7.2 Integration By Parts

Integration by parts is a technique of integration used for integrands involving the products of algebraic, exponential or logarithmic functions

Let u and v be differentiable functions of x.

$$\int u\,dv = uv - \int v\,du$$

$\int xe^{-x}\,dx$

$u = x \quad dv = e^{-x}$

$du = 1\,dx \quad v = -e^{-x}$

$(x)(-e^{-x}) - \int -e^{-x}\,dx$

$-xe^{-x} - (-e^{-x})(-1) + C$

$-xe^{-x} - e^{-x} + C$

$\int x^2 e^{2x}\,dx$

$u = x^2 \quad dv = e^{2x}$

$du = 2x\,dx \quad v = \frac{1}{2}e^{2x}$

$(x^2)(\frac{1}{2}e^{2x}) - \int 2xe^{2x}\,dx$

$u = 2x \quad dv = e^{2x}$

$du = 2dx \quad v = \frac{1}{2}e^{2x}$

$\frac{1}{2}x^2e^{2x} - [(2x)(\frac{1}{2}e^{2x}) - \int e^{2x} \cdot 2dx]$

$\frac{1}{2}x^2e^{2x} - xe^{2x} + e^{2x} + C$

What to think about when using Integration by Parts

1. Let dv be the more complicated portion that has a basic integration rule.

2. Let u be the portion of the integrand where it's derivative is simpler than u.

Examples

$\int lnx^2\, dx$

$u = \ln x^2$ $\quad dv = dx$

$du = \frac{1}{x^2} \cdot 2x\, dx$ $\quad v = x$

$du = \frac{2}{x} dx$

$(\ln x^2)(x) - \int x \left(\frac{2}{x}\right) dx$

$x \ln x^2 - 2x + C$

$\int x^2 lnx\, dx$

$u = \ln x$ $\quad dv = x^2 dx$

$du = \frac{1}{x} dx$ $\quad v = \frac{1}{3}x^3$

$(\ln x)\left(\frac{1}{3}x^3\right) - \int \left(\frac{1}{3}x^3\right)\left(\frac{1}{x}\right) dx$

$\frac{1}{3}x^3 \ln x - \frac{1}{3}\left(\frac{x^3}{3}\right) + C$

$\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + C$

$\int \frac{lnx}{x^2}\, dx$

$u = \ln x$ $\quad dv = x^{-2}$

$du = \frac{1}{x} dx$ $\quad v = -x^{-1}$

$\left(-\frac{1}{x}\right)(\ln x) - \int \left(-\frac{1}{x}\right)\left(\frac{1}{x}\right) dx$

$-\frac{1}{x}\ln x + \int x^{-2} dx$

$-\frac{1}{x}\ln x + \frac{x^{-1}}{-1} + C$

$-\frac{1}{x}\ln x - \frac{1}{x} + C$

$\int_0^1 \ln(1 + 2x)\, dx$

$u = \ln(1+2x)$ $\quad dv = dx$

$du = \frac{1}{1+2x} \cdot 2\, dx$ $\quad v = x$

$du = \frac{2}{1+2x} dx$

$[\ln(1+2x)](x) - \int \frac{2x}{1+2x} dx$

$u = 1+2x$ $\quad 2x = u-1$

$du = 2dx$

$x\ln(1+2x) - \int \frac{u-1}{u} \cdot \frac{du}{2}$

$x\ln(1+2x) - \frac{1}{2}\int 1 - \frac{1}{u}\, du$

$x\ln(1+2x) - \frac{1}{2}u + \frac{1}{2}\ln|u|$

$x\ln(1+2x) - \frac{1}{2}(1+2x) + \frac{1}{2}\ln|1+2x| \Big|_0^1$

$\left[1(\ln 3) - \frac{1}{2}(3) + \frac{1}{2}\ln 3\right] - \left[0 - \frac{1}{2}(1) + \frac{1}{2}\ln 1\right]$

$\ln 3 - \frac{3}{2} + \frac{1}{2}\ln 3 + \frac{1}{2}$

$\frac{3}{2}\ln 3 - 1$