Section 1.2 Finding Limits Graphically and Numerically

1.

x	3.9	3.99	3.999	4	4.001	4.01	4.1
$f(x)$	0.3448	0.3344	0.3334	?	0.3332	0.3322	0.3226

$$\lim_{x \to 4} \frac{x - 4}{x^2 - 5x + 4} = 0.3333 \qquad \left(\text{Actual limit is } \frac{1}{3}.\right)$$

2.

x	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
$f(x)$	0.5132	0.5013	0.5001	?	0.4999	0.4988	0.4881

$$\lim_{x\to 0} \frac{\sqrt{x+1}-1}{x} = 0.5000 \quad \left(\text{Actual limit is } \frac{1}{2}.\right)$$

3.

x	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
$f(x)$	0.9983	0.99998	1.0000	?	1.0000	0.99998	0.9983

$$\lim_{x\to 0} \frac{\sin x}{x} = 1.0000 \qquad (\text{Actual limit is } 1.)\ (\text{Make sure you use radian mode.})$$

4.

x	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
$f(x)$	0.0500	0.0050	0.0005	?	-0.0005	-0.0050	-0.0500

$$\lim_{x\to 0} \frac{\cos x - 1}{x} = 0.0000 \qquad (\text{Actual limit is } 0.)\ (\text{Make sure you use radian mode.})$$

5.

x	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
$f(x)$	0.9516	0.9950	0.9995	?	1.0005	1.0050	1.0517

$$\lim_{x\to 0} \frac{e^x - 1}{x} = 1.0000 \qquad (\text{Actual limit is } 1.)$$

6.

x	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
$f(x)$	1.0536	1.0050	1.0005	?	0.9995	0.9950	0.9531

$$\lim_{x\to 0} \frac{\ln(x+1)}{x} = 1.0000 \qquad (\text{Actual limit is } 1.)$$

7.

x	0.9	0.99	0.999	1	1.001	1.01	1.1
$f(x)$	0.2564	0.2506	0.2501	?	0.2499	0.2494	0.2439

$$\lim_{x\to 1} \frac{x-2}{x^2+x-6} = 0.2500 \qquad \left(\text{Actual limit is } \frac{1}{4}.\right)$$

8.

x	-4.1	-4.01	-4.001	-4	-3.999	-3.99	-3.9
$f(x)$	1.1111	1.0101	1.0010	?	0.9990	0.9901	0.9091

$$\lim_{x\to -4} \frac{x+4}{x^2+9x+20} = 1.0000 \qquad (\text{Actual limit is } 1.)$$

9.

x	0.9	0.99	0.999	1	1.001	1.01	1.1
$f(x)$	0.7340	0.6733	0.6673	?	0.6660	0.6600	0.6015

$\lim_{x \to 1} \dfrac{x^4 - 1}{x^6 - 1} \approx 0.6666$ $\quad \left(\text{Actual limit is } \dfrac{2}{3}.\right)$

10.

x	-3.1	-3.01	-3.001	-3	-2.999	-2.99	-2.9
$f(x)$	27.91	27.0901	27.0090	?	26.9910	26.9101	26.11

$\lim_{x\to -3} \frac{x^3+27}{x+3} = 27.0000$ (Actual limit is 27.)

11.

x	-6.1	-6.01	-6.001	-6	-5.999	-5.99	-5.9
$f(x)$	-0.1248	-0.1250	-0.1250	?	-0.1250	-0.1250	-0.1252

$\lim_{x\to -6} \frac{\sqrt{10-x}-4}{x+6} = -0.1250$ $\left(\text{Actual limit is } -\frac{1}{8}.\right)$

12.

x	1.9	1.99	1.999	2	2.001	2.01	2.1
$f(x)$	0.1149	0.115	0.1111	?	0.1111	0.1107	0.1075

$\lim_{x\to 2} \frac{x/(x+1)-2/3}{x-2} = 0.1111$ $\left(\text{Actual limit is } \frac{1}{9}.\right)$

13.

x	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
$f(x)$	1.9867	1.9999	2.0000	?	2.0000	1.9999	1.9867

$\lim_{x\to 0} \frac{\sin 2x}{x} = 2.0000$ (Actual limit is 2.) (Make sure you use radian mode.)

14.

x	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
$f(x)$	0.4950	0.5000	0.5000	?	0.5000	0.5000	0.4950

$\lim_{x\to 0} \frac{\tan x}{\tan 2x} = 0.5000$ $\left(\text{Actual limit is } \frac{1}{2}.\right)$

15.

x	1.9	1.99	1.999	2	2.001	2.01	2.1
$f(x)$	0.5129	0.5013	0.5001	?	0.4999	0.4988	0.4879

$\lim_{x\to 2} \frac{\ln x - \ln 2}{x-2} = 0.5000$ $\left(\text{Actual limit is } \frac{1}{2}.\right)$

16.

x	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
$f(x)$	3.99982	4	4	?	0	0	0.00018

$\lim_{x\to 0} \frac{4}{1+e^{1/x}}$ does not exist.

