Section 3.4 Concavity and the Second Derivative Test

1. $y = x^2 - x - 2$

   $y' = 2x - 1$

   $y'' = 2$

   $y'' > 0$ for all x.

   Concave upward: $(-\infty, \infty)$

2. $g(x) = 3x^2 - x^3$

$g'(x) = 6x - 3x^2$

$g''(x) = 6 - 6x$

$g''(x) = 0$ when $x = 1$.

Concave upward: $(-\infty, 1)$

Concave downward: $(1, \infty)$

Intervals:	$-\infty < x < 1$	$1 < x < \infty$
Sign of g'':	$g'' > 0$	$g'' < 0$
Conclusion:	Concave upward	Concave downward

3. $f(x) = -x^3 + 6x^2 - 9x - 1$

$f'(x) = -3x^2 + 12x - 9$

$f''(x) = -6x + 12 = -6(x - 2)$

$f''(x) = 0$ when $x = 2$.

Concave upward: $(-\infty, 2)$

Concave downward: $(2, \infty)$

Intervals:	$-\infty < x < 2$	$2 < x < \infty$
Sign of f'':	$f'' > 0$	$f'' < 0$
Conclusion:	Concave upward	Concave downward

4. $h(x) = x^5 - 5x + 2$

$h'(x) = 5x^4 - 5$

$h''(x) = 20x^3$

$h''(x) = 0$ when $x = 0$.

Concave upward: $(0, \infty)$

Concave downward: $(-\infty, 0)$

Intervals:	$-\infty < x < 0$	$0 < x < \infty$
Sign of h'':	$h'' < 0$	$h'' > 0$
Conclusion:	Concave downward	Concave upward

5. $f(x) = \dfrac{24}{x^2 + 12}$

$f'(x) = \dfrac{-48x}{(x^2 + 12)^2}$

$f''(x) = \dfrac{-144(4 - x^2)}{(x^2 + 12)^3}$

$f''(x) = 0$ when $x = \pm 2$.

Concave upward: $(-\infty, -2), (2, \infty)$

Concave downward: $(-2, 2)$

Intervals:	$-\infty < x < -2$	$-2 < x < 2$	$2 < x < \infty$
Sign of f'':	$f'' > 0$	$f'' < 0$	$f'' > 0$
Conclusion:	Concave upward	Concave downward	Concave upward

6. $f(x) = \dfrac{2x^2}{3x^2 + 1}$

$f'(x) = \dfrac{4x}{(3x^2 + 1)^2}$

$f''(x) = \dfrac{-4(3x - 1)(3x + 1)}{(3x^2 + 1)^3}$

$f''(x) = 0$ when $x = \pm\dfrac{1}{3}$.

Concave upward: $\left(-\dfrac{1}{3}, \dfrac{1}{3}\right)$

Concave downward: $\left(-\infty, -\dfrac{1}{3}\right), \left(\dfrac{1}{3}, \infty\right)$

Intervals:	$-\infty < x < -\frac{1}{3}$	$-\frac{1}{3} < x < \frac{1}{3}$	$\frac{1}{3} < x < \infty$
Sign of f'':	$f'' < 0$	$f'' > 0$	$f'' < 0$
Conclusion:	Concave downward	Concave upward	Concave downward

7. $f(x) = \dfrac{x^2+1}{x^2-1}$

$f' = \dfrac{-4x}{(x^2-1)^2}$

$f'' = \dfrac{4(3x^2+1)}{(x^2-1)^3}$

f is not continuous at $x = \pm 1$.

Intervals:	$-\infty < x < -1$	$-1 < x < 1$	$1 < x < \infty$
Sign of f'':	$f'' > 0$	$f'' < 0$	$f'' > 0$
Conclusion:	Concave upward	Concave downward	Concave upward

Concave upward: $(-\infty, -1), (1, \infty)$

Concave downward: $(-1, 1)$

8. $y = \frac{1}{270}(-3x^5 + 40x^3 + 135x)$

$y' = \frac{1}{270}(-15x^4 + 120x^2 + 135)$

$y'' = -\frac{2}{9}x(x-2)(x+2)$

$y'' = 0$ when $x = 0, \ \pm 2$.

Intervals:	$-\infty < x < -2$	$-2 < x < 0$	$0 < x < 2$	$2 < x < \infty$
Sign of y'':	$y'' > 0$	$y'' < 0$	$y'' > 0$	$y'' < 0$
Conclusion:	Concave upward	Concave downward	Concave upward	Concave downward

Concave upward: $(-\infty, -2), (0, 2)$

Concave downward: $(-2, 0), (2, \infty)$

9. $g(x) = \dfrac{x^2+4}{4-x^2}$

$g'(x) = \dfrac{16x}{(4-x^2)^2}$

$g''(x) = \dfrac{16(3x^2+4)}{(4-x^2)^3} = \dfrac{16(3x^2+4)}{(2-x)^3(2+x)^3}$

f is not continuous at $x = \pm 2$.

Intervals:	$-\infty < x < -2$	$-2 < x < 2$	$2 < x < \infty$
Sign of g'':	$g'' < 0$	$g'' > 0$	$g'' < 0$
Conclusion:	Concave downward	Concave upward	Concave downward

Concave upward: $(-2, 2)$

Concave downward: $(-\infty, -2), (2, \infty)$

10. $h(x) = \dfrac{x^2 - 1}{2x - 1}$

$h'(x) = \dfrac{2(x^2 - x + 1)}{(2x - 1)^2}$

$h''(x) = \dfrac{-6}{(2x - 1)^3}$

Intervals:	$-\infty < x < \frac{1}{2}$	$\frac{1}{2} < x < \infty$
Sign of h'':	$h'' > 0$	$h'' < 0$
Conclusion:	Concave upward	Concave downward

f'' is not continuous at $x = \dfrac{1}{2}$.

Concave upward: $\left(-\infty, \dfrac{1}{2}\right)$

Concave downward: $\left(\dfrac{1}{2}, \infty\right)$

11. $y = 2x - \tan x, \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$

$y' = 2 - \sec^2 x$

$y'' = -2\sec^2 x \tan x$

$y'' = 0$ when $x = 0$.

Intervals:	$-\frac{\pi}{2} < x < 0$	$0 < x < \frac{\pi}{2}$
Sign of y'':	$y'' > 0$	$y'' < 0$
Conclusion:	Concave upward	Concave downward

Concave upward: $\left(-\dfrac{\pi}{2}, 0\right)$

Concave downward: $\left(0, \dfrac{\pi}{2}\right)$

12. $y = x + 2\csc x, \quad (-\pi, \pi)$

$y' = 1 - 2\csc x \cot x$

$y'' = -2\csc x(-\csc^2 x) - 2\cot x(-\csc x \cot x)$

$= 2(\csc^3 x + \csc x \cot^2 x)$

$y'' = 0$ when $x = 0$.

Intervals:	$-\pi < x < 0$	$0 < x < \pi$
Sign of y'':	$y'' < 0$	$y'' > 0$
Conclusion:	Concave downward	Concave upward

Concave upward: $(0, \pi)$

Concave downward: $(-\pi, 0)$

13. $f(x) = x^3 - 9x^2 + 24x - 18$

$f'(x) = 3x^2 - 18x + 24$

$f''(x) = 6x - 18 = 0$ when $x = 3$.

