Section 7.1 Basic Integration Rules

1. $\int (5x - 3)^4\, dx$

$u = 5x - 3,\ du = 5\, dx,\ n = 4$

Use $\int u^n\, du$.

2. $\int \frac{2t + 1}{t^2 + t - 4}\, dt$

$u = t^2 + t - 4,\ du = (2t + 1)\, dt$

Use $\int \frac{du}{u}$.

3. $\int \frac{1}{\sqrt{x}(1 - 2\sqrt{x})}\, dx$

$u = 1 - 2\sqrt{x},\ du = -\frac{1}{\sqrt{x}}\, dx$

Use $\int \frac{du}{u}$.

4. $\int \frac{2}{(2t - 1)^2 + 4}\, dt$

$u = 2t - 1,\ du = 2\, dt,\ a = 2$

Use $\int \frac{du}{u^2 + a^2}$.

5. $\int \frac{3}{\sqrt{1 - t^2}}\, dt$

$u = t,\ du = dt,\ a = 1$

Use $\int \frac{du}{\sqrt{a^2 - u^2}}$.

6. $\int \frac{-2x}{\sqrt{x^2 - 4}}\, dx$

$u = x^2 - 4,\ du = 2x\, dx,\ n = -\frac{1}{2}$

Use $\int u^n\, du$.

7. $\int t \sin t^2\, dt$

$u = t^2,\ du = 2t\, dt$

Use $\int \sin u\, du$.

8. $\int \sec 5x \tan 5x\, dx$

$u = 5x,\ du = 5\, dx$

Use $\int \sec u \tan u\, du$.

9. $\int (\cos x) e^{\sin x}\, dx$

$u = \sin x,\ du = \cos x\, dx$

Use $\int e^u\, du$.

10. $\int \frac{1}{x\sqrt{x^2 - 4}}\, dx$

$u = x,\ du = dx,\ a = 2$

Use $\int \frac{du}{u\sqrt{u^2 - a^2}}$.

11. Let $u = x - 5,\ du = dx$.

$$\int 14(x - 5)^6\, dx = 14\int (x - 5)^6\, dx = 2(x - 5)^7 + C$$

12. Let $u = t + 6,\ du = dt$.

$$\int \frac{5}{(t + 6)^3}\, dt = 5\int (t + 6)^{-3}\, dt$$

$$= 5 \cdot \frac{(t + 6)^{-2}}{-2} + C$$

$$= \frac{-5}{2(t + 6)^2} + C$$

13. Let $u = z - 10,\ du = dz$.

$$\int \frac{7}{(z - 10)^7}\, dz = 7\int (z - 10)^{-7}\, dz = -\frac{7}{6(z - 10)^6} + C$$

14. Let $u = t^4 + 1,\ du = 4t^3\, dt$.

$$\int t^3\sqrt{t^4 + 1}\, dt = \frac{1}{4}\int (t^4 + 1)^{1/2}(4t^3)\, dt$$

$$= \frac{1}{4} \cdot \frac{(t^4 + 1)^{3/2}}{(3/2)} + C$$

$$= \frac{1}{6}(t^4 + 1)^{3/2} + C$$

15. $\int \left[v + \frac{1}{(3v-1)^3}\right] dv = \int v\,dv + \frac{1}{3}\int (3v-1)^{-3}(3)\,dv$

$$= \frac{1}{2}v^2 - \frac{1}{6(3v-1)^2} + C$$

16. $\int \left[4x - \frac{2}{(2x+3)^2}\right] dx = \int 4x\,dx - \int 2(2x+3)^{-2}\,dx$

$$= 2x^2 - \frac{(2x+3)^{-1}}{-1} + C$$

$$= 2x^2 + \frac{1}{2x+3} + C$$

17. Let $u = -t^3 + 9t + 1$,

$du = (-3t^2 + 9)\,dt = -3(t^2 - 3)\,dt.$

$$\int \frac{t^2 - 3}{-t^3 + 9t + 1}\,dt = -\frac{1}{3}\int \frac{-3(t^2-3)}{-t^3 + 9t + 1}\,dt$$

$$= -\frac{1}{3}\ln\left|-t^3 + 9t + 1\right| + C$$

18. Let $u = 3x^2 + 6x$, $du = (6x + 6)\,dx = 6(x+1)\,dx$.

$$\int \frac{x+1}{\sqrt{3x^2 + 6x}}\,dx = \frac{1}{6}\int (3x^2 + 6x)^{-1/2} 6(x+1)\,dx$$

$$= \frac{1}{6}\cdot\frac{(3x^2 + 6x)^{1/2}}{(1/2)} + C$$

$$= \frac{1}{3}\sqrt{3x^2 + 6x} + C$$

19. $\int \frac{x^2}{x-1}\,dx = \int (x+1)\,dx + \int \frac{1}{x-1}\,dx$

$$= \frac{1}{2}x^2 + x + \ln|x - 1| + C$$

20. $\int \frac{3x}{x+4}\,dx = \int \left(3 - \frac{12}{x+4}\right) dx$

$$= 3x - 12\ln|x + 4| + C$$

21. Let $u = 1 + e^x$, $du = e^x\,dx$.

$$\int \frac{e^x}{1 + e^x}\,dx = \ln(1 + e^x) + C$$

22. $\int \left(\frac{1}{2x+5} - \frac{1}{2x-5}\right) dx = \frac{1}{2}\int \frac{1}{2x+5}(2)\,dx - \frac{1}{2}\int \frac{1}{2x-5}(2)\,dx$