17. $\lim_{x\to 3}(4 - x) = 1$

18. $\lim_{x\to 0} \sec x = 1$

19. $\lim_{x\to 2} f(x) = \lim_{x\to 2}(4 - x) = 2$

20. $\lim_{x\to 1} f(x) = \lim_{x\to 1}(x^2 + 3) = 4$

21. $\lim_{x\to 2} \frac{|x - 2|}{x - 2}$ does not exist. The function approach
from the right side of 2, but it approaches -1 from
left side of 2.

22. $\lim_{x \to 0} \dfrac{4}{2 + e^{1/x}}$ does not exist. The function approaches 2 from the left side of 0, but it approaches 0 from the right side of 0.

23. $\lim_{x \to 0} \cos(1/x)$ does not exist. The function oscillates between -1 and 1 as x approaches 0.

24. $\lim_{x \to \pi/2} \tan x$ does not exist. The function increases without bound as x approaches $\dfrac{\pi}{2}$ from the left and decreases without bound as x approaches $\dfrac{\pi}{2}$ from the right.

25. (a) $f(1)$ exists. The closed circle at $(1, 2)$ indicates that $f(1) = 2$.

(b) $\lim_{x \to 1} f(x)$ does not exist. As x approaches 1 from the left, $f(x)$ approaches 3.5, whereas as x approaches 1 from the right, $f(x)$ approaches 1.

(c) $f(4)$ does not exist. The open circle at $(4, 2)$ indicates that $f(x)$ is not defined at $x = 4$.

(d) $\lim_{x \to 4} f(x)$ exists. As x approaches 4, $f(x)$ approaches 2. $\lim_{x \to 4} f(x) = 2$

26. (a) $f(-2)$ does not exist. The vertical dotted line indicates that f is not defined at -2.

(b) $\lim_{x \to -2} f(x)$ does not exist. As x approaches -2, the values of $f(x)$ do not approach a specific number.

(c) $f(0)$ exists. The closed circle at $(0, 4)$ indicates that $f(0) = 4$.

(d) $\lim_{x \to 0} f(x)$ does not exist. As x approaches 0 from the left, $f(x)$ approaches $\frac{1}{2}$, whereas as x approaches 0 from the right, $f(x)$ approaches 4.

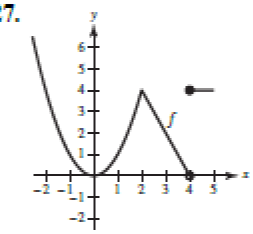
(e) $f(2)$ does not exist. The open circle at $\left(2, \frac{1}{2}\right)$ indicates that $f(2)$ is not defined.

(f) $\lim_{x \to 2} f(x)$ exists. As x approaches 2, $f(x)$ approaches $\frac{1}{2}$. $\lim_{x \to 2} f(x) = \frac{1}{2}$

(g) $f(4)$ exists. The closed circle at $(4, 2)$ indicates that $f(4) = 2$.

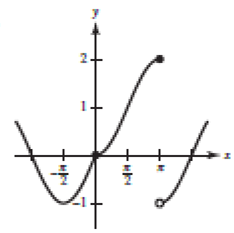
(h) $\lim_{x \to 4} f(x)$ does not exist. As x approaches 4, the values of $f(x)$ do not approach a specific number.

27.



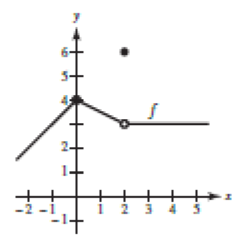
$\lim_{x \to c} f(x)$ exists for all values of $c \neq 4$.

28.

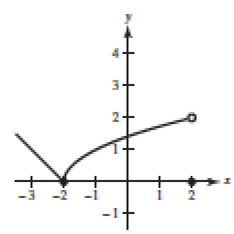


$\lim_{x \to c} f(x)$ exists for all values of $c \neq \pi$.

29. Answers will vary. Sample answer:



30. Answers will vary. Sample answer:



31. You need $|f(x) - 3| = |(x + 1) - 3| = |x - 2| < 0.4$. So, take $\delta = 0.4$. If $0 < |x - 2| < 0.4$, then $|x - 2| = |(x + 1) - 3| = |f(x) - 3| < 0.4$, as desired.