Intervals:	$-\infty < x < 3$	$3 < x < \infty$
Sign of f'':	$f'' < 0$	$f'' > 0$
Conclusion:	Concave downward	Concave upward

Concave upward: $(3, \infty)$

Concave downward: $(-\infty, 3)$

Point of inflection: $(3, 0)$

14. $f(x) = -x^3 + 6x^2 - 5$

$f'(x) = -3x^2 + 12x$

$f''(x) = -6x + 12 = -6(x - 2) = 0$ when $x = 2$.

Intervals:	$-\infty < x < 2$	$2 < x < \infty$
Sign of f'':	$f'' > 0$	$f'' < 0$
Conclusion:	Concave upward	Concave downward

Concave upward: $(-\infty, 2)$

Concave downward: $(2, \infty)$

Point of inflection: $(2, 11)$

15. $f(x) = \frac{1}{2}x^4 + 2x^3$

$f'(x) = 2x^3 + 6x^2$

$f''(x) = 6x^2 + 12x = 6x(x + 2)$

$f''(x) = 0$ when $x = 0, -2$

Intervals:	$-\infty < x < -2$	$-2 < x < 2$	$0 < x < \infty$
Sign of f'':	$f'' > 0$	$f'' < 0$	$f'' > 0$
Conclusion:	Concave upward	Concave downward	Concave upward

Concave upward: $(-\infty, -2), (0, \infty)$

Concave downward: $(-2, 0)$

Points of inflection: $(-2, -8)$ and $(0, 0)$

16. $f(x) = 4 - x - 3x^4$

$f'(x) = -1 - 12x^3$

$f''(x) = -36x^2 = 0$ when $x = 0$.

Intervals:	$-\infty < x < 0$	$0 < x < \infty$
Sign of f'':	$f'' < 0$	$f'' < 0$
Conclusion:	Concave downward	Concave downward

Concave downward: $(-\infty, \infty)$

No points of inflection

17. $f(x) = x(x - 4)^3$

$f'(x) = x\left[3(x - 4)^2\right] + (x - 4)^3 = (x - 4)^2(4x - 4)$

$f''(x) = 4(x - 1)\left[2(x - 4)\right] + 4(x - 4)^2 = 4(x - 4)\left[2(x - 1) + (x - 4)\right] = 4(x - 4)(3x - 6) = 12(x - 4)(x - 2)$

$f''(x) = 12(x - 4)(x - 2) = 0$ when $x = 2, 4$.

Intervals:	$-\infty < x < 2$	$2 < x < 4$	$4 < x < \infty$
Sign of $f''(x)$:	$f''(x) > 0$	$f''(x) < 0$	$f''(x) > 0$
Conclusion:	Concave upward	Concave downward	Concave upward

Concave upward: $(-\infty, 2), (4, \infty)$

Concave downward: $(2, 4)$

Points of inflection: $(2, -16), (4, 0)$

18. $f(x) = (x-2)^3(x-1)$

$f'(x) = (x-2)^2(4x-5)$

$f''(x) = 6(x-2)(2x-3)$

$f''(x) = 0$ when $x = \frac{3}{2}, 2$.

Intervals:	$-\infty < x < \frac{3}{2}$	$\frac{3}{2} < x < 2$	$2 < x < \infty$
Sign of f'':	$f'' > 0$	$f'' < 0$	$f'' > 0$
Conclusion:	Concave upward	Concave downward	Concave upward

Concave upward: $\left(-\infty, \frac{3}{2}\right), (2, \infty)$

Concave downward: $\left(\frac{3}{2}, 2\right)$

Points of inflection: $\left(\frac{3}{2}, -\frac{1}{16}\right), (2, 0)$

19. $f(x) = x\sqrt{x+3}$, Domain: $[-3, \infty)$

$f'(x) = x\left(\frac{1}{2}\right)(x+3)^{-1/2} + \sqrt{x+3} = \frac{3(x+2)}{2\sqrt{x+3}}$

$f''(x) = \frac{6\sqrt{x+3} - 3(x+2)(x+3)^{-1/2}}{4(x+3)}$

$= \frac{3(x+4)}{4(x+3)^{3/2}} = 0$ when $x = -4$.

$x = -4$ is not in the domain. f'' is not continuous at $x = -3$.

Interval:	$-3 < x < \infty$
Sign of f'':	$f'' > 0$
Conclusion:	Concave upward

Concave upward: $(-3, \infty)$

There are no points of inflection.

20. $f(x) = x\sqrt{9-x}$, Domain: $x \le 9$

$f'(x) = \frac{3(6-x)}{2\sqrt{9-x}}$

$f''(x) = \frac{3(x-12)}{4(9-x)^{3/2}} = 0$ when $x = 12$.

$x = 12$ is not in the domain. f'' is not continuous at $x = 9$.

Interval:	$-\infty < x < 9$
Sign of f'':	$f'' < 0$
Conclusion:	Concave downward

Concave downward: $(-\infty, 9)$

No point of inflection

21. $f(x) = \frac{4}{x^2+1}$

$f'(x) = \frac{-8x}{(x^2+1)^2}$

$f''(x) = \frac{8(3x^2-1)}{(x^2+1)^3}$

$f''(x) = 0$ for $x = \pm\frac{\sqrt{3}}{3}$

Intervals:	$-\infty < x < -\frac{\sqrt{3}}{3}$	$-\frac{\sqrt{3}}{3} < x < \frac{\sqrt{3}}{3}$	$\frac{\sqrt{3}}{3} < x < \infty$
Sign of f'':	$f'' > 0$	$f'' < 0$	$f'' > 0$
Conclusion:	Concave upward	Concave downward	Concave upward

Concave upward: $\left(-\infty, -\frac{\sqrt{3}}{3}\right), \left(\frac{\sqrt{3}}{3}, \infty\right)$

Concave downward: $\left(-\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right)$

Points of inflection: $\left(-\frac{\sqrt{3}}{3}, 3\right)$ and $\left(\frac{\sqrt{3}}{3}, 3\right)$

22. $f(x) = \dfrac{x+3}{\sqrt{x}}$, Domain: $x > 0$

$f'(x) = \dfrac{x-3}{2x^{3/2}}$

$f''(x) = \dfrac{9-x}{4x^{5/2}} = 0$ when $x = 9$

Intervals:	$0 < x < 9$	$9 < x < \infty$
Sign of f'':	$f'' > 0$	$f'' < 0$
Conclusion:	Concave upward	Concave downward

Concave upward: $(0, 9)$

Concave downward: $(9, \infty)$

Points of inflection: $(9, 4)$

23. $f(x) = \sin\dfrac{x}{2}$, $0 \le x \le 4\pi$

$f'(x) = \dfrac{1}{2}\cos\left(\dfrac{x}{2}\right)$

$f''(x) = -\dfrac{1}{4}\sin\left(\dfrac{x}{2}\right)$

$f''(x) = 0$ when $x = 0, 2\pi, 4\pi$.

Intervals:	$0 < x < 2\pi$	$2\pi < x < 4\pi$
Sign of f'':	$f'' < 0$	$f'' > 0$
Conclusion:	Concave downward	Concave upward

Concave upward: $(2\pi, 4\pi)$

Concave downward: $(0, 2\pi)$

Point of inflection: $(2\pi, 0)$

24. $f(x) = 2\csc\dfrac{3x}{2}$, $0 < x < 2\pi$

$f'(x) = -3\csc\dfrac{3x}{2}\cot\dfrac{3x}{2}$

$f''(x) = \dfrac{9}{2}\left(\csc^3\dfrac{3x}{2} + \csc\dfrac{3x}{2}\cot^2\dfrac{3x}{2}\right) \neq 0$ for any x in the domain of f.

f'' is not continuous at $x = \dfrac{2\pi}{3}$ and $x = \dfrac{4\pi}{3}$.