$$= \frac{1}{2}\ln|2x + 5| - \frac{1}{2}\ln|2x - 5| + C$$

$$= \frac{1}{2}\ln\left|\frac{2x+5}{2x-5}\right| + C$$

23. $\int (5 + 4x^2)^2\,dx = \int (25 + 40x^2 + 16x^4)\,dx$

$$= 25x + \frac{40}{3}x^3 + \frac{16}{5}x^5 + C$$

$$= \frac{x}{15}(48x^4 + 200x^2 + 375) + C$$

24. $\int x\left(3 + \frac{2}{x}\right)^2 dx = \int \left(9x + 12 + \frac{4}{x}\right) dx$

$$= \frac{9}{2}x^2 + 12x + 4\ln|x| + C$$

25. Let $u = 2\pi x^2$, $du = 4\pi x\,dx$.

$$\int x(\cos 2\pi x^2)\,dx = \frac{1}{4\pi}\int (\cos 2\pi x^2)(4\pi x)\,dx$$

$$= \frac{1}{4\pi}\sin 2\pi x^2 + C$$

26. Let $u = \pi x$, $du = \pi\,dx$.

$$\int \csc \pi x \cot \pi x\,dx = \frac{1}{\pi}\int (\csc \pi x)(\cot \pi x)\,\pi\,dx$$

$$= -\frac{1}{\pi}\csc \pi x + C$$

27. Let $u = \cos x$, $du = -\sin x\,dx$.

$$\int \frac{\sin x}{\sqrt{\cos x}}\,dx = -\int (\cos x)^{-1/2}(-\sin x)\,dx$$

$$= -2\sqrt{\cos x} + C$$

28. Let $u = \cot x$, $du = -\csc^2 x\,dx$.

$$\int \csc^2 x e^{\cot x}\,dx = -\int e^{\cot x}(-\csc^2 x)\,dx = -e^{\cot x} + C$$

29. Let $u = 1 + e^x$, $du = e^x\,dx$.

$$\int \frac{2}{e^{-x} + 1}\,dx = 2\int \left(\frac{2}{e^{-x} + 1}\right)\left(\frac{e^x}{e^x}\right) dx$$

$$= 2\int \frac{e^x}{1 + e^x}\,dx = 2\ln(1 + e^x) + C$$

30. $$\int \frac{2}{7e^x + 4}\,dx = 2\int \frac{1}{7e^x + 4}\left(\frac{e^{-x}}{e^{-x}}\right)dx$$
$$= 2\int \frac{e^{-x}}{7 + 4e^{-x}}\,dx$$
$$= 2\left(-\frac{1}{4}\right)\int \frac{1}{\left(7 + 4e^{-x}\right)}\left(-4e^{-x}\right)dx$$
$$= -\frac{1}{2}\ln\left|7 + 4e^{-x}\right| + C$$

31. $$\int \frac{\ln x^2}{x}\,dx = 2\int (\ln x)\frac{1}{x}\,dx = 2\,\frac{(\ln x)^2}{2} + C = (\ln x)^2 + C$$

32. Let $u = \ln(\cos x)$, $du = \dfrac{-\sin x}{\cos x}\,dx = -\tan x\,dx$.

$$\int (\tan x)(\ln \cos x)\,dx = -\int (\ln \cos x)(-\tan x)\,dx$$
$$= \frac{-\left[\ln(\cos x)\right]^2}{2} + C$$

33. $$\int \frac{1 + \cos\alpha}{\sin\alpha}\,d\alpha = \int \csc\alpha\,d\alpha + \int \cot\alpha\,d\alpha$$
$$= -\ln\left|\csc\alpha + \cot\alpha\right| + \ln\left|\sin\alpha\right| + C$$

34. $$\frac{1}{\cos\theta - 1} = \frac{1}{\cos\theta - 1}\cdot\frac{\cos\theta + 1}{\cos\theta + 1} = \frac{\cos\theta + 1}{\cos^2\theta - 1}$$
$$= \frac{\cos\theta + 1}{-\sin^2\theta} = -\csc\theta\cdot\cot\theta - \csc^2\theta$$

$$\int \frac{1}{\cos\theta - 1}\,d\theta = \int\left(-\csc\theta\cot\theta - \csc^2\theta\right)d\theta$$
$$= \csc\theta + \cot\theta + C$$
$$= \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta} + C$$
$$= \frac{1 + \cos\theta}{\sin\theta} + C$$

35. Let $u = 4t + 1$, $du = 4\,dt$.

$$\int \frac{-1}{\sqrt{1 - (4t + 1)^2}}\,dt = -\frac{1}{4}\int \frac{4}{\sqrt{1 - (4t + 1)^2}}\,dt$$
$$= -\frac{1}{4}\arcsin(4t + 1) + C$$

