Section 3.3 Increasing and Decreasing Functions and the First Derivative Test

1. (a) Increasing: $(0, 6)$ and $(8, 9)$. Largest: $(0, 6)$

(b) Decreasing: $(6, 8)$ and $(9, 10)$. Largest: $(6, 8)$

2. (a) Increasing: $(4, 5), (6, 7)$. Largest: $(4, 5), (6, 7)$

(b) Decreasing: $(-3, 1), (1, 4), (5, 6)$. Largest: $(-3, 1)$

3. $y = -(x + 1)^2$

From the graph, f is increasing on $(-\infty, -1)$ and decreasing on $(-1, \infty)$.

Analytically, $y' = -2(x + 1)$.

Critical number: $x = -1$

Test intervals:	$-\infty < x < -1$	$-1 < x < \infty$
Sign of y':	$y' > 0$	$y' < 0$
Conclusion:	Increasing	Decreasing

4. $f(x) = x^2 - 6x + 8$

From the graph, f is decreasing on $(-\infty, 3)$ and increasing on $(3, \infty)$.

Analytically, $f'(x) = 2x - 6$.

Critical number: $x = 3$

Test intervals:	$-\infty < x < 3$	$3 < x < \infty$
Sign of $f'(x)$:	$f' < 0$	$f' > 0$
Conclusion:	Decreasing	Increasing

5. $y = \dfrac{x^3}{4} - 3x$

From the graph, y is increasing on $(-\infty, -2)$ and $(2, \infty)$, and decreasing on $(-2, 2)$.

Analytically, $y' = \dfrac{3x^2}{4} - 3 = \dfrac{3}{4}(x^2 - 4) = \dfrac{3}{4}(x - 2)(x + 2)$

Critical numbers: $x = \pm 2$

Test intervals:	$-\infty < x < -2$	$-2 < x < 2$	$2 < x < \infty$
Sign of y':	$y' > 0$	$y' < 0$	$y' > 0$
Conclusion:	Increasing	Decreasing	Increasing

6. $f(x) = x^4 - 2x^2$

From the graph, f is decreasing on $(-\infty, -1)$ and $(0, 1)$, and increasing on $(-1, 0)$ and $(1, \infty)$.

Analytically, $f'(x) = 4x^3 - 4x = 4x(x - 1)(x + 1)$.

Critical numbers: $x = 0, \pm 1$.

Test intervals:	$-\infty < x < -1$	$-1 < x < 0$	$0 < x < 1$	$1 < x < \infty$
Sign of f':	$f' < 0$	$f' > 0$	$f' < 0$	$f' > 0$
Conclusion:	Decreasing	Increasing	Decreasing	Increasing

7. $f(x) = \dfrac{1}{(x+1)^2}$

From the graph, f is increasing on $(-\infty, -1)$ and decreasing on $(-1, \infty)$.

Analytically, $f'(x) = \dfrac{-2}{(x+1)^3}$.

No critical numbers. Discontinuity: $x = -1$

Test intervals:	$-\infty < x < -1$	$-1 < x < \infty$
Sign of $f'(x)$:	$f' > 0$	$f' < 0$
Conclusion:	Increasing	Decreasing

8. $y = \dfrac{x^2}{2x-1}$

From the graph, y is increasing on $(-\infty, 0)$ and $(1, \infty)$, and decreasing on $(0, 1/2)$ and $(1/2, 1)$.

Analytically, $y' = \dfrac{(2x-1)2x - x^2(2)}{(2x-1)^2} = \dfrac{2x^2 - 2x}{(2x-1)^2} = \dfrac{2x(x-1)}{(2x-1)^2}$

Critical numbers: $x = 0, 1$

Discontinuity: $x = 1/2$

Test intervals:	$-\infty < x < 0$	$0 < x < 1/2$	$1/2 < x < 1$	$1 < x < \infty$
Sign of y':	$y' > 0$	$y' < 0$	$y' < 0$	$y' > 0$
Conclusion:	Increasing	Decreasing	Decreasing	Increasing

9. $g(x) = x^2 - 2x - 8$

$g'(x) = 2x - 2$

Critical number: $x = 1$

Test intervals:	$-\infty < x < 1$	$1 < x < \infty$
Sign of $g'(x)$:	$g' < 0$	$g' > 0$
Conclusion:	Decreasing	Increasing

Increasing on: $(1, \infty)$

Decreasing on: $(-\infty, 1)$

10. $h(x) = 12x - x^3$

$h'(x) = 12 - 3x^2 = 3(4 - x^2) = 3(2 - x)(2 + x)$

Critical numbers: $x = \pm 2$

Test intervals:	$-\infty < x < -2$	$-2 < x < 2$	$2 < x < \infty$
Sign of $h'(x)$:	$h' < 0$	$h' > 0$	$h' < 0$
Conclusion:	Decreasing	Increasing	Decreasing

Increasing on: $(-2, 2)$

Decreasing on: $(-\infty, -2), (2, \infty)$

11. $y = x\sqrt{16 - x^2}$ Domain: $[-4, 4]$

$$y' = \frac{-2(x^2 - 8)}{\sqrt{16 - x^2}} = \frac{-2}{\sqrt{16 - x^2}}\left(x - 2\sqrt{2}\right)\left(x + 2\sqrt{2}\right)$$

Critical numbers: $x = \pm 2\sqrt{2}$

Test intervals:	$-4 < x < -2\sqrt{2}$	$-2\sqrt{2} < x < 2\sqrt{2}$	$2\sqrt{2} < x < 4$
Sign of y':	$y' < 0$	$y' > 0$	$y' < 0$
Conclusion:	Decreasing	Increasing	Decreasing

Increasing on: $\left(-2\sqrt{2}, 2\sqrt{2}\right)$

Decreasing on: $\left(-4, -2\sqrt{2}\right), \left(2\sqrt{2}, 4\right)$

12. $y = x + \dfrac{9}{x}$

$$y' = \frac{1 - 9}{x^2} = \frac{x^2 - 9}{x^2} = \frac{(x - 3)(x + 3)}{x^2}$$

Critical numbers: $x = \pm 3$

Discontinuity: $x = 0$

Test intervals:	$-\infty < x < -3$	$-3 < x < 0$	$0 < x < 3$	$3 < x < \infty$
Sign of y':	$y' > 0$	$y' < 0$	$y' < 0$	$y' > 0$
Conclusion:	Increasing	Decreasing	Decreasing	Increasing

Increasing on: $(-\infty, -3), (3, \infty)$

Decreasing on: $(-3, 0), (0, 3)$

13. $f(x) = \sin x - 1, \quad 0 < x < 2\pi$

$f'(x) = \cos x$

Critical numbers: $x = \dfrac{\pi}{2}, \dfrac{3\pi}{2}$

Test intervals:	$0 < x < \dfrac{\pi}{2}$	$\dfrac{\pi}{2} < x < \dfrac{3\pi}{2}$	$\dfrac{3\pi}{2} < x < 2\pi$
Sign of $f'(x)$:	$f' > 0$	$f' < 0$	$f' > 0$
Conclusion:	Increasing	Decreasing	Increasing

Increasing on: $\left(0, \dfrac{\pi}{2}\right), \left(\dfrac{3\pi}{2}, 2\pi\right)$

Decreasing on: $\left(\dfrac{\pi}{2}, \dfrac{3\pi}{2}\right)$

14. $h(x) = \cos\dfrac{x}{2}, \qquad 0 < x < 2\pi$

$h'(x) = -\dfrac{1}{2}\sin\dfrac{x}{2}$

Critical numbers: none

Test interval:	$0 < x < 2\pi$
Sign of $h'(x)$:	$h' < 0$
Conclusion:	Decreasing

Decreasing on $0 < x < 2\pi$

15. $y = x - 2\cos x, \quad 0 < x < 2\pi$

$y' = 1 + 2\sin x$

$y' = 0$: $\sin x = -\frac{1}{2}$

Critical numbers: $x = \frac{7\pi}{6}, \frac{11\pi}{6}$

Test intervals:	$0 < x < \frac{7\pi}{6}$	$\frac{7\pi}{6} < x < \frac{11\pi}{6}$	$\frac{11\pi}{6} < x < 2\pi$
Sign of y':	$y' > 0$	$y' < 0$	$y' > 0$
Conclusion:	Increasing	Decreasing	Increasing

Increasing on: $\left(0, \frac{7\pi}{6}\right), \left(\frac{11\pi}{6}, 2\pi\right)$

Decreasing on: $\left(\frac{7\pi}{6}, \frac{11\pi}{6}\right)$

16. $f(x) = \sin^2 x + \sin x, 0 < x < 2\pi$

$f'(x) = 2\sin x\cos x + \cos x = \cos x(2\sin x + 1)$

$2\sin x + 1 = 0 \Rightarrow \sin x = -\frac{1}{2} \Rightarrow x = \frac{7\pi}{6}, \frac{11\pi}{6}$

$\cos x = 0 \Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$

Critical numbers: $\frac{\pi}{2}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$

Test intervals:	$0 < x < \frac{\pi}{2}$	$\frac{\pi}{2} < x < \frac{7\pi}{6}$	$\frac{7\pi}{6} < x < \frac{3\pi}{2}$	$\frac{3\pi}{2} < x < \frac{11\pi}{6}$	$\frac{11\pi}{6} < x < 2\pi$
Sign of $f'(x)$:	$f' > 0$	$f' < 0$	$f' > 0$	$f' < 0$	$f' > 0$
Conclusion:	Increasing	Decreasing	Increasing	Decreasing	Increasing

