## Section 2.3 Product and Quotient Rules and Higher-Order Derivatives

**1.** $g(x) = (2x - 3)(1 - 5x)$

$$\begin{aligned} g'(x) &= (2x - 3)(-5) + (1 - 5x)(2) \\ &= -10x + 15 + 2 - 10x \\ &= -20x + 17 \end{aligned}$$

**2.** $y = (3x - 4)(x^3 + 5)$

$$\begin{aligned} y' &= (3x - 4)(3x^2) + (x^3 + 5)(3) \\ &= 9x^3 - 12x^2 + 3x^3 + 15 \\ &= 12x^3 - 12x^2 + 15 \end{aligned}$$

**3.** $h(t) = \sqrt{t}(1 - t^2) = t^{1/2}(1 - t^2)$

$$\begin{aligned} h'(t) &= t^{1/2}(-2t) + (1 - t^2)\frac{1}{2}t^{-1/2} \\ &= -2t^{3/2} + \frac{1}{2t^{1/2}} - \frac{1}{2}t^{3/2} \\ &= -\frac{5}{2}t^{3/2} + \frac{1}{2t^{1/2}} \\ &= \frac{1 - 5t^2}{2t^{1/2}} = \frac{1 - 5t^2}{2\sqrt{t}} \end{aligned}$$

**4.** $g(s) = \sqrt{s}(s^2 + 8) = s^{1/2}(s^2 + 8)$

$$\begin{aligned} g'(s) &= s^{1/2}(2s) + (s^2 + 8)\frac{1}{2}s^{-1/2} \\ &= 2s^{3/2} + \frac{1}{2}s^{3/2} + 4s^{-1/2} \\ &= \frac{5}{2}s^{3/2} + \frac{4}{s^{1/2}} \\ &= \frac{5s^2 + 8}{2\sqrt{s}} \end{aligned}$$

**5.** $f(x) = e^x \cos x$

$$\begin{aligned} f'(x) &= e^x(-\sin x) + e^x \cos x \\ &= e^x(\cos x - \sin x) \end{aligned}$$

**6.** $g(x) = \sqrt{x} \sin x$

$$\begin{aligned} g'(x) &= \sqrt{x} \cos x + \sin x\left(\frac{1}{2\sqrt{x}}\right) \\ &= \sqrt{x} \cos x + \frac{1}{2\sqrt{x}} \sin x \end{aligned}$$

**7.** $f(x) = \dfrac{x}{x - 5}$



$$f'(x) = \frac{(x - 5)(1) - x(1)}{(x - 5)^2} = \frac{x - 5 - x}{(x - 5)^2} = -\frac{5}{(x - 5)^2}$$

**8.** $g(t) = \dfrac{3t^2 - 1}{2t + 5}$

$$\begin{aligned} g'(t) &= \frac{(2t + 5)(6t) - (3t^2 - 1)(2)}{(2t + 5)^2} \\ &= \frac{12t^2 + 30t - 6t^2 + 2}{(2t + 5)^2} \\ &= \frac{6t^2 + 30t + 2}{(2t + 5)^2} \end{aligned}$$

**9.** $h(x) = \dfrac{\sqrt{x}}{x^3 + 1} = \dfrac{x^{1/2}}{x^3 + 1}$

$$\begin{aligned} h'(x) &= \frac{(x^3 + 1)\frac{1}{2}x^{-1/2} - x^{1/2}(3x^2)}{(x^3 + 1)^2} \\ &= \frac{x^3 + 1 - 6x^3}{2x^{1/2}(x^3 + 1)^2} \\ &= \frac{1 - 5x^3}{2\sqrt{x}(x^3 + 1)^2} \end{aligned}$$

**10.** $f(x) = \dfrac{x^2}{2\sqrt{x} + 1}$

$$\begin{aligned} f'(x) &= \frac{(2\sqrt{x} + 1)(2x) - x^2(x^{-1/2})}{(2\sqrt{x} + 1)^2} \\ &= \frac{4x^{3/2} + 2x - x^{3/2}}{(2\sqrt{x} + 1)^2} \\ &= \frac{3x^{3/2} + 2x}{(2\sqrt{x} + 1)^2} \\ &= \frac{x(3\sqrt{x} + 2)}{(2\sqrt{x} + 1)^2} \end{aligned}$$

**11.** $g(x) = \dfrac{\sin x}{e^x}$

$$\begin{aligned} g'(x) &= \frac{e^x \cos x - \sin x(e^x)}{(e^x)^2} \\ &= \frac{\cos x - \sin x}{e^x} \end{aligned}$$

12. $f(t) = \dfrac{\cos t}{t^3}$

$$f'(t) = \frac{t^3(-\sin t) - \cos t(3t^2)}{(t^3)^2} = -\frac{t\sin t + 3\cos t}{t^4}$$

13. $f(x) = (x^3 + 4x)(3x^2 + 2x - 5)$

$$\begin{aligned} f'(x) &= (x^3 + 4x)(6x + 2) + (3x^2 + 2x - 5)(3x^2 + 4) \\ &= 6x^4 + 24x^2 + 2x^3 + 8x + 9x^4 + 6x^3 - 15x^2 + 12x^2 + 8x - 20 \\ &= 15x^4 + 8x^3 + 21x^2 + 16x - 20 \\ f'(0) &= -20 \end{aligned}$$

14. $y = (x^2 - 3x + 2)(x^3 + 1)$

$$\begin{aligned} y' &= (x^2 - 3x + 2)(3x^2) + (x^3 + 1)(2x - 3) \\ &= 3x^4 - 9x^3 + 6x^2 + 2x^4 - 3x^3 + 2x - 3 \\ &= 5x^4 - 12x^3 + 6x^2 + 2x - 3 \\ y'(2) &= 5(2^4) - 12(2^3) + 6(2^2) + 2(2) - 3 = 9 \end{aligned}$$

15. $f(x) = \dfrac{x^2 - 4}{x - 3}$

$$\begin{aligned} f'(x) &= \frac{(x - 3)(2x) - (x^2 - 4)(1)}{(x - 3)^2} \\ &= \frac{2x^2 - 6x - x^2 + 4}{(x - 3)^2} \\ &= \frac{x^2 - 6x + 4}{(x - 3)^2} \\ f'(1) &= \frac{1 - 6 + 4}{(1 - 3)^2} = -\frac{1}{4} \end{aligned}$$

16. $f(x) = \dfrac{x - 4}{x + 4}$

$$\begin{aligned} f'(x) &= \frac{(x + 4)(1) - (x - 4)(1)}{(x + 4)^2} \\ &= \frac{x + 4 - x + 4}{(x + 4)^2} \\ &= \frac{8}{(x + 4)^2} \\ f'(3) &= \frac{8}{(3 + 4)^2} = \frac{8}{49} \end{aligned}$$

17. $f(x) = x\cos x$

$$\begin{aligned} f'(x) &= (x)(-\sin x) + (\cos x)(1) = \cos x - x\sin x \\ f'\left(\frac{\pi}{4}\right) &= \frac{\sqrt{2}}{2} - \frac{\pi}{4}\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{8}(4 - \pi) \end{aligned}$$

18. $f(x) = \dfrac{\sin x}{x}$

$$\begin{aligned} f'(x) &= \frac{(x)(\cos x) - (\sin x)(1)}{x^2} \\ &= \frac{x\cos x - \sin x}{x^2} \\ f'\left(\frac{\pi}{6}\right) &= \frac{(\pi/6)(\sqrt{3}/2) - (1/2)}{\pi^2/36} \\ &= \frac{3\sqrt{3}\pi - 18}{\pi^2} \\ &= \frac{3(\sqrt{3}\pi - 6)}{\pi^2} \end{aligned}$$