32. You need $\left|f(x) - 1\right| = \left|\frac{1}{x-1} - 1\right| = \left|\frac{2-x}{x-1}\right| < 0.01.$

Let $\delta = \frac{1}{101}$. If $0 < |x - 2| < \frac{1}{101}$, then

$$-\frac{1}{101} < x - 2 < \frac{1}{101} \Rightarrow 1 - \frac{1}{101} < x - 1 < 1 + \frac{1}{101}$$

$$\Rightarrow \frac{100}{101} < x - 1 < \frac{102}{101}$$

$$\Rightarrow |x - 1| > \frac{100}{101}$$

and you have

$$\left|f(x) - 1\right| = \left|\frac{1}{x-1} - 1\right| = \left|\frac{2-x}{x-1}\right| < \frac{1/101}{100/101} = \frac{1}{100}$$
$$= 0.01.$$

33. You need to find δ such that $0 < |x - 1| < \delta$ implies

$\left|f(x) - 1\right| = \left|\frac{1}{x} - 1\right| < 0.1.$ That is,

$$-0.1 < \frac{1}{x} - 1 < 0.1$$
$$1 - 0.1 < \frac{1}{x} < 1 + 0.1$$
$$\frac{9}{10} < \frac{1}{x} < \frac{11}{10}$$
$$\frac{10}{9} > x > \frac{10}{11}$$
$$\frac{10}{9} - 1 > x - 1 > \frac{10}{11} - 1$$
$$\frac{1}{9} > x - 1 > -\frac{1}{11}.$$

So take $\delta = \frac{1}{11}$. Then $0 < |x - 1| < \delta$ implies

$$-\frac{1}{11} < x - 1 < \frac{1}{11}$$
$$-\frac{1}{11} < x - 1 < \frac{1}{9}.$$

Using the first series of equivalent inequalities, you obtain

$$\left|f(x) - 1\right| = \left|\frac{1}{x} - 1\right| < 0.1.$$

34. You need to find δ such that $0 < |x - 2| < \delta$ implies $\left|f(x) - 3\right| = \left|x^2 - 1 - 3\right| = \left|x^2 - 4\right| < 0.2$. That is,

$$-0.2 < x^2 - 4 < 0.2$$
$$4 - 0.2 < x^2 < 4 + 0.2$$
$$3.8 < x^2 < 4.2$$
$$\sqrt{3.8} < x < \sqrt{4.2}$$
$$\sqrt{3.8} - 2 < x - 2 < \sqrt{4.2} - 2$$

So take $\delta = \sqrt{4.2} - 2 \approx 0.0494$.

Then $0 < |x - 2| < \delta$ implies

$$-\left(\sqrt{4.2} - 2\right) < x - 2 < \sqrt{4.2} - 2$$
$$\sqrt{3.8} - 2 < x - 2 < \sqrt{4.2} - 2.$$

Using the first series of equivalent inequalities, you obtain

$$\left|f(x) - 3\right| = \left|x^2 - 4\right| < 0.2.$$

35. $\lim_{x\to 2} (3x + 2) = 3(2) + 2 = 8 = L$

$$\left|(3x + 2) - 8\right| < 0.01$$
$$|3x - 6| < 0.01$$
$$3|x - 2| < 0.01$$
$$0 < |x - 2| < \tfrac{0.01}{3} \approx 0.0033 = \delta$$

So, if $0 < |x - 2| < \delta = \frac{0.01}{3}$, you have

$$3|x - 2| < 0.01$$
$$|3x - 6| < 0.01$$
$$\left|(3x + 2) - 8\right| < 0.01$$
$$\left|f(x) - L\right| < 0.01.$$

36. $\lim_{x\to 6}\left(6 - \frac{x}{3}\right) = 6 - \frac{6}{3} = 4 = L$

$$\left|\left(6 - \frac{x}{3}\right) - 4\right| < 0.01$$
$$\left|2 - \frac{x}{3}\right| < 0.01$$
$$\left|-\tfrac{1}{3}(x - 6)\right| < 0.01$$
$$|x - 6| < 0.03$$
$$0 < |x - 6| < 0.03 = \delta$$

So, if $0 < |x - 6| < \delta = 0.03$, you have

$$\left|-\tfrac{1}{3}(x - 6)\right| < 0.01$$
$$\left|2 - \frac{x}{3}\right| < 0.01$$
$$\left|\left(6 - \frac{x}{3}\right) - 4\right| < 0.01$$
$$\left|f(x) - L\right| < 0.01.$$

37. $\lim_{x\to 2}(x^2 - 3) = 2^2 - 3 = 1 = L$

$$\left|(x^2 - 3) - 1\right| < 0.01$$
$$\left|x^2 - 4\right| < 0.01$$
$$\left|(x + 2)(x - 2)\right| < 0.01$$
$$\left|x + 2\right|\left|x - 2\right| < 0.01$$
$$\left|x - 2\right| < \frac{0.01}{\left|x + 2\right|}$$

If you assume $1 < x < 3$, then $\delta \approx 0.01/5 = 0.002$.

