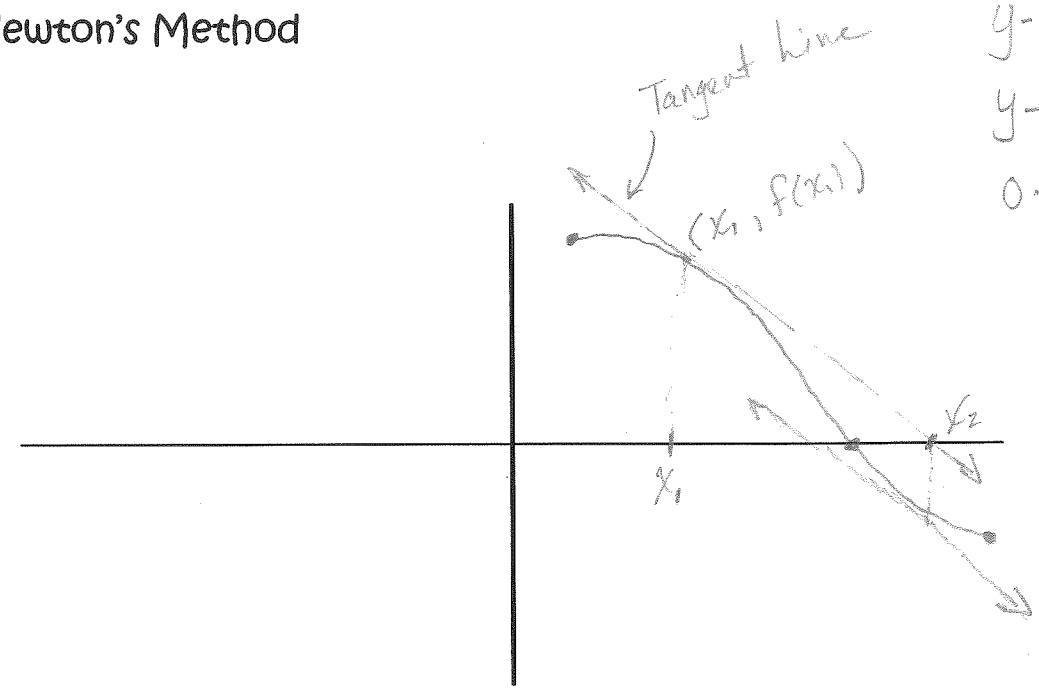
2.7 Newton's Method



$y - y_1 = m(x - x_1)$

$y - f(x_1) = f'(x_1)(x - x_1)$

$0 - f(x_1) = f'(x_1)(x - x_1)$

$\frac{-f(x_1)}{f'(x_1)} + x_1 = x$

Newton's Method

1. Make an initial guess x_1

2. Determine a new approximation

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

3. Stop when $|x_n - x_{n+1}|$ is within the desired accuracy OR you have completed the number of iterations.

Examples: Newton's Method

Calculate four iterations of Newton's Method to approximate the zero of $f(x) = x^2 - 5$. Use $x_1 = 2$ as the initial guess.

$f'(x) = 2x$

$x_1 = 2$

$x_2 = 2 - \frac{f(2)}{f'(2)} = 2 - \frac{(-1)}{(4)} = 2 + \frac{1}{4} = 2.25$

$x_3 = 2.25 - \frac{f(2.25)}{f'(2.25)} = 2.25 - \frac{.0625}{4.5} \approx 2.2361$

$x_4 =$

$x_5 =$

Use Newton's Method to approximate the zero(s) of $f(x) = e^{-3x} - x^2$. Continue the iterations until two successive approximations differ by less than 0.0001.