19. $f(x) = e^x \sin x$

$$\begin{aligned} f'(x) &= e^x\cos x + e^x\sin x \\ &= e^x(\cos x + \sin x) \\ f'(0) &= 1 \end{aligned}$$

20. $f(x) = \dfrac{\cos x}{e^x}$

$$\begin{aligned} f'(x) &= \frac{e^x(-\sin x) - \cos x(e^x)}{(e^x)^2} \\ &= \frac{-\sin x - \cos x}{e^x} \\ f'(0) &= \frac{0 - 1}{1} = -1 \end{aligned}$$

| *Function* | *Rewrite* | *Differentiate* | *Simplify* |
| --- | --- | --- | --- |
| **21.** $y = \dfrac{x^3 + 6x}{3}$ | $y = \frac{1}{3}x^3 + 2x$ | $y' = x^2 + 2$ | $y' = x^2 + 2$ |
| **22.** $y = \dfrac{5x^2 - 3}{4}$ | $y = \frac{5}{4}x^2 - \frac{3}{4}$ | $y' = \frac{10}{4}x$ | $y' = \frac{5x}{2}$ |
| **23.** $y = \dfrac{6}{7x^2}$ | $y = \frac{6}{7}x^{-2}$ | $y' = -\frac{12}{7}x^{-3}$ | $y' = -\frac{12}{7x^3}$ |
| **24.** $y = \dfrac{10}{3x^3}$ | $y = \frac{10}{3}x^{-3}$ | $y' = -\frac{30}{3}x^{-4}$ | $y' = -\frac{10}{x^4}$ |
| **25.** $y = \dfrac{4x^{3/2}}{x}$ | $y = 4x^{1/2}, x > 0$ | $y' = 2x^{-1/2}$ | $y' = \frac{2}{\sqrt{x}}, x > 0$ |
| **26.** $y = \dfrac{2x}{x^{1/3}}$ | $y = 2x^{2/3}$ | $y' = \frac{4}{3}x^{-1/3}$ | $y' = \frac{4}{3x^{1/3}}$ |

**27.** $f(x) = \dfrac{4 - 3x - x^2}{x^2 - 1}$

$$f'(x) = \frac{(x^2 - 1)(-3 - 2x) - (4 - 3x - x^2)(2x)}{(x^2 - 1)^2}$$

$$= \frac{-3x^2 + 3 - 2x^3 + 2x - 8x + 6x^2 + 2x^3}{(x^2 - 1)^2}$$

$$= \frac{3x^2 - 6x + 3}{(x^2 - 1)^2}$$

$$= \frac{3(x^2 - 2x + 1)}{(x^2 - 1)^2}$$

$$= \frac{3(x - 1)^2}{(x - 1)^2(x + 1)^2} = \frac{3}{(x + 1)^2}, x \neq 1$$

**28.** $f(x) = \dfrac{x^2 + 5x + 6}{x^2 - 4}$

$$f'(x) = \frac{(x^2 - 4)(2x + 5) - (x^2 + 5x + 6)(2x)}{(x^2 - 4)^2}$$

$$= \frac{2x^3 + 5x^2 - 8x - 20 - 2x^3 - 10x^2 - 12x}{(x^2 - 4)^2}$$

$$= \frac{-5x^2 - 20x - 20}{(x^2 - 4)^2}$$

$$= \frac{-5(x^2 + 4x + 4)}{(x - 2)^2(x + 2)^2}$$

$$= \frac{-5(x + 2)^2}{(x - 2)^2(x + 2)^2}$$

$$= -\frac{5}{(x - 2)^2}, x \neq 2, -2$$

**Alternate solution:**

$$f(x) = \frac{x^2 + 5x + 6}{x^2 - 4}$$

$$= \frac{(x + 3)(x + 2)}{(x + 2)(x - 2)}$$

$$= \frac{x + 3}{x - 2}, \ x \neq -2$$

$$f'(x) = \frac{(x - 2)(1) - (x + 3)(1)}{(x - 2)^2}$$

$$= -\frac{5}{(x - 2)^2}$$

**29.** $f(x) = x\left(1 - \dfrac{4}{x+3}\right) = x - \dfrac{4x}{x+3}$

$$f'(x) = 1 - \frac{(x+3)4 - 4x(1)}{(x+3)^2}$$
$$= \frac{\left(x^2 + 6x + 9\right) - 12}{(x+3)^2}$$
$$= \frac{x^2 + 6x - 3}{(x+3)^2}$$

**30.** $f(x) = x^4\left[1 - \dfrac{2}{x+1}\right] = x^4\left[\dfrac{x-1}{x+1}\right]$

$$f'(x) = x^4\left[\frac{(x+1) - (x-1)}{(x+1)^2}\right] + \left[\frac{x-1}{x+1}\right]\left(4x^3\right)$$
$$= x^4\left[\frac{2}{(x+1)^2}\right] + \left[\frac{x^2-1}{(x+1)^2}\right]\left(4x^3\right)$$
$$= 2x^3\left[\frac{2x^2 + x - 2}{(x+1)^2}\right]$$

**31.** $f(x) = \dfrac{3x-1}{\sqrt{x}} = 3x^{1/2} - x^{-1/2}$

$$f'(x) = \frac{3}{2}x^{-1/2} + \frac{1}{2}x^{-3/2} = \frac{3x+1}{2x^{3/2}}$$

**Alternate solution:**

$$f(x) = \frac{3x-1}{\sqrt{x}} = \frac{3x-1}{x^{1/2}}$$
$$f'(x) = \frac{x^{1/2}(3) - (3x-1)\left(\frac{1}{2}\right)\left(x^{-1/2}\right)}{x}$$
$$= \frac{\frac{1}{2}x^{-1/2}(3x+1)}{x}$$
$$= \frac{3x+1}{2x^{3/2}}$$

**32.** $f(x) = \sqrt[3]{x}\left(\sqrt{x} + 3\right) = x^{1/3}\left(x^{1/2} + 3\right)$

$$f'(x) = x^{1/3}\left(\frac{1}{2}x^{-1/2}\right) + \left(x^{1/2} + 3\right)\left(\frac{1}{3}x^{-2/3}\right)$$
$$= \frac{5}{6}x^{-1/6} + x^{-2/3}$$
$$= \frac{5}{6x^{1/6}} + \frac{1}{x^{2/3}}$$

**Alternate solution:**

$$f(x) = \sqrt[3]{x}\left(\sqrt{x} + 3\right)$$
$$= x^{5/6} + 3x^{1/3}$$
$$f'(x) = \frac{5}{6}x^{-1/6} + x^{-2/3}$$
$$= \frac{5}{6x^{1/6}} + \frac{1}{x^{2/3}}$$

**33.** $h(s) = \left(s^3 - 2\right)^2 = s^6 - 4s^3 + 4$

$$h'(s) = 6s^5 - 12s^2 = 6s^2\left(s^3 - 2\right)$$

**34.** $h(x) = \left(x^2 + 3\right)^3 = x^6 + 9x^4 + 27x^2 + 27$

$$h'(x) = 6x^5 + 36x^3 + 54x$$
$$= 6x\left(x^4 + 6x^2 + 9\right)$$
$$= 6x\left(x^2 + 3\right)^2$$

**35.** $f(x) = \dfrac{2 - (1/x)}{x - 3} = \dfrac{2x - 1}{x(x-3)} = \dfrac{2x-1}{x^2 - 3x}$

$$f'(x) = \frac{\left(x^2 - 3x\right)2 - (2x-1)(2x-3)}{\left(x^2 - 3x\right)^2}$$
$$= \frac{2x^2 - 6x - 4x^2 + 8x - 3}{\left(x^2 - 3x\right)^2}$$
$$= \frac{-2x^2 + 2x - 3}{\left(x^2 - 3x\right)^2} = \frac{2x^2 - 2x + 3}{x^2(x-3)^2}$$

