3.5 A Summary of Curve Sketching

Guidelines for analyzing the graph of a function

$f(x)$

Domain

Range

- * X intercepts
- * y-intercepts
- * asymptotes

Symmetry

$f'(x)$

- * Increasing
- * Decreasing
- * Relative Minima & Maxima

$f''(x)$

- * Concave Up
- * Concave Down
- * Points of Inflection

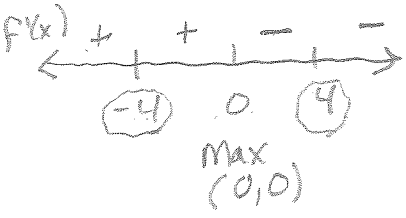
Analyze and sketch the graph of $f(x) = \frac{x^2}{2(x^2-16)}$

$f(x)$ $(0,0)$

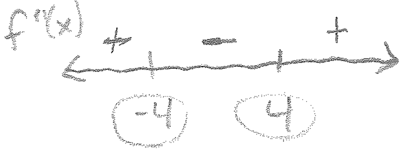
$x = \pm 4$

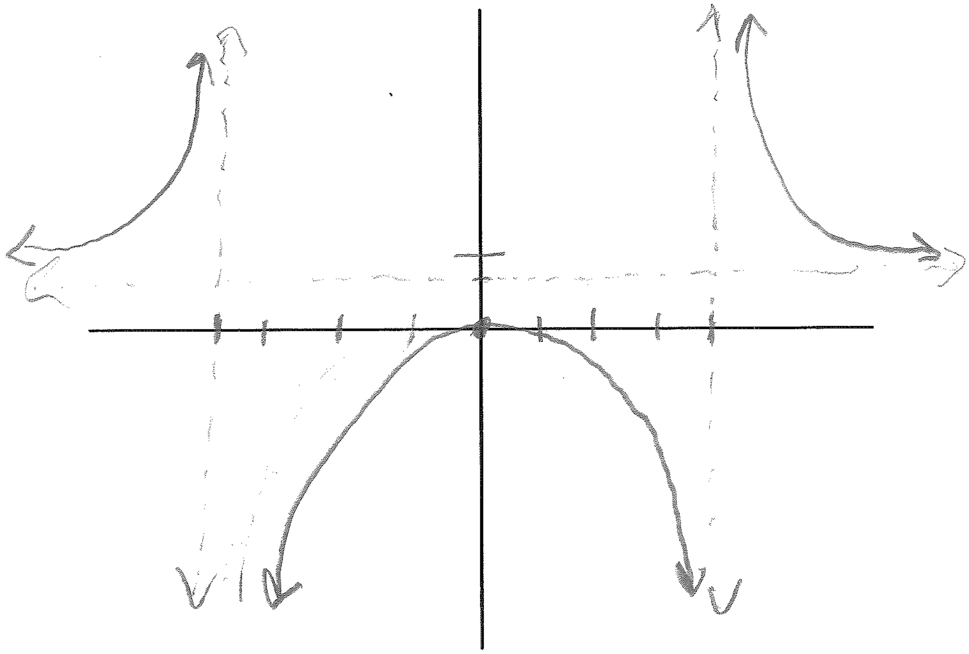
$y = \frac{1}{2}$

$f'(x) = \frac{-16x}{(x^2-16)^2}$ $x = 0$ $x = \pm 4$



$f''(x) = \frac{-16(-3x^2-16)}{(x^2-16)^3}$ $x = \pm 4$





Analyze and sketch the graph of $f(x) = \frac{-x^2-3x+9}{x-3}$ $\quad - \frac{x^2-3x+9}{x-3} = \frac{-x^2+3x-9}{x-3}$

$f(x) \quad (0, 3)$

$-x^2+3x-9=0$

$x = \frac{-3 \pm \sqrt{9-4(-1)(-9)}}{2(-1)}$ = imaginary

$$x-3 \overline{\big)\, -x^2+3x-9} \quad \text{quotient } -x$$

$$\underline{-x^2+3x}$$

$y=-x$

$x=3$

$$f'(x) = \frac{(x-3)(-2x+3)-(-x^2+3x-9)(1)}{(x-3)^2} = \frac{-2x^2+3x+6x-9+x^2-3x+9}{(x-3)^2}$$

$$= \frac{-x^2+6x}{(x-3)^2} = \frac{-x(x-6)}{(x-3)^2} \quad x=0 \quad x=6 \quad x=3$$