Increasing on: $\left(0, \frac{\pi}{2}\right), \left(\frac{7\pi}{6}, \frac{3\pi}{2}\right), \left(\frac{11\pi}{6}, 2\pi\right)$

Decreasing on: $\left(\frac{\pi}{2}, \frac{7\pi}{6}\right), \left(\frac{3\pi}{2}, \frac{11\pi}{6}\right)$

17. $g(x) = e^{-x} + e^{3x}$

$g'(x) = -e^{-x} + 3e^{3x}$

Critical number: $x = -\frac{1}{4}\ln 3$

Test intervals:	$-\infty < x < -\frac{1}{4}\ln 3$	$-\frac{1}{4}\ln 3 < x < \infty$
Sign of $g'(x)$:	$g' < 0$	$g' > 0$
Conclusion:	Decreasing	Increasing

Increasing on: $\left(-\frac{1}{4}\ln 3, \infty\right)$

Decreasing on: $\left(-\infty, -\frac{1}{4}\ln 3\right)$

18. $h(x) = \sqrt{x}\,e^{-x}, \quad x \geq 0$

$$h'(x) = -\sqrt{x}\,e^{-x} + \frac{1}{2\sqrt{x}}e^{-x} = e^{-x}\left(\frac{1}{2\sqrt{x}} - \sqrt{x}\right) = e^{-x} - \frac{1-2x}{2\sqrt{x}}$$

Critical number: $x = \frac{1}{2}$ $\quad$ ($x = 0$ is an endpoint)

Test intervals:	$0 < x < \frac{1}{2}$	$\frac{1}{2} < x < \infty$
Sign of $h'(x)$:	$h' > 0$	$h' < 0$
Conclusion:	Increasing	Decreasing

Increasing on: $\left(0, \frac{1}{2}\right)$

Decreasing on: $\left(\frac{1}{2}, \infty\right)$

19. $f(x) = x^2 \ln\left(\frac{x}{2}\right), \quad x > 0$

$$f'(x) = 2x \ln\left(\frac{x}{2}\right) + \frac{x^2}{x} = 2x \ln\left(\frac{x}{2}\right) + x$$

Critical number: $x = \frac{2}{\sqrt{e}}$

Test intervals:	$0 < x < \frac{2}{\sqrt{e}}$	$\frac{2}{\sqrt{e}} < x < \infty$
Sign of $f'(x)$:	$f' < 0$	$f' > 0$
Conclusion:	Decreasing	Increasing

Increasing on: $\left(\frac{2}{\sqrt{e}}, \infty\right)$

Decreasing on: $\left(0, \frac{2}{\sqrt{e}}\right)$

20. $f(x) = \frac{\ln x}{\sqrt{x}}, \quad x > 0$

$$f'(x) = \frac{\frac{\sqrt{x}}{x} - \ln x \frac{1}{2\sqrt{x}}}{x} = \frac{2 - \ln x}{2x^{3/2}}$$

Critical number: $x = e^2$

Test intervals:	$0 < x < e^2$	$e^2 < x < \infty$
Sign of $f'(x)$:	$f' > 0$	$f' < 0$
Conclusion:	Increasing	Decreasing

Increasing on: $\left(0, e^2\right)$

Decreasing on: $\left(e^2, \infty\right)$

21. (a) $f(x) = x^2 - 6x$

$f'(x) = 2x - 6$

Critical number: $x = 3$

(b)

Test intervals:	$-\infty < x < 3$	$3 < x <$
Sign of f':	$f' < 0$	$f' > 0$
Conclusion:	Decreasing	Increasing

Increasing on: $(3, \infty)$

Decreasing on: $(-\infty, 3)$

(c) Relative minimum: $(3, -9)$

22. (a) $f(x) = x^2 + 6x + 10$

$f'(x) = 2x + 6$

Critical number: $x = -3$

(b)

Test intervals:	$-\infty < x < -3$	$-3 < x <$
Sign of f':	$f' < 0$	$f' > 0$
Conclusion:	Decreasing	Increasing

Increasing on: $(-3, \infty)$

Decreasing on: $(-\infty, -3)$

(c) Relative minimum: $(-3, 1)$

23. (a) $f(x) = -2x^2 + 4x + 3$

$f'(x) = -4x + 4 = 0$

Critical number: $x = 1$

(b)

Test intervals:	$-\infty < x < 1$	$1 < x < \infty$
Sign of $f'(x)$:	$f' > 0$	$f' < 0$
Conclusion:	Increasing	Decreasing

Increasing on: $(-\infty, 1)$

Decreasing on: $(1, \infty)$

(c) Relative maximum: $(1, 5)$

24. (a) $f(x) = -3x^2 - 4x - 2$

$f'(x) = -6x - 4 = 0$

Critical number: $x = -\frac{2}{3}$

(b)

Test intervals:	$-\infty < x < -\frac{2}{3}$	$-\frac{2}{3} < x <$
Sign of $f'(x)$:	$f' > 0$	$f' < 0$
Conclusion:	Increasing	Decreasing

Increasing on: $\left(-\infty, -\frac{2}{3}\right)$

Decreasing on: $\left(-\frac{2}{3}, \infty\right)$

(c) Relative maximum: $\left(-\frac{2}{3}, -\frac{2}{3}\right)$

25. (a) $f(x) = 2x^3 + 3x^2 - 12x$

$f'(x) = 6x^2 + 6x - 12 = 6(x + 2)(x - 1) = 0$

Critical numbers: $x = -2, 1$

(b)

Test intervals:	$-\infty < x < -2$	$-2 < x < 1$	$1 < x < \infty$
Sign of $f'(x)$:	$f' > 0$	$f' < 0$	$f' > 0$
Conclusion:	Increasing	Decreasing	Increasing

Increasing on: $(-\infty, -2), (1, \infty)$

Decreasing on: $(-2, 1)$

(c) Relative maximum: $(-2, 20)$

Relative minimum: $(1, -7)$

26. (a) $f(x) = x^3 - 6x^2 + 15$

$f'(x) = 3x^2 - 12x = 3x(x - 4)$

Critical numbers: $x = 0, 4$

(b)

Test intervals:	$-\infty < x < 0$	$0 < x < 4$	$4 < x < \infty$
Sign of $f'(x)$:	$f' > 0$	$f' < 0$	$f' > 0$
Conclusion:	Increasing	Decreasing	Increasing

Increasing on: $(-\infty, 0), (4, \infty)$

Decreasing on: $(0, 4)$

(c) Relative maximum: $(0, 15)$

Relative minimum: $(4, -17)$

27. (a) $f(x) = (x-1)^2(x+3) = x^3 + x^2 - 5x + 3$

$f'(x) = 3x^2 + 2x - 5 = (x-1)(3x+5)$

Critical numbers: $x = 1, -\frac{5}{3}$

(b)

Test intervals:	$-\infty < x < -\frac{5}{3}$	$-5/3 < x < 1$	$1 < x < \infty$
Sign of f':	$f' > 0$	$f' < 0$	$f' > 0$
Conclusion:	Increasing	Decreasing	Increasing

Increasing on: $\left(-\infty, -\frac{5}{3}\right)$ and $(1, \infty)$

Decreasing on: $\left(-\frac{5}{3}, 1\right)$

(c) Relative maximum: $\left(-\frac{5}{3}, \frac{256}{27}\right)$

Relative minimum: $(1, 0)$

28. (a) $f(x) = (x+2)^2(x-1)$

$f'(x) = 3x(x+2)$

Critical numbers: $x = -2, 0$

(b)

Test intervals:	$-\infty < x < -2$	$-2 < x < 0$	$0 < x < \infty$
Sign of $f'(x)$:	$f' > 0$	$f' < 0$	$f' > 0$
Conclusion:	Increasing	Decreasing	Increasing

Increasing on: $(-\infty, -2), (0, \infty)$

Decreasing on: $(-2, 0)$

(c) Relative maximum: $(-2, 0)$

Relative minimum: $(0, -4)$

29. (a) $f(x) = \dfrac{x^5 - 5x}{5}$

$f'(x) = x^4 - 1$

Critical numbers: $x = -1, 1$

(b)

Test intervals:	$-\infty < x < -1$	$-1 < x < 1$	$1 < x < \infty$
Sign of $f'(x)$:	$f' > 0$	$f' < 0$	$f' > 0$
Conclusion:	Increasing	Decreasing	Increasing

Increasing on: $(-\infty, -1), (1, \infty)$

Decreasing on: $(-1, 1)$

(c) Relative maximum: $\left(-1, \dfrac{4}{5}\right)$

Relative minimum: $\left(1, -\dfrac{4}{5}\right)$

30. (a) $f(x) = x^4 - 32x + 4$

$f'(x) = 4x^3 - 32 = 4(x^3 - 8)$

Critical number: $x = 2$

(b)

Test intervals:	$-\infty < x < 2$	$2 < x < \infty$
Sign of $f'(x)$:	$f' < 0$	$f' > 0$
Conclusion:	Decreasing	Increasing

Increasing on: $(2, \infty)$

Decreasing on: $(-\infty, 2)$

(c) Relative minimum: $(2, -44)$

31. (a) $f(x) = x^{1/3} + 1$

$f'(x) = \frac{1}{3}x^{-2/3} = \frac{1}{3x^{2/3}}$

Critical number: $x = 0$

(b)