**36.** $g(x) = x^2\left(\dfrac{2}{x} - \dfrac{1}{x+1}\right) = 2x - \dfrac{x^2}{x+1}$

$$g'(x) = 2 - \frac{(x+1)2x - x^2(1)}{(x+1)^2} = \frac{2\left(x^2 + 2x + 1\right) - x^2 - 2x}{(x+1)^2} = \frac{x^2 + 2x + 2}{(x+1)^2}$$

**37.** $f(x) = \left(2x^3 + 5x\right)(x-3)(x+2)$

$$f'(x) = \left(6x^2 + 5\right)(x-3)(x+2) + \left(2x^3 + 5x\right)(1)(x+2) + \left(2x^3 + 5x\right)(x-3)(1)$$
$$= \left(6x^2 + 5\right)\left(x^2 - x - 6\right) + \left(2x^3 + 5x\right)(x+2) + \left(2x^3 + 5x\right)(x-3)$$
$$= \left(6x^4 + 5x^2 - 6x^3 - 5x - 36x^2 - 30\right) + \left(2x^4 + 4x^3 + 5x^2 + 10x\right) + \left(2x^4 + 5x^2 - 6x^3 - 15x\right)$$
$$= 10x^4 - 8x^3 - 21x^2 - 10x - 30$$

**Note:** You could simplify first: $f(x) = \left(2x^3 + 5x\right)\left(x^2 - x - 6\right)$

38. $f(x) = (x^3 - x)(x^2 + 2)(x^2 + x - 1)$

$$\begin{aligned} f'(x) &= (3x^2 - 1)(x^2 + 2)(x^2 + x - 1) + (x^3 - x)(2x)(x^2 + x - 1) + (x^3 - x)(x^2 + 2)(2x + 1) \\ &= (3x^4 + 5x^2 - 2)(x^2 + x - 1) + (2x^4 - 2x^2)(x^2 + x - 1) + (x^5 + x^3 - 2x)(2x + 1) \\ &= (3x^6 + 5x^4 - 2x^2 + 3x^5 + 5x^3 - 2x - 3x^4 - 5x^2 + 2) + (2x^6 - 2x^4 + 2x^5 - 2x^3 - 2x^4 + 2x^2) \\ &\quad + (2x^6 + 2x^4 - 4x^2 + x^5 + x^3 - 2x) \\ &= 7x^6 + 6x^5 + 4x^3 - 9x^2 - 4x + 2 \end{aligned}$$

39. $f(x) = \dfrac{x^2 + c^2}{x^2 - c^2}$

$$\begin{aligned} f'(x) &= \frac{(x^2 - c^2)(2x) - (x^2 + c^2)(2x)}{(x^2 - c^2)^2} \\ &= -\frac{4xc^2}{(x^2 - c^2)^2} \end{aligned}$$

40. $f(x) = \dfrac{c^2 - x^2}{c^2 + x^2}$

$$\begin{aligned} f'(x) &= \frac{(c^2 + x^2)(-2x) - (c^2 - x^2)(2x)}{(c^2 + x^2)^2} \\ &= -\frac{4xc^2}{(c^2 + x^2)^2} \end{aligned}$$

41. $f(t) = t^2 \sin t$

$f'(t) = t^2 \cos t + 2t \sin t = t(t \cos t + 2 \sin t)$

42. $f(\theta) = (\theta + 1) \cos \theta$

$$\begin{aligned} f'(\theta) &= (\theta + 1)(-\sin \theta) + (\cos \theta)(1) \\ &= \cos \theta - (\theta + 1) \sin \theta \end{aligned}$$

43. $f(t) = \dfrac{\cos t}{t}$

$f'(t) = \dfrac{-t \sin t - \cos t}{t^2} = -\dfrac{t \sin t + \cos t}{t^2}$

44. $f(x) = \dfrac{\sin x}{x^3}$

$f'(x) = \dfrac{x^3 \cos x - \sin x(3x^2)}{(x^3)^2} = \dfrac{x \cos x - 3 \sin x}{x^4}$

45. $f(x) = -e^x + \tan x$

$f'(x) = -e^x + \sec^2 x$

46. $y = e^x - \cot x$

$y' = e^x + \csc^2 x$

47. $g(t) = \sqrt[4]{t} + 6 \csc t = t^{1/4} + 6 \csc t$

$g'(t) = \dfrac{1}{4} t^{-3/4} - 6 \csc t \cot t = \dfrac{1}{4t^{3/4}} - 6 \csc t \cot t$

48. $h(x) = \dfrac{1}{x} - 12 \sec x = x^{-1} - 12 \sec x$

$h'(x) = -x^{-2} - 12 \sec x \tan x = \dfrac{-1}{x^2} - 12 \sec x \tan x$

49. $y = \dfrac{3(1 - \sin x)}{2 \cos x} = \dfrac{3 - 3 \sin x}{2 \cos x}$

$$\begin{aligned} y' &= \frac{(-3 \cos x)(2 \cos x) - (3 - 3 \sin x)(-2 \sin x)}{(2 \cos x)^2} \\ &= \frac{-6 \cos^2 x + 6 \sin x - 6 \sin^2 x}{4 \cos^2 x} \\ &= \frac{3}{2}(-1 + \tan x \sec x - \tan^2 x) \\ &= \frac{3}{2} \sec x(\tan x - \sec x) \end{aligned}$$

50. $y = \dfrac{\sec x}{x}$

$y' = \dfrac{x \sec x \tan x - \sec x}{x^2} = \dfrac{\sec x(x \tan x - 1)}{x^2}$

51. $y = -\csc x - \sin x$

$$\begin{aligned} y' &= \csc x \cot x - \cos x \\ &= \frac{\cos x}{\sin^2 x} - \cos x \\ &= \cos x(\csc^2 x - 1) \\ &= \cos x \cot^2 x \end{aligned}$$

52. $y = x \sin x + \cos x$

$y' = x \cos x + \sin x - \sin x = x \cos x$

53. $f(x) = x^2 \tan x$

$f'(x) = x^2 \sec^2 x + 2x \tan x = x(x \sec^2 x + 2 \tan x)$

54. $f(x) = \sin x \cos x$

$f'(x) = \sin x(-\sin x) + \cos x(\cos x) = \cos 2x$

55. $y = 2x \sin x + x^2 e^x$

$$\begin{aligned} y' &= 2x(\cos x) + 2 \sin x + x^2 e^x + 2xe^x \\ &= 2x \cos x + 2 \sin x + xe^x(x + 2) \end{aligned}$$

56. $h(x) = 2e^x \cos x$

$h'(x) = 2(e^x \cos x - e^x \sin x) = 2e^x(\cos x - \sin x)$

57. $y = \dfrac{e^x}{4\sqrt{x}}$

$$y' = \frac{4\sqrt{x}e^x - e^x\left(4/2\sqrt{x}\right)}{\left(4\sqrt{x}\right)^2}$$

$$= \frac{e^x\left[4\sqrt{x} - \left(2/\sqrt{x}\right)\right]}{16x}$$

$$= \frac{e^x(4x - 2)}{16x^{3/2}}$$

$$= \frac{e^x(2x - 1)}{8x^{3/2}}$$

58. $y = \dfrac{2e^x}{x^2 + 1}$

$$y' = \frac{(x^2 + 1)2e^x - 2e^x(2x)}{(x^2 + 1)^2} = \frac{2e^x(x^2 - 2x + 1)}{(x^2 + 1)^2}$$

59. The derivative of $\cos x$ is $-\sin x$.

$$f'(x) = \frac{6(21x^2 + 2\cos x) - (7x^3 - 2\cos x)(0)}{36}$$

$$= \frac{21x^2 + 2\cos x}{6}$$

$$= \frac{7x^2}{2} + \frac{\sin x}{3}$$

60. The derivative of $\csc x$ is $-\csc x \cot x$.

$$g'(x) = (-4x)(-\csc x \cot x) + (\csc x)(-4)$$

$$= 4x \csc x \cot x - 4 \csc x$$

$$= 4 \csc x(x \cot x - 1)$$

61. $g(x) = \left(\dfrac{x+1}{x+2}\right)(2x - 5)$

$$g'(x) = \left(\frac{x+1}{x+2}\right)(2) + (2x - 5)\left[\frac{(x+2)(1) - (x+1)(1)}{(x+2)^2}\right]$$

$$= \frac{2x^2 + 8x - 1}{(x+2)^2}$$

(Form of answer may vary.)