Intervals:	$0 < x < \dfrac{2\pi}{3}$	$\dfrac{2\pi}{3} < x < \dfrac{4\pi}{3}$	$\dfrac{4\pi}{3} < x < 2\pi$
Sign of $f''(x)$:	$f'' > 0$	$f'' < 0$	$f'' > 0$
Conclusion:	Concave upward	Concave downward	Concave upward

Concave upward: $\left(0, \dfrac{2\pi}{3}\right), \left(\dfrac{4\pi}{3}, 2\pi\right)$

Concave downward: $\left(\dfrac{2\pi}{3}, \dfrac{4\pi}{3}\right)$

No point of inflection

25. $f(x) = \sec\left(x - \frac{\pi}{2}\right), 0 < x < 4\pi$

$f'(x) = \sec\left(x - \frac{\pi}{2}\right)\tan\left(x - \frac{\pi}{2}\right)$

$f''(x) = \sec^3\left(x - \frac{\pi}{2}\right) + \sec\left(x - \frac{\pi}{2}\right)\tan^2\left(x - \frac{\pi}{2}\right) \neq 0$ for any x in the domain of f.

f'' is not continuous at $x = \pi$, $x = 2\pi$, and $x = 3\pi$.

Intervals:	$0 < x < \pi$	$\pi < x < 2\pi$	$2\pi < x < 3\pi$	$3\pi < x < 4\pi$
Sign of f'':	$f'' > 0$	$f'' < 0$	$f'' > 0$	$f'' < 0$
Conclusion:	Concave upward	Concave downward	Concave upward	Concave upward

Concave upward: $(0, \pi), (2\pi, 3\pi)$

Concave downward: $(\pi, 2\pi), (3\pi, 4\pi)$

No point of inflection

26. $f(x) = \sin x + \cos x, 0 \le x \le 2\pi$

$f'(x) = \cos x - \sin x$

$f''(x) = \sin x - \cos x$

$f''(x) = 0$ when $x = \frac{3\pi}{4}, \frac{7\pi}{4}$.

Intervals:	$0 < x < \frac{3\pi}{4}$	$\frac{3\pi}{4} < x < \frac{7\pi}{4}$	$\frac{7\pi}{4} < x < 2\pi$
Sign of f'':	$f''(x) < 0$	$f''(x) > 0$	$f''(x) < 0$
Conclusion:	Concave downward	Concave upward	Concave downward

Concave upward: $\left(\frac{3\pi}{4}, \frac{7\pi}{4}\right)$

Concave downward: $\left(0, \frac{3\pi}{4}\right), \left(\frac{7\pi}{4}, 2\pi\right)$

Points of inflection: $\left(\frac{3\pi}{4}, 0\right), \left(\frac{7\pi}{4}, 0\right)$

27. $f(x) = 2\sin x + \sin 2x, 0 \le x \le 2\pi$

$f'(x) = 2\cos x + 2\cos 2x$

$f''(x) = -2\sin x - 4\sin 2x = -2\sin x(1 + 4\cos x)$

$f''(x) = 0$ when $x = 0, 1.823, \pi, 4.460$.

Intervals:	$0 < x < 1.823$	$1.823 < x < \pi$	$\pi < x < 4.460$	$4.460 < x < 2\pi$
Sign of f'':	$f'' < 0$	$f'' > 0$	$f'' < 0$	$f'' > 0$
Conclusion:	Concave downward	Concave upward	Concave downward	Concave upward

Concave upward: $(1.823, \pi), (4.460, 2\pi)$

Concave downward: $(0, 1.823), (\pi, 4.460)$

Points of inflection: $(1.823, 1.452), (\pi, 0), (4.46, -1.452)$

28. $f(x) = x + 2\cos x, [0, 2\pi]$

$f'(x) = 1 - 2\sin x$

$f''(x) = -2\cos x$

$f''(x) = 0$ when $x = \frac{\pi}{2}, \frac{3\pi}{2}$.

Intervals:	$0 < x < \frac{\pi}{2}$	$\frac{\pi}{2} < x < \frac{3\pi}{2}$	$\frac{3\pi}{2} < x < 2\pi$
Sign of f'':	$f'' < 0$	$f'' > 0$	$f'' < 0$
Conclusion:	Concave downward	Concave upward	Concave downward

Concave upward: $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

Concave downward: $\left(0, \frac{\pi}{2}\right), \left(\frac{3\pi}{2}, 2\pi\right)$

Points of inflection: $\left(\frac{\pi}{2}, \frac{\pi}{2}\right), \left(\frac{3\pi}{2}, \frac{3\pi}{2}\right)$

29. $y = e^{-3/x}$

$y' = \frac{3}{x^2}e^{-3/x}$

$y'' = \frac{e^{-3/x}(9 - 6x)}{x^4}$

$y'' = 0$ when $x = \frac{3}{2}$. y is not defined at $x = 0$.

Test intervals:	$-\infty < x < 0$	$0 < x < \frac{3}{2}$	$\frac{3}{2} < x < \infty$
Sign of y'':	$y'' > 0$	$y'' > 0$	$y'' < 0$
Conclusion:	Concave upward	Concave upward	Concave downward

Point of inflection: $\left(\frac{3}{2}, e^{-2}\right)$

Concave upward: $(-\infty, 0), \left(0, \frac{3}{2}\right)$

Concave downward: $\left(\frac{3}{2}, \infty\right)$

30. $y = \frac{1}{2}(e^x - e^{-x})$

$y' = \frac{1}{2}(e^x + e^{-x})$

$y'' = \frac{1}{2}(e^x - e^{-x})$

$y'' = 0$ when $x = 0$.

Test interval:	$-\infty < x < 0$	$0 < x < \infty$
Sign of y'':	$y'' < 0$	$y'' > 0$
Conclusion:	Concave downward	Concave upward

Point of inflection: $(0, 0)$

Concave upward: $(0, \infty)$

Concave downward: $(-\infty, 0)$

31. $f(x) = x - \ln x$, Domain: $x > 0$

$f'(x) = 1 - \frac{1}{x}$

$f''(x) = \frac{1}{x^2}$

$f''(x) > 0$ on the entire domain of f. There are no points of inflection.

Concave upward: $(0, \infty)$

32. $y = \ln\sqrt{x^2 + 9} = \frac{1}{2}\ln(x^2 + 9)$

$y' = \frac{x}{x^2 + 9}$

$y'' = \frac{9 - x^2}{(x^2 + 9)^2}$

$y'' = 0$ when $x = \pm 3$.

Test interval:	$-\infty < x < -3$	$-3 < x < 3$	$3 < x < \infty$
Sign of y'':	$y'' < 0$	$y'' > 0$	$y'' < 0$
Conclusion:	Concave downward	Concave upward	Concave downward

Points of inflection: $\left(\pm 3, \frac{1}{2}\ln 18\right)$

Concave upward: $(-3, 3)$

Concave downward: $(-\infty, -3), (3, \infty)$

33. $f(x) = \arcsin x^{4/5}, \quad -1 \le x \le 1$

$$f'(x) = \frac{4}{5x^{1/5}\sqrt{1 - x^{8/5}}}$$

$$f''(x) = \frac{20x^{8/5} - 4}{25x^{6/5}\left(1 - x^{8/5}\right)^{3/2}}$$

$f''(x) = 0$ when $20x^{8/5} = 4 \Rightarrow x^{8/5} = \frac{1}{5} \Rightarrow x = \pm\left(\frac{1}{5}\right)^{5/8} \approx \pm 0.3657.$

f'' is undefined at $x = 0$.