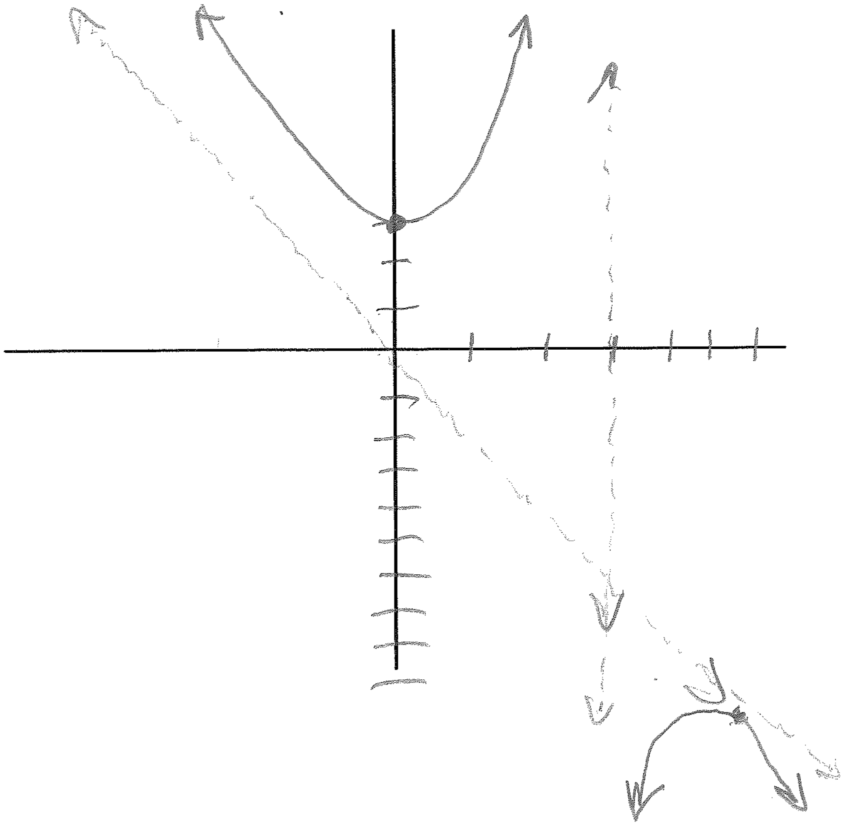
$f'(x)$: $-$ | $+$ | $+$ | $-$ at 0, $\boxed{3}$, 6

$(0,3) \qquad (6,-9)$

$$f''(x) = \frac{\overset{(x-3)^2(-2)(x-3)}{(x-3)^2(-2x+6)}-(-x^2+6x)(2)(x-3)}{(x-3)^4} = \frac{2(x-3)\left[(-1)(x-3)^2-(-x^2+6x)\right]}{(x-3)^4}$$

$$= \frac{2\left[(-1)(x^2-6x+9)+x^2-6x\right]}{(x-3)^3} = \frac{2[-9]}{(x-3)^3} = \frac{-18}{(x-3)^3}$$

$f''(x)$: $+$ | $-$ at $\boxed{3}$



Analyze and sketch the graph of $f(x) = -2x^{\frac{8}{3}} + 5x^{\frac{5}{3}}$

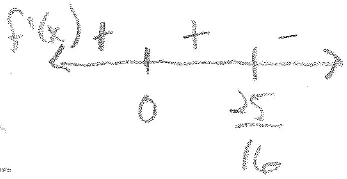
$f(x) \quad x^{5/3}(-2x+5) = 0$

$x = 0 \quad x = \frac{5}{2} \qquad (0,0) \quad (\frac{5}{2}, 0)$

$f'(x) = -\frac{16}{3}x^{5/3} + \frac{25}{3}x^{2/3}$

$0 = x^{2/3}\left(-\frac{16}{3}x + \frac{25}{3}\right) \qquad -\frac{16}{3}x = \frac{-25}{3}$