62. $f(x) = \left(\dfrac{x^2 - x - 3}{x^2 + 1}\right)(x^2 + x + 1)$

$$f'(x) = 2\frac{x^5 + 2x^3 + 2x^2 - 2}{(x^2 + 1)^2}$$

(Form of answer may vary.)

63. $g(\theta) = \dfrac{\theta}{1 - \sin\theta}$

$$g'(\theta) = \frac{1 - \sin\theta + \theta\cos\theta}{(1 - \sin\theta)^2}$$

(Form of answer may vary.)

64. $f(\theta) = \dfrac{\sin\theta}{1 - \cos\theta}$

$$f'(\theta) = \frac{1}{\cos\theta - 1} = \frac{\cos\theta - 1}{(1 - \cos\theta)^2}$$

(Form of answer may vary.)

65. $y = \dfrac{1 + \csc x}{1 - \csc x}$

$$y' = \frac{(1 - \csc x)(-\csc x \cot x) - (1 + \csc x)(\csc x \cot x)}{(1 - \csc x)^2} = \frac{-2\csc x \cot x}{(1 - \csc x)^2}$$

$$y'\left(\frac{\pi}{6}\right) = \frac{-2(2)(\sqrt{3})}{(1 - 2)^2} = -4\sqrt{3}$$

66. $f(x) = \tan x \cot x = 1$

$f'(x) = 0$

$f'(1) = 0$

67. $h(t) = \dfrac{\sec t}{t}$

$$h'(t) = \frac{t(\sec t \tan t) - (\sec t)(1)}{t^2} = \frac{\sec t(t \tan t - 1)}{t^2}$$

$$h'(\pi) = \frac{\sec\pi(\pi\tan\pi - 1)}{\pi^2} = \frac{1}{\pi^2}$$

68. $f(x) = \sin x(\sin x + \cos x)$

$$f'(x) = \sin x(\cos x - \sin x) + (\sin x + \cos x)\cos x$$

$$= \sin x \cos x - \sin^2 x + \sin x \cos x + \cos^2 x$$

$$= \sin 2x + \cos 2x$$

$$f'\left(\frac{\pi}{4}\right) = \sin\frac{\pi}{2} + \cos\frac{\pi}{2} = 1$$

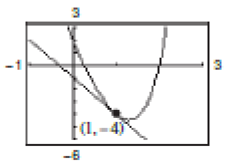
**69.** (a) $f(x) = (x^3 + 4x - 1)(x - 2), \quad (1, -4)$

$$f'(x) = (x^3 + 4x - 1)(1) + (x - 2)(3x^2 + 4)$$
$$= x^3 + 4x - 1 + 3x^3 - 6x^2 + 4x - 8$$
$$= 4x^3 - 6x^2 + 8x - 9$$

$f'(1) = -3$; Slope at $(1, -4)$

Tangent line: $y + 4 = -3(x - 1) \Rightarrow y = -3x - 1$

(b)




(c) Graphing utility confirms $\dfrac{dy}{dx} = -3$ at $(1, -4)$.

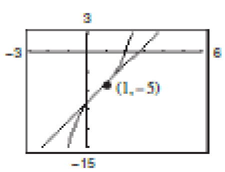
**70.** (a) $f(x) = (x - 2)(x^2 + 4), (1, -5)$

$$f'(x) = (x - 2)(2x) + (x^2 + 4)(1)$$
$$= 2x^2 - 4x + x^2 + 4$$
$$= 3x^2 - 4x + 4$$

$f'(1) = -3$; Slope at $(1, -5)$

Tangent line: $y - (-5) = 3(x - 1) \Rightarrow y = 3x - 8$

(b)




(c) Graphing utility confirms $\dfrac{dy}{dx} = 3$ at $(1, -5)$.

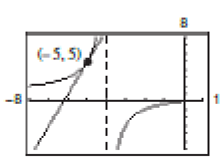
**71.** (a) $f(x) = \dfrac{x}{x + 4}, \quad (-5, 5)$

$$f'(x) = \frac{(x + 4)(1) - x(1)}{(x + 4)^2} = \frac{4}{(x + 4)^2}$$

$f'(-5) = \dfrac{4}{(-5 + 4)^2} = 4$; Slope at $(-5, 5)$

Tangent line: $y - 5 = 4(x + 5) \Rightarrow y = 4x + 25$

(b)




(c) Graphing utility confirms $\dfrac{dy}{dx} = 4$ at $(-5, 5)$.

**72.** (a) $f(x) = \dfrac{x + 3}{x - 3}, \quad (4, 7)$

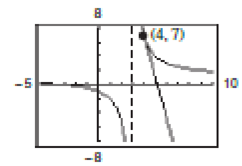
$$f'(x) = \frac{(x - 3)(1) - (x + 3)(1)}{(x - 3)^2} = -\frac{6}{(x - 3)^2}$$

$f'(4) = \dfrac{-6}{1} = -6$; Slope at $(4, 7)$

Tangent line:

$y - 7 = -6(x - 4) \Rightarrow y = -6x + 31$

(b)




(c) Graphing utility confirms $\dfrac{dy}{dx} = -6$ at $(4, 7)$.

**73.** (a) $f(x) = \tan x, \quad \left(\dfrac{\pi}{4}, 1\right)$

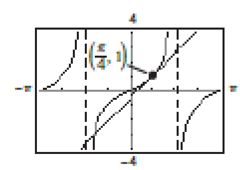
$$f'(x) = \sec^2 x$$

$f'\left(\dfrac{\pi}{4}\right) = 2$; Slope at $\left(\dfrac{\pi}{4}, 1\right)$

Tangent line: $\quad y - 1 = 2\left(x - \dfrac{\pi}{4}\right)$

$$y - 1 = 2x - \frac{\pi}{2}$$
$$4x - 2y - \pi + 2 = 0$$

(b)




(c) Graphing utility confirms $\dfrac{dy}{dx} = 2$ at $\left(\dfrac{\pi}{4}, 1\right)$.

**74.** (a) $f(x) = \sec x, \quad \left(\dfrac{\pi}{3}, 2\right)$

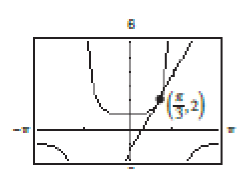
$$f'(x) = \sec x \tan x$$

$f'\left(\dfrac{\pi}{3}\right) = 2\sqrt{3}$; Slope at $\left(\dfrac{\pi}{3}, 2\right)$

Tangent line:

$$y - 2 = 2\sqrt{3}\left(x - \frac{\pi}{3}\right)$$
$$6\sqrt{3}x - 3y + 6 - 2\sqrt{3}\pi = 0$$

(b)




(c) Graphing utility confirms $\dfrac{dy}{dx} = 2\sqrt{3}$ at $\left(\dfrac{\pi}{3}, 2\right)$.

