B2

1.3 Evaluating Limits Analytically

Common Limits

Constant Function $\lim_{x\to c} b = b$

Identity Function $\lim_{x\to c} x = c$

Power Function $\lim_{x\to c} x^n = c^n$

Properties of Limits

Let b and c be real numbers, let n be a positive integer, and let f and g be functions with limits $\lim_{x\to c} f(x) = L$ and $\lim_{x\to c} g(x) = K$.

$\lim_{x\to c}[bf(x)] = bL$

$\lim_{x\to c}[f(x) \pm g(x)] = \lim_{x\to c} f(x) \pm \lim_{x\to c} g(x) = L \pm K$

$\lim_{x\to c}[f(x) \cdot g(x)] = \lim_{x\to c} f(x) \cdot \lim_{x\to c} g(x) = L \cdot K$

$\lim_{x\to c}\left[\frac{f(x)}{g(x)}\right] = \frac{\lim_{x\to c} f(x)}{\lim_{x\to c} g(x)} = \frac{L}{K}$ $\lim_{x\to c} g(x) \neq 0$

$\lim_{x\to c}[f(x)]^n = \left[\lim_{x\to c} f(x)\right]^n = L^n$

Limit of a Polynomial Function

If $p(x)$ is a polynomial function and c is a real number $\lim_{x\to c} p(x) = p(c)$

Limit of a Rational Function

If $r(x)$ is a rational function, $r(x) = \frac{p(x)}{q(x)}$ and c is a real number such that $q(c) \neq 0$

$\lim_{x\to c} r(x) = \frac{p(c)}{q(c)}$

Direct Substitution

Limit of a Radical Function

Let n be a positive integer, for all c when n is odd and for $c > 0$ when n is even

$\lim_{x\to c} \sqrt[n]{x} = \sqrt[n]{c}$

Limit of a Composite Function

If f and g are functions such that $\lim_{x \to c} g(x) = L$ and $\lim_{x \to L} f(x) = f(L)$ then

$\lim_{x \to c} f(g(x)) =$ $f\left[\lim_{x \to c} g(x)\right]$

Limits of Transcendental Functions

$\lim_{x \to c} sinx =$ sinc $\quad \lim_{x \to c} cosx =$ cosc $\quad \lim_{x \to c} tanx =$ tanc

$\lim_{x \to c} secx =$ sec(c) $\quad \lim_{x \to c} cscx =$ csc(c) $\quad \lim_{x \to c} cotx =$ cotc

$\lim_{x \to c} a^x =$ a^c $\quad \lim_{x \to c} lnx =$ lnc

** c must be in the domain!

Examples – Evaluating Limits

$\lim_{x \to 16} \sqrt[4]{x} =$ $\sqrt[4]{16} = 2$

$\lim_{x \to 2}(x^2 + 5x + 4) =$ 18

$2^2 + 10 + 4 = 18$

$\lim_{x \to 2} x^3 =$ $2^3 = 8$

$\lim_{x \to 2}\left(\frac{x^2+5x+4}{x+4}\right) =$ 3

$\frac{2^2 + 10 + 4}{2+4}$

$\lim_{x \to 4} 8x =$ $8 \cdot 4 = 32$

$\lim_{x \to 2} e^{3x} =$ e^6

$\lim_{x \to \frac{3\pi}{4}} tanx =$ $\tan\frac{3\pi}{4} = -1$

$\lim_{x \to -4} \sqrt[3]{2x^2 - 5} =$ $\sqrt[3]{32-5} = 3$

Functions that Agree at all but One Point

Let c be a real number, and let $f(x) = g(x)$ for all $x \neq c$ in an open interval containing c. If the limit of $g(x)$ as x approaches c exists, then the limit of $f(x)$ also exists and

$\lim_{x \to c} f(x) = \lim_{x \to c} g(x)$

Examples: Evaluating Limits, Beyond Substitution

Given $f(x) = \begin{cases} x^3 + 2 & x \neq -3 \\ 5 & x = -3 \end{cases}$ *then* $\lim\limits_{x \to -3} f(x) =$ $(-3)^3 + 2 = -27 + 2 = -25$

$\lim\limits_{x \to 2} \frac{x^2+2x-8}{x-2} =$ $\lim\limits_{x \to 2} \frac{(x+4)\cancel{(x-2)}}{\cancel{x-2}} = 6$

$\lim\limits_{x \to 0} \frac{\sqrt{x+25}-5}{x} \frac{(\sqrt{x+25}+5)}{(\sqrt{x+25}+5)}$

$= \lim\limits_{x \to 0} \frac{\cancel{x}+\cancel{25}-\cancel{25}}{\cancel{x}(\sqrt{x+25}+5)}$

$= \lim\limits_{x \to 0} \frac{1}{\sqrt{x+25}+5} = \frac{1}{10}$

Squeeze Theorem

Refer to the proof on pg. 82

Special Limits

$\lim_{x \to 0} \frac{sinx}{x} = 1$

$\lim_{x \to 0} \frac{1-cosx}{x} = 0$

$\lim_{x \to 0} (1+x)^{\frac{1}{x}} = e$

Example: Limits with Trig Functions

$\lim_{x \to 0} \frac{2-2cosx}{3x} =$

$$\lim_{x \to 0} \frac{2(1-\cos x)}{3x}$$

$$\lim_{x \to 0} \left(\frac{2}{3}\right)\left(\frac{1-\cos x}{x}\right)$$

$$\left(\frac{2}{3}\right)(0) = 0$$

$\lim_{x \to 0} \frac{tan4x}{6x} =$

$$\lim_{x \to 0} \frac{\frac{\sin 4x}{\cos 4x}}{6x}$$

$$= \lim_{x \to 0} \frac{\sin 4x}{\cos 4x} \cdot \frac{1}{6x}$$

$$= \lim_{x \to 0} \frac{\sin 4x}{4x} \cdot \frac{1}{\cos 4x} \cdot \frac{4}{6}$$

$$= (1)(1)\left(\frac{4}{6}\right) = \frac{2}{3}$$

$$\frac{\sin 4x}{x} \cancel{=} \frac{4\sin x}{x}$$

① Plug it in

② Algebra, Rationalize, Trig Identities ...