$x = \frac{-25}{3} \cdot \frac{-3}{16} = \frac{25}{16}$

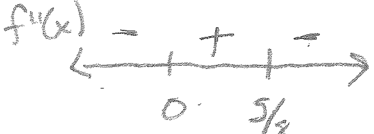


$(\frac{25}{16}, 3.944)$

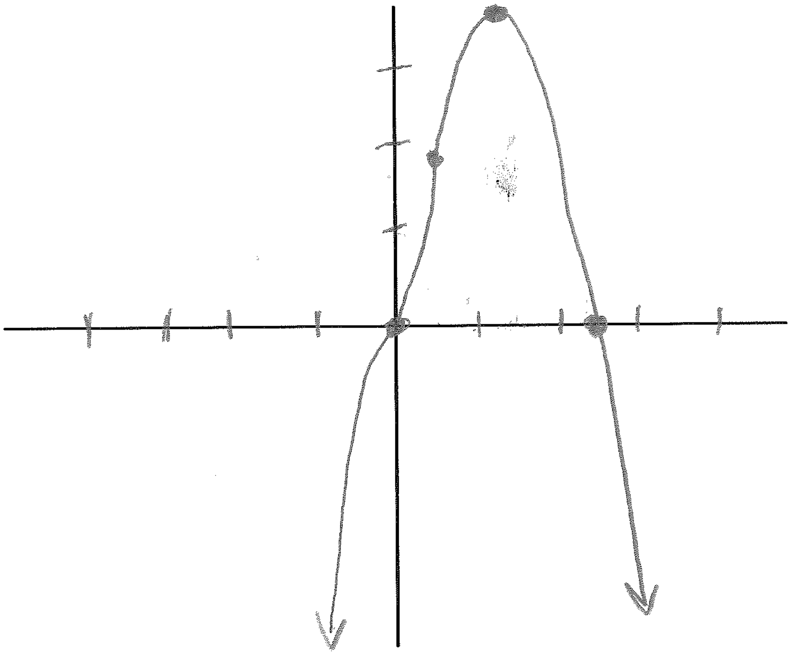
$f''(x) = -\frac{80}{9}x^{2/3} + \frac{50}{9}x^{-\frac{1}{3}}$

$= -\frac{80}{9}x^{2/3} + \frac{50}{9x^{1/3}} = \frac{-80x+50}{9x^{1/3}} \qquad x = \frac{5}{8}$

$x = 0$



$(0,0) \quad (5/8, 1.713)$



Analyze and sketch the graph of $y = x^3 - 6x^2 + 9x$

$f(x)$ $\quad x(x^2-6x+9)=0$ $\quad$ Double

$x(x-3)^2=0$ $\quad (0,0)$ $(3,0)$

$f'(x)= 3x^2-12x+9$

$0=x^2-4x+3$

$0=(x-3)(x-1)$

$f'(x)$: $+$ | $-$ | $+$

1 $\quad$ 3

$(1,4)$ $\quad (3,0)$

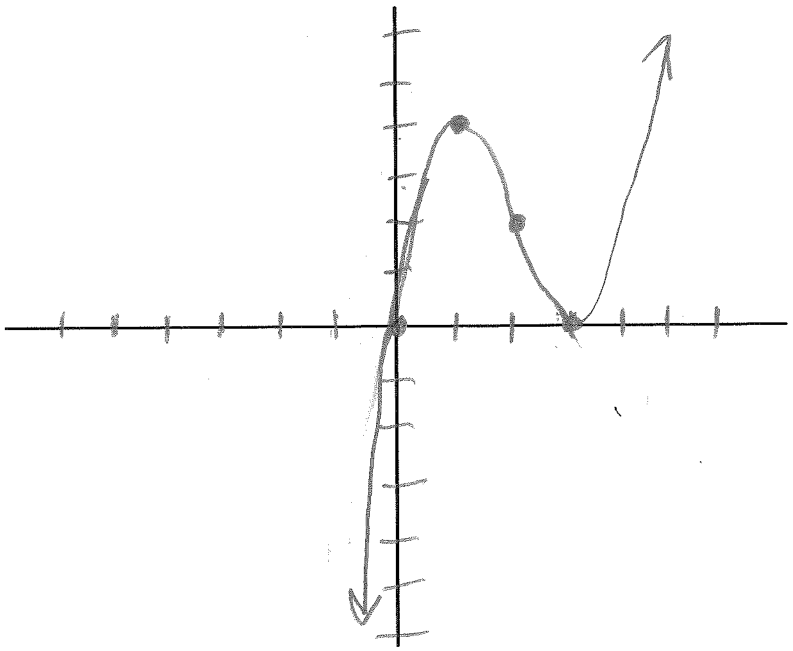
$f''(x)= 6x-12$

$0=6(x-2)$

$f''(x)$: $-$ | $+$

2

$(2,2)$



Analyze and sketch the graph of $f(x) = \frac{sinx}{1-cosx}$

f(x) $\sin x = 0$

$x = \pi n$ $(\pi n, 0)$

$1 - \cos x = 0$

$1 = \cos x$ $x = 0 + 2\pi n$

$0 = x$

$$f'(x) = \frac{(1-\cos x)(\cos x) - (\sin x)(-(-\sin x))}{(1-\cos x)^2} = \frac{\cos x - \cos^2 x - \sin^2 x}{(1-\cos x)^2}$$