75. (a) $f(x) = (x-1)e^x, \quad (1, 0)$

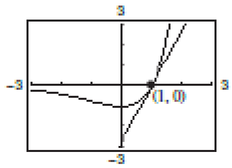
$$f'(x) = (x-1)e^x + e^x = e^x$$

$$f'(1) = e$$

Tangent line: $y - 0 = e(x-1)$

$$y = e(x-1)$$

(b)




(c) Graphing utility confirms $\dfrac{dy}{dx} = e$ at $(1, 0)$.

76. (a) $f(x) = \dfrac{e^x}{x+4}, \quad \left(0, \dfrac{1}{4}\right)$

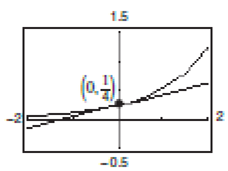
$$f'(x) = \frac{(x+4)e^x - e^x}{(x+4)^2} = \frac{e^x(x+3)}{(x+4)^2}$$

$$f'(0) = \frac{3}{16}$$

Tangent line: $y - \dfrac{1}{4} = \dfrac{3}{16}(x-0)$

$$y = \frac{3}{16}x + \frac{1}{4}$$

(b)




(c) Graphing utility confirms $\dfrac{dy}{dx} = \dfrac{3}{16}$ at $\left(0, \dfrac{1}{4}\right)$.

77. $f(x) = \dfrac{8}{x^2+4}; \quad (2, 1)$

$$f'(x) = \frac{(x^2+4)(0) - 8(2x)}{(x^2+4)^2} = \frac{-16x}{(x^2+4)^2}$$

$$f'(2) = \frac{-16(2)}{(4+4)^2} = -\frac{1}{2}$$

$$y - 1 = -\frac{1}{2}(x-2)$$

$$y = -\frac{1}{2}x + 2$$

$$2y + x - 4 = 0$$

78. $f(x) = \dfrac{4x}{x^2+6}; \left(2, \dfrac{4}{5}\right)$

$$f'(x) = \frac{(x^2+6)(4) - 4x(2x)}{(x^2+6)^2} = \frac{24-4x^2}{(x^2+6)^2}$$

$$f'(2) = \frac{24-16}{10^2} = \frac{2}{25}$$

$$y - \frac{4}{5} = \frac{2}{25}(x-2)$$

$$y = \frac{2}{25}x + \frac{16}{25}$$

$$25y - 2x - 16 = 0$$

79. $f(x) = \dfrac{x^2}{x-1} = x^2(x-1)^{-1}$

$$f'(x) = x^2\left[-(x-1)^{-2}\right] + (x-1)^{-1}(2x)$$

$$= -\frac{x^2}{(x-1)^2} + \frac{2x}{(x-1)}$$

$$= \frac{-x^2 + 2x(x-1)}{(x-1)^2}$$

$$= \frac{x^2-2x}{(x-1)^2} = \frac{x(x-2)}{(x-1)^2}$$

$f'(x) = 0$ when $x = 0$ and $x = 2$.

So, $f(0) = 0$ and $f(2) = 4$.

Horizontal tangents: $(0, 0), (2, 4)$

80. $f(x) = \dfrac{x^2}{x^2+1}$

$$f'(x) = \frac{(x^2+1)(2x) - (x^2)(2x)}{(x^2+1)^2} = \frac{2x}{(x^2+1)^2}$$

$f'(x) = 0$ when $x = 0$.

So, $f(0) = 0$.

Horizontal tangent: $(0, 0)$

81. $g(x) = \dfrac{8(x-2)}{e^x}$

$$g'(x) = \frac{e^x(8) - 8(x-2)e^x}{e^{2x}} = \frac{24-8x}{e^x}$$

$g'(x) = 0$ when $x = 3$.

So, $f(3) = \dfrac{8}{e^3} = 8e^{-3}$.

Horizontal tangent: $\left(3, 8e^{-3}\right)$

**82.** $f(x) = e^x \sin x, \quad 0 \le x \le \pi$

$f'(x) = e^x \cos x + e^x \sin x = e^x(\cos x + \sin x)$

$f'(x) = 0$ when $\cos x = -\sin x \Rightarrow x = \dfrac{3\pi}{4}.$

So, $f\left(\dfrac{3\pi}{4}\right) = e^{3\pi/4} \sin \dfrac{\pi}{4} = \dfrac{\sqrt{2}}{2} e^{3\pi/4}.$

Horizontal tangent: $\left(\dfrac{3\pi}{4}, \dfrac{\sqrt{2}}{2} e^{3\pi/4}\right)$

**83.** $f(x) = \dfrac{x+1}{x-1}$

$f'(x) = \dfrac{(x-1)-(x+1)}{(x-1)^2} = \dfrac{-2}{(x-1)^2}$

$2y + x = 6 \Rightarrow y = -\dfrac{1}{2}x + 3$; Slope: $-\dfrac{1}{2}$

$$\frac{-2}{(x-1)^2} = -\frac{1}{2}$$

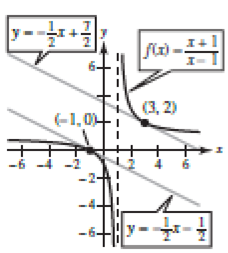
$$(x-1)^2 = 4$$

$$x - 1 = \pm 2$$

$$x = -1, 3;\ f(-1) = 0,\ f(3) = 2$$

$$y - 0 = -\frac{1}{2}(x+1) \Rightarrow y = -\frac{1}{2}x - \frac{1}{2}$$

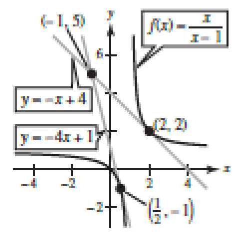
$$y - 2 = -\frac{1}{2}(x-3) \Rightarrow y = -\frac{1}{2}x + \frac{7}{2}$$




**84.** $f(x) = \dfrac{x}{x-1}$

$f'(x) = \dfrac{(x-1)-x}{(x-1)^2} = \dfrac{-1}{(x-1)^2}$

Let $(x, y) = (x, x/(x-1))$ be a point of tangency on the graph of $f$.




$$\frac{5 - (x/(x-1))}{-1-x} = \frac{-1}{(x-1)^2}$$

$$\frac{4x-5}{(x-1)(x+1)} = \frac{1}{(x-1)^2}$$

$$(4x-5)(x-1) = x+1$$

$$4x^2 - 10x + 4 = 0$$

$$(x-2)(2x-1) = 0 \Rightarrow x = \frac{1}{2}, 2$$

$$f\left(\frac{1}{2}\right) = -1,\ f(2) = 2;\ f'\left(\frac{1}{2}\right) = -4,\ f'(2) = -1$$

Two tangent lines:

$$y + 1 = -4\left(x - \frac{1}{2}\right) \Rightarrow y = -4x + 1$$

$$y - 2 = -1(x-2) \Rightarrow y = -x + 4$$

**85.** $f'(x) = \dfrac{(x+2)3 - 3x(1)}{(x+2)^2} = \dfrac{6}{(x+2)^2}$

$g'(x) = \dfrac{(x+2)5 - (5x+4)(1)}{(x+2)^2} = \dfrac{6}{(x+2)^2}$

$g(x) = \dfrac{5x+4}{(x+2)} = \dfrac{3x}{(x+2)} + \dfrac{2x+4}{(x+2)} = f(x) + 2$

$f$ and $g$ differ by a constant.

**86.** $f'(x) = \dfrac{x(\cos x - 3) - (\sin x - 3x)(1)}{x^2} = \dfrac{x \cos x - \sin x}{x^2}$

$g'(x) = \dfrac{x(\cos x + 2) - (\sin x + 2x)(1)}{x^2} = \dfrac{x \cos x - \sin x}{x^2}$

$g(x) = \dfrac{\sin x + 2x}{x} = \dfrac{\sin x - 3x + 5x}{x} = f(x) + 5$

$f$ and $g$ differ by a constant.