So, if $0 < \left|x - 2\right| < \delta \approx 0.002$, you have

$$\left|x - 2\right| < 0.002 = \frac{1}{5}(0.01) < \frac{1}{\left|x + 2\right|}(0.01)$$
$$\left|x + 2\right|\left|x - 2\right| < 0.01$$
$$\left|x^2 - 4\right| < 0.01$$
$$\left|(x^2 - 3) - 1\right| < 0.01$$
$$\left|f(x) - L\right| < 0.01.$$

38. $\lim_{x\to 4}(x^2 + 6) = 4^2 + 6 = 22 = L$

$$\left|(x^2 + 6) - 22\right| < 0.01$$
$$\left|x^2 - 16\right| < 0.01$$
$$\left|(x + 4)(x - 4)\right| < 0.01$$
$$\left|x - 4\right| < \frac{0.01}{\left|x + 4\right|}$$

If you assume $3 < x < 5$, then $\delta = \frac{0.01}{9} \approx 0.00111$.

So, if $0 < \left|x - 4\right| < \delta \approx \frac{0.01}{9}$, you have

$$\left|x - 4\right| < \frac{0.01}{9} < \frac{0.01}{\left|x + 4\right|}$$
$$\left|(x + 4)(x - 4)\right| < 0.01$$
$$\left|x^2 - 16\right| < 0.01$$
$$\left|(x^2 + 6) - 22\right| < 0.01$$
$$\left|f(x) - L\right| < 0.01.$$

39. $\lim_{x\to 4}(x + 2) = 4 + 2 = 6$

Given $\varepsilon > 0$:

$$\left|(x + 2) - 6\right| < \varepsilon$$
$$\left|x - 4\right| < \varepsilon = \delta$$

So, let $\delta = \varepsilon$. So, if $0 < \left|x - 4\right| < \delta = \varepsilon$, you have

$$\left|x - 4\right| < \varepsilon$$
$$\left|(x + 2) - 6\right| < \varepsilon$$
$$\left|f(x) - L\right| < \varepsilon.$$

40. $\lim_{x\to -2}(4x + 5) = 4(-2) + 5 = -3$

Given $\varepsilon > 0$:

$$\left|(4x + 5) - (-3)\right| < \varepsilon$$
$$\left|4x + 8\right| < \varepsilon$$
$$4\left|x + 2\right| < \varepsilon$$
$$\left|x + 2\right| < \frac{\varepsilon}{4} = \delta$$

So, let $\delta = \frac{\varepsilon}{4}$.

So, if $0 < \left|x + 2\right| < \delta = \frac{\varepsilon}{4}$, you have

$$\left|x + 2\right| < \frac{\varepsilon}{4}$$
$$\left|4x + 8\right| < \varepsilon$$
$$\left|(4x + 5) - (-3)\right| < \varepsilon$$
$$\left|f(x) - L\right| < \varepsilon.$$

41. $\lim_{x\to -4}\left(\frac{1}{2}x - 1\right) = \frac{1}{2}(-4) - 1 = -3$

Given $\varepsilon > 0$:

$$\left|\left(\tfrac{1}{2}x - 1\right) - (-3)\right| < \varepsilon$$
$$\left|\tfrac{1}{2}x + 2\right| < \varepsilon$$
$$\tfrac{1}{2}\left|x - (-4)\right| < \varepsilon$$
$$\left|x - (-4)\right| < 2\varepsilon$$

So, let $\delta = 2\varepsilon$.

So, if $0 < \left|x - (-4)\right| < \delta = 2\varepsilon$, you have

$$\left|x - (-4)\right| < 2\varepsilon$$
$$\left|\tfrac{1}{2}x + 2\right| < \varepsilon$$
$$\left|\left(\tfrac{1}{2}x - 1\right) + 3\right| < \varepsilon$$
$$\left|f(x) - L\right| < \varepsilon.$$

42. $\lim_{x\to 3}\left(\frac{3}{4}x + 1\right) = \frac{3}{4}(3) + 1 = \frac{13}{4}$

Given $\varepsilon > 0$:

$$\left|\left(\frac{3}{4}x + 1\right) - \frac{13}{4}\right| < \varepsilon$$
$$\left|\frac{3}{4}x - \frac{9}{4}\right| < \varepsilon$$
$$\frac{3}{4}\left|x - 3\right| < \varepsilon$$
$$\left|x - 3\right| < \frac{4}{3}\varepsilon$$

So, let $\delta = \frac{4}{3}\varepsilon$.