Test intervals:	$-\infty < x < 0$	$0 < x < \infty$
Sign of $f'(x)$:	$f' > 0$	$f' > 0$
Conclusion:	Increasing	Increasing

Increasing on: $(-\infty, \infty)$

(c) No relative extrema

32. (a) $f(x) = x^{2/3} - 4$

$f'(x) = \frac{2}{3}x^{-1/3} = \frac{2}{3x^{1/3}}$

Critical number: $x = 0$

(b)

Test intervals:	$-\infty < x < 0$	$0 < x < \infty$
Sign of $f'(x)$:	$f' < 0$	$f' > 0$
Conclusion:	Decreasing	Increasing

Increasing on: $(0, \infty)$

Decreasing on: $(-\infty, 0)$

(c) Relative minimum: $(0, -4)$

33. (a) $f(x) = (x + 2)^{2/3}$

$f'(x) = \frac{2}{3}(x + 2)^{-1/3} = \frac{2}{3(x + 2)^{1/3}}$

Critical number: $x = -2$

(b)

Test intervals:	$-\infty < x < -2$	$-2 < x < \infty$
Sign of f':	$f' < 0$	$f' > 0$
Conclusion:	Decreasing	Increasing

Decreasing on: $(-\infty, -2)$

Increasing on: $(-2, \infty)$

(c) Relative minimum: $(-2, 0)$

34. (a) $f(x) = (x - 3)^{1/3}$

$f'(x) = \frac{1}{3}(x - 3)^{-2/3} = \frac{1}{3(x - 3)^{2/3}}$

Critical number: $x = 3$

(b)

Test intervals:	$-\infty < x < 3$	$3 < x < \infty$
Sign of f':	$f' > 0$	$f' > 0$
Conclusion:	Increasing	Increasing

Increasing on: $(-\infty, \infty)$

(c) No relative extrema

35. (a) $f(x) = 5 - |x - 5|$

$f'(x) = -\frac{x - 5}{|x - 5|} = \begin{cases} 1, & x < 5 \\ -1, & x > 5 \end{cases}$

Critical number: $x = 5$

(b)

Test intervals:	$-\infty < x < 5$	$5 < x < \infty$
Sign of $f'(x)$:	$f' > 0$	$f' < 0$
Conclusion:	Increasing	Decreasing

Increasing on: $(-\infty, 5)$

Decreasing on: $(5, \infty)$

(c) Relative maximum: $(5, 5)$

36. (a) $f(x) = |x + 3| - 1$

$$f'(x) = \frac{x+3}{|x+3|} = \begin{cases} 1, & x > -3 \\ -1, & x < -3 \end{cases}$$

Critical number: $x = -3$

(b)

Test intervals:	$-\infty < x < -3$	$-3 < x < \infty$
Sign of $f'(x)$:	$f' < 0$	$f' > 0$
Conclusion:	Decreasing	Increasing

Increasing on: $(-3, \infty)$

Decreasing on: $(-\infty, -3)$

(c) Relative minimum: $(-3, -1)$

37. (a) $f(x) = 2x + \dfrac{1}{x}$

$$f'(x) = 2 - \frac{1}{x^2} = \frac{2x^2 - 1}{x^2}$$

Critical numbers: $x = \pm\dfrac{\sqrt{2}}{2}$

Discontinuity: $x = 0$

(b)

Test intervals:	$-\infty < x < -\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2} < x < 0$	$0 < x < \frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2} < x < \infty$
Sign of f':	$f' > 0$	$f' < 0$	$f' < 0$	$f' > 0$
Conclusion:	Increasing	Decreasing	Decreasing	Increasing

Increasing on: $\left(-\infty, -\dfrac{\sqrt{2}}{2}\right)$ and $\left(\dfrac{\sqrt{2}}{2}, \infty\right)$

Decreasing on: $\left(-\dfrac{\sqrt{2}}{2}, 0\right)$ and $\left(0, \dfrac{\sqrt{2}}{2}\right)$

(c) Relative maximum: $\left(-\dfrac{\sqrt{2}}{2}, -2\sqrt{2}\right)$

Relative minimum: $\left(\dfrac{\sqrt{2}}{2}, 2\sqrt{2}\right)$

38. (a) $f(x) = \dfrac{x}{x-5}$

$$f'(x) = \frac{(x-5) - x}{(x-5)^2} = \frac{-5}{(x-5)^2}$$

No critical numbers

Discontinuity: $x = 5$

(b)

Test intervals:	$-\infty < x < 5$	$5 < x < \infty$
Sign of $f'(x)$:	$f' < 0$	$f' < 0$
Conclusion:	Decreasing	Decreasing

39. (a) $f(x) = \dfrac{x^2}{x^2 - 9}$

$$f'(x) = \frac{(x^2 - 9)(2x) - (x^2)(2x)}{(x^2 - 9)^2} = \frac{-18x}{(x^2 - 9)^2}$$

Critical number: $x = 0$

Discontinuities: $x = -3, 3$

(b)

Test intervals:	$-\infty < x < -3$	$-3 < x < 0$	$0 < x < 3$	$3 < x < \infty$
Sign of $f'(x)$:	$f' > 0$	$f' > 0$	$f' < 0$	$f' < 0$
Conclusion:	Increasing	Increasing	Decreasing	Decreasing

Increasing on: $(-\infty, -3), (-3, 0)$

Decreasing on: $(0, 3), (3, \infty)$

(c) Relative maximum: $(0, 0)$

40. (a) $f(x) = \dfrac{x^2 - 2x + 1}{x + 1}$

$$f'(x) = \frac{(x + 1)(2x - 2) - (x^2 - 2x + 1)(1)}{(x + 1)^2} = \frac{x^2 + 2x - 3}{(x + 1)^2} = \frac{(x + 3)(x - 1)}{(x + 1)^2}$$

Critical numbers: $x = -3, 1$

Discontinuity: $x = -1$

(b)

Test intervals:	$-\infty < x < -3$	$-3 < x < -1$	$-1 < x < 1$	$1 < x < \infty$
Sign of $f'(x)$:	$f' > 0$	$f' < 0$	$f' < 0$	$f' > 0$
Conclusion:	Increasing	Decreasing	Decreasing	Increasing

Increasing on: $(-\infty, -3), (1, \infty)$

Decreasing on: $(-3, -1), (-1, 1)$

(c) Relative maximum: $(-3, -8)$

Relative minimum: $(1, 0)$

41. (a) $f(x) = \begin{cases} 4 - x^2, & x \le 0 \\ -2x, & x > 0 \end{cases}$

$$f'(x) = \begin{cases} -2x, & x < 0 \\ -2, & x > 0 \end{cases}$$

Critical number: $x = 0$

(b)

Test intervals:	$-\infty < x < 0$	$0 < x < \infty$
Sign of f':	$f' > 0$	$f' < 0$
Conclusion:	Increasing	Decreasing

Increasing on: $(-\infty, 0)$

Decreasing on: $(0, \infty)$

(c) Relative maximum: $(0, 4)$

42. (a) $f(x) = \begin{cases} 2x + 1, & x \le -1 \\ x^2 - 2, & x > -1 \end{cases}$

$f'(x) = \begin{cases} 2, & x < -1 \\ 2x, & x > -1 \end{cases}$

Critical numbers: $x = -1, 0$

(b)

Test intervals:	$-\infty < x < -1$	$-1 < x < 0$	$0 < x < \infty$
Sign of f':	$f' > 0$	$f' < 0$	$f' > 0$
Conclusion:	Increasing	Decreasing	Increasing

Increasing on: $(-\infty, -1)$ and $(0, \infty)$

Decreasing on: $(-1, 0)$

(c) Relative maximum: $(-1, -1)$

Relative minimum: $(0, -2)$

43. (a) $f(x) = \begin{cases} 3x + 1, & x \le 1 \\ 5 - x^2, & x > 1 \end{cases}$

$f'(x) = \begin{cases} 3, & x < 1 \\ -2x, & x > 1 \end{cases}$

Critical number: $x = 1$

(b)

Test intervals:	$-\infty < x < 1$	$1 < x < \infty$
Sign of f':	$f' > 0$	$f' < 0$
Conclusion:	Increasing	Decreasing

Increasing on: $(-\infty, 1)$

Decreasing on: $(1, \infty)$

(c) Relative maximum: $(1, 4)$

44. (a) $f(x) = \begin{cases} -x^3 + 1, & x \le 0 \\ -x^2 + 2x, & x > 0 \end{cases}$

$f'(x) = \begin{cases} -3x^2, & x < 0 \\ -2x + 2, & x > 0 \end{cases}$

Critical numbers: $x = 0, 1$

(b)

Test intervals:	$-\infty < x < 0$	$0 < x < 1$	$1 < x < \infty$
Sign of f':	$f' < 0$	$f' > 0$	$f' < 0$
Conclusion:	Decreasing	Increasing	Decreasing

Increasing on: $(0, 1)$

Decreasing on: $(-\infty, 0)$ and $(1, \infty)$

(c) Relative maximum: $(1, 1)$

Note: $(0, 1)$ is not a relative minimum

45. $f(x) = (3 - x)e^{x-3}$

$f'(x) = (3 - x)e^{x-3} - e^{x-3}$

$\quad = e^{x-3}(2 - x)$

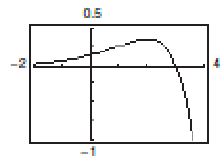
Critical number: $x = 2$

Test intervals:	$-\infty < x < 2$	$2 < x < \infty$
Sign of $f'(x)$:	$f' > 0$	$f' < 0$
Conclusion:	Increasing	Decreasing