Test intervals:	$-1 < x < -\left(\frac{1}{5}\right)^{5/8}$	$-\left(\frac{1}{5}\right)^{5/8} < x < 0$	$0 < x < \left(\frac{1}{5}\right)^{5/8}$	$\left(\frac{1}{5}\right)^{5/8} < x < 1$
Sign of f'':	$f'' > 0$	$f'' < 0$	$f'' < 0$	$f'' > 0$
Conclusion:	Concave upward	Concave downward	Concave downward	Concave upward

Points of inflection: $\left(\pm\left(\frac{1}{5}\right)^{5/8}, \arcsin\sqrt{\frac{1}{5}}\right) \approx (\pm 0.3657, 0.4636)$

Concave upward: $\left(-1, -\left(\frac{1}{5}\right)^{5/8}\right), \left(\left(\frac{1}{5}\right)^{5/8}, 1\right)$

Concave downward: $\left(-\left(\frac{1}{5}\right)^{5/8}, 0\right), \left(0, \left(\frac{1}{5}\right)^{5/8}\right)$

34. $f(x) = \arctan\left(x^2\right)$

$$f'(x) = \frac{2x}{x^4 + 1}$$

$$f''(x) = \frac{2\left(1 - 3x^4\right)}{\left(x^4 + 1\right)^2}$$

$f''(x) = 0$ when $3x^4 = 1 \Rightarrow x = \pm\sqrt[4]{\frac{1}{3}} \approx \pm 0.7598.$

Test interval:	$-\infty < x < -\sqrt[4]{\frac{1}{3}}$	$-\sqrt[4]{\frac{1}{3}} < x < \sqrt[4]{\frac{1}{3}}$	$\sqrt[4]{\frac{1}{3}} < x < \infty$
Sign of $f''(x)$:	$f'' < 0$	$f'' > 0$	$f'' < 0$
Conclusion:	Concave downward	Concave upward	Concave downward

Points of inflection: $\left(\pm\sqrt[4]{\frac{1}{3}}, \arctan\sqrt{\frac{1}{3}}\right) \approx (\pm 0.7598, 0.5236)$

Concave upward: $\left(-\sqrt[4]{\frac{1}{3}}, \sqrt[4]{\frac{1}{3}}\right)$

Concave downward: $\left(-\infty, -\sqrt[4]{\frac{1}{3}}\right), \left(\sqrt[4]{\frac{1}{3}}, \infty\right)$

35. $f(x) = 6x - x^2$

$f'(x) = 6 - 2x$

$f''(x) = -2$

Critical number: $x = 3$

$f''(3) = -2 < 0$

Therefore, $(3, 9)$ is a relative maximum.

36. $f(x) = x^2 + 3x - 8$

$f'(x) = 2x + 3$

$f''(x) = 2$

Critical number: $x = -\frac{3}{2}$

$f''\left(-\frac{3}{2}\right) = 2 > 0$

Therefore, $\left(-\frac{3}{2}, -\frac{41}{4}\right)$ is a relative minimum.

37. $f(x) = x^3 - 3x^2 + 3$

$f'(x) = 3x^2 - 6x = 3x(x - 2)$

$f''(x) = 6x - 6 = 6(x - 1)$

Critical numbers: $x = 0, x = 2$

$f''(0) = -6 < 0$

Therefore, $(0, 3)$ is a relative maximum.

$f''(2) = 6 > 0$

Therefore, $(2, -1)$ is a relative minimum.

38. $f(x) = -x^3 + 7x^2 - 15x$

$f'(x) = -3x^2 + 14x - 15 = -(x - 3)(3x - 5)$

$f''(x) = -6x + 14 = -2(3x - 7)$

Critical numbers: $x = 3, \frac{5}{3}$

$f''(3) = -4 < 0$

Therefore, $(3, 9)$ is a relative maximum.

$f''\left(\frac{5}{3}\right) = 4 > 0$

Therefore, $\left(\frac{5}{3}, -\frac{275}{27}\right)$ is a relative minimum.

39. $f(x) = x^4 - 4x^3 + 2$

$f'(x) = 4x^3 - 12x^2 = 4x^2(x - 3)$

$f''(x) = 12x^2 - 24x = 12x(x - 2)$

Critical numbers: $x = 0, x = 3$

However, $f''(0) = 0$, so you must use the First Derivative Test. $f'(x) < 0$ on the intervals $(-\infty, 0)$ and $(0, 3)$; so, $(0, 2)$ is not an extremum. $f''(3) > 0$ so $(3, -25)$ is a relative minimum.

40. $f(x) = -x^4 + 4x^3 + 8x^2$

$f'(x) = -4x^3 + 12x^2 + 16x = -4x(x - 4)(x + 1)$

$f''(x) = -12x^2 + 24x + 16 = -4(3x^2 - 6x - 4)$

Critical numbers: $x = -1, 0, 4$

$f''(-1) = -20 < 0$

Therefore $(-1, 3)$ is a relative maximum.

$f''(0) = 16 > 0$

Therefore, $(0, 0)$ is a relative minimum.

$f''(4) = -80 < 0$

Therefore, $(4, 128)$ is a relative maximum.

41. $f(x) = x^{2/3} - 3$

$f'(x) = \dfrac{2}{3x^{1/3}}$

$f''(x) = -\dfrac{2}{9x^{4/3}}$

Critical number: $x = 0$

However, $f''(0)$ is undefined, so you must use the First Derivative Test. Because $f'(x) < 0$ on $(-\infty, 0)$ and $f'(x) > 0$ on $(0, \infty)$, $(0, -3)$ is a relative minimum.

42. $f(x) = \sqrt{x^2 + 1}$

$f'(x) = \dfrac{x}{\sqrt{x^2 + 1}}$

$f''(x) = \dfrac{1}{(x^2 + 1)^{3/2}}$

Critical number: $x = 0$

$f''(0) = 1 > 0$

Therefore, $(0, 1)$ is a relative minimum.

43. $f(x) = x + \dfrac{4}{x}$

$f'(x) = 1 - \dfrac{4}{x^2} = \dfrac{x^2 - 4}{x^2}$

$f''(x) = \dfrac{8}{x^3}$

Critical numbers: $x = \pm 2$

$f''(-2) = -1 < 0$

Therefore, $(-2, -4)$ is a relative maximum.

$f''(2) = 1 > 0$

Therefore, $(2, 4)$ is a relative minimum.

44. $f(x) = \dfrac{x}{x-1}$

$f'(x) = \dfrac{-1}{(x-1)^2}$

There are no critical numbers and $x = 1$ is not in the domain. There are no relative extrema.

45. $f(x) = \cos x - x, 0 \le x \le 4\pi$

$f'(x) = -\sin x - 1 \le 0$

Therefore, f is non-increasing and there are no relative extrema.

46. $f(x) = 2\sin x + \cos 2x, 0 \le x \le 2\pi$

$f'(x) = 2\cos x - 2\sin 2x = 2\cos x - 4\sin x \cos x$

$= 2\cos x(1 - 2\sin x) = 0$ when $x = \dfrac{\pi}{6}, \dfrac{\pi}{2}, \dfrac{5\pi}{6}, \dfrac{3\pi}{2}$.

$f''(x) = -2\sin x - 4\cos 2x$

$f''\left(\dfrac{\pi}{6}\right) = -3 < 0$

Therefore, $\left(\dfrac{\pi}{6}, \dfrac{3}{2}\right)$ is a relative maximum.