36. Let $u = 2x$, $du = 2\,dx$, $a = 5$.

$$\int \frac{1}{25 + 4x^2}\,dx = \frac{1}{2}\int \frac{1}{5^2 + (2x)^2}(2)\,dx$$
$$= \frac{1}{10}\arctan\frac{2x}{5} + C$$

37. Let $u = \cos\left(\dfrac{2}{t}\right)$, $du = \dfrac{2\sin(2/t)}{t^2}\,dt$.

$$\int \frac{\tan(2/t)}{t^2}\,dt = \frac{1}{2}\int \frac{1}{\cos(2/t)}\left[\frac{2\sin(2/t)}{t^2}\right]dt$$
$$= \frac{1}{2}\ln\left|\cos\left(\frac{2}{t}\right)\right| + C$$

38. Let $u = \dfrac{1}{t}$, $du = \dfrac{-1}{t^2}\,dt$.

$$\int \frac{e^{1/t}}{t^2}\,dt = -\int e^{1/t}\left(\frac{-1}{t^2}\right)dt = -e^{1/t} + C$$

39. Note: $10x - x^2 = 25 - \left(25 - 10x + x^2\right)$
$$= 25 - (5 - x)^2$$

$$\int \frac{6}{\sqrt{10x - x^2}}\,dx = 6\int \frac{1}{\sqrt{25 - (5 - x)^2}}\,dx$$
$$= -6\int \frac{-1}{\sqrt{5^2 - (5 - x)^2}}\,dx$$
$$= -6\arcsin\frac{(5 - x)}{5} + C$$
$$= 6\arcsin\left(\frac{x - 5}{5}\right) + C$$

40. $$\int \frac{1}{(x - 1)\sqrt{4x^2 - 8x + 3}}\,dx = \int \frac{2}{\left[2(x - 1)\right]\sqrt{\left[2(x - 1)\right]^2 - 1}}\,dx = \operatorname{arcsec}\left|2(x - 1)\right| + C$$

41. $\displaystyle\int \frac{4}{4x^2 + 4x + 65}\,dx = \int \frac{1}{[x + (1/2)]^2 + 16}\,dx$

$$= \frac{1}{4}\arctan\left[\frac{x + (1/2)}{4}\right] + C$$

$$= \frac{1}{4}\arctan\left(\frac{2x + 1}{8}\right) + C$$

42. $\displaystyle\int \frac{1}{x^2 - 4x + 9}\,dx = \int \frac{1}{x^2 - 4x + 4 + 5}\,dx$

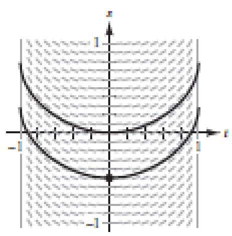
$$= \int \frac{1}{(x - 2)^2 + \left(\sqrt{5}\right)^2}\,dx$$

$$= \frac{1}{\sqrt{5}}\arctan\left(\frac{x - 2}{\sqrt{5}}\right) + C$$

$$= \frac{\sqrt{5}}{5}\arctan\left(\frac{\sqrt{5}}{5}(x - 2)\right) + C$$

43. $\displaystyle\frac{ds}{dt} = \frac{t}{\sqrt{1 - t^4}}, \quad \left(0, -\frac{1}{2}\right)$

(a)



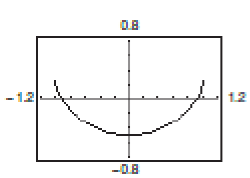
(b) $u = t^2,\ du = 2t\,dt$

$$\int \frac{t}{\sqrt{1 - t^4}}\,dt = \frac{1}{2}\int \frac{2t}{\sqrt{1 - \left(t^2\right)^2}}\,dt$$

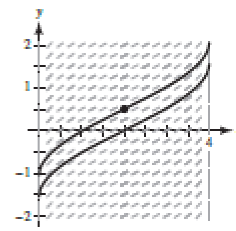
$$= \frac{1}{2}\arcsin t^2 + C$$

$$\left(0, -\frac{1}{2}\right): -\frac{1}{2} = \frac{1}{2}\arcsin 0 + C \Rightarrow C = -\frac{1}{2}$$

$$s = \frac{1}{2}\arcsin t^2 - \frac{1}{2}$$



44. (a) $\displaystyle\frac{dy}{dx} = \frac{1}{\sqrt{4x - x^2}}, \quad \left(2, \frac{1}{2}\right)$



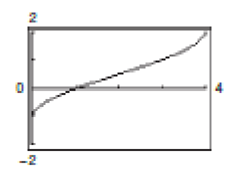
(b) $\displaystyle y = \int \frac{1}{\sqrt{4x - x^2}}\,dx$

$$= \int \frac{1}{\sqrt{4 - \left(x^2 - 4x + 4\right)}}\,dx$$

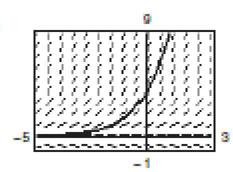
$$= \int \frac{1}{\sqrt{4 - (x - 2)^2}}\,dx = \arcsin\left(\frac{x - 2}{2}\right) + C$$

$$\left(2, \frac{1}{2}\right): \frac{1}{2} = \arcsin(0) + C \Rightarrow C = \frac{1}{2}$$

$$y = \arcsin\left(\frac{x - 2}{2}\right) + \frac{1}{2}$$

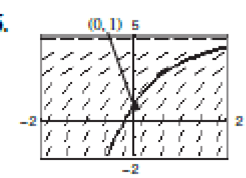


45.