So, if $0 < \left|x - 3\right| < \delta = \frac{4}{3}\varepsilon$, you have

$$\left|x - 3\right| < \frac{4}{3}\varepsilon$$
$$\frac{3}{4}\left|x - 3\right| < \varepsilon$$
$$\left|\frac{3}{4}x - \frac{9}{4}\right| < \varepsilon$$
$$\left|\left(\frac{3}{4}x + 1\right) - \frac{13}{4}\right| < \varepsilon$$
$$\left|f(x) - L\right| < \varepsilon.$$

43. $\lim_{x\to 6} 3 = 3$

Given $\varepsilon > 0$:

$$\left|3 - 3\right| < \varepsilon$$
$$0 < \varepsilon$$

So, any $\delta > 0$ will work.

So, for any $\delta > 0$, you have

$$\left|3 - 3\right| < \varepsilon$$
$$\left|f(x) - L\right| < \varepsilon.$$

44. $\lim_{x\to 2} (-1) = -1$

Given $\varepsilon > 0$:

$$\left|-1 - (-1)\right| < \varepsilon$$
$$0 < \varepsilon$$

So, any $\delta > 0$ will work.

So, for any $\delta > 0$, you have

$$\left|(-1) - (-1)\right| < \varepsilon$$
$$\left|f(x) - L\right| < \varepsilon.$$

45. $\lim_{x\to 0} \sqrt[3]{x} = 0$

Given $\varepsilon > 0$:

$$\left|\sqrt[3]{x} - 0\right| < \varepsilon$$
$$\left|\sqrt[3]{x}\right| < \varepsilon$$
$$\left|x\right| < \varepsilon^3 = \delta$$

So, let $\delta = \varepsilon^3$.

So, for $0\left|x - 0\right|\delta = \varepsilon^3$, you have

$$\left|x\right| < \varepsilon^3$$
$$\left|\sqrt[3]{x}\right| < \varepsilon$$
$$\left|\sqrt[3]{x} - 0\right| < \varepsilon$$
$$\left|f(x) - L\right| < \varepsilon.$$

46. $\lim_{x\to 4} \sqrt{x} = \sqrt{4} = 2$

Given $\varepsilon > 0$:

$$\left|\sqrt{x} - 2\right| < \varepsilon$$
$$\left|\sqrt{x} - 2\right|\left|\sqrt{x} + 2\right| < \varepsilon\left|\sqrt{x} + 2\right|$$
$$\left|x - 4\right| < \varepsilon\left|\sqrt{x} + 2\right|$$

Assuming $1 < x < 9$, you can choose $\delta = 3\varepsilon$. Then,

$$0 < \left|x - 4\right| < \delta = 3\varepsilon \Rightarrow \left|x - 4\right| < \varepsilon\left|\sqrt{x} + 2\right|$$
$$\Rightarrow \left|\sqrt{x} - 2\right| < \varepsilon.$$

47. $\lim_{x\to -5} \left|x - 5\right| = \left|(-5) - 5\right| = \left|-10\right| = 10$

Given $\varepsilon > 0$:

$$\left|\left|x - 5\right| - 10\right| < \varepsilon$$
$$\left|-(x - 5) - 10\right| < \varepsilon \qquad (x - 5 < 0)$$
$$\left|-x - 5\right| < \varepsilon$$
$$\left|x - (-5)\right| < \varepsilon$$

So, let $\delta = \varepsilon$.

So for $\left|x - (-5)\right| < \delta = \varepsilon$, you have

$$\left|-(x + 5)\right| < \varepsilon$$
$$\left|-(x - 5) - 10\right| < \varepsilon$$
$$\left|\left|x - 5\right| - 10\right| < \varepsilon \qquad (\text{because } x - 5 < 0)$$
$$\left|f(x) - L\right| < \varepsilon.$$

48. $\lim_{x \to 3} |x - 3| = |3 - 3| = 0$

Given $\varepsilon > 0$:

$$\big||x - 3| - 0\big| < \varepsilon$$
$$|x - 3| < \varepsilon$$

So, let $\delta = \varepsilon$.