$f''\left(\dfrac{\pi}{2}\right) = 2 > 0$

Therefore, $\left(\dfrac{\pi}{2}, 1\right)$ is a relative minimum.

$f''\left(\dfrac{5\pi}{6}\right) = -3 < 0$

Therefore, $\left(\dfrac{5\pi}{6}, \dfrac{3}{2}\right)$ is a relative maximum.

$f''\left(\dfrac{3\pi}{2}\right) = 6 > 0$

Therefore, $\left(\dfrac{3\pi}{2}, -3\right)$ is a relative minimum.

47. $y = f(x) = 8x^2 - \ln x$

$f'(x) = 16x - \dfrac{1}{x}$

$f''(x) = 16 + \dfrac{1}{x^2}$

$f'(x) = 0 \Rightarrow 16x = \dfrac{1}{x} \Rightarrow 16x^2 = 1 \Rightarrow x = \pm\dfrac{1}{4}$

Critical number:

$x = \dfrac{1}{4} \quad \left(x = -\dfrac{1}{4} \text{ is not in the domain.}\right)$

$f''\left(\dfrac{1}{4}\right) > 0$

Therefore, $\left(\dfrac{1}{4}, \dfrac{1}{2} - \ln\dfrac{1}{4}\right) = \left(\dfrac{1}{4}, \dfrac{1}{2} + \ln 4\right)$ is a relat

minimum.

48. $y = f(x) = x \ln x$

$f'(x) = \ln x + 1$

$f''(x) = \dfrac{1}{x}$

Critical number: $\ln x + 1 = 0 \Rightarrow \ln x = -1$

$\Rightarrow x = e^{-1} = \dfrac{1}{e}$

$f''\left(\dfrac{1}{e}\right) > 0$

Therefore, $\left(\dfrac{1}{e}, -\dfrac{1}{e}\right)$ is a relative minimum.

49. $y = f(x) = \dfrac{x}{\ln x}$

Domain: $0 < x < 1, x > 1$

$f'(x) = \dfrac{(\ln x)(1) - (x)(1/x)}{(\ln x)^2} = \dfrac{\ln x - 1}{(\ln x)^2}$

$f''(x) = \dfrac{2 - \ln x}{x(\ln x)}$

Critical number: $x = e$

$f''(e) > 0$

Therefore, (e, e) is a relative minimum.

50. $y = f(x) = x^2 \ln \frac{x}{4}$, Domain: $x > 0$

$$f'(x) = x^2\left(\frac{1}{x}\right) + 2x \ln \frac{x}{4} = x\left(1 + 2 \ln \frac{x}{4}\right)$$

$$f''(x) = 1 + 2 \ln \frac{x}{4} + 2x\left(\frac{1}{x}\right) = 3 + 2 \ln \frac{x}{4}$$

Critical number: $x = 4e^{-1/2}$

$f''\left(4e^{-1/2}\right) > 0$

Therefore, $\left(4e^{-1/2}, -8e^{-1}\right)$ is a relative minimum.

51. $f(x) = \dfrac{e^x + e^{-x}}{2}$

$$f'(x) = \frac{e^x - e^{-x}}{2}$$

$$f''(x) = \frac{e^x + e^{-x}}{2}$$

Critical number: $x = 0$

$f''(0) > 0$

Therefore, $(0, 1)$ is a relative minimum.

52. $g(x) = \dfrac{1}{\sqrt{2\pi}}e^{-(x-3)^2/2}$

$$g'(x) = \frac{-1}{\sqrt{2\pi}}(x - 3)e^{-(x-3)^2/2}$$

$$g''(x) = \frac{1}{\sqrt{2\pi}}(x - 2)(x - 4)e^{-(x-3)^2/2}$$

Critical number: $x = 3$

$g''(3) < 0$

Therefore, $\left(3, \dfrac{1}{\sqrt{2\pi}}\right) \approx (3, 0.399)$ is a relative minimum.

53. $f(x) = x^2e^{-x}$

$$f'(x) = -x^2e^{-x} + 2xe^{-x} = xe^{-x}(2 - x)$$

$$f''(x) = -e^{-x}\left(2x - x^2\right) + e^{-x}(2 - 2x)$$

$$= e^{-x}\left(x^2 - 4x + 2\right)$$

Critical numbers: $x = 0, 2$

$f''(0) > 0$

Therefore, $(0, 0)$ is a relative minimum.

$f''(2) < 0$

Therefore, $\left(2, 4e^{-2}\right)$ is a relative maximum.

54. $f(x) = xe^{-x}$

$$f'(x) = -xe^{-x} + e^{-x} = e^{-x}(1 - x)$$

$$f''(x) = -e^{-x} + \left(-e^{-x}\right)(1 - x) = e^{-x}(x - 2)$$

Critical number: $x = 1$

$f''(1) < 0$

Therefore, $\left(1, e^{-1}\right)$ is a relative maximum.

55. $f(x) = 8x\left(4^{-x}\right)$

$$f'(x) = -8\left(4^{-x}\right)(x \ln 4 - 1)$$

$$f''(x) = 8\left(4^{-x}\right) \ln 4(x \ln 4 - 2)$$

Critical number: $x = \dfrac{1}{\ln 4} = \dfrac{1}{2 \ln 2}$

$f''\left(\dfrac{1}{2 \ln 2}\right) < 0$

Therefore, $\left(\dfrac{1}{2 \ln 2}, \dfrac{4e^{-1}}{\ln 2}\right)$ is a relative maximum.

56. $y = f(x) = x^2 \log_3 x = x^2 \dfrac{\ln x}{\ln 3}$

$$f'(x) = \frac{x(2 \ln x + 1)}{\ln 3}$$

$$f''(x) = \frac{2 \ln x + 3}{\ln 3}$$

Critical number: $\ln x = -\dfrac{1}{2} \Rightarrow x = e^{-1/2}$

$f''\left(e^{-1/2}\right) > 0$

Therefore, $\left(e^{-1/2}, -0.1674\right)$ is a relative minimum.

57. $f(x) = \operatorname{arcsec} x - x$

$f'(x) = \dfrac{1}{|x|\sqrt{x^2 - 1}} - 1 = 0$ when $|x|\sqrt{x^2 - 1} =$

$$x^2\left(x^2 - 1\right) = 1$$

$x^4 - x^2 - 1 = 0$ when $x^2 = \dfrac{1 + \sqrt{5}}{2}$

or $x = \pm\sqrt{\dfrac{1 + \sqrt{5}}{2}} = \pm 1.272.$

$$f''(x) = -\frac{1}{x\sqrt{x^2 - 1}|x|} - \frac{x}{\left(x^2 - 1\right)^{3/2}|x|}$$

$f''(1.272) < 0$

Therefore, $(1.272, -0.606)$ is a relative maximum.

$f''(-1.272) > 0$

Therefore, $(-1.272, 3.747)$ is a relative minimum.

58. $f(x) = \arcsin x - 2x$

$$f'(x) = \frac{1}{\sqrt{1 - x^2}} - 2$$

$$f''(x) = \frac{x}{(1 - x^2)^{3/2}}$$

Critical numbers: $x = \pm\frac{\sqrt{3}}{2}$

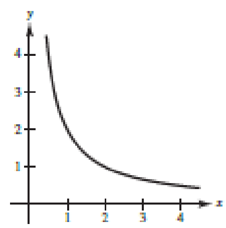
$$f''\left(\frac{\sqrt{3}}{2}\right) > 0$$

$\left(\frac{\sqrt{3}}{2}, -0.68\right)$ is a relative minimum.

$$f''\left(-\frac{\sqrt{3}}{2}\right) < 0$$

$\left(-\frac{\sqrt{3}}{2}, 0.68\right)$ is a relative maximum.