$y = 4e^{0.8x}$

46.



$y = 5 - 4e^{-x}$

47. $\displaystyle\frac{dy}{dx} = \left(e^x + 5\right)^2 = e^{2x} + 10e^x + 25$

$$y = \int \left(e^{2x} + 10e^x + 25\right)dx$$

$$= \frac{1}{2}e^{2x} + 10e^x + 25x + C$$

48. $\frac{dy}{dx} = \left(4 - e^{2x}\right)^2 = 16 - 8e^{2x} + e^{4x}$

$$y = \int \left(16 - 8e^{2x} + e^{4x}\right) dx$$
$$= 16x - 4e^{2x} + \frac{1}{4}e^{4x} + C$$

49. $\frac{dr}{dt} = \frac{10e^t}{\sqrt{1 - e^{2t}}}$

$$r = \int \frac{10e^t}{\sqrt{1 - \left(e^t\right)^2}}\, dt$$
$$= 10 \arcsin\left(e^t\right) + C$$

50. $\frac{dr}{dt} = \frac{\left(1 + e^t\right)^2}{e^{3t}} = \frac{1 + 2e^t + e^{2t}}{e^{3t}} = e^{-3t} + 2e^{-2t} + e^{-t}$

$$r = \int \left(e^{-3t} + 2e^{-2t} + e^{-t}\right) dt$$
$$= -\frac{1}{3}e^{-3t} - e^{-2t} - e^{-t} + C$$

51. $\frac{dy}{dx} = \frac{\sec^2 x}{4 + \tan^2 x}$

Let $u = \tan x$, $du = \sec^2 x\, dx$.

$$y = \int \frac{\sec^2 x}{4 + \tan^2 x}\, dx = \frac{1}{2}\arctan\left(\frac{\tan x}{2}\right) + C$$

52. $y' = \frac{1}{x\sqrt{4x^2 - 9}}$

Let $u = 2x$, $du = 2\, dx$, $a = 3$.

$$y = \int \frac{1}{x\sqrt{4x^2 - 9}}\, dx = \int \frac{1}{(2x)\sqrt{(2x)^2 - 3^2}}(2)\, dx$$
$$= \frac{1}{3}\operatorname{arcsec}\frac{|2x|}{3} + C$$

53. Let $u = 2x$, $du = 2\, dx$.

$$\int_0^{\pi/4} \cos 2x\, dx = \tfrac{1}{2}\int_0^{\pi/4} \cos 2x(2)\, dx$$
$$= \left[\tfrac{1}{2}\sin 2x\right]_0^{\pi/4} = \tfrac{1}{2}$$

54. Let $u = \sin t$, $du = \cos t\, dt$.

$$\int_0^{\pi} \sin^2 t \cos t\, dt = \left[\tfrac{1}{3}\sin^3 t\right]_0^{\pi} = 0$$

55. Let $u = -x^2$, $du = -2x\, dx$.

$$\int_0^1 xe^{-x^2}\, dx = -\tfrac{1}{2}\int_0^1 e^{-x^2}(-2x)\, dx = \left[-\tfrac{1}{2}e^{-x^2}\right]_0^1$$
$$= \tfrac{1}{2}\left(1 - e^{-1}\right) \approx 0.316$$

56. Let $u = 1 - \ln x$, $du = \frac{-1}{x}\, dx$.

$$\int_1^e \frac{1 - \ln x}{x}\, dx = -\int_1^e (1 - \ln x)\left(\frac{-1}{x}\right) dx$$
$$= \left[-\frac{1}{2}(1 - \ln x)^2\right]_1^e = \frac{1}{2}$$

57. Let $u = x^2 + 36$, $du = 2x\, dx$.

$$\int_0^8 \frac{2x}{\sqrt{x^2 + 36}}\, dx = \int_0^8 \left(x^2 + 36\right)^{-1/2}(2x)\, dx$$
$$= 2\left[\left(x^2 + 36\right)^{1/2}\right]_0^8 = 8$$

58. $\int_1^3 \frac{2x^2 + 3x - 2}{x}\, dx = \int_1^3 \left(2x + 3 - \frac{2}{x}\right) dx$

$$= \left[x^2 + 3x - 2\ln|x|\right]_1^3$$
$$= (9 + 9 - 2\ln 3) - (1 + 3 - ($$
$$= 14 - 2\ln 3$$

59. Let $u = 3x$, $du = 3\, dx$.

$$\int_0^{2/\sqrt{3}} \frac{1}{4 + 9x^2}\, dx = \frac{1}{3}\int_0^{2/\sqrt{3}} \frac{3}{4 + (3x)^2}\, dx$$
$$= \left[\frac{1}{6}\arctan\left(\frac{3x}{2}\right)\right]_0^{2/\sqrt{3}}$$
$$= \frac{\pi}{18} \approx 0.175$$