Increasing on: $(-\infty, 2)$

Decreasing on: $(2, \infty)$

Relative minimum: $(2, e^{-1})$



46. $f(x) = (x - 1)e^x$

$f'(x) = (x - 1)e^x + e^x = xe^x$

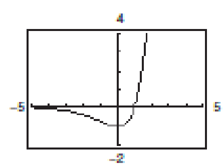
Critical number: $x = 0$

Test intervals:	$-\infty < x < 0$	$0 < x < \infty$
Sign of $f'(x)$:	$f' < 0$	$f' > 0$
Conclusion:	Decreasing	Increasing

Increasing on: $(0, \infty)$

Decreasing on: $(-\infty, 0)$

Relative minimum: $(0, -1)$



47. $f(x) = 4(x - \arcsin x), -1 \le x \le 1$

$$f'(x) = 4 - \frac{4}{\sqrt{1 - x^2}}$$

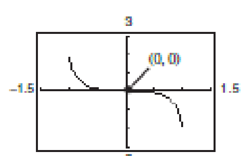
Critical number: $x = 0$

Test intervals:	$-1 \le x < 0$	$0 < x \le 1$
Sign of $f'(x)$:	$f' < 0$	$f' < 0$
Conclusion:	Decreasing	Decreasing

Decreasing on: $(-1, 1)$

No relative extrema

(Absolute maximum at $x = -1$, absolute minimum at $x = 1$)



48. $f(x) = x \arctan x$

$$f'(x) = \frac{x}{1 + x^2} + \arctan x$$

$f'(x) = 0$

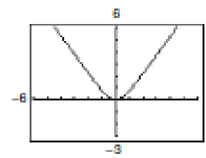
Critical number: $x = 0$

Test intervals:	$-\infty < x < 0$	$0 < x < \infty$
Sign of $f'(x)$:	$f' < 0$	$f' > 0$
Conclusion:	Decreasing	Increasing

Increasing on: $(0, \infty)$

Decreasing on: $(-\infty, 0)$

Relative minimum: $(0, 0)$



49. $g(x) = (x)3^{-x}$

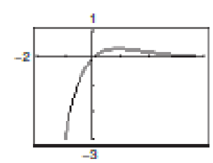
$g'(x) = (1 - x \ln 3)3^{-x}$

Critical number: $x = \dfrac{1}{\ln 3} \approx 0.9102$

Test intervals:	$-\infty < x < \dfrac{1}{\ln 3}$	$\dfrac{1}{\ln 3} < x < \infty$
Sign of $f'(x)$:	$f' > 0$	$f' < 0$
Conclusion:	Increasing	Decreasing

Increasing on: $\left(-\infty, \dfrac{1}{\ln 3}\right)$

Decreasing on: $\left(\dfrac{1}{\ln 3}, \infty\right)$



Relative maximum: $\left(\dfrac{1}{\ln 3}, \dfrac{1}{e \ln 3}\right) \approx (0.9102, 0.3349)$

50. $f(x) = 2^{x^2-3}$

$f'(x) = (\ln 2)2^{x^2-3}(2x)$

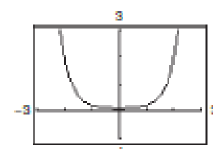
Critical number: $x = 0$

Test intervals:	$-\infty < x < 0$	$0 < x < \infty$
Sign of $f'(x)$:	$f' < 0$	$f' > 0$
Conclusion:	Decreasing	Increasing

Increasing on: $(0, \infty)$

Decreasing on: $(-\infty, 0)$

Relative minimum: $\left(0, \frac{1}{8}\right)$



51. $f(x) = x - \log_4 x = x - \dfrac{\ln x}{\ln 4}$

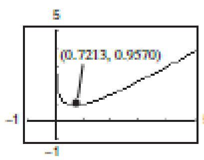
$f'(x) = 1 - \dfrac{1}{x \ln 4} = 0 \Rightarrow x \ln 4 = 1 \Rightarrow x = \dfrac{1}{\ln 4}$

Critical number: $x = \dfrac{1}{\ln 4}$

Test intervals:	$0 < x < \dfrac{1}{\ln 4}$	$\dfrac{1}{\ln 4} < x < \infty$
Sign of $f'(x)$:	$f' < 0$	$f' > 0$
Conclusion:	Decreasing	Increasing

Increasing on: $\left(\dfrac{1}{\ln 4}, \infty\right)$

Decreasing on: $\left(0, \dfrac{1}{\ln 4}\right)$



Relative maximum:

$\left(\dfrac{1}{\ln 4}, \dfrac{1}{\ln 4} - \log_4\left(\dfrac{1}{\ln 4}\right)\right) = \left(\dfrac{1}{\ln 4}, \dfrac{\ln(\ln 4) + 1}{\ln 4}\right)$

$\approx (0.7213, 0.9570)$

52. $f(x) = \dfrac{x^3}{3} - \ln x$

Domain: $x > 0$

$f'(x) = x^2 - \dfrac{1}{x} = \dfrac{x^2 - 1}{x}$

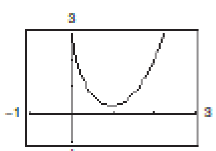
Critical number: $x = 1$

Test intervals:	$0 < x < 1$	$1 < x < \infty$
Sign of $f'(x)$:	$f' < 0$	$f' > 0$
Conclusion:	Decreasing	Increasing

Increasing on: $(1, \infty)$

Decreasing on: $(0, 1)$

Relative minimum: $\left(1, \dfrac{1}{3}\right)$



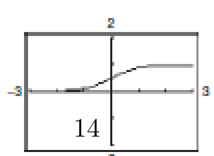
53. $g(x) = \dfrac{e^{2x}}{e^{2x} + 1}$

$g'(x) = \dfrac{(e^{2x} + 1)2e^{2x} - e^{2x}(2e^{2x})}{(e^{2x} + 1)^2} = \dfrac{2e^{2x}}{(e^{2x} + 1)^2}$

No critical numbers.

Increasing on: $(-\infty, \infty)$

No relative extrema.



54. $h(x) = \ln(2 - \ln x)$

Domain: $x > 0$ and $2 - \ln x > 0 \Rightarrow 0 < x < e^2$

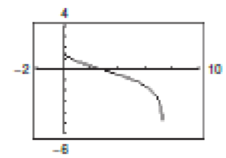
$h'(x) = \dfrac{1}{2 - \ln x}\left(-\dfrac{1}{x}\right) = \dfrac{1}{x \ln x - 2x} = \dfrac{1}{x(\ln x - 2)}$

No critical numbers.

$h'(x) < 0$ on entire domain.

Decreasing on: $(0, e^2)$

No relative extrema.



55. $f(x) = e^{-1/(x-2)} = e^{1/(2-x)}, x \neq 2$

$f'(x) = e^{1/(2-x)}\left(\dfrac{1}{(2 - x)^2}\right)$

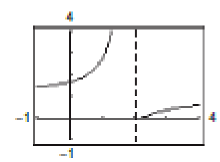
No critical numbers.

$x = 2$ is a vertical asymptote.

Test intervals:	$-\infty < x < 2$	$2 < x < \infty$
Sign of $f'(x)$:	$f' > 0$	$f' > 0$
Conclusion:	Increasing	Increasing

Increasing on: $(-\infty, 2), (2, \infty)$

No relative extrema.



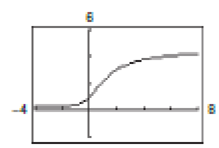
56. $f(x) = e^{\arctan x}$

$f'(x) = e^{\arctan x}\left(\dfrac{1}{1 + x^2}\right) \neq 0$

No critical numbers.

Increasing on: $(-\infty, \infty)$

No relative extrema.



57. For $(4, f(4))$ to be a relative minimum, $f'(x)$ should change from negative to positive at $x = 4$, but $f'(x)$ is positive on both sides of $x = 4$.

If $f'(x) > 0$ for $2 < x < 4$ and $f'(x) > 0$ for $4 < x < 6$, then $(4, f(4))$ is neither a relative minimum nor a relative maximum of f.

58. For $(4, f(4))$ to be a relative maximum, $f'(x)$ should change from positive to negative at $x = 4$, but $f'(x)$ changes from negative to positive at $x = 4$.

If $f'(x) < 0$ for $2 < x < 4$ and $f'(x) > 0$ for $4 < x < 6$, then $(4, f(4))$ is a relative minimum of f.