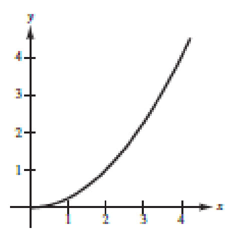
59. (a)



$f' < 0$ means f decreasing

f' increasing means concave upward

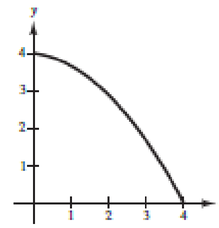
(b)



$f' > 0$ means f increasing

f' increasing means concave upward

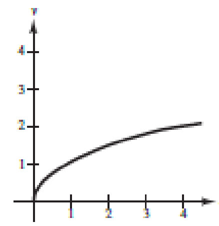
60. (a)



$f' < 0$ means f decreasing

f' decreasing means concave downward

(b)



$f' > 0$ means f increasing

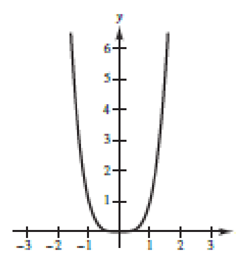
f' decreasing means concave downward

61. Answers will vary. *Sample answer*:

Let $f(x) = x^4$.

$f''(x) = 12x^2$

$f''(0) = 0$, but $(0, 0)$ is not a point of inflection.



62. (a) The rate of change of sales is increasing.

$S'' > 0$

(b) The rate of change of sales is decreasing.

$S' > 0, S'' < 0$

(c) The rate of change of sales is constant.

$S' = C, S'' = 0$

(d) Sales are steady.

$S = C, S' = 0, S'' = 0$

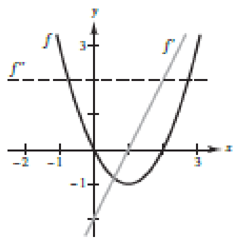
(e) Sales are declining, but at a lower rate.

$S' < 0, S'' > 0$

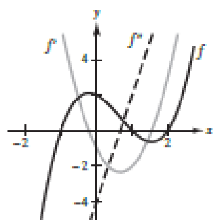
(f) Sales have bottomed out and have started to rise.

$S' > 0, S'' > 0$ Answers will vary.

63. (a)



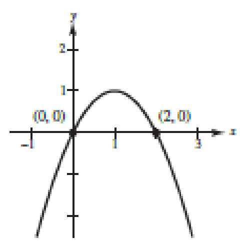
(b)



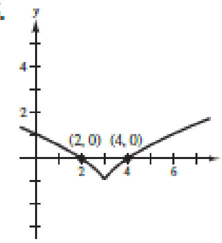
64. (a) The graph of f is increasing and concave downward: $f' > 0,\ f'' < 0.$

(b) The graph of f is decreasing and concave upward: $f' < 0,\ f'' > 0.$

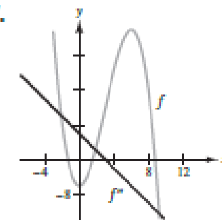
65.



66.



67.



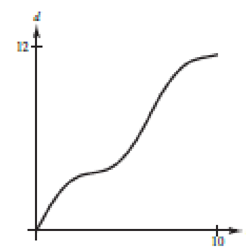
f'' is linear.

f' is quadratic.

f is cubic.

f concave upward on $(-\infty, 3)$, downward on $(3, \infty)$.

68. (a)

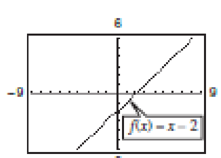


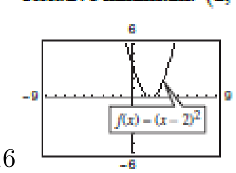
(b) Because the depth d is always increasing, there are no relative extrema. $f'(x) > 0$

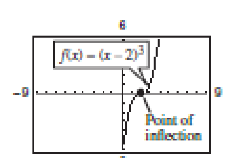
(c) The rate of change of d is decreasing until you reach the widest point of the jug, then the rate increases until you reach the narrowest part of the jug's neck, then the rate decreases until you reach the top of the jug.

69. (a)

$n = 1$:	$n = 2$:	$n = 3$:	$n = 4$:
$f(x) = x - 2$	$f(x) = (x - 2)^2$	$f(x) = (x - 2)^3$	$f(x) = (x - 2)^4$
$f'(x) = 1$	$f'(x) = 2(x - 2)$	$f'(x) = 3(x - 2)^2$	$f'(x) = 4(x - 2)^3$
$f''(x) = 0$	$f''(x) = 2$	$f''(x) = 6(x - 2)$	$f''(x) = 12(x - 2)^2$
No point of inflection	No point of inflection	Point of inflection: $(2, 0)$	No point of inflection
	Relative minimum: $(2, 0)$		Relative minimum: $(2, 0)$







Conclusion: If $n \geq 3$ and n is odd, then $(2, 0)$ is point of inflection. If $n \geq 2$ and n is even, then $(2, 0)$ is a relative minimum

(b) Let $f(x) = (x-2)^n$, $f'(x) = n(x-2)^{n-1}$, $f''(x) = n(n-1)(x-2)^{n-2}$.

For $n \geq 3$ and odd, $n-2$ is also odd and the concavity changes at $x = 2$.

For $n \geq 4$ and even, $n-2$ is also even and the concavity does not change at $x = 2$.

So, $x = 2$ is point of inflection if and only if $n \geq 3$ is odd.

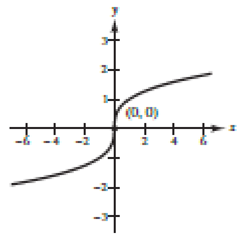
70. (a) $f(x) = \sqrt[3]{x}$

$f'(x) = \frac{1}{3}x^{-2/3}$

$f''(x) = -\frac{2}{9}x^{-5/3}$

Point of inflection: $(0, 0)$

(b) $f''(x)$ does not exist at $x = 0$.



71. $f(x) = ax^3 + bx^2 + cx + d$

Relative maximum: $(3, 3)$

Relative minimum: $(5, 1)$

Point of inflection: $(4, 2)$

$f'(x) = 3ax^2 + 2bx + c,\ f''(x) = 6ax + 2b$

$$\left.\begin{aligned} f(3) &= 27a + 9b + 3c + d = 3 \\ f(5) &= 125a + 25b + 5c + d = 1 \end{aligned}\right\} 98a + 16b + 2c = -2 \Rightarrow 49a + 8b + c = -1$$

$f'(3) = 27a + 6b + c = 0,\ f''(4) = 24a + 2b = 0$

$$\begin{array}{ll} 49a + 8b + c = -1 & 24a + 2b = 0 \\ \underline{27a + 6b + c = 0} & \underline{22a + 2b = -1} \\ 22a + 2b = -1 & 2a = 1 \end{array}$$

$a = \frac{1}{2},\ b = -6,\ c = \frac{45}{2},\ d = -24$

$f(x) = \frac{1}{2}x^3 - 6x^2 + \frac{45}{2}x - 24$

72. $f(x) = ax^3 + bx^2 + cx + d$

Relative maximum: $(2, 4)$

Relative minimum: $(4, 2)$

Point of inflection: $(3, 3)$

$f'(x) = 3ax^2 + 2bx + c,\ f''(x) = 6ax + 2b$

$$\left.\begin{aligned} f(2) &= 8a + 4b + 2c + d = 4 \\ f(4) &= 64a + 16b + 4c + d = 2 \end{aligned}\right\} 56a + 12b + 2c = -2 \Rightarrow 28a + 6b + c = -1$$