**87.** (a) $p'(x) = f'(x)g(x) + f(x)g'(x)$

$$p'(1) = f'(1)g(1) + f(1)g'(1) = 1(4) + 6\left(-\frac{1}{2}\right) = 1$$

(b) $q'(x) = \dfrac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$

$$q'(4) = \frac{3(-1) - 7(0)}{3^2} = -\frac{1}{3}$$

**88.** (a) $p'(x) = f'(x)g(x) + f(x)g'(x)$

$$p'(4) = \frac{1}{2}(8) + 1(0) = 4$$

(b) $q'(x) = \dfrac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$

$$q'(7) = \frac{4(2) - 4(-1)}{4^2} = \frac{12}{16} = \frac{3}{4}$$

**89.** Area $= A(t) = (6t + 5)\sqrt{t} = 6t^{3/2} + 5t^{1/2}$

$$A'(t) = 9t^{1/2} + \frac{5}{2}t^{-1/2} = \frac{18t + 5}{2\sqrt{t}}\text{ cm}^2/\text{sec}$$

**90.** $V = \pi r^2 h = \pi(t + 2)\left(\frac{1}{2}\sqrt{t}\right) = \frac{1}{2}\left(t^{3/2} + 2t^{1/2}\right)\pi$

$$V'(t) = \frac{1}{2}\left(\frac{3}{2}t^{1/2} + t^{-1/2}\right)\pi = \frac{3t + 2}{4t^{1/2}}\pi\text{ in.}^3/\text{sec}$$

**91.** $P(t) = 500\left[1 + \dfrac{4t}{50 + t^2}\right]$

$$P'(t) = 500\left[\frac{(50 + t^2)(4) - (4t)(2t)}{(50 + t^2)^2}\right]$$

$$= 500\left[\frac{200 - 4t^2}{(50 + t^2)^2}\right]$$

$$= 2000\left[\frac{50 - t^2}{(50 + t^2)^2}\right]$$

$P'(2) \approx 31.55$ bacteria/h

**92.** $C = 100\left(\dfrac{200}{x^2} + \dfrac{x}{x + 30}\right),\ 1 \le x$

$$\frac{dC}{dx} = 100\left(-\frac{400}{x^3} + \frac{30}{(x + 30)^2}\right)$$

(a) When $x = 10$: $\dfrac{dC}{dx} = -\$38.13$ thousand/100 components

(b) When $x = 15$: $\dfrac{dC}{dx} = -\$10.37$ thousand/100 components

(c) When $x = 20$: $\dfrac{dC}{dx} = -\$3.80$ thousand/100 components

As the order size increases, the cost per item decreases.

**93.** (a) $\cot x = \dfrac{\cos x}{\sin x}$

$$\frac{d}{dx}[\cot x] = \frac{d}{dx}\left[\frac{\cos x}{\sin x}\right] = \frac{\sin x(-\sin x) - (\cos x)(\cos x)}{(\sin x)^2} = -\frac{\sin^2 x + \cos^2 x}{\sin^2 x} = -\frac{1}{\sin^2 x} = -\csc^2 x$$

(b) $\sec x = \dfrac{1}{\cos x}$

$$\frac{d}{dx}[\sec x] = \frac{d}{dx}\left[\frac{1}{\cos x}\right] = \frac{(\cos x)(0) - (1)(-\sin x)}{(\cos x)^2} = \frac{\sin x}{\cos x\cos x} = \frac{1}{\cos x}\cdot\frac{\sin x}{\cos x} = \sec x\tan x$$

(c) $\csc x = \dfrac{1}{\sin x}$



$$\frac{d}{dx}[\csc x] = \frac{d}{dx}\left[\frac{1}{\sin x}\right] = \frac{(\sin x)(0) - (1)(\cos x)}{(\sin x)^2} = -\frac{\cos x}{\sin x\sin x} = -\frac{1}{\sin x}\cdot\frac{\cos x}{\sin x} = -\csc x\cot x$$

**94.** $f(x) = \sec x$

$g(x) = \csc x,\ [0, 2\pi)$

$f'(x) = g'(x)$

$$\sec x \tan x = -\csc x \cot x \Rightarrow \frac{\sec x \tan x}{\csc x \cot x} = -1 \Rightarrow \frac{\frac{1}{\cos x} \cdot \frac{\sin x}{\cos x}}{\frac{1}{\sin x} \cdot \frac{\cos x}{\sin x}} = -1 \Rightarrow \frac{\sin^3 x}{\cos^3 x} = -1 \Rightarrow \tan^3 x = -1 \Rightarrow \tan x = -1$$

$$x = \frac{3\pi}{4}, \frac{7\pi}{4}$$

**95.** $f(x) = x^2 + 7x - 4$

$f'(x) = 2x + 7$

$f''(x) = 2$

**96.** $f(x) = 4x^5 - 2x^3 + 5x^2$

$f'(x) = 20x^4 - 6x^2 + 10x$

$f''(x) = 80x^3 - 12x + 10$

**97.** $f(x) = 4x^{3/2}$

$f'(x) = 6x^{1/2}$

$f''(x) = 3x^{-1/2} = \dfrac{3}{\sqrt{x}}$

**98.** $f(x) = x^2 + 3x^{-3}$

$f'(x) = 2x - 9x^{-4}$

$f''(x) = 2 + 36x^{-5} = 2 + \dfrac{36}{x^5}$

**99.** $f(x) = \dfrac{x}{x - 1}$

$f'(x) = \dfrac{(x-1)(1) - x(1)}{(x-1)^2} = \dfrac{-1}{(x-1)^2}$

$f''(x) = \dfrac{2}{(x-1)^3}$

**100.** $f(x) = \dfrac{x^2 + 3x}{x - 4}$

$$f'(x) = \frac{(x-4)(2x+3) - (x^2+3x)(1)}{(x-4)^2}$$

$$= \frac{2x^2 - 5x - 12 - x^2 - 3x}{(x-4)^2} = \frac{x^2 - 8x - 12}{x^2 - 8x + 16}$$

$$f''(x) = \frac{(x-4)^2(2x-8) - (x^2 - 8x - 12)(2x - 8)}{(x-4)^4}$$

$$= \frac{(x-4)\left[(x-4)(2x-8) - 2(x^2 - 8x - 12)\right]}{(x-4)^4}$$

$$= \frac{(x-4)(2x-8) - 2(x^2 - 8x - 12)}{(x-4)^3}$$

$$= \frac{2x^2 - 16x + 32 - 2x^2 + 16x + 24}{(x-4)^3}$$

$$= \frac{56}{(x-4)^3}$$

**101.** $f(x) = x \sin x$

$f'(x) = x \cos x + \sin x$

$f''(x) = x(-\sin x) + \cos x + \cos x$

$= -x \sin x + 2 \cos x$

**102.** $f(x) = \sec x$

$f'(x) = \sec x \tan x$

$f''(x) = \sec x(\sec^2 x) + \tan x(\sec x \tan x)$

$= \sec x(\sec^2 x + \tan^2 x)$

**103.** $g(x) = \dfrac{e^x}{x}$

$g'(x) = \dfrac{xe^x - e^x}{x^2}$



$$g''(x) = \frac{x^2(xe^x + e^x - e^x) - 2x(xe^x - e^x)}{x^4} = \frac{e^x}{x^3}(x^2 - 2x + 2)$$

**104.** $h(t) = e^t \sin t$

$$h'(t) = e^t \cos t + e^t \sin t = e^t(\cos t + \sin t)$$
$$h''(t) = e^t(-\sin t + \cos t) + e^t(\cos t + \sin t)$$
$$= 2e^t \cos t$$