60. $\int_0^7 \frac{1}{\sqrt{100 - x^2}}\, dx = \left[\arcsin\left(\frac{x}{10}\right)\right]_0^7 = \arcsin\left(\frac{7}{10}\right)$

61. $A = \int_0^{3/2} (-4x + 6)^{3/2}\, dx$

$$= -\tfrac{1}{4}\int_0^{3/2} (6 - 4x)^{3/2}(-4)\, dx$$
$$= -\tfrac{1}{4}\left[\tfrac{2}{5}(6 - 4x)^{5/2}\right]_0^{3/2}$$
$$= -\tfrac{1}{10}\left(0 - 6^{5/2}\right)$$
$$= \tfrac{18}{5}\sqrt{6} \approx 8.8182$$

62. $A = \int_0^5 \frac{3x + 2}{x^2 + 9}\, dx$

$$= \int_0^5 \frac{3x}{x^2 + 9}\, dx + \int_0^5 \frac{2}{x^2 + 9}\, dx$$
$$= \left[\frac{3}{2}\ln\left|x^2 + 9\right| + \frac{2}{3}\arctan\left(\frac{x}{3}\right)\right]_0^5$$
$$= \frac{3}{2}\ln(34) + \frac{2}{3}\arctan\left(\frac{5}{3}\right) - \frac{3}{2}\ln 9$$
$$= \frac{3}{2}\ln\left(\frac{34}{9}\right) + \frac{2}{3}\arctan\left(\frac{5}{3}\right)$$
$$\approx 2.6806$$

63. $y^2 = x^2(1 - x^2)$

$y = \pm\sqrt{x^2(1 - x^2)}$

$A = 4\int_0^1 x\sqrt{1 - x^2}\, dx$

$= -2\int_0^1 (1 - x^2)^{1/2}(-2x)\, dx$

$= -\frac{4}{3}\left[(1 - x^2)^{3/2}\right]_0^1$

$= -\frac{4}{3}(0 - 1) = \frac{4}{3}$

64. $A = \int_0^{\pi/2} \sin 2x\, dx = -\frac{1}{2}[\cos 2x]_0^{\pi/2} = -\frac{1}{2}(-1 - 1) = 1$

65. Power Rule: $\int u^n\, du = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1$

$u = x^2 + 1, n = 3$

66. $\int \sec u \tan u\, du = \sec u + C$

67. Log Rule: $\int \frac{du}{u} = \ln|u| + C, \quad u = x^2 + 1$

68. Arctan Rule: $\int \frac{du}{a^2 + u^2} = \frac{1}{a}\arctan\left(\frac{u}{a}\right) + C$

69. $\sin x + \cos x = a\sin(x + b)$

$\sin x + \cos x = a\sin x\cos b + a\cos x\sin b$

$\sin x + \cos x = (a\cos b)\sin x + (a\sin b)\cos x$

Equate coefficients of like terms to obtain the following.

$1 = a\cos b$ and $1 = a\sin b$

So, $a = 1/\cos b$. Now, substitute for a in $1 = a\sin b$.

$1 = \left(\frac{1}{\cos b}\right)\sin b$

$1 = \tan b \Rightarrow b = \frac{\pi}{4}$

Because $b = \frac{\pi}{4}$, $a = \frac{1}{\cos(\pi/4)} = \sqrt{2}$. So, $\sin x + \cos x = \sqrt{2}\sin\left(x + \frac{\pi}{4}\right)$.

$$\int \frac{dx}{\sin x + \cos x} = \int \frac{dx}{\sqrt{2}\sin(x + (\pi/4))} = \frac{1}{\sqrt{2}}\int \csc\left(x + \frac{\pi}{4}\right) dx = -\frac{1}{\sqrt{2}}\ln\left|\csc\left(x + \frac{\pi}{4}\right) + \cot\left(x + \frac{\pi}{4}\right)\right| + C$$

70. $\frac{\sin x}{\cos x} + \frac{\cos x}{1 + \sin x} = \frac{\sin x(1 + \sin x) + \cos^2 x}{\cos x(1 + \sin x)}$

$= \frac{\sin x + \sin^2 x + \cos^2 x}{\cos x(1 + \sin x)}$

$= \frac{\sin x + 1}{\cos x(1 + \sin x)}$

$= \frac{1}{\cos x} = \sec x$

So,

$\int \sec x\, dx = \int\left[\frac{\sin x}{\cos x} + \frac{\cos x}{1 + \sin x}\right] dx$

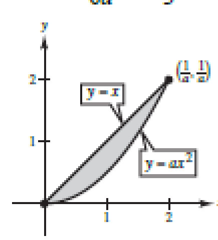
$= -\ln|\cos x| + \ln|1 + \sin x| + C$

$= \ln\left|\frac{1 + \sin x}{\cos x}\right| + C$

$= \ln|\sec x + \tan x| + C$

71. $\int_0^{1/a} (x - ax^2)\, dx = \left[\frac{1}{2}x^2 - \frac{a}{3}x^3\right]_0^{1/a} = \frac{1}{6a^2}$

Let $\frac{1}{6a^2} = \frac{2}{3}$, $12a^2 = 3$, $a = \frac{1}{2}$.