$f'(2) = 12a + 4b + c = 0,\ f'(4) = 48a + 8b + c = 0,\ f''(3) = 18a + 2b = 0$

$$\begin{array}{ll} 28a + 6b + c = -1 & 18a + 2b = 0 \\ 12a + 4b + c = 0 & 16a + 2b = -1 \\ 16a + 2b = -1 & 2a = 1 \end{array}$$

$a = \frac{1}{2},\ b = -\frac{9}{2},\ c = 12,\ d = -6$

$f(x) = \frac{1}{2}x^3 - \frac{9}{2}x^2 + 12x - 6$

73. $f(x) = ax^3 + bx^2 + cx + d$

Maximum: $(-4, 1)$

Minimum: $(0, 0)$

(a) $f'(x) = 3ax^2 + 2bx + c, \quad f''(x) = 6ax + 2b$

$$\begin{aligned} f(0) = 0 &\Rightarrow d = 0 \\ f(-4) = 1 &\Rightarrow -64a + 16b - 4c = 1 \\ f'(-4) = 0 &\Rightarrow 48a - 8b + c = 0 \\ f'(0) = 0 &\Rightarrow c = 0 \end{aligned}$$

Solving this system yields $a = \frac{1}{32}$ and $b = 6a = \frac{3}{16}$.

$f(x) = \frac{1}{32}x^3 + \frac{3}{16}x^2$

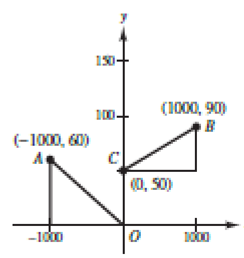
(b) The plane would be descending at the greatest rate at the point of inflection.

$f''(x) = 6ax + 2b = \frac{3}{16}x + \frac{3}{8} = 0 \Rightarrow x = -2.$

Two miles from touchdown.

74. (a) line OA: $y = -0.06x$ slope: -0.06

line CB: $y = 0.04x + 50$ slope: 0.04



$f(x) = ax^3 + bx^2 + cx + d$

$f'(x) = 3ax^2 + 2bx + c$

$(-1000, 60)$: $\quad 60 = (-1000)^3 a + (1000)^2 b - 1000c + d$

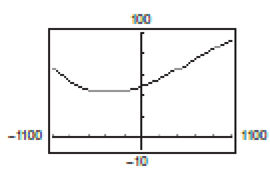
$\quad -0.06 = (1000)^2\, 3a - 2000b + c$

$(1000, 90)$: $\quad 90 = (1000)^3 a + (1000)^2 b + 1000c + d$

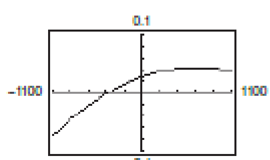
$\quad 0.04 = (1000)^2\, 3a + 2000b + c$

The solution to this system of four equations is $a = -1.25 \times 10^{-8}$, $b = 0.000025$, $c = 0.0275$, and $d = 50$.

(b) $y = -1.25 \times 10^{-8}x^3 + 0.000025x^2 + 0.0275x + 50$



(c)



(d) The steepest part of the road is 6% at the point A.

75. $C = 0.5x^2 + 15x + 5000$

$\overline{C} = \dfrac{C}{x} = 0.5x + 15 + \dfrac{5000}{x}$

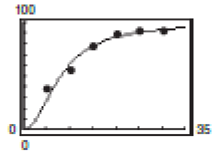
$\overline{C}$ = average cost per unit

$\dfrac{d\overline{C}}{dx} = 0.5 - \dfrac{5000}{x^2} = 0$ when $x = 100$

By the First Derivative Test, $\overline{C}$ is minimized when $x = 100$ units.

76. $S = \dfrac{100t^2}{65 + t^2}, t > 0$

(a)



(b) $S'(t) = \dfrac{13{,}000t}{\left(65 + t^2\right)^2}$

$$S''(t) = \frac{13{,}000\left(65 - 3t^2\right)}{\left(65 + t^2\right)^3} = 0 \Rightarrow t = 4.65$$

S is concave upwards on $(0, 4.65)$, concave downwards on $(4.65, 30)$.

(c) $S'(t) > 0$ for $t > 0$.

As t increases, the speed increases, but at a slower rate.

77. $f(x) = 2(\sin x + \cos x), \qquad f\left(\dfrac{\pi}{4}\right) = 2\sqrt{2}$

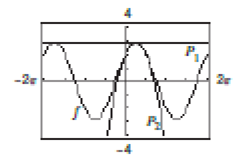
$f'(x) = 2(\cos x - \sin x), \qquad f'\left(\dfrac{\pi}{4}\right) = 0$

$f''(x) = 2(-\sin x - \cos x), \quad f''\left(\dfrac{\pi}{4}\right) = -2\sqrt{2}$

$P_1(x) = 2\sqrt{2} + 0\left(x - \dfrac{\pi}{4}\right) = 2\sqrt{2}$

$P_1'(x) = 0$

$P_2(x) = 2\sqrt{2} + 0\left(x - \dfrac{\pi}{4}\right) + \dfrac{1}{2}\left(-2\sqrt{2}\right)\left(x - \dfrac{\pi}{4}\right)^2 = 2\sqrt{2} - \sqrt{2}\left(x - \dfrac{\pi}{4}\right)^2$



$P_2'(x) = -2\sqrt{2}\left(x - \dfrac{\pi}{4}\right)$

$P_2''(x) = -2\sqrt{2}$

The values of f, P_1, P_2, and their first derivatives are equal at $x = \pi/4$. The values of the second derivatives of f and P_2 are equal at $x = \pi/4$. The approximations worsen as you move away from $x = \pi/4$.

78. $f(x) = 2(\sin x + \cos x), \qquad f(0) = 2$

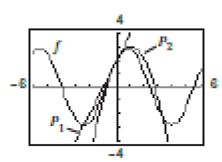
$f'(x) = 2(\cos x - \sin x), \qquad f'(0) = 2$

$f''(x) = 2(-\sin x - \cos x), \qquad f''(0) = -2$

$P_1(x) = 2 + 2(x - 0) = 2(1 + x)$

$P_1'(x) = 2$

$P_2(x) = 2 + 2(x - 0) + \frac{1}{2}(-2)(x - 0)^2 = 2 + 2x - x^2$



$P_2'(x) = 2 - 2x$

$P_2''(x) = -2$

The values of f, P_1, P_2, and their first derivatives are equal at $x = 0$. The values of the second derivatives of f and P_2 are equal at $x = 0$. The approximations worsen as you move away from $x = 0$.

79. $f(x) = \arctan x,\ a = -1, \qquad f(-1) = -\dfrac{\pi}{4}$

$$f'(x) = \frac{1}{1+x^2}, \qquad f'(-1) = \frac{1}{2}$$

$$f''(x) = -\frac{2x}{(1+x^2)^2}, \qquad f''(-1) = \frac{1}{2}$$

$$P_1(x) = f(-1) + f'(-1)(x+1) = -\frac{\pi}{4} + \frac{1}{2}(x+1)$$

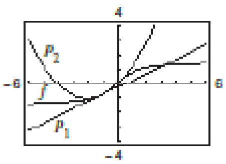
$$P_1'(x) = \frac{1}{2}$$

$$P_2(x) = f(-1) + f'(-1)(x+1) + \frac{1}{2}f''(-1)(x+1)^2 = -\frac{\pi}{4} + \frac{1}{2}(x+1) + \frac{1}{4}(x+1)^2$$

$$P_2'(x) = \frac{1}{2} + \frac{1}{2}(x+1)$$

$$P_2''(x) = \frac{1}{2}$$

The values of f, P_1, P_2, and their first derivatives are equal when $x = -1$. The approximations worsen as you move away from $x = -1$.