**105.** $f'(x) = 2x^2$

$$f''(x) = 4x$$

**106.** $f''(x) = 2 - 2x^{-1}$

$$f'''(x) = 2x^{-2} = \frac{2}{x^2}$$

**107.** $f'''(x) = 2\sqrt{x}$

$$f^{(4)}(x) = \frac{1}{2}(2)x^{-1/2} = \frac{1}{\sqrt{x}}$$

**108.** $f^{(4)}(x) = 2x + 1$

$$f^{(5)}(x) = 2$$
$$f^{(6)}(x) = 0$$

**109.** $f(x) = 2g(x) + h(x)$

$$f'(x) = 2g'(x) + h'(x)$$
$$f'(2) = 2g'(2) + h'(2)$$
$$= 2(-2) + 4$$
$$= 0$$

**110.** $f(x) = 4 - h(x)$

$$f'(x) = -h'(x)$$
$$f'(2) = -h'(2) = -4$$

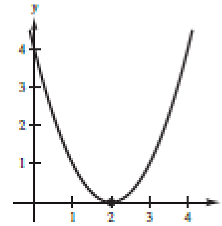
**111.** $f(x) = \dfrac{g(x)}{h(x)}$

$$f'(x) = \frac{h(x)g'(x) - g(x)h'(x)}{[h(x)]^2}$$
$$f'(2) = \frac{h(2)g'(2) - g(2)h'(2)}{[h(2)]^2}$$
$$= \frac{(-1)(-2) - (3)(4)}{(-1)^2}$$
$$= -10$$

**112.** $f(x) = g(x)h(x)$

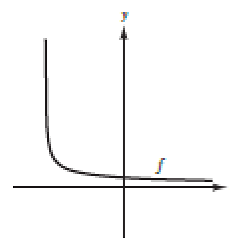
$$f'(x) = g(x)h'(x) + h(x)g'(x)$$
$$f'(2) = g(2)h'(2) + h(2)g'(2)$$
$$= (3)(4) + (-1)(-2)$$
$$= 14$$

**113.** The graph of a differentiable function $f$ such that $f(2) = 0$, $f' < 0$ for $-\infty < x < 2$, and $f' > 0$ for $2 < x < \infty$ would, in general, look like the graph below.

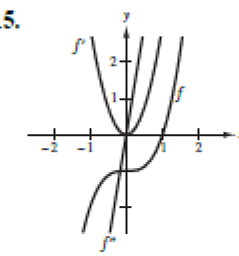


One such function is $f(x) = (x - 2)^2$.

**114.** The graph of a differentiable function $f$ such that $f > 0$ and $f' < 0$ for all real numbers $x$ would, in general, look like the graph below.

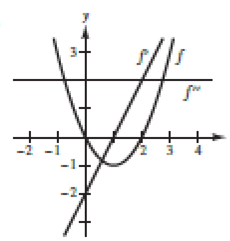



**115.**



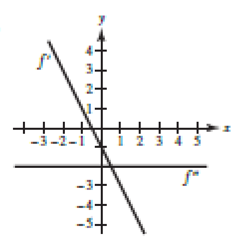

It appears that $f$ is cubic, so $f'$ would be quadratic and $f''$ would be linear.

**116.**

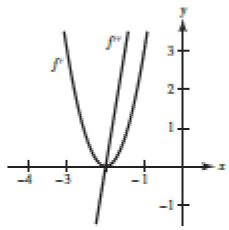



It appears that $f$ is quadratic so $f'$ would be linear and $f''$ would be constant.

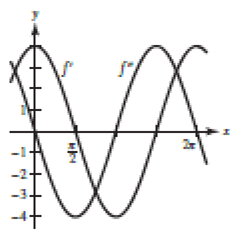
**117.**



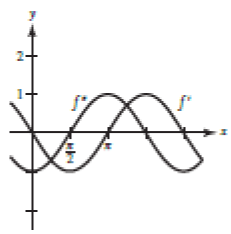
**118.**




**119.**




**120.**




**121.** $v(t) = 36 - t^2, 0 \le t \le 6$

$a(t) = v'(t) = -2t$

$v(3) = 27$ m/sec

$a(3) = -6$ m/sec$^2$

The speed of the object is decreasing.

**122.** $v(t) = \dfrac{100t}{2t+15}$

$$a(t) = v'(t) = \frac{(2t+15)(100) - (100t)(2)}{(2t+15)^2} = \frac{1500}{(2t+15)^2}$$

(a) $a(5) = \dfrac{1500}{\left[2(5) + 15\right]^2} = 2.4$ ft/sec$^2$

(b) $a(10) = \dfrac{1500}{\left[2(10) + 15\right]^2} \approx 1.2$ ft/sec$^2$

(c) $a(20) = \dfrac{1500}{\left[2(20) + 15\right]^2} \approx 0.5$ ft/sec$^2$

**123.** $s(t) = -8.25t^2 + 66t$

$v(t) = s'(t) = 16.50t + 66$

$a(t) = v'(t) = -16.50$

| $t$(sec) | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $s(t)$ (ft) | 0 | 57.75 | 99 | 123.75 | 132 |
| $v(t) = s'(t)$ (ft/sec) | 66 | 49.5 | 33 | 16.5 | 0 |
| $a(t) = v'(t)$ (ft/sec$^2$) | −16.5 | −16.5 | −16.5 | −16.5 | −16.5 |

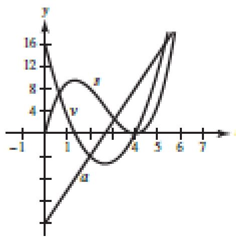
Average velocity on:

$[0, 1]$ is $\dfrac{57.75 - 0}{1 - 0} = 57.75$

$[1, 2]$ is $\dfrac{99 - 57.75}{2 - 1} = 41.25$

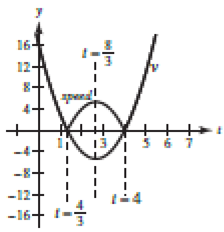
$[2, 3]$ is $\dfrac{123.75 - 99}{3 - 2} = 24.75$

$[3, 4]$ is $\dfrac{132 - 123.75}{4 - 3} = 8.25$

**124.** (a) 


$s$ position function
$v$ velocity function
$a$ acceleration function

(b) The speed of the particle is the absolute value of its velocity. So, the particle's speed is slowing down on the intervals $(0, 4/3)$ and $(8/3, 4)$, and it speeds up on the intervals $(4/3, 8/3)$ and $(4, 6)$.