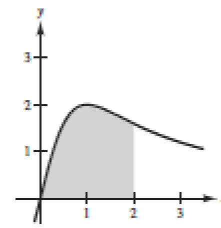
72. No. When $u = x^2$, it does not follow that $x = \sqrt{u}$ because x is negative on $[-1, 0)$.

73. (a) They are equivalent because
$$e^{x+C_1} = e^x \cdot e^{C_1} = Ce^x, C = e^{C_1}.$$
(b) They differ by a constant.
$$\sec^2 x + C_1 = (\tan^2 x + 1) + C_1 = \tan^2 x + C$$

74. $\int_0^5 f(x)\,dx < 0$ because there is more area below the x-axis than above.

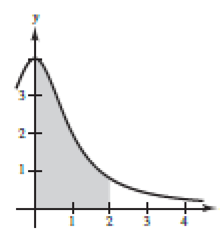
75. $\int_0^2 \frac{4x}{x^2+1}\,dx \approx 3$

Matches (a).

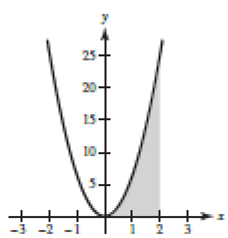


76. $\int_0^2 \frac{4}{x^2+1}\,dx \approx 4$

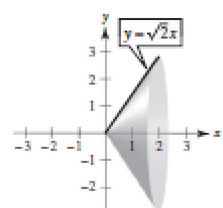
Matches (d).



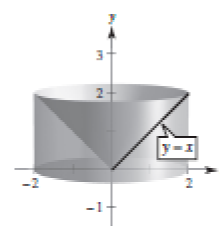
77. (a) $y = 2\pi x^2, \quad 0 \le x \le 2$



(b) $y = \sqrt{2}x, \quad 0 \le x \le 2$

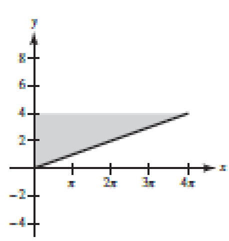


(c) $y = x, \quad 0 \le x \le 2$



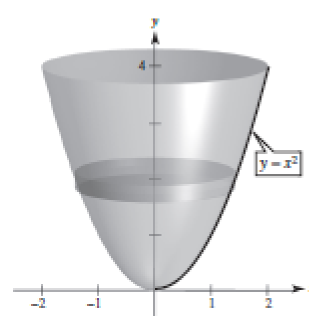
78. (a) $x = \pi y, \quad 0 \le y \le 4$

$y = \frac{1}{\pi}x, \quad 0 \le x \le 4\pi$



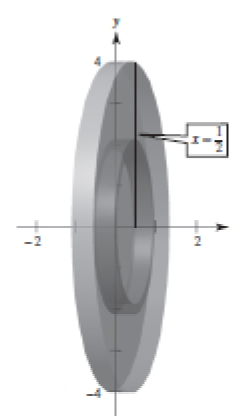
(b) $x = \sqrt{y}, \quad 0 \le y \le 4$

$y = x^2, \quad 0 \le x \le 2$



(c) $x = \frac{1}{2}, \quad 0 \le y \le 4$

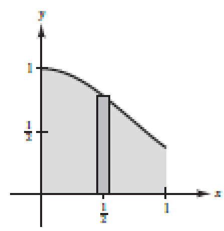
$2\pi\int_0^4 y\left(\frac{1}{2}\right) dy$



79. (a) **Shell Method:**

Let $u = -x^2$, $du = -2x\,dx$.

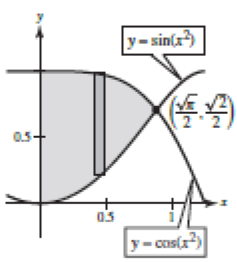
$$V = 2\pi \int_0^1 x e^{-x^2}\,dx$$
$$= -\pi \int_0^1 e^{-x^2}(-2x)\,dx$$
$$= \left[-\pi e^{-x^2}\right]_0^1$$
$$= \pi\left(1 - e^{-1}\right) \approx 1.986$$



(b) **Shell Method:**

$$V = 2\pi \int_0^b x e^{-x^2}\,dx$$
$$= \left[-\pi e^{-x^2}\right]_0^b$$
$$= \pi\left(1 - e^{-b^2}\right) = \frac{4}{3}$$
$$e^{-b^2} = \frac{3\pi - 4}{3\pi}$$
$$b = \sqrt{\ln\left(\frac{3\pi}{3\pi - 4}\right)} \approx 0.743$$

80.



