + C$

$v(0) = \frac{250}{36} \Rightarrow v(t) = at + \frac{250}{36}$

$v(13) = \frac{800}{36} = 13a + \frac{250}{36}$

$\frac{550}{36} = 13a$

$a = \frac{550}{468} = \frac{275}{234} \approx 1.175 \text{ m/sec}^2$

(b) $s(t) = a\frac{t^2}{2} + \frac{250}{36}t \quad (s(0) = 0)$

$s(13) = \frac{275}{234}\frac{(13)^2}{2} + \frac{250}{36}(13) \approx 189.58$ m

64. $v(0) = 45 \text{ mi/h} = 66 \text{ ft/sec}$

$30 \text{ mi/h} = 44 \text{ ft/sec}$

$15 \text{ mi/h} = 22 \text{ ft/sec}$

$a(t) = -a$

$v(t) = -at + 66$

$s(t) = -\frac{a}{2}t^2 + 66t$ (Let $s(0) = 0$.)

$v(t) = 0$ after the car moves 132 feet.

$-at + 66 = 0$ when $t = \frac{66}{a}$.

$$s\left(\frac{66}{a}\right) = -\frac{a}{2}\left(\frac{66}{a}\right)^2 + 66\left(\frac{66}{a}\right)$$

$$= 132 \text{ when } a = \frac{33}{2} = 16.5.$$

$a(t) = -16.5$

$v(t) = -16.5t + 66$

$s(t) = -8.25t^2 + 66t$

(a) $-16.5t + 66 = 44$

$$t = \frac{22}{16.5} \approx 1.333$$

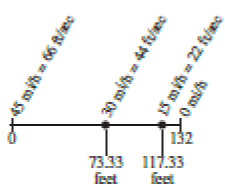
$$s\left(\frac{22}{16.5}\right) \approx 73.33 \text{ ft}$$

$-16.5t + 66 = 22$

$$t = \frac{44}{16.5} \approx 2.667$$

$$s\left(\frac{44}{16.5}\right) \approx 117.33 \text{ ft}$$

(b)



It takes 1.333 seconds to reduce the speed from 45 miles per hour to 30 miles per hour, 1.333 seconds to reduce the speed from 30 miles per hour to 15 miles per hour, and 1.333 seconds to reduce the speed from 15 miles per hour to 0 miles per hour. Each time, less distance is needed to reach the next speed reduction.

65. $f''(x) = 2x$

$f'(x) = x^2 + C$

$f'(2) = 0 \Rightarrow 4 + C = 0 \Rightarrow C = -4$

$f(x) = \frac{x^3}{3} - 4x + C_1$

$f(2) = 0 \Rightarrow \frac{8}{3} - 8 + C_1 = 0 \Rightarrow C_1 = \frac{16}{3}$

$f(x) = \frac{x^3}{3} - 4x + \frac{16}{3}$

66. $f'(x) = \begin{cases} -1, & 0 \le x < 2 \\ 2, & 2 < x < 3 \\ 0, & 3 < x \le 4 \end{cases}$

$f(x) = \begin{cases} -x + C_1, & 0 \le x < 2 \\ 2x + C_2, & 2 < x < 3 \\ C_3, & 3 < x \le 4 \end{cases}$

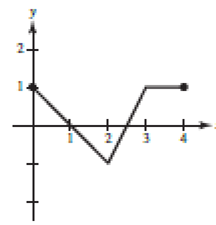
$f(0) = 1 \Rightarrow C_1 = 1$

f continuous at

$x = 2 \Rightarrow -2 + 1 = 4 + C_2 \Rightarrow C_2 = -5$

f continuous at $x = 3 \Rightarrow 6 - 5 = C_3 = 1$

$f(x) = \begin{cases} -x + 1, & 0 \le x < 2 \\ 2x - 5, & 2 \le x < 3 \\ 1, & 3 \le x \le 4 \end{cases}$



67. $\frac{d}{dx}\left(\ln|Cx|\right) = \frac{d}{dx}\left(\ln|C| + \ln|x|\right) = 0 + \frac{1}{x} = \frac{1}{x}$

68. $\frac{d}{dx}\left(\ln|x| + C\right) = \frac{1}{x} + 0 = \frac{1}{x}$

69. $\int \sqrt{x}\,(10x - 3)\,dv = \int \left(10x^{3/2} - 3x^{1/2}\right)dx$

$$= \frac{10x^{5/2}}{5/2} - \frac{3x^{3/2}}{3/2} + C$$

$$= 4x^{5/2} - 2x^{3/2} + C$$

So, the answer is D.

70. The equation of the tangent line is $f'(x) = -\frac{2}{5}x + 4$.

$f(x) = \int f'(x)\,dx = \int \left(-\frac{2}{5}x + 4\right)dx$

$= -\frac{1}{5}x^2 + 4x + C.$

Use $f(0) = 3$ to find C.

$3 = -\frac{1}{5}(0)^2 + 4(0) + C \Rightarrow C = 3$

$f(x) = -\frac{1}{5}x^2 + 4x + 3$

$f(10) = -\frac{1}{5}(10)^2 + 4(10) + 3$

$= 23$

So, the answer is D.

71. (a) Because $\frac{dP}{dt} = 20t^3 - 35t^{4/3} < 0$ when $t = 1$, $\frac{dP}{dt}$ is not always positive. So, the population is not always increasing.

(b) $\frac{dP}{dt} = 20t^3 - 35t^{4/3}$

$0 = 20t^3 - 35t^{4/3}$

$0 = 5t^{4/3}(4t^{5/3} - 7)$

$5t^{4/3} = 0 \qquad 4t^{5/3} - 7 = 0$

$t = 0 \qquad t^{5/3} = \frac{7}{4}$

$t = \left(\frac{7}{4}\right)^{3/5}$

$t \approx 1.399$

Because $\frac{dP}{dt} < 0$ when $0 < t < 1.4$, the population is at its lowest point when $t = 1.4$ years.

(c) $P(t) = \int P'(t)dt$

$= \int \left(20t^3 - 35t^{4/3}\right)dt$

$= \frac{20}{4}t^4 - \frac{35t^{7/3}}{7/3} + C$

$= 5t^4 - 15t^{7/3} + C$

Because $P(0) = 8000$, $P(t) = 5t^4 - 15t^{7/3} + 8000$. So, the population after 10 years will be $P(10) = 5(10)^4 - 15(10)^{7/3} + 8000 \approx 54{,}768$ people.