**125.** $f(x) = x^n$

$$f^{(n)}(x) = n(n-1)(n-2)\cdots(2)(1) = n!$$

**Note:** $n! = n(n-1)\cdots 3 \cdot 2 \cdot 1$ (read "$n$ factorial")

**126.** $f(x) = \dfrac{1}{x}$

$$f^{(n)}(x) = \frac{(-1)^n(n)(n-1)(n-2)\cdots(2)(1)}{x^{n+1}} = \frac{(-1)^n n!}{x^{n+1}}$$

**127.** $f(x) = g(x)h(x)$

(a)
$$\begin{aligned}
f'(x) &= g(x)h'(x) + h(x)g'(x)\\
f''(x) &= g(x)h''(x) + g'(x)h'(x) + h(x)g''(x) + h'(x)g'(x)\\
&= g(x)h''(x) + 2g'(x)h'(x) + h(x)g''(x)\\
f'''(x) &= g(x)h'''(x) + g'(x)h''(x) + 2g'(x)h''(x) + 2g''(x)h'(x) + h(x)g'''(x) + h'(x)g''(x)\\
&= g(x)h'''(x) + 3g'(x)h''(x) + 3g''(x)h'(x) + g'''(x)h(x)\\
f^{(4)}(x) &= g(x)h^{(4)}(x) + g'(x)h'''(x) + 3g'(x)h'''(x) + 3g''(x)h''(x) + 3g''(x)h''(x) + 3g'''(x)h'(x)\\
&\quad + g'''(x)h'(x) + g^{(4)}(x)h(x)\\
&= g(x)h^{(4)}(x) + 4g'(x)h'''(x) + 6g''(x)h''(x) + 4g'''(x)h'(x) + g^{(4)}(x)h(x)
\end{aligned}$$

(b)
$$\begin{aligned}
f^{(n)}(x) &= g(x)h^{(n)}(x) + \frac{n(n-1)(n-2)\cdots(2)(1)}{1[(n-1)(n-2)\cdots(2)(1)]}g'(x)h^{(n-1)}(x) + \frac{n(n-1)(n-2)\cdots(2)(1)}{(2)(1)[(n-2)(n-3)\cdots(2)(1)]}g''(x)h^{(n-2)}(x)\\
&\quad + \frac{n(n-1)(n-2)\cdots(2)(1)}{(3)(2)(1)[(n-3)(n-4)\cdots(2)(1)]}g'''(x)h^{(n-3)}(x) + \cdots\\
&\quad + \frac{n(n-1)(n-2)\cdots(2)(1)}{[(n-1)(n-2)\cdots(2)(1)](1)}g^{(n-1)}(x)h'(x) + g^{(n)}(x)h(x)\\
&= g(x)h^{(n)}(x) + \frac{n!}{1!(n-1)!}g'(x)h^{(n-1)}(x) + \frac{n!}{2!(n-2)!}g''(x)h^{(n-2)}(x) + \cdots\\
&\quad + \frac{n!}{(n-1)!1!}g^{(n-1)}(x)h'(x) + g^{(n)}(x)h(x)
\end{aligned}$$



**Note:** $n! = n(n-1)\cdots 3 \cdot 2 \cdot 1$ (read "$n$ factorial")

**128.** $[xf(x)]' = xf'(x) + f(x)$

$[xf(x)]'' = xf''(x) + f'(x) + f'(x) = xf''(x) + 2f'(x)$

$[xf(x)]''' = xf'''(x) + f''(x) + 2f''(x) = xf'''(x) + 3f''(x)$

In general, $[xf(x)]^{(n)} = xf^{(n)}(x) + nf^{(n-1)}(x)$.

**129.** $f(x) = x^n \sin x$

$f'(x) = x^n \cos x + nx^{n-1} \sin x$

When $n = 1$: $f'(x) = x \cos x + \sin x$

When $n = 2$: $f'(x) = x^2 \cos x + 2 \sin x$

When $n = 3$: $f'(x) = x^3 \cos x + 3x^2 \sin x$

When $n = 4$: $f'(x) = x^4 \cos x + 4x^3 \sin x$

For general $n$, $f'(x) = x^n \cos x + nx^{n-1} \sin x$.

**130.** $f(x) = \dfrac{\cos x}{x^n} = x^{-n} \cos x$

$$\begin{aligned} f'(x) &= -x^{-n} \sin x - nx^{-n-1} \cos x \\ &= -x^{-n-1}(x \sin x + n \cos x) \\ &= -\frac{x \sin x + n \cos x}{x^{n+1}} \end{aligned}$$

When $n = 1$: $f'(x) = -\dfrac{x \sin x + \cos x}{x^2}$

When $n = 2$: $f'(x) = -\dfrac{x \sin x + 2 \cos x}{x^3}$

When $n = 3$: $f'(x) = -\dfrac{x \sin x + 3 \cos x}{x^4}$

When $n = 4$: $f'(x) = -\dfrac{x \sin x + 4 \cos x}{x^5}$

For general $n$, $f'(x) = -\dfrac{x \sin x + n \cos x}{x^{n+1}}$.

**131.** True

$$\begin{aligned} h'(c) &= f(c)g'(c) + g(c)f'(c) \\ &= f(c)(0) + g(c)(0) \\ &= 0 \end{aligned}$$

**132.** True

**133.** True

**134.** True. If $v(t) = c$, then $a(t) = v'(t) = 0$.

**135.** $f(x) = x|x| = \begin{cases} x^2, & x \ge 0 \\ -x^2, & x < 0 \end{cases}$

$f'(x) = \begin{cases} 2x, & x > 0 \\ -2x, & x < 0 \end{cases} = 2|x|$

$f''(x) = \begin{cases} 2, & x > 0 \\ -2, & x < 0 \end{cases}$

$f''(0)$ does not exist because the left and right derivatives do not agree at $x = 0$.

**136.** (a) $(fg' - f'g)' = fg'' + f'g' - f'g' - f''g$
$= fg'' - f''g$ True

(b) $(fg)'' = (fg' + f'g)'$
$= fg'' + f'g' + f'g' + f''g$
$= fg'' + 2f'g' + f''g$
$\neq fg'' + f''g$ False

**137.**
$$\begin{aligned} \frac{d}{dx}[f(x)g(x)h(x)] &= \frac{d}{dx}[(f(x)g(x))h(x)] \\ &= \frac{d}{dx}[f(x)g(x)]h(x) + f(x)g(x)h'(x) \\ &= [f(x)g'(x) + g(x)f'(x)]h(x) + f(x)g(x)h'(x) \\ &= f'(x)g(x)h(x) + f(x)g'(x)h(x) + f(x)g(x)h'(x) \end{aligned}$$

**138.** Evaluate each statement.

A: "$f$ is continuous at $x = c$" is a true statement by Theorem 2.1.

B: "$\lim_{x \to c} f(x)$ exists" is a true statement by the definition of a limit.

C: "$f'(c)$ is defined" is a true statement by the definition of a derivative.

D: "$f''(c)$ is defined" could be a false statement because $f'$ may not be differentiable at $x = c$.

So, the answer is D.

**139.** $y = 4e^x \cot x$

$$\frac{dy}{dx} = 4\left[e^x\left(-\csc^2 x\right) + (\cot x)\left(e^x\right)\right] = 4e^x\left(-\csc^2 x + \cot x\right) = 4e^x\left(\cot x - \csc^2 x\right)$$

So, the answer is C.

**140.** $h(x) = \dfrac{x^2 - x}{x + 5} = \left(x^2 - x\right)(x + 5)^{-1}$

$$\begin{aligned} h'(x) &= \left(x^2 - x\right)\left[-(x + 5)^{-2}\right] + (x + 5)^{-1}(2x - 1) \\ &= (x + 5)^{-2}\left[-x^2 + x + (2x - 1)(x + 5)\right] \\ &= (x + 5)^{-2}\left(x^2 + 10x - 5\right) \end{aligned}$$

$$\begin{aligned} h''(x) &= (x + 5)^{-2}(2x + 10) + \left(x^2 + 10x - 5\right)\left[-2(x + 5)^{-3}\right] \\ &= (x + 5)^{-3}\left[(2x + 10)(x + 5) - 2\left(x^2 + 10x - 5\right)\right] \\ &= (x + 5)^{-3}\left(2x^2 + 20x + 50 - 2x^2 - 20x + 10\right) \\ &= \frac{60}{(x + 5)^3} \end{aligned}$$

So, the answer is A.

**141.** $f(x) = \sin x - \cos x$

$f'(x) = \cos x + \sin x = \sin x + \cos x$

$f''(x) = \cos x - \sin x = -\sin x + \cos x$

$f'''(x) = -\cos x - \sin x = -\sin x - \cos x$

$f^{(4)}(x) = -\cos x + \sin x = \sin x - \cos x = f(x)$

Because $n = 4$, the answer is B.