59. (a) $f(x) = x - 2\sin x,\ 0 < x < 2\pi$

$f'(x) = 1 - 2\cos x$

Critical numbers: $\frac{\pi}{3}, \frac{5\pi}{3}$

Test intervals:	$0 < x < \frac{\pi}{3}$	$\frac{\pi}{3} < x < \frac{5\pi}{3}$	$\frac{5\pi}{3} < x < 2\pi$
Sign of f':	$f' < 0$	$f' > 0$	$f' < 0$
Conclusion:	Decreasing	Increasing	Decreasing

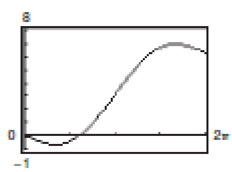
Increasing on: $\left(\frac{\pi}{3}, \frac{5\pi}{3}\right)$

Decreasing on: $\left(0, \frac{\pi}{3}\right), \left(\frac{5\pi}{3}, 2\pi\right)$

(b) Relative maximum: $\left(\frac{5\pi}{3}, \frac{5\pi}{3} + \sqrt{3}\right)$

Relative minimum: $\left(\frac{\pi}{3}, \frac{\pi}{3} - \sqrt{3}\right)$

(c)



60. (a) $f(x) = \sin x \cos x + 5 = \frac{1}{2}\sin 2x + 5,\ 0 < x < 2\pi$

$f'(x) = \cos 2x$

Critical numbers: $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

Test intervals:	$0 < x < \frac{\pi}{4}$	$\frac{\pi}{4} < x < \frac{3\pi}{4}$	$\frac{3\pi}{4} < x < \frac{5\pi}{4}$	$\frac{5\pi}{4} < x < \frac{7\pi}{4}$	$\frac{7\pi}{4} < x < 2\pi$
Sign of f':	$f' > 0$	$f' < 0$	$f' > 0$	$f' < 0$	$f' > 0$
Conclusion:	Increasing	Decreasing	Increasing	Decreasing	Increasing

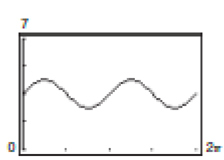
Increasing on: $\left(0, \frac{\pi}{4}\right), \left(\frac{3\pi}{4}, \frac{5\pi}{4}\right), \left(\frac{7\pi}{4}, 2\pi\right)$

Decreasing on: $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right), \left(\frac{5\pi}{4}, \frac{7\pi}{4}\right)$

(b) Relative maxima: $\left(\frac{\pi}{4}, \frac{11}{2}\right), \left(\frac{5\pi}{4}, \frac{11}{2}\right)$

Relative minima: $\left(\frac{3\pi}{4}, \frac{9}{2}\right), \left(\frac{7\pi}{4}, \frac{9}{2}\right)$

(c)



61. (a) $f(x) = \sin x + \cos x, \quad 0 < x < 2\pi$

$f'(x) = \cos x - \sin x = 0 \Rightarrow \sin x = \cos x$

Critical numbers: $x = \frac{\pi}{4}, \frac{5\pi}{4}$

Test intervals:	$0 < x < \frac{\pi}{4}$	$\frac{\pi}{4} < x < \frac{5\pi}{4}$	$\frac{5\pi}{4} < x < 2\pi$
Sign of $f'(x)$:	$f' > 0$	$f' < 0$	$f' > 0$
Conclusion:	Increasing	Decreasing	Increasing

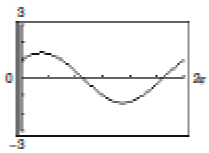
Increasing on: $\left(0, \frac{\pi}{4}\right), \left(\frac{5\pi}{4}, 2\pi\right)$

Decreasing on: $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$

(b) Relative maximum: $\left(\frac{\pi}{4}, \sqrt{2}\right)$

Relative minimum: $\left(\frac{5\pi}{4}, -\sqrt{2}\right)$

(c)



62. (a) $f(x) = \frac{x}{2} + \cos x, 0 < x < 2\pi$

$f'(x) = \frac{1}{2} - \sin x = 0$

Critical numbers: $x = \frac{\pi}{6}, \frac{5\pi}{6}$

Test intervals:	$0 < x < \frac{\pi}{4}$	$\frac{\pi}{4} < x < \frac{5\pi}{4}$	$\frac{5\pi}{4} < x < 2\pi$
Sign of $f'(x)$:	$f' > 0$	$f' < 0$	$f' > 0$
Conclusion:	Increasing	Decreasing	Increasing

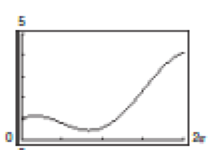
Increasing on: $\left(0, \frac{\pi}{6}\right), \left(\frac{5\pi}{6}, 2\pi\right)$

Decreasing on: $\left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$

(b) Relative maximum: $\left(\frac{\pi}{6}, \frac{\pi + 6\sqrt{3}}{12}\right)$

Relative minimum: $\left(\frac{5\pi}{6}, \frac{5\pi - 6\sqrt{3}}{12}\right)$

(c)



63. (a) $f(x) = \cos^2(2x), \qquad 0 < x < 2\pi$

$f'(x) = -4\cos 2x \sin 2x = 0 \Rightarrow \cos 2x = 0$ or $\sin 2x = 0$

Critical numbers: $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

Test intervals:	$0 < x < \frac{\pi}{4}$	$\frac{\pi}{4} < x < \frac{\pi}{2}$	$\frac{\pi}{2} < x < \frac{3\pi}{4}$	$\frac{3\pi}{4} < x < \pi$
Sign of $f'(x)$:	$f' < 0$	$f' > 0$	$f' < 0$	$f' > 0$
Conclusion:	Decreasing	Increasing	Decreasing	Increasing

Test intervals:	$\pi < x < \frac{5\pi}{4}$	$\frac{5\pi}{4} < x < \frac{3\pi}{2}$	$\frac{3\pi}{2} < x < \frac{7\pi}{4}$	$\frac{7\pi}{4} < x < 2\pi$
Sign of $f'(x)$:	$f' < 0$	$f' > 0$	$f' < 0$	$f' > 0$
Conclusion:	Decreasing	Increasing	Decreasing	Increasing

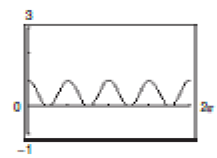
Increasing on: $\left(\frac{\pi}{4}, \frac{\pi}{2}\right), \left(\frac{3\pi}{4}, \pi\right), \left(\frac{5\pi}{4}, \frac{3\pi}{2}\right), \left(\frac{7\pi}{4}, 2\pi\right)$

Decreasing on: $\left(0, \frac{\pi}{4}\right), \left(\frac{\pi}{2}, \frac{3\pi}{4}\right), \left(\pi, \frac{5\pi}{4}\right), \left(\frac{3\pi}{2}, \frac{7\pi}{4}\right)$

(b) Relative maxima: $\left(\frac{\pi}{2}, 1\right), (\pi, 1), \left(\frac{3\pi}{2}, 1\right)$

Relative minima: $\left(\frac{\pi}{4}, 0\right), \left(\frac{3\pi}{4}, 0\right), \left(\frac{5\pi}{4}, 0\right), \left(\frac{7\pi}{4}, 0\right)$

(c)



64. (a) $f(x) = \sin x - \sqrt{3}\cos x, 0 < x < 2\pi$

$f'(x) = \cos x + \sqrt{3}\sin x = 0 \Rightarrow \sqrt{3}\sin x = -\cos x$

$\tan x = \frac{-1}{\sqrt{3}} = \frac{-\sqrt{3}}{3}$

Critical numbers: $x = \frac{5\pi}{6}, \frac{11\pi}{6}$

Test intervals:	$0 < x < \frac{5\pi}{6}$	$\frac{5\pi}{6} < x < \frac{11\pi}{6}$	$\frac{11\pi}{6} < x < 2\pi$
Sign of $f'(x)$:	$f' > 0$	$f' < 0$	$f' > 0$
Conclusion:	Increasing	Decreasing	Increasing

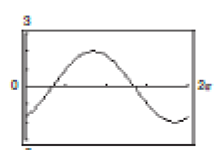
Increasing on: $\left(0, \frac{5\pi}{6}\right), \left(\frac{11\pi}{6}, 2\pi\right)$

Decreasing on: $\left(\frac{5\pi}{6}, \frac{11\pi}{6}\right)$

(b) Relative maximum: $\left(\frac{5\pi}{6}, 2\right)$

Relative minimum: $\left(\frac{11\pi}{6}, -2\right)$

(c)



65. (a) $f(x) = \sin^2 x + \sin x, \quad 0 < x < 2\pi$

$f'(x) = 2\sin x \cos x + \cos x = \cos x(2\sin x + 1) = 0$

Critical numbers: $x = \dfrac{\pi}{2}, \dfrac{7\pi}{6}, \dfrac{3\pi}{2}, \dfrac{11\pi}{6}$

Test intervals:	$0 < x < \frac{\pi}{2}$	$\frac{\pi}{2} < x < \frac{7\pi}{6}$	$\frac{7\pi}{6} < x < \frac{3\pi}{2}$	$\frac{3\pi}{2} < x < \frac{11\pi}{6}$	$\frac{11\pi}{6} < x < 2\pi$
Sign of $f'(x)$:	$f' > 0$	$f' < 0$	$f' > 0$	$f' < 0$	$f' > 0$
Conclusion:	Increasing	Decreasing	Increasing	Decreasing	Increasing

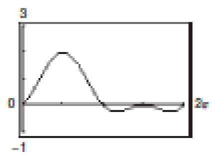
Increasing on: $\left(0, \dfrac{\pi}{2}\right), \left(\dfrac{7\pi}{6}, \dfrac{3\pi}{2}\right), \left(\dfrac{11\pi}{6}, 2\pi\right)$

Decreasing on: $\left(\dfrac{\pi}{2}, \dfrac{7\pi}{6}\right), \left(\dfrac{3\pi}{2}, \dfrac{11\pi}{6}\right)$