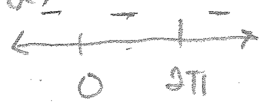
$$= \frac{\cos x - 1(\cos^2 x + \sin^2 x)}{(1-\cos x)^2} = \frac{\cos x - 1}{(1-\cos x)^2} = \frac{-1}{1-\cos x}$$

$\cos x - 1 = 0$

$\cos x = 1$

$x = 0 + 2\pi n$

f'(x)

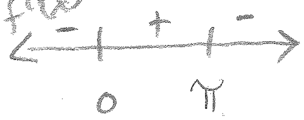


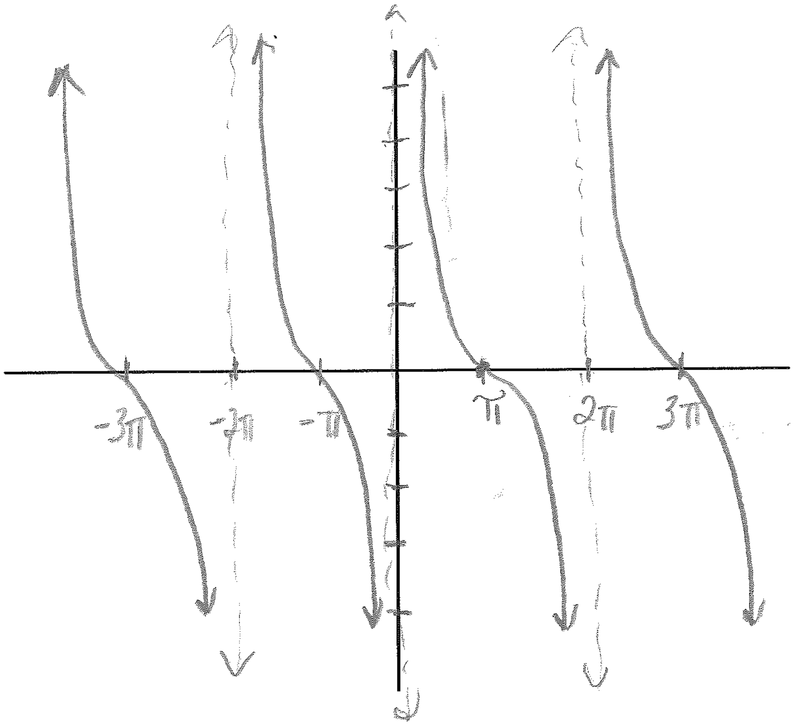
$$f''(x) = (-1)(-1)(1-\cos x)^{-2} = \frac{\sin x}{(1-\cos x)^2}$$

$x = \pi n$

$x = 0 + 2\pi n$

f''(x)





Analyze and sketch the graph of $y = \ln(x^2 - 2x + 4)$

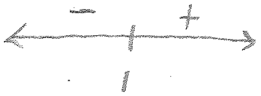
$f(x)$ $\quad 0 = \ln(x^2-2x+4)$ $\qquad \ln(0-0+4)$

