B2

# 1.2, Finding Limits Graphically and Numerically

It is estimated that $t$ years from now, the population of a certain community will be

$P(t) = \frac{11t+12}{2t+3}$ thousand people.

a. What is the current population of the community? 4 thousand people

b. What will the population be in 6 years? 5,200 people

c. When will there be 6000 people in the community, explain your answer.

Never, the graph will never reach 6000.

**Informal Definition of a Limit**

If $f(x)$ becomes arbitrarily close to a single number, $L$ as $x$ approaches $c$ from either side, then the limit of $f(x)$ as $x$ approaches $c$ is $L$.

Notation: $\lim_{x \to c} f(x) = L$

**Exploration** (pg. 65)

| $x$ | 1.75 | 1.9 | 1.99 | 1.999 | 2 | 2.001 | 2.01 | 2.1 | 2.25 |
|---|---|---|---|---|---|---|---|---|---|
| $f(x)$ | .75 | .9 | .99 | .999 | | 1.001 | 1.01 | 1.1 | 1.25 |

$$\lim_{x \to 2} \frac{x^2 - 3x + 2}{x-2} = 1$$

**Examples: Finding Limits from Tables and Graphs**

$$\lim_{x \to 0} \frac{x}{\sqrt{x+9}-3} = 6$$

| $x$ | -1 | -.1 | -.01 | -.001 | 0 | .001 | .01 | .1 | |
|---|---|---|---|---|---|---|---|---|---|
| $f(x)$ | | | | | | | | | |

$\lim_{x \to 5} f(x)$ if $f(x) = \begin{cases} 1 & x \neq 5 \\ -2 & x = 5 \end{cases}$ $= 1$

| $x$ | 4 | 4.9 | 4.99 | 4.999 | 5 | 5.001 | 5.01 | 5.1 | 6 |
|---|---|---|---|---|---|---|---|---|---|
| $f(x)$ | 1 | 1 | 1 | 1 | -2 | 1 | 1 | 1 | 1 |

**Common Behaviors for a Nonexistent Limit**

1. f(x) approaches different values from the left and right.
2. f(x) increases or decreases without bound.
3. f(x) oscillates.

## Examples: Nonexistent Limits

$$\lim_{x \to 0} \frac{x}{|x|} = \text{DNE}$$

Limit from the left is different from the limit from the right.

| $x$ | $-.1$ | $-.01$ | $-.001$ | $-.0001$ | $0$ | $.0001$ | $.001$ | $.01$ | $.1$ |
|---|---|---|---|---|---|---|---|---|---|
| $f(x)$ | $-1$ | $-1$ | $-1$ | $-1$ | | $1$ | $1$ | $1$ | $1$ |

$$\lim_{x \to 0} \frac{1}{x^4} = \text{DNE}$$

Unbounded behavior

| $x$ | $-.1$ | $-.01$ | $-.001$ | $-.0001$ | $0$ | $.0001$ | $.001$ | $.01$ | $.1$ |
|---|---|---|---|---|---|---|---|---|---|
| $f(x)$ | $10{,}000$ | $1\times10^{8}$ | $1\times10^{12}$ | $1\times10^{16}$ | | $1\times10^{16}$ | $1\times10^{12}$ | $1\times10^{8}$ | $10{,}000$ |

$$\lim_{x \to 0} \cos\frac{1}{x^2} = \text{DNE}$$

Oscillating

| $x$ | $-.1$ | $-.01$ | $-.001$ | $-.0001$ | $0$ | $.0001$ | $.001$ | $.01$ | $.1$ |
|---|---|---|---|---|---|---|---|---|---|
| $f(x)$ | $.86232$ | $-.9522$ | $.93675$ | $-.2634$ | | $-.2634$ | $.93675$ | $-.9522$ | $.86232$ |

**$\varepsilon - \delta$ Definition of a Limit**

Let $f$ be a function defined on an open interval containing $c$ (except possibly at $c$) and let $L$ be a real number. The statement:

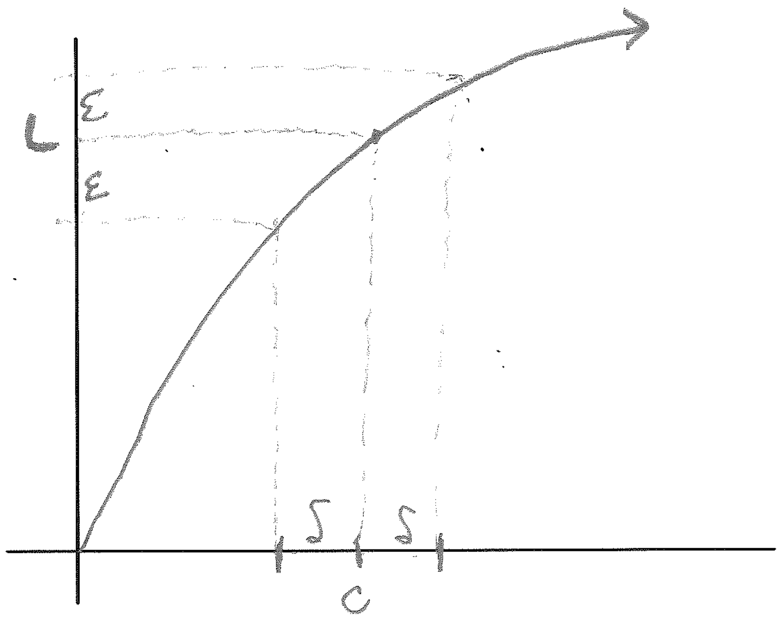
$$\lim_{x \to c} f(x) = L$$

Means that for each $\varepsilon < 0$ there exists a $\delta > 0$ such that if

$$0 < |x - c| < \delta$$

Then

$$|f(x) - L| < \varepsilon$$



**Examples: Using the $\varepsilon - \delta$ Definition**

Given $\lim_{x \to 1}(5 - 4x) = 1$ find $\delta$ such that $|5 - 4x - 1| < 0.01$ whenever $0 < |x - 1| < \delta$

$(\varepsilon)$

$|5-4x-1| < .01$

$|-4x+4| < .01$

$|-4(x-1)| < .01$

$\frac{4|x-1|}{4} < \frac{.01}{4}$

$|x-1| < \frac{.01}{4} = .0025$

$\delta = .0025$

Use the $\varepsilon - \delta$ definition of a limit to prove that $\lim_{x \to -2}(2x + 7) = 3$

$|(2x+7)-3| < \varepsilon$

$|2x+4| < \varepsilon$

$|2(x+2)| < \varepsilon$

$2|x-(-2)| < \varepsilon$

$|x-(-2)| < \frac{\varepsilon}{2}$

$\delta = \frac{\varepsilon}{2}$