(b) Relative minima: $\left(\dfrac{7\pi}{6}, -\dfrac{1}{4}\right), \left(\dfrac{11\pi}{6}, -\dfrac{1}{4}\right)$

Relative maxima: $\left(\dfrac{\pi}{2}, 2\right), \left(\dfrac{3\pi}{2}, 0\right)$

(c)



66. (a) $f(x) = \dfrac{\sin x}{1 + \cos^2 x}, 0 < x < 2\pi$

$f'(x) = \dfrac{\cos x(2 + \sin^2 x)}{(1 + \cos^2 x)^2} = 0$

Critical numbers: $x = \dfrac{\pi}{2}, \dfrac{3\pi}{2}$

Test intervals:	$0 < x < \frac{\pi}{2}$	$\frac{\pi}{2} < x < \frac{3\pi}{2}$	$\frac{3\pi}{2} < x < 2\pi$
Sign of $f'(x)$:	$f' > 0$	$f' < 0$	$f' > 0$
Conclusion:	Increasing	Decreasing	Increasing

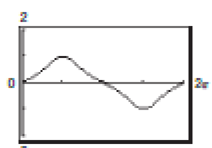
Increasing on: $\left(0, \dfrac{\pi}{2}\right), \left(\dfrac{3\pi}{2}, 2\pi\right)$

Decreasing on: $\left(\dfrac{\pi}{2}, \dfrac{3\pi}{2}\right)$

(b) Relative maximum: $\left(\dfrac{\pi}{2}, 1\right)$

Relative minimum: $\left(\dfrac{3\pi}{2}, -1\right)$

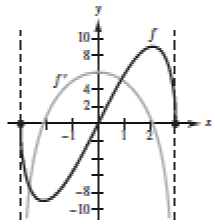
(c)



67. $f(x) = 2x\sqrt{9 - x^2}$, $[-3, 3]$

(a) $f'(x) = \dfrac{2(9 - 2x^2)}{\sqrt{9 - x^2}}$

(b)



(c) $\dfrac{2(9 - 2x^2)}{\sqrt{9 - x^2}} = 0$

Critical numbers: $x = \pm\dfrac{3}{\sqrt{2}} = \pm\dfrac{3\sqrt{2}}{2}$

(d) Intervals:

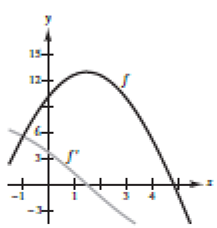
$\left(-3, -\dfrac{3\sqrt{2}}{2}\right)$	$\left(-\dfrac{3\sqrt{2}}{2}, \dfrac{3\sqrt{2}}{2}\right)$	$\left(\dfrac{3\sqrt{2}}{2}, 3\right)$
$f'(x) < 0$	$f'(x) > 0$	$f'(x) < 0$
Decreasing	Increasing	Decreasing

f is increasing when f' is positive and decreasing when f' is negative.

68. $f(x) = 10\left(5 - \sqrt{x^2 - 3x + 16}\right)$, $[0, 5]$

(a) $f'(x) = -\dfrac{5(2x - 3)}{\sqrt{x^2 - 3x + 16}}$

(b)



(c) $-\dfrac{5(2x - 3)}{\sqrt{x^2 - 3x + 16}} = 0$

Critical number: $x = \dfrac{3}{2}$

(d) Intervals:

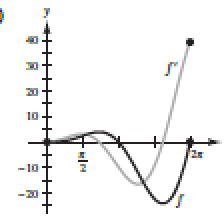
$\left(0, \dfrac{3}{2}\right)$	$\left(\dfrac{3}{2}, 5\right)$
$f'(x) > 0$	$f'(x) < 0$
Increasing	Decreasing

f is increasing when f' is positive and decreasing when f' is negative.

69. $f(t) = t^2 \sin t$, $[0, 2\pi]$

(a) $f'(t) = t^2 \cos t + 2t \sin t = t(t \cos t + 2 \sin t)$

(b)



(c) $t(t \cos t + 2 \sin t) = 0$

$t = 0$ or $t = -2 \tan t$

$t \cot t = -2$

$t \approx 2.2889, 5.0870$ (graphing utility)

Critical numbers: $t = 2.2889, 5.0870$

(d) Intervals:

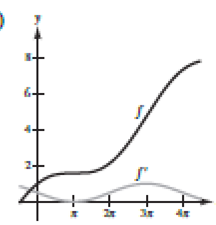
$(0, 2.2889)$	$(2.2889, 5.0870)$	$(5.0870, 2\pi)$
$f'(t) > 0$	$f'(t) < 0$	$f'(t) > 0$
Increasing	Decreasing	Increasing

f is increasing when f' is positive and decreasing when f' is negative.

70. $f(x) = \dfrac{x}{2} + \cos\dfrac{x}{2}$, $[0, 4\pi]$

(a) $f'(x) = \dfrac{1}{2} - \dfrac{1}{2}\sin\dfrac{x}{2}$

(b)



(c) $\dfrac{1}{2} - \dfrac{1}{2}\sin\dfrac{x}{2} = 0$

$\sin\dfrac{x}{2} = 1$

$\dfrac{x}{2} = \dfrac{\pi}{2}$

Critical number: $x = \pi$

(d) Intervals:

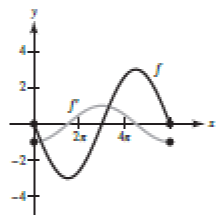
$(0, \pi)$	$(\pi, 4\pi)$
$f'(x) > 0$	$f'(x) > 0$
Increasing	Increasing

f is increasing when f' is positive.

71. (a) $f(x) = -3\sin\dfrac{x}{3}, [0, 6\pi]$

$f'(x) = -\cos\dfrac{x}{3}$

(b)



(c) Critical numbers: $x = \dfrac{3\pi}{2}, \dfrac{9\pi}{2}$

(d) Intervals:

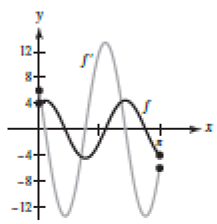
$\left(0, \frac{3\pi}{2}\right)$	$\left(\frac{3\pi}{2}, \frac{9\pi}{2}\right)$	$\left(\frac{9\pi}{2}, 6\pi\right)$
$f' < 0$	$f' > 0$	$f' < 0$
Decreasing	Increasing	Decreasing

f is increasing when f' is positive, and decreasing when f' is negative.

72. (a) $f(x) = 2\sin 3x + 4\cos 3x, [0, \pi]$

$f'(x) = 6\cos 3x - 12\sin 3x$

(b)



(c) $f'(x) = 0 \Rightarrow \tan 3x = \dfrac{1}{2}$

Critical numbers: $x \approx 0.1545, 1.2017, 2.2489$

(d) Intervals:

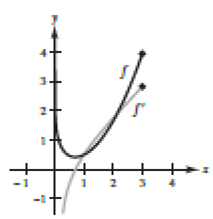
$(0, 0.1545)$	$(0.1545, 1.2017)$	$(1.2017, 2.2489)$	$(2.2489, \pi)$
$f' > 0$	$f' < 0$	$f' > 0$	$f' < 0$
Increasing	Decreasing	Increasing	Decreasing

f is increasing when f' is positive, and decreasing when f' is negative.

73. $f(x) = \dfrac{1}{2}\left(x^2 - \ln x\right), (0, 3]$

(a) $f'(x) = \dfrac{2x^2 - 1}{2x}$

(b)



(c) $\dfrac{2x^2 - 1}{2x} = 0$

Critical number: $x = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$

(d) Intervals:

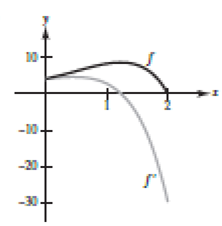
$\left(0, \frac{\sqrt{2}}{2}\right)$	$\left(\frac{\sqrt{2}}{2}, 3\right)$
$f'(x) < 0$	$f'(x) > 0$
Decreasing	Increasing

(e) f is increasing when f' is positive, and decreasing when f' is negative.

74. $f(x) = \left(4 - x^2\right)e^x, [0, 2]$

(a) $f'(x) = \left(4 - 2x - x^2\right)e^x$

(b)



(c) $\left(4 - 2x - x^2\right)e^x = 0$

Critical number: $x \approx 1.2361 \quad \left(x = -1 + \sqrt{5}\right)$

(d) Intervals:

$(0, 1.2361)$	$(1.2361, 2)$
$f'(x) > 0$	$f'(x) < 0$
Increasing	Decreasing

(e) f is increasing when f' is positive, and decreasing when f' is negative.

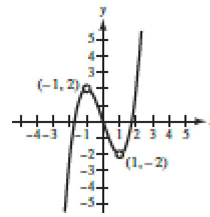
75. $f(x) = \dfrac{x^5 - 4x^3 + 3x}{x^2 - 1} = \dfrac{(x^2 - 1)(x^3 - 3x)}{x^2 - 1} = x^3 - 3x,\ x \neq \pm 1$

$f(x) = g(x) = x^3 - 3x$ for all $x \neq \pm 1$.

$f'(x) = 3x^2 - 3 = 3(x^2 - 1),\ x \neq \pm 1 \Rightarrow f'(x) \neq 0$

f symmetric about origin

zeros of f: $(0, 0), (\pm\sqrt{3}, 0)$



$g(x)$ is continuous on $(-\infty, \infty)$ and $f(x)$ has holes at $(-1, 2)$ and $(1, -2)$.