So, for $0 < |x - 3| < \delta = \varepsilon$, you have

$$|x - 3| < \varepsilon$$
$$\big||x - 3| - 0\big| < \varepsilon$$
$$|f(x) - L| < \varepsilon.$$

49. $\lim_{x \to 1} (x^2 + 1) = 1^2 + 1 = 2$

Given $\varepsilon > 0$:

$$|(x^2 + 1) - 2| < \varepsilon$$
$$|x^2 - 1| < \varepsilon$$
$$|(x + 1)(x - 1)| < \varepsilon$$
$$|x - 1| < \frac{\varepsilon}{|x + 1|}$$

If you assume $0 < x < 2$, then $\delta = \varepsilon/3$.

So for $0 < |x - 1| < \delta = \frac{\varepsilon}{3}$, you have

$$|x - 1| < \frac{1}{3}\varepsilon < \frac{1}{|x + 1|}\varepsilon$$
$$|x^2 - 1| < \varepsilon$$
$$|(x^2 + 1) - 2| < \varepsilon$$
$$|f(x) - 2| < \varepsilon.$$

50. $\lim_{x \to -4} (x^2 + 4x) = (-4)^2 + 4(-4) = 0$

Given $\varepsilon > 0$:

$$|(x^2 + 4x) - 0| < \varepsilon$$
$$|x(x + 4)| < \varepsilon$$
$$|x + 4| < \frac{\varepsilon}{|x|}$$

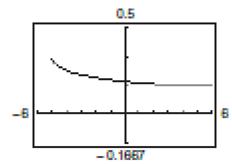
If you assume $-5 < x < -3$, then $\delta = \frac{\varepsilon}{5}$.

So for $0 < |x - (-4)| < \delta = \frac{\varepsilon}{5}$, you have

$$|x + 4| < \frac{\varepsilon}{5} < \frac{1}{|x|}\varepsilon$$
$$|x(x + 4)| < \varepsilon$$
$$|(x^2 + 4x) - 0| < \varepsilon$$
$$|f(x) - L| < \varepsilon.$$

51. $f(x) = \dfrac{\sqrt{x + 5} - 3}{x - 4}$

$\lim_{x \to 4} f(x) = \frac{1}{6}$

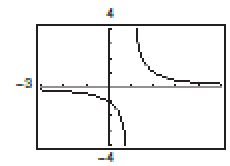


The domain is $[-5, 4) \cup (4, \infty)$.

The graphing utility does not show the hole at $\left(4, \frac{1}{6}\right)$.

52. $f(x) = \dfrac{x - 3}{x^2 - 4x + 3}$

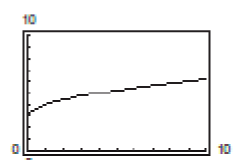
$\lim_{x \to 3} f(x) = \frac{1}{2}$



The domain is $(-\infty, 1) \cup (1, 3) \cup (3, \infty)$. The graphing utility does not show the hole at $\left(3, \frac{1}{2}\right)$.

53. $f(x) = \dfrac{x - 9}{\sqrt{x} - 3}$

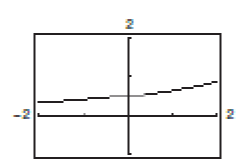
$\lim_{x \to 9} f(x) = 6$



The domain is $[0, 9) \cup (9, \infty)$. The graphing utility does not show the hole at $(9, 6)$.

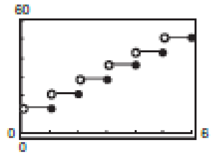
54. $f(x) = \dfrac{e^{x/2} - 1}{x}$

$\lim_{x \to 0} f(x) = \frac{1}{2}$



The domain is $(-\infty, 0) \cup (0, \infty)$. The graphing utility does not show the hole at $\left(0, \frac{1}{2}\right)$.

55. $C(t) = 14 - 7.5[\![-(t-1)]\!]$

(a) 

(b)

t	3	3.3	3.4	3.5	3.6	3.7	4
C	29	36.5	36.5	36.5	36.5	36.5	36.5

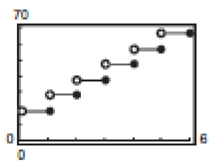
$\lim_{t\to 3.5} C(t) = 36.5$

(c)

t	2	2.5	2.9	3	3.1	3.5	4
C	21.5	29	29	36.5	36.5	36.5	36.5

The limit does not exist because the limits from the right and left are not equal.

56. $C(t) = 18 - 9.75[\![-(t-1)]\!]$

(a) 

(b)

t	3	3.3	3.4	3.5	3.6	3.7	4
C	37.5	47.25	47.25	47.25	47.25	47.25	47.25

$\lim_{t\to 3.5} C(t) = 47.25$

(c)

t	2	2.5	2.9	3	3.1	3.5	4
C	27.75	37.5	37.5	37.5	47.25	47.25	47.25

The limit does not exist because the limits from the right and left are not equal.