Shell Method:

$$V = 2\pi \int_0^{\sqrt{\pi}/2} x\left(\cos x^2 - \sin x^2\right)dx$$
$$= \pi\left[\sin x^2 + \cos x^2\right]_0^{\sqrt{\pi}/2}$$
$$= \pi\left[\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right) - (0 + 1)\right]$$
$$= \pi\left(\sqrt{2} - 1\right)$$

81. $y = f(x) = \ln(\sin x)$

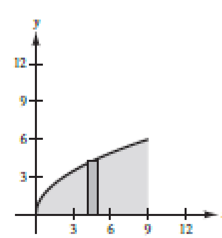
$$f'(x) = \frac{\cos x}{\sin x}$$
$$s = \int_{\pi/4}^{\pi/2} \sqrt{1 + \frac{\cos^2 x}{\sin^2 x}}\,dx = \int_{\pi/4}^{\pi/2} \sqrt{\frac{\sin^2 x + \cos^2 x}{\sin^2 x}}$$
$$= \int_{\pi/4}^{\pi/2} \frac{1}{\sin x}\,dx = \int_{\pi/4}^{\pi/2} \csc x\,dx$$
$$= \left[-\ln\left|\csc x + \cot x\right|\right]_{\pi/4}^{\pi/2}$$
$$= -\ln(1) + \ln\left(\sqrt{2} + 1\right)$$
$$= \ln\left(\sqrt{2} + 1\right) \approx 0.881$$

82. $y = \ln(\cos x)$, $\quad 0 \le x \le \pi/3$

$$y' = \frac{-\sin x}{\cos x} = -\tan x$$
$$1 + (y')^2 = 1 + \tan^2 x = \sec^2 x$$
$$s = \int_0^{\pi/3} \sqrt{1 + (y')^2}\,dx = \int_0^{\pi/3} \sec x\,dx$$
$$= \left[\ln\left|\sec x + \tan x\right|\right]_0^{\pi/3}$$
$$= \ln\left(2 + \sqrt{3}\right) - \ln(1) = \ln\left(2 + \sqrt{3}\right) \approx 1.317$$

83. $y = 2\sqrt{x}$

$$y' = \frac{1}{\sqrt{x}}$$
$$1 + (y')^2 = 1 + \frac{1}{x} = \frac{x + 1}{x}$$
$$S = 2\pi \int_0^9 2\sqrt{x}\sqrt{\frac{x + 1}{x}}\,dx$$
$$= 2\pi \int_0^9 2\sqrt{x + 1}\,dx$$
$$= \left[4\pi\left(\frac{2}{3}\right)(x + 1)^{3/2}\right]_0^9 = \frac{8\pi}{3}\left(10\sqrt{10} - 1\right) \approx 256.54$$



84.
$$y = 36 - x^2$$
$$y' = -2x$$
$$1 + (y')^2 = 1 + (-2x)^2 = 1 + 4x^2$$
$$S = 2\pi \int_0^6 x\sqrt{1 - 4x^2}\, dx$$
$$= \frac{2\pi}{8} \int_0^6 (1 + 4x^2)^{1/2}(8x)\, dx$$
$$= \frac{\pi}{4}\left[\frac{2}{3}(1 + 4x^2)^{3/2}\right]_0^6$$
$$= \frac{\pi}{6}(145^{3/2} - 1)$$
$$\approx 913.696$$

85. Average value $= \dfrac{1}{b-a}\displaystyle\int_a^b f(x)\, dx$
$$= \frac{1}{3 - (-3)}\int_{-3}^{3} \frac{1}{1 + x^2}\, dx$$
$$= \frac{1}{6}\left[\arctan x\right]_{-3}^{3}$$
$$= \frac{1}{6}\left[\arctan 3 - \arctan(-3)\right]$$
$$= \frac{1}{3}\arctan 3 \approx 0.4163$$

86. Average value $= \dfrac{1}{b-a}\displaystyle\int_a^b f(x)\, dx$
$$= \frac{1}{(\pi/n) - 0}\int_0^{\pi/n} \sin nx\, dx$$
$$= \frac{n}{\pi}\left[\frac{-1}{n}\cos nx\right]_0^{\pi/n}$$
$$= -\frac{1}{\pi}(\cos \pi - \cos 0) = \frac{2}{\pi}$$

87.
$$y = \tan \pi x$$
$$y' = \pi \sec^2 \pi x$$
$$1 + (y')^2 = 1 + \pi^2 \sec^4 \pi x$$
$$s = \int_0^{1/4} \sqrt{1 + \pi^2 \sec^4 \pi x}\, dx \approx 1.0320$$

88.
$$y = x^{2/3}$$
$$y' = \frac{2}{3x^{1/3}}$$
$$1 + (y')^2 = 1 + \frac{4}{9x^{2/3}}$$
$$s = \int_1^8 \sqrt{1 + \frac{4}{9x^{2/3}}}\, dx \approx 7.6337$$

89. (a) $\displaystyle\int \cos^3 x\, dx = \int (1 - \sin^2 x)\cos x\, dx = \sin x - \frac{\sin^3 x}{3} + C = \frac{1}{3}\sin x(\cos^2 x + 2) + C$

(b) $\displaystyle\int \cos^5 x\, dx = \int (1 - \sin^2 x)^2 \cos x\, dx = \int (1 - 2\sin^2 x + \sin^4 x)\cos x\, dx$
$$= \sin x - \frac{2}{3}\sin^3 x + \frac{\sin^5 x}{5} + C = \frac{1}{15}\sin x(3\cos^4 x + 4\cos^2 x + 8) + C$$

