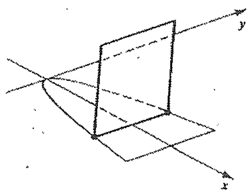
# 6.2 Volume: The Disk and Washer Method (Day 3)

**Another method for finding the volumes of solids is using known cross sections. Some common cross-sections are squares, rectangles, semi-circles and trapezoids.**




**Volumes of Solids with Known Cross Sections**

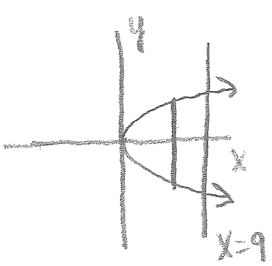
**Perpendicular to the x-axis**

$$\int_a^b A(x)\,dx$$ Area of cross section

**Perpendicular to the y-axis**

$$\int_a^b A(y)\,dy$$ Area of cross section

Let $R$ be the region bounded by the graphs of $x = y^2$ and $x = 9$. Find the volume of the solid that has $R$ as its base and every cross section is a semi-circle perpendicular to the $x - axis$.

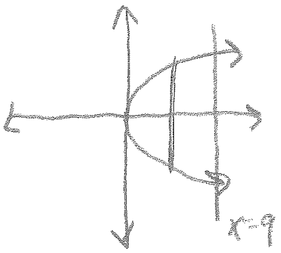


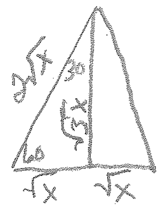

$$\int_0^9 \pi(\sqrt{x})^2\,dx = \pi\int_0^9 x\,dx = \pi\left(\tfrac{1}{2}x^2\right)\Big|_0^9$$

$$= \pi\left[\tfrac{1}{2}(81) - 0\right] = \frac{81}{2}\pi$$

**Examples – Volumes with Known Cross-Sections**

Let $R$ be the region bounded by the graphs of $x = y^2$ and $x = 9$. Find the volume of the solid that has $R$ as its base and every cross section is an equilateral triangle perpendicular to the $x - axis$.

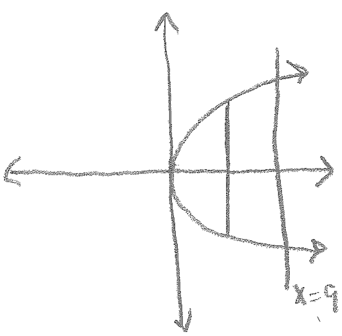



$$\int_0^9 \left(\frac{1}{2}\right)(2\sqrt{x})(\sqrt{3x})\,dx = \int_0^9 x\sqrt{3}\,dx = \sqrt{3}\left(\frac{x^2}{2}\right)\Big|_0^9$$

$$= \frac{\sqrt{3}}{2}(81) - 0 = \frac{81\sqrt{3}}{2}$$

Let $R$ be the region bounded by the graphs of $x = y^2$ and $x = 9$. Find the volume of the solid that has $R$ as its base and every cross section is a trapezoid with lower base in the $xy$ −plane, upper base equal to $\frac{1}{2}$ the length of the lower base, and the height equal to $\frac{1}{4}$ the length of the lower base perpendicular to the $x - axis$.



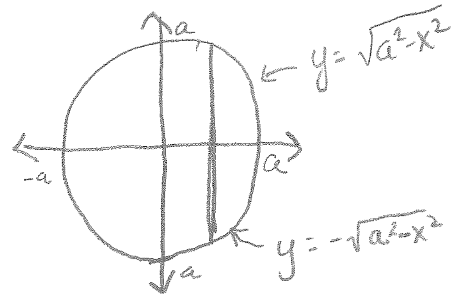

$$\int_0^9 \left(\frac{1}{2}\right)\left(\frac{1}{4}\right)(2\sqrt{x})\left(2\sqrt{x} + \frac{1}{2}(2\sqrt{x})\right)dx$$

$$\frac{1}{4}\int_0^9 \sqrt{x}\left(2\sqrt{x} + \sqrt{x}\right)dx$$

$$\frac{1}{4}\int_0^9 3x\,dx = \left(\frac{1}{4}\right)(3)\left(\frac{x^2}{2}\right)\Big|_0^9$$

$$= \frac{3}{8}(9)^2 = \frac{243}{8}$$

A solid has as its base the circular region in the $xy$ plane bounded by the graph of $x^2 + y^2 = a^2$ with $a > 0$. Find the volume of the solid if every cross section by a plane perpendicular to the $x-axis$ is a square.

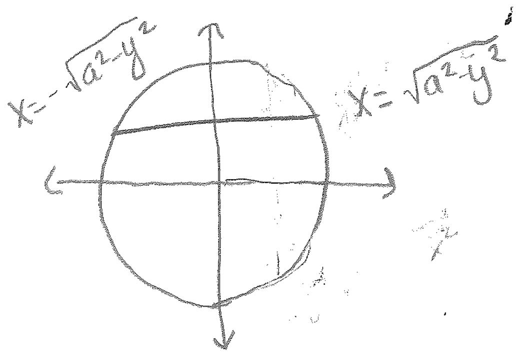


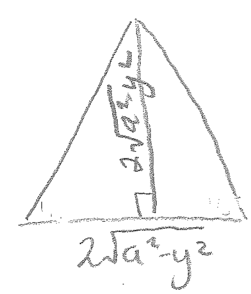

$$2\int_0^a \left(2\sqrt{a^2-x^2}\right)^2 dx$$

$$2\int_0^a 4(a^2-x^2)\,dx = 8\left(a^2x - \frac{1}{3}x^3\right)\Big|_0^a$$

$$= 8a^2(a) - \frac{8}{3}a^3 = 8a^3 - \frac{8}{3}a^3 = \frac{16}{3}a^3$$

A solid has as its base the circular region in the $xy$ plane bounded by the graph of $x^2 + y^2 = a^2$ with $a > 0$. Find the volume of the solid if every cross section by a plane perpendicular to the $y-axis$ is an isosceles triangle with base on the $xy$-plane and altitude equal to the length of the base.




$$2\int_0^a \left(\frac{1}{2}\right)\left(2\sqrt{a^2-y^2}\right)\left(2\sqrt{a^2-y^2}\right) dy = 2\int_0^a 2(a^2-y^2)\,dy$$

$$= 4\left(a^2y - \frac{1}{3}y^3\right)\Big|_0^a = 4\left(a^3 - \frac{1}{3}a^3\right) - 0 = 4a^3 - \frac{4}{3}a^3 = \frac{8}{3}a^3$$