# 1.6 Limits at Infinity

**Definition of a Horizontal Asymptote**

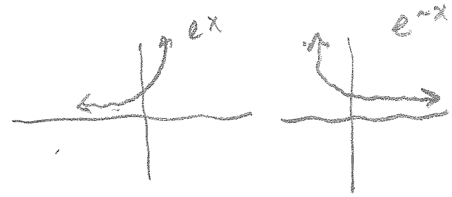
The line $y = L$ is a horizontal asymptote of the graph of $f$ when $\lim\limits_{x\to-\infty} f(x) = L$ or $\lim\limits_{x\to\infty} f(x) = L$

**Limits at Infinity**

1. If $r$ is a positive rational number and $c$ is any real number, then

$$\lim_{x\to\infty} \frac{c}{x^r} = 0 \quad \text{OR} \quad \lim_{x\to-\infty} \frac{c}{x^r} = 0$$

2. $\lim\limits_{x\to-\infty} e^x = 0$ and $\lim\limits_{x\to\infty} e^{-x} = 0$




**Examples – Evaluating a limit at infinity**

$\lim\limits_{x\to-\infty} \left(7 - \frac{1}{x^2}\right) = 7$

$\lim\limits_{x\to-\infty} (e^x - 6) = -6$

$$\lim_{x\to\infty} \frac{3x}{x-1} = \lim_{x\to\infty} \frac{\frac{3x}{x}}{\frac{x}{x} - \frac{1}{x}} = \lim_{x\to\infty} \frac{3}{1-\frac{1}{x}} = 3$$

**Guidelines for Finding Limits at $\pm\infty$ of Rational Functions**

1. Degree of numerator < Degree of Denominator

$$\lim_{x\to\infty} f(x) = 0 \quad \text{OR} \quad \lim_{x\to-\infty} f(x) = 0$$

2. Degree of numerator = Degree of Denominator

$$\lim_{x\to\infty} f(x) = \frac{\text{leading coefficient}}{\text{leading coefficient}}$$

3. Degree of numerator > Degree of Denominator

DNE

**Examples: More Evaluating Limits at Infinity**

$\lim_{x\to\infty} \frac{-x+4}{5x^2+2} = 0$

$\lim_{x\to\infty} \frac{-x^2+4}{5x^2+2} = -\frac{1}{5}$

$\lim_{x\to\infty} \frac{-x^3+4}{5x^2+2} = \text{DNE}$

$\lim_{x\to\infty} \frac{|2x|}{3x+1} = \frac{2}{3}$

$$\begin{cases} \frac{2x}{3x+1} & x>0 \\ \frac{-2x}{3x+1} & x<0 \end{cases}$$

$\lim_{x\to\infty} \frac{\cos x+3x}{x} = 3$

$\lim_{x\to\infty} (-x^5) = \text{DNE}$ or $-\infty$

$\lim_{x\to-\infty} (-x^5)$ DNE or $\infty$

$\lim_{x\to\infty} \frac{3x^2+x}{x-1}$ DNE or $\infty$

$\lim_{x\to-\infty} \frac{3x^2+x}{x-1} = -\infty$ OR DNE

$$\lim_{x\to\infty} \frac{\cos x}{x} + \frac{3\cancel{x}}{\cancel{x}} = 0+3 = 3$$

$$\lim_{x\to\infty} \quad \frac{-1}{x} \le \frac{\cos x}{x} \le \frac{1}{x}$$

$$\lim_{x\to\infty} \frac{\cos x}{x} = 0$$

You are manufacturing greeting cards that cost \$0.65 per card to produce. Your initial investment 4500, which implies that the total cost, $C$ of producing $x$ cards is given by $C(x) = 0.65x + 4500$.

A. What would represent the average cost, $\bar{C}$?

$$\bar{C} = \frac{.65x + 4500}{x}$$

B. Find the average cost per card for $x = 5000$, $x = 50{,}000$ and $x = 500{,}000$.

$\bar{C}(5000) = 1.55$ $\quad \bar{C}(50{,}000) = .74$ $\quad \bar{C}(500{,}000) = .66$

C. What is the limit of $\bar{C}$ as $x$ approaches infinity? What does this mean?

$$\lim_{x \to \infty} \frac{.65x + 4500}{x} = .65$$

If you produce larger number of cards, the average cost approaches \$.65 per card