(c) $\displaystyle\int \cos^7 x\, dx = \int (1 - \sin^2 x)^3 \cos x\, dx$
$$= \int (1 - 3\sin^2 x + 3\sin^4 x - \sin^6 x)\cos x\, dx$$
$$= \sin x - \sin^3 x + \frac{3}{5}\sin^5 x - \frac{1}{7}\sin^7 x + C$$
$$= \frac{1}{35}\sin x(5\cos^6 x + 6\cos^4 x + 8\cos^2 x + 16) + C$$

(d) $\displaystyle\int \cos^{15} x\, dx = \int (1 - \sin^2 x)^7 \cos x\, dx$

You would expand $(1 - \sin^2 x)^7$.

90. (a) $\int \tan^3 x\,dx = \int(\sec^2 x - 1)\tan x\,dx = \int \sec^2 x \tan x\,dx - \int \tan x\,dx = \dfrac{\tan^2 x}{2} - \int \tan x\,dx$

$\int \tan^3 x\,dx = \dfrac{\tan^2 x}{2} + \ln|\cos x| + C$

(b) $\int \tan^5 x\,dx = \int(\sec^2 x - 1)\tan^3 x\,dx = \dfrac{\tan^4 x}{4} - \int \tan^3 x\,dx$

(c) $\int \tan^{2k+1} x\,dx = \int(\sec^2 x - 1)\tan^{2k-1} x\,dx = \dfrac{\tan^{2k} x}{2k} - \int \tan^{2k-1} x\,dx$

(d) You would use these formulas recursively.

91. (A) $\dfrac{d}{dx}\left[2\sqrt{x^2+1} + C\right] = 2\left(\dfrac{1}{2}\right)(x^2+1)^{-1/2}(2x) = \dfrac{2x}{\sqrt{x^2+1}}$

(B) $\dfrac{d}{dx}\left[\sqrt{x^2+1} + C\right] = \dfrac{1}{2}(x^2+1)^{-1/2}(2x) = \dfrac{x}{\sqrt{x^2+1}}$

(C) $\dfrac{d}{dx}\left[\dfrac{1}{2}\sqrt{x^2+1} + C\right] = \dfrac{1}{2}\left(\dfrac{1}{2}\right)(x^2+1)^{-1/2}(2x) = \dfrac{x}{2\sqrt{x^2+1}}$

(D) $\dfrac{d}{dx}\left[\ln(x^2+1) + C\right] = \dfrac{2x}{x^2+1}$

So, the answer is B.

92. (A) $\dfrac{d}{dx}\left[\ln\sqrt{x^2+1} + C\right] = \dfrac{1}{2}\left(\dfrac{2x}{x^2+1}\right) = \dfrac{x}{x^2+1}$

(B) $\dfrac{d}{dx}\left[\dfrac{2x}{(x^2+1)^2} + C\right] = \dfrac{(x^2+1)^2(2) - (2x)(2)(x^2+1)(2x)}{(x^2+1)^4} = \dfrac{2(1-3x^2)}{(x^2+1)^3}$

(C) $\dfrac{d}{dx}[\arctan x + C] = \dfrac{1}{1+x^2}$

(D) $\dfrac{d}{dx}\left[\ln(x^2+1) + C\right] = \dfrac{2x}{x^2+1}$

So, the answer is A.

93. (A) $\dfrac{d}{dx}\left[\ln\sqrt{x^2+1} + C\right] = \dfrac{1}{2}\left(\dfrac{2x}{x^2+1}\right) = \dfrac{x}{x^2+1}$

(B) $\dfrac{d}{dx}\left[\dfrac{2x}{(x^2+1)^2} + C\right] = \dfrac{(x^2+1)^2(2) - (2x)(2)(x^2+1)(2x)}{(x^2+1)^4} = \dfrac{2(1-3x^2)}{(x^2+1)^3}$

(C) $\dfrac{d}{dx}[\arctan x + C] = \dfrac{1}{1+x^2}$

(D) $\dfrac{d}{dx}\left[\ln(x^2+1) + C\right] = \dfrac{2x}{x^2+1}$

So, the answer is C.

94. (A) $\frac{d}{dx}\left[2x\sin(x^2+1)+C\right] = 2x\left[\cos(x^2+1)(2x)\right] + 2\sin(x^2+1) = 2\left[2x^2\cos(x^2+1)+\sin(x^2+1)\right]$

(B) $\frac{d}{dx}\left[-\frac{1}{2}\sin(x^2+1)+C\right] = -\frac{1}{2}\cos(x^2+1)(2x) = -x\cos(x^2+1)$

(C) $\frac{d}{dx}\left[\frac{1}{2}\sin(x^2+1)+C\right] = \frac{1}{2}\cos(x^2+1)(2x) = x\cos(x^2+1)$

(D) $\frac{d}{dx}\left[-2x\sin(x^2+1)+C\right] = -2x\left[\cos(x^2+1)(2x)\right] - 2\sin(x^2+1) = -2\left[2x^2\cos(x^2+1)+\sin(x^2+1)\right]$

So, the answer is C.