80. $f(x) = \dfrac{\sqrt{x}}{x-1}, \qquad f(2) = \sqrt{2}$

$$f'(x) = \frac{-(x+1)}{2\sqrt{x}(x-1)^2}, \qquad f'(2) = -\frac{3}{2\sqrt{2}} = -\frac{3\sqrt{2}}{4}$$

$$f''(x) = \frac{3x^2 + 6x - 1}{4x^{3/2}(x-1)^3}, \qquad f''(2) = \frac{23}{8\sqrt{2}} = \frac{23\sqrt{2}}{16}$$

$$P_1(x) = \sqrt{2} + \left(-\frac{3\sqrt{2}}{4}\right)(x-2) = -\frac{3\sqrt{2}}{4}x + \frac{5\sqrt{2}}{2}$$

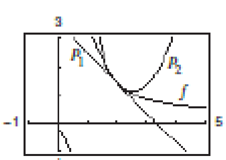
$$P_1'(x) = -\frac{3\sqrt{2}}{4}$$

$$P_2(x) = \sqrt{2} + \left(-\frac{3\sqrt{2}}{4}\right)(x-2) + \frac{1}{2}\left(\frac{23\sqrt{2}}{16}\right)(x-2)^2 = \sqrt{2} - \frac{3\sqrt{2}}{4}(x-2) + \frac{23\sqrt{2}}{32}(x-2)^2$$

$$P_2'(x) = -\frac{3\sqrt{2}}{4} + \frac{23\sqrt{2}}{16}(x-2)$$

$$P_2''(x) = \frac{23\sqrt{2}}{16}$$

The values of f, P_1, P_2 and their first derivatives are equal at $x = 2$. The values of the second derivatives of f and P_2 are equal at $x = 2$. The approximations worsen as you move away from $x = 2$.

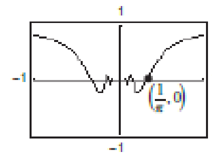


81. $f(x) = x\sin\left(\frac{1}{x}\right)$

$$f'(x) = x\left[-\frac{1}{x^2}\cos\left(\frac{1}{x}\right)\right] + \sin\left(\frac{1}{x}\right) = -\frac{1}{x}\cos\left(\frac{1}{x}\right) + \sin\left(\frac{1}{x}\right)$$

$$f''(x) = -\frac{1}{x}\left[\frac{1}{x^2}\sin\left(\frac{1}{x}\right)\right] + \frac{1}{x^2}\cos\left(\frac{1}{x}\right) - \frac{1}{x^2}\cos\left(\frac{1}{x}\right) = -\frac{1}{x^3}\sin\left(\frac{1}{x}\right) = 0$$

$$x = \frac{1}{\pi}$$



Point of inflection: $\left(\frac{1}{\pi}, 0\right)$

When $x > 1/\pi$, $f'' < 0$, so the graph is concave downward.

82. $f(x) = x(x - 6)^2 = x^3 - 12x^2 + 36x$

$f'(x) = 3x^2 - 24x + 36 = 3(x - 2)(x - 6) = 0$

$f''(x) = 6x - 24 = 6(x - 4) = 0$

Relative extrema: $(2, 32)$ and $(6, 0)$

Point of inflection $(4, 16)$ is midway between the relative extrema of f.

83. True. Let $y = ax^3 + bx^2 + cx + d$, $a \neq 0$. Then $y'' = 6ax + 2b = 0$ when $x = -(b/3a)$, and the concavity changes at this point.

84. False. For example, let $f(x) = (x - 2)^4$.

85. f and g are concave upward on (a, b) implies that f' and g' are increasing on (a, b), and $f'' > 0$ and $g'' > 0$.

So, $(f + g)'' > 0 \Rightarrow f + g$ is concave upward on (a, b) by Theorem 4.7.

86. f, g are positive, increasing, and concave upward on $(a, b) \Rightarrow f(x) > 0$, $f'(x) \geq 0$ and $f''(x) > 0$, and $g(x) > 0$, $g'(x) \geq 0$ and $g''(x) > 0$ on (a, b). For $x \in (a, b)$,

$(fg)'(x) = f'(x)g(x) + f(x)g'(x)$

$(fg)''(x) = f''(x)g(x) + 2f'(x)g'(x) + f(x)g''(x) > 0$

So, fg is concave upward on (a, b).

87. Evaluate each point.

A: $\frac{dy}{dx} < 0$

$\frac{d^2y}{dx^2} > 0$

B: $\frac{dy}{dx} > 0$

$\frac{d^2y}{dx^2} > 0$

C: $\frac{dy}{dx} > 0$

$\frac{d^2y}{dx^2} < 0$

D: $\frac{dy}{dx} < 0$

$\frac{d^2y}{dx^2} < 0$

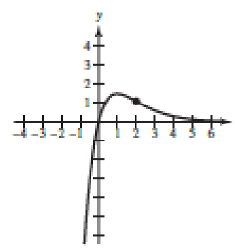
So, the answer is C.

88. $h(x) = 4xe^{-x}$

$h'(x) = 4e^{-x} - 4xe^{-x}$

$h''(x) = -4e^{-x}(4e^{-x} - 4xe^{-x})$

$= -4e^{-x}(2 - x) = 0$ when $x = 2$.



Because $h'' < 0$ on $(-\infty, 2)$, $h'' > 0$ on $(2, \infty)$, and $x = 2$ is a point of inflection, the graph of $h(x)$ is decreasing and concave upward on $(2, \infty)$.

So, the answer is A.

89. (a) $f(x) = 2\sin x - x$

$f'(x) = 2\cos x - 1$

$f''(x) = -2\sin x$

(b) $f'(x) = 2\cos x - 1$

$0 = 2\cos x - 1$

$1 = 2\cos x$

$\cos x = \dfrac{1}{2}$

$x = \dfrac{\pi}{3}, \dfrac{5\pi}{3}$

So, the critical numbers of f are $x = \dfrac{\pi}{3}$ and $x = \dfrac{5\pi}{3}$.

(c) When $x = \dfrac{\pi}{3}, f\left(\dfrac{\pi}{3}\right) = 2\sin\left(\dfrac{\pi}{3}\right) - \dfrac{\pi}{3} = \sqrt{3} - \dfrac{\pi}{3} \approx 0.6849$.

When $x = \dfrac{5\pi}{3}, f\left(\dfrac{5\pi}{3}\right) = 2\sin\left(\dfrac{5\pi}{3}\right) - \dfrac{5\pi}{3} = -\sqrt{3} - \dfrac{5\pi}{3} \approx -6.968$.

So, the relative maximum is $\left(\dfrac{\pi}{3}, \sqrt{3} - \dfrac{\pi}{3}\right)$ and the relative minimum is $\left(\dfrac{5\pi}{3}, -\sqrt{3} - \dfrac{5\pi}{3}\right)$.

(d) $f''(x) = -2\sin x$

$0 = -2\sin x$

$0 = \sin x$

$x = 0, \pi$

The points of inflection occur at $x = 0$ and $x = \pi$.