76. $f(t) = \cos^2 t - \sin^2 t = 1 - 2\sin^2 t = g(t)$

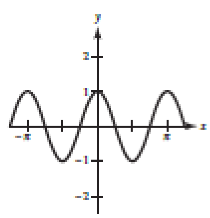
$f'(t) = -4 \sin t \cos t = -2 \sin 2t$

f symmetric with respect to y-axis

zeros of f: $\pm\dfrac{\pi}{4}$

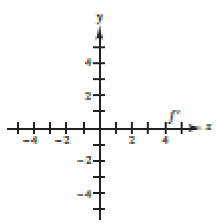
Relative maximum: $(0, 1)$

Relative minimum: $\left(-\dfrac{\pi}{2}, -1\right), \left(\dfrac{\pi}{2}, -1\right)$

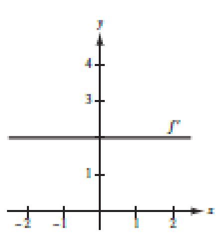


The graphs of $f(x)$ and $g(x)$ are the same.

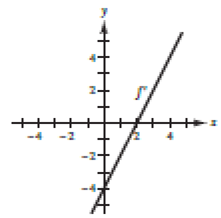
77. $f(x) = c$ is constant $\Rightarrow f'(x) = 0$.



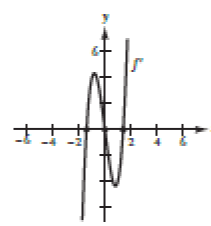
78. $f(x)$ is a line of slope $\approx 2 \Rightarrow f'(x) = 2$.



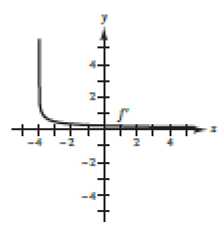
79. f is quadratic $\Rightarrow f'$ is a line.



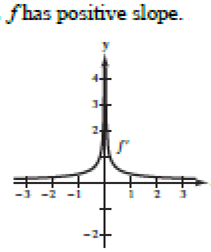
80. f is a 4th degree polynomial $\Rightarrow f'$ is a cubic polynomial.



81. f has positive, but decreasing slope.



82. f has positive slope.



In Exercises 83–88, $f'(x) > 0$ on $(-\infty, -4)$, $f'(x) < 0$ on $(-4, 6)$ and $f'(x) > 0$ on $(6, \infty)$.

83. $g(x) = f(x) + 5$
$g'(x) = f'(x)$
$g'(0) = f'(0) < 0$

84. $g(x) = 3f(x) - 3$
$g'(x) = 3f'(x)$
$g'(-5) = 3f'(-5) > 0$

85. $g(x) = -f(x)$
$g'(x) = -f'(x)$
$g'(-6) = -f'(-6) < 0$

86. $g(x) = -f(x)$
$g'(x) = -f'(x)$
$g'(0) = -f'(0) > 0$

87. $g(x) = f(x - 10)$
$g'(x) = f'(x - 10)$
$g'(0) = f'(-10) > 0$

88. $g(x) = f(x - 10)$
$g'(x) = f'(x - 10)$
$g'(8) = f'(-2) < 0$

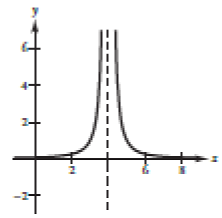
89. No. f does have a horizontal tangent line at $x = c$, but f could be increasing (or decreasing) on both sides of the point. For example, $f(x) = x^3$ at $x = 0$.

90. Yes. An example is $f(x) = e^{-x}, f'(x) = -e^{-x}$.

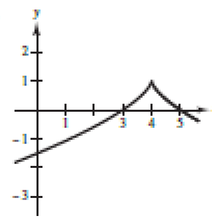
91. $f'(x)\begin{cases} > 0, & x < 4 \Rightarrow f \text{ is increasing on } (-\infty, 4). \\ \text{undefined}, & x = 4 \\ < 0, & x > 4 \Rightarrow f \text{ is decreasing on } (4\ \infty). \end{cases}$

Two possibilities for $f(x)$ are given below.

(a)



(b)



92. (i) (a) Critical numbers: $x = -1, 0, 1$

(Because $f'(-1) = f'(0) = f'(1) = 0$)

(b) f increasing on $(-\infty, -1)$ and $(0, 1)$

(Because $f' > 0$ on these intervals)

f decreasing on $(-1, 0)$ and $(1, \infty)$

(Because $f' < 0$ on these intervals)

(c) f has a relative maximum at $x = -1$ and $x = 1$. f has a relative minimum at $x = 0$.

(ii) (a) Critical numbers: $x = -3, 1, 5$

(Because $f'(-3) = f'(1) = f'(s) = 0$)

(b) f increasing on $(-3, 1)$ and $(1, 5)$

(Because $f' > 0$ on these intervals). In fact, f is increasing on $(-3, 5)$.

f decreasing on $(-\infty, -3)$ and $(5, \infty)$

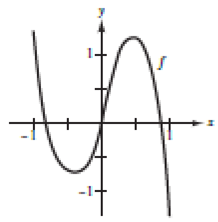
(Because $f' < 0$ on these intervals)

(c) f has a relative minimum at $x = -3$, and a relative maximum at $x = 5$.
$x = 1$ is not a relative extremum.

In Exercises 93 and 94, answers will vary.

Sample answers:

93. (a)

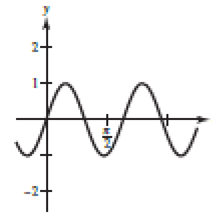


(b) The critical numbers are in intervals $(-0.50, -0.25)$ and $(0.25, 0.50)$ because the sign of f' changes in these intervals. f is decreasing on approximately $(-1, -0.40)$, $(0.48, 1)$, and increasing on $(-0.40, 0.48)$.

(c) Relative minimum when $x \approx -0.40$: $(-0.40, 0.75)$

Relative maximum when $x \approx 0.48$: $(0.48, 1.25)$

94. (a)



(b) The critical numbers are in the intervals $\left(0, \frac{\pi}{6}\right), \left(\frac{\pi}{3}, \frac{\pi}{2}\right)$, and $\left(\frac{3\pi}{4}, \frac{5\pi}{6}\right)$ because the sign of f' changes in these intervals. f is increasing on approximately $\left(0, \frac{\pi}{7}\right)$ and $\left(\frac{3\pi}{7}, \frac{6\pi}{7}\right)$ and decreasing on $\left(\frac{\pi}{7}, \frac{3\pi}{7}\right)$ and $\left(\frac{6\pi}{7}, \pi\right)$.

(c) Relative minima when $x \approx \frac{3\pi}{7}, \pi$

Relative maxima when $x \approx \frac{\pi}{7}, \frac{6\pi}{7}$

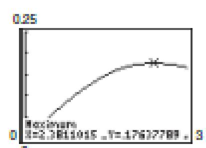
95. $C = \dfrac{3t}{27 + t^3}, t \ge 0$

(a)

t	0	0.5	1	1.5	2	2.5	3
$C(t)$	0	0.055	0.107	0.148	0.171	0.176	0.167

The concentration seems greatest near $t = 2.5$ hours.

(b)



The concentration is greatest when $t \approx 2.38$ hours.

(c) $C' = \dfrac{(27 + t^3)(3) - (3t)(3t^2)}{(27 + t^3)^2} = \dfrac{3(27 - 2t^3)}{(27 + t^3)^2}$

$C' = 0$ when $t = 3/\sqrt[3]{2} \approx 2.38$ hours.

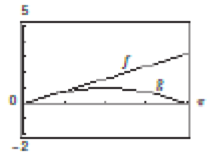
By the First Derivative Test, this is a maximum.

96. $f(x) = x, g(x) = \sin x, 0 < x < \pi$

(a)

x	0.5	1	1.5	2	2.5	3
$f(x)$	0.5	1	1.5	2	2.5	3
$g(x)$	0.479	0.841	0.997	0.909	0.598	0.141

$f(x)$ seems greater than $g(x)$ on $(0, \pi)$.

(b) 

$x > \sin x$ on $(0, \pi)$, so $f(x) > g(x)$.

(c) Let $h(x) = f(x) - g(x) = x - \sin x$

$h'(x) = 1 - \cos x > 0$ on $(0, \pi)$.

Therefore, $h(x)$ is increasing on $(0, \pi)$. Because $h(0) = 0$ and $h'(x) > 0$ on $(0, \pi)$,

$$h(x) > 0$$
$$x - \sin x > 0$$
$$x > \sin x$$
$$f(x) > g(x) \text{ on } (0, \pi)$$

97. $v = k(R - r)r^2 = k(Rr^2 - r^3)$

$v' = k(2Rr - 3r^2)$

$= kr(2R - 3r) = 0$

$r = 0$ or $\frac{2}{3}R$

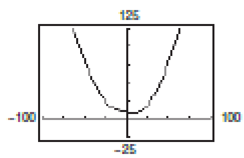
Maximum when $r = \frac{2}{3}R$

98. $R = \sqrt{0.001T^4 - 4T + 100}$

(a) $R' = \dfrac{0.004T^3 - 4}{2\sqrt{0.001T^4 - 4T + 100}} = 0$

Critical number: $T = 10°$

Minimum resistance: $R \approx 8.3666$ ohms

(b) 

The minimum resistance is approximately $R \approx 8.37$ ohms at $T = 10°$.