57. In the definition of $\lim_{x\to c} f(x)$, f must be defined on both sides of c, but does not have to be defined at c itself. The value of f at c has no bearing on the limit as x approaches c.

58. (a) No. The fact that $f(2) = 4$ has no bearing on the existence of the limit of $f(x)$ as x approaches 2.

(b) No. The fact that $\lim_{x\to 2} f(x) = 4$ has no bearing on the value of $f(x)$ at 2.

59. (a) $C = 2\pi r$

$r = \frac{C}{2\pi} = \frac{6}{2\pi} = \frac{3}{\pi} \approx 0.9549$ cm

(b) When $C = 5.5$: $r = \frac{5.5}{2\pi} \approx 0.87535$ cm

When $C = 6.5$: $r = \frac{6.5}{2\pi} \approx 1.03451$ cm

So, $0.87535 < r < 1.03451$.

(c) $\lim_{x\to 3/\pi} 2\pi r = 6$

$\varepsilon = 0.5$

$\delta \approx 0.0796$

60. $V = \frac{4}{3}\pi r^3, V = 2.48$

(a) $2.48 = \frac{4}{3}\pi r^3$

$$r^3 = \frac{1.86}{\pi}$$

$$r \approx 0.8397 \text{ in.}$$

(b) $2.45 \le V \le 2.51$

$$2.45 \le \frac{4}{3}\pi r^3 \le 2.51$$

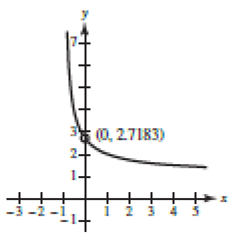
$$0.5849 \le r^3 \le 0.5992$$

$$0.8363 \le r \le 0.8431$$

(c) For $\varepsilon = 2.51 - 2.48 = 0.03, \delta \approx 0.003$.

61. $f(x) = (1 + x)^{1/x}$

$\lim_{x \to 0} (1 + x)^{1/x} = e \approx 2.71828$



x	$f(x)$
-0.1	2.867972
-0.01	2.731999
-0.001	2.719642
-0.0001	2.718418
-0.00001	2.718295
-0.000001	2.718283

x	$f(x)$
0.1	2.593742
0.01	2.704814
0.001	2.716942
0.0001	2.718146
0.00001	2.718268
0.000001	2.718280

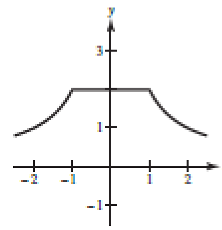
62. $f(x) = \frac{|x + 1| - |x - 1|}{x}$

x	-1	-0.5	-0.1	0	0.1	0.5	1.0
$f(x)$	2	2	2	Undef.	2	2	2

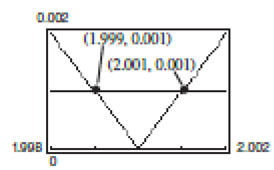
$\lim_{x \to 0} f(x) = 2$

Note that for

$-1 < x < 1, x \ne 0, f(x) = \frac{(x + 1) + (x - 1)}{x} = 2.$



63.



Using the zoom and trace feature, $\delta = 0.001$. So $(2 - \delta, 2 + \delta) = (1.999, 2.001)$.

Note: $\frac{x^2 - 4}{x - 2} = x + 2$ for $x \ne 2$.

64. (a) $\lim_{x \to c} f(x)$ exists for all $c \ne -3$.

(b) $\lim_{x \to c} f(x)$ exists for all $c \ne -2, 0$.

65. False. The existence or nonexistence of $f(x)$ at $x = c$ has no bearing on the existence of the limit of $f(x)$ as $x \to c$.

66. True

67. False. Let

$$f(x) = \begin{cases} x - 4, & x \ne 2 \\ 0, & x = 2 \end{cases}.$$

So, $f(2) = 0$ and $\lim_{x \to 2} f(x) = \lim_{x \to 2} (x - 4) = 2 \ne 0$.

68. False. Let

$$f(x) = \begin{cases} x - 4, & x \ne 2 \\ 0, & x = 2 \end{cases}.$$

So, $\lim_{x \to 2} f(x) = \lim_{x \to 2} (x - 4) = 2$ and $f(2) = 0 \ne 2$.

69. $f(x) = \sqrt{x}$

$\lim_{x\to 0.25} \sqrt{x} = 0.5$ is true.

As x approaches $0.25 = \frac{1}{4}$ from either side,

$f(x) = \sqrt{x}$ approaches $\frac{1}{2} = 0.5$.

70. $f(x) = \sqrt{x}$

$\lim_{x\to 0} \sqrt{x} = 0$ is false.