$e^0 = x^2-2x+4$ $\qquad (0, \ln 4)$

$0 = x^2-2x+3$

$4-4(1)(3)$ = negative, no zeros

$f'(x) = \frac{1}{x^2-2x+4} \cdot (2x-2) = \frac{2(x-1)}{x^2-2x+4}$ $\quad x=1$



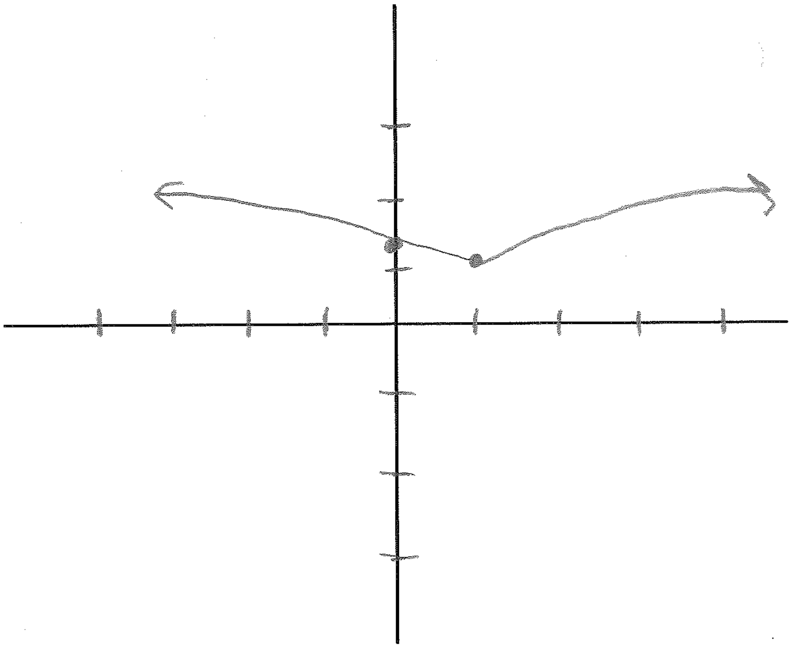
$(1, \ln 3)$

$$f''(x) = \frac{(x^2-2x+4)(2) - (2x-2)(2x-2)}{(x^2-2x+4)^2} = \frac{2(x^2-2x+4 - (x-1)(2x-2))}{(x^2-2x+4)^2}$$

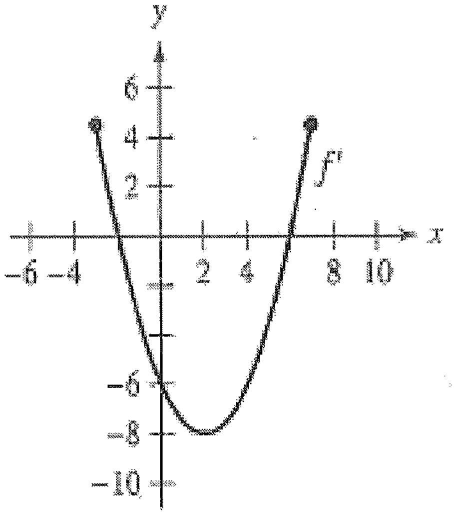
$$= \frac{2(x^2-2x+4-(2x^2-2x-2x+2))}{(x^2-2x+4)^2} = \frac{2(-x^2+2x+2)}{(x^2-2x+4)^2}$$

$$= \frac{-2(x^2-2x-2)}{(x^2-2x+4)^2}$$

None



Given the following graph of the derivative of a function f on the interval [-3,7], answer the questions below.



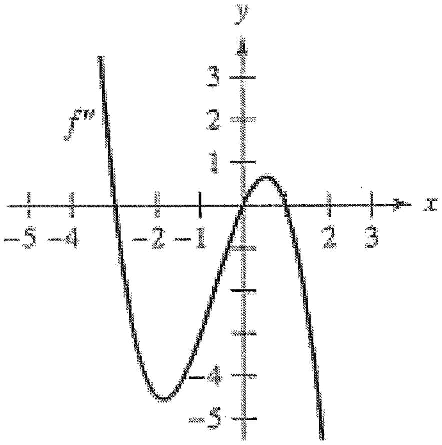
A. On what interval(s) is f decreasing? $(-2, 6)$ because $f'(x) < 0$

B. On what interval(s) is the graph of f concave up? $(2, 7)$ because $f'(x)$ is increasing

C. At what x-value(s) does f have relative extrema? $X = -2$ $X = 6$ because $f'(2) = 0$, $f'(x) > 0$ $(-4, -2)$ and $f'(x) < 0$ $(-2, 6)$
$f'(6) = 0$, $f'(x) < 0$, $(-2, 6)$ and $f'(x) > 0$ $(6, 7)$

D. At what x-value(s) does the graph of f have a point of inflection? $X = 2$ because $f'(x)$ changes from decreasing to increasing at $x = 2$.

Use the graph of the second derivative of a function f to answer the questions below.



A. On what interval(s) is the graph of f concave up? $(-\infty,-3) \cup (0,1)$ because $f''(x) > 0$

B. On what interval(s) is f' decreasing? $(-3,0) \cup (1,\infty)$ because $f''(x) < 0$

To Think About:

Can a graph cross its horizontal asymptote? Yes

Can a graph cross its vertical asymptote? no