99. (a) $s(t) = 6t - t^2, t \ge 0$

$v(t) = 6 - 2t$

(b) $v(t) = 0$ when $t = 3$.

Moving in positive direction for $0 \le t < 3$ because $v(t) > 0$ on $0 \le t < 3$.

(c) Moving in negative direction when $t > 3$.

(d) The particle changes direction at $t = 3$.

100. (a) $s(t) = t^2 - 7t + 10, t \ge 0$

$v(t) = 2t - 7$

(b) $v(t) = 0$ when $t = \frac{7}{2}$

Particle moving in positive direction for $t > \frac{7}{2}$ because $v'(t) > 0$ on $\left(\frac{7}{2}, \infty\right)$.

(c) Particle moving in negative direction on $\left[0, \frac{7}{2}\right)$.

(d) The particle changes direction at $t = \frac{7}{2}$.

101. (a) $s(t) = t^3 - 5t^2 + 4t, t \ge 0$

$v(t) = 3t^2 - 10t + 4$

(b) $v(t) = 0$ for $t = \dfrac{10 \pm \sqrt{100 - 48}}{6} = \dfrac{5 \pm \sqrt{13}}{3}$

Particle is moving in a positive direction on $\left[0, \dfrac{5 - \sqrt{13}}{3}\right) \approx [0, 0.4648)$ and $\left(\dfrac{5 + \sqrt{13}}{3}, \infty\right) \approx (2.8685, \infty)$ because $v > 0$ on these intervals.

(c) Particle is moving in a negative direction on $\left(\dfrac{5 - \sqrt{13}}{3}, \dfrac{5 + \sqrt{13}}{3}\right) \approx (0.4648, 2.8685)$

(d) The particle changes direction at $t = \dfrac{5 \pm \sqrt{13}}{3}$.

102. (a) $s(t) = t^3 - 20t^2 + 128t - 280$

$v(t) = 3t^2 - 40t + 128$

(b) $v(t) = (3t - 16)(t - 8)$

$v(t) = 0$ when $t = \frac{16}{3}, 8$

$v(t) > 0$ for $\left[0, \frac{16}{3}\right)$ and $(8, \infty)$

(c) $v(t) < 0$ for $\left(\frac{16}{3}, 8\right)$

(d) The particle changes direction at $t = \frac{16}{3}$ and 8.

103. Answers will vary.

104. Answers will vary.

105. True.

Let $h(x) = f(x) + g(x)$ where f and g are increasing. Then $h'(x) = f'(x) + g'(x) > 0$ because $f'(x) > 0$ and $g'(x) > 0$.

106. False.

Let $h(x) = f(x)g(x)$, where $f(x) = g(x) = x$. Then $h(x) = x^2$ is decreasing on $(-\infty, 0)$.

107. False.

Let $f(x) = x^3$, then $f'(x) = 3x^2$ and f only has one critical number. Or, let $f(x) = x^3 + 3x + 1$, then $f'(x) = 3(x^2 + 1)$ has no critical numbers.

108. True.

If $f(x)$ is an nth-degree polynomial, then the degree of $f'(x)$ is $n - 1$.

109. False. For example, $f(x) = x^3$ does not have a relative extremum at the critical number $x = 0$.

110. False. The function might not be continuous on the interval.

111. Assume that $f'(x) < 0$ for all x in the interval (a, b) and let $x_1 < x_2$ be any two points in the interval. By the Mean Value Theorem, you know there exists a number c such that $x_1 < c < x_2$, and

$$f'(c) = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Because $f'(c) < 0$ and $x_2 - x_1 > 0$, then $f(x_2) - f(x_1) < 0$, which implies that $f(x_2) < f(x_1)$. So, f is decreasing on the interval.

112. Suppose $f'(x)$ changes from positive to negative at c. Then there exists a and b in I such that $f'(x) > 0$ for all x in (a, c) and $f'(x) < 0$ for all x in (c, b). By Theorem 4.5, f is increasing on (a, c) and decreasing on (c, b). Therefore, $f(c)$ is a maximum of f on (a, b) and so, a relative maximum of f.

113. Let $f(x) = (1 + x)^n - nx - 1$. Then

$f'(x) = n(1 + x)^{n-1} - n = n\left[(1 + x)^{n-1} - 1\right] > 0$

because $x > 0$ and $n > 1$.

So, $f(x)$ is increasing on $(0, \infty)$. Because

$f(0) = 0 \Rightarrow f(x) > 0$ on $(0, \infty)$

$(1 + x)^n - nx - 1 > 0 \Rightarrow (1 + x)^n > 1 + nx$.

114. Let x_1 and x_2 be two real numbers, $x_1 < x_2$. Then $x_1^3 < x_2^3 \Rightarrow f(x_1) < f(x_2)$. So f is increasing on $(-\infty, \infty)$.

115. Let x_1 and x_2 be two positive real numbers, $0 < x_1 < x_2$. Then

$$\frac{1}{x_1} > \frac{1}{x_2}$$

$$f(x_1) > f(x_2)$$

So, f is decreasing on $(0, \infty)$.

116. $f(x) = axe^{bx^2}$

$f'(x) = ax(2bx)e^{bx^2} + ae^{bx^2} = ae^{bx^2}(1 + 2bx^2)$

$f(4) = 2$: $2 = 4ae^{16b} \Rightarrow 2a = \frac{1}{e^{16b}} \Rightarrow a = \frac{1}{2}e^{-16b}$

Relative maximum at $x = 4$:

$f'(4) = 0 \Rightarrow 1 + 2b(16) = 0 \Rightarrow b = -\frac{1}{32}$

So, $a = \frac{1}{2}e^{1/2} = \frac{\sqrt{e}}{2}$,

$f(x) = \frac{\sqrt{e}}{2}xe^{-x^2/32}$.

Notice the f is increasing on $(0, 4)$ and decreasing on $(4, \infty)$, so $(4, 2)$ is a relative maximum.

117. By Theorem 3.5, if $h'(x) < 0$ for all x in $(-1, 3)$, then h is decreasing on $[-1, 3]$, and $-1 < x < 3$.

So, the answer is D.

118. $g(x) = \sqrt{2}x - 2\cos x$

$g'(x) = \sqrt{2} + 2\sin x$

$0 = \sqrt{2} + 2\sin x$

$\sin x = -\dfrac{\sqrt{2}}{2}$

$x = \dfrac{5\pi}{4}, \dfrac{7\pi}{4}$ (critical numbers)

(a) Because $g'(x) > 0$ on the intervals $0 < x < \dfrac{5\pi}{4}$ and $\dfrac{7\pi}{4} < x < 2\pi$, $g(x)$ is increasing on $\left[0, \dfrac{5\pi}{4}\right)$ and $\left(\dfrac{7\pi}{4}, 2\pi\right]$.

(b) Because $g'(x) < 0$ on the interval $\dfrac{5\pi}{4} < x < \dfrac{7\pi}{4}$, $g(x)$ is decreasing on $\left(\dfrac{5\pi}{4}, \dfrac{7\pi}{4}\right)$.

(c) When $x = \dfrac{5\pi}{4}$,

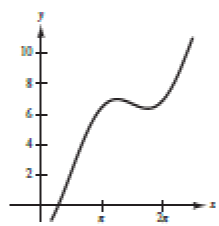
$g\left(\dfrac{5\pi}{4}\right) = \sqrt{2}\left(\dfrac{5\pi}{4}\right) - 2\cos\left(\dfrac{5\pi}{4}\right) \approx 6.97.$

When $x = \dfrac{7\pi}{4}$,

$g\left(\dfrac{7\pi}{4}\right) = \sqrt{2}\left(\dfrac{7\pi}{4}\right) - 2\cos\left(\dfrac{7\pi}{4}\right) \approx 6.36.$

So, the relative minimum is at $x = \dfrac{7\pi}{4}$ and the relative maximum is at $x = \dfrac{5\pi}{4}$.

(d)



119. (a) $\lim_{x\to-\infty} (x^2 - 1)e^x = 0$

$\lim_{x\to\infty} (x^2 - 1)e^x = \infty$

(b) $f(x) = (x^2 - 1)e^x$

$f'(x) = (x^2 - 1)e^x + 2xe^x$

$0 = (x^2 - 1)e^x + 2xe^x$

$0 = e^x(x^2 - 1 + 2x)$

$e^x = 0$ undef. and $x^2 + 2x - 1 = 0$

$x^2 + 2x + 1 = 1 + 1$

$(x + 1)^2 = 2$

$x = -1 \pm \sqrt{2}$

So, $x = -1 \pm \sqrt{2}$ are the critical numbers of f.

(c) Because $f'(x) > 0$ on $-\infty < x < -1 - \sqrt{2}$ and $-1 + \sqrt{2} < x < \infty$, f is increasing on $(-\infty, -1 - \sqrt{2})$ and $(-1 + \sqrt{2}, \infty)$.

Because $f'(x) < 0$ on $-1 - \sqrt{2} < x < -1 + \sqrt{2}$, f is decreasing on $(-1 - \sqrt{2}, -1 + \sqrt{2})$.

(d) $f(-1 - \sqrt{2}) \approx 0.4318$

$f(-1 + \sqrt{2}) \approx -1.2536$

So, $(-1 - \sqrt{2}, 0.4318)$ is a relative maximum and $(-1 + \sqrt{2}, -1.2536)$ is a relative minimum.