$f(x) = \sqrt{x}$ is not defined on an open interval containing 0 because the domain of f is $x \geq 0$.

71. Using a graphing utility, you can see that

$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

$$\lim_{x\to 0} \frac{\sin 2x}{x} = 2, \text{ etc.}$$

So, $\lim_{x\to 0} \frac{\sin nx}{x} = n$.

72. Using a graphing utility, you can see that

$$\lim_{x\to 0} \frac{\tan x}{x} = 1$$

$$\lim_{x\to 0} \frac{\tan 2x}{x} = 2, \quad \text{etc.}$$

So, $\lim_{x\to 0} \frac{\tan nx}{x} = n$.

73. If $\lim_{x\to c} f(x) = L_1$ and $\lim_{x\to c} f(x) = L_2$, then for every $\varepsilon > 0$, there exists $\delta_1 > 0$ and $\delta_2 > 0$ such that $|x - c| < \delta_1 \Rightarrow |f(x) - L_1| < \varepsilon$ and $|x - c| < \delta_2 \Rightarrow |f(x) - L_2| < \varepsilon$. Let δ equal the smaller of δ_1 and δ_2. Then for $|x - c| < \delta$, you have $|L_1 - L_2| = |L_1 - f(x) + f(x) - L_2| \leq |L_1 - f(x)| + |f(x) - L_2| < \varepsilon + \varepsilon$. Therefore, $|L_1 - L_2| < 2\varepsilon$. Because $\varepsilon > 0$ is arbitrary, it follows that $L_1 = L_2$.

74. $f(x) = mx + b, m \neq 0$. Let $\varepsilon > 0$ be given. Take

$\delta = \frac{\varepsilon}{|m|}$.

If $0 < |x - c| < \delta = \frac{\varepsilon}{|m|}$, then

$$|m||x - c| < \varepsilon$$
$$|mx - mc| < \varepsilon$$
$$|(mx + b) - (mc + b)| < \varepsilon$$

which shows that $\lim_{x\to c} (mx + b) = mc + b$.

75. $\lim_{x\to c}[f(x) - L] = 0$ means that for every $\varepsilon > 0$ there exists $\delta > 0$ such that if

$0 < |x - c| < \delta$,

then

$|(f(x) - L) - 0| < \varepsilon$.

This means the same as $|f(x) - L| < \varepsilon$ when

$0 < |x - c| < \delta$.

So, $\lim_{x\to c} f(x) = L$.

76. (a) $$(3x + 1)(3x - 1)x^2 + 0.01 = (9x^2 - 1)x^2 + \frac{1}{100}$$
$$= 9x^4 - x^2 + \frac{1}{100}$$
$$= \frac{1}{100}(10x^2 - 1)(90x^2 - 1)$$

So, $(3x + 1)(3x - 1)x^2 + 0.01 > 0$ if

$10x^2 - 1 < 0$ and $90x^2 - 1 < 0$.

Let $(a, b) = \left(-\frac{1}{\sqrt{90}}, \frac{1}{\sqrt{90}}\right)$.

For all $x \neq 0$ in (a, b), the graph is positive. You can verify this with a graphing utility.

(b) You are given $\lim_{x\to c} g(x) = L > 0$. Let $\varepsilon = \frac{1}{2}L$.

There exists $\delta > 0$ such that $0 < |x - c| < \delta$

implies that $|g(x) - L| < \varepsilon = \frac{L}{2}$. That is,

$$-\frac{L}{2} < g(x) - L < \frac{L}{2}$$
$$\frac{L}{2} < \quad g(x) \quad < \frac{3L}{2}$$

For x in the interval $(c - \delta, c + \delta)$, $x \neq c$, you have $g(x) > \frac{L}{2} > 0$, as desired.

77. $\lim_{x \to \pi} x = \pi$

So, the answer is *C*.

78. The function $f(x) = \frac{10}{x^4}$ increases without bound as x approaches 0 from the left and as x approaches 0 from the right. So, the limit is nonexistent, which is answer D.

79. As x approaches 0 from the left and right, the function approaches 2. So, $\lim_{x \to 0} f(x) = 2$, which is answer B.

80. Evaluate each statement.

I: As x approaches 3 from the left and right, the function approaches 1. So, $\lim_{x \to 3} \sqrt{x - 2} = 1$ is a true statement.

II: As x approaches 3 from the left and right, the function approaches 0. So, $\lim_{x \to 3} (6 - 2x) = 0$ is a true statement.

III: As x approaches 3 from the left, the function approaches 0 and as x approaches 3 from the right, the function approaches 1. So, the limit $\lim_{x \to 3} f(x)$ does not exist is a true statement.

Because I, II, and III are true statements, the answer is D.