Section 4.7 Inverse Trigonometric Functions: Integration

1. $\displaystyle\int \frac{dx}{\sqrt{9 - x^2}} = \arcsin\frac{x}{3} + C$

2. $\displaystyle\int \frac{dx}{\sqrt{1 - 4x^2}} = \frac{1}{2}\int \frac{2}{\sqrt{1 - 4x^2}}\,dx = \frac{1}{2}\arcsin 2x + C$

3. $\displaystyle\int \frac{1}{x\sqrt{4x^2 - 1}}\,dx = \int \frac{2}{2x\sqrt{(2x)^2 - 1}}\,dx = \operatorname{arcsec}|2x| + C$

4. $\displaystyle\int \frac{12}{1 + 9x^2}\,dx = 4\int \frac{3}{1 + 9x^2}\,dx = 4\arctan 3x + C$

5. $\displaystyle\int \frac{1}{\sqrt{1 - (x + 1)^2}}\,dx = \arcsin(x + 1) + C$

6. $\displaystyle\int \frac{1}{4 + (x - 3)^2}\,dx = \frac{1}{2}\arctan\frac{x - 3}{2} + C$

7. Let $u = t^2$, $du = 2t\,dt$.

$$\int \frac{t}{\sqrt{1 - t^4}}\,dt = \frac{1}{2}\int \frac{1}{\sqrt{1 - (t^2)^2}}(2t)\,dt = \frac{1}{2}\arcsin t^2 + C$$

8. Let $u = x^2$, $du = 2x\,dx$.

$$\int \frac{1}{x\sqrt{x^4 - 4}}\,dx = \frac{1}{2}\int \frac{1}{x^2\sqrt{(x^2)^2 - 2^2}}(2x)\,dx$$

$$= \frac{1}{4}\operatorname{arcsec}\frac{x^2}{2} + C$$

9. $\displaystyle\int \frac{t}{t^4 + 25}\,dt = \frac{1}{2}\int \frac{1}{(t^2)^2 + 5^2}(2)\,dt$

$$= \frac{1}{2}\frac{1}{5}\arctan\frac{t^2}{5} + C$$

$$= \frac{1}{10}\arctan\frac{t^2}{5} + C$$

10. $\displaystyle\int \frac{1}{x\sqrt{1-(\ln x)^2}}\,dx = \int \frac{1}{\sqrt{1-(\ln x)^2}}\cdot\frac{1}{x}\,dx$

$\displaystyle = \arcsin(\ln x) + C$

11. Let $u = e^{2x}$, $du = 2e^{2x}\,dx$.

$$\int \frac{e^{2x}}{4+e^{4x}}\,dx = \frac{1}{2}\int \frac{2e^{2x}}{4+\left(e^{2x}\right)^2}\,dx = \frac{1}{4}\arctan\frac{e^{2x}}{2} + C$$

12. $u = 3x$, $du = 3\,dx$, $a = 5$

$$\int \frac{2}{x\sqrt{9x^2-25}}\,dx = 2\int \frac{1}{(3x)\sqrt{(3x)^2-5^2}}\,3\,dx$$

$$= \frac{2}{5}\operatorname{arcsec}\frac{|3x|}{5} + C$$

13. $\displaystyle\int \frac{\sec^2 x}{\sqrt{25-\tan^2 x}}\,dx = \int \frac{\sec^2 x}{\sqrt{5^2-(\tan x)^2}}\,dx$

$\displaystyle = \arcsin\left(\frac{\tan x}{5}\right) + C$

14. $\displaystyle\int \frac{\sin x}{7+\cos^2 x}\,dx = \int \frac{-1}{\left(\sqrt{7}\right)^2+\cos^2 x}(-\sin x)\,dx$

$\displaystyle = -\frac{1}{\sqrt{7}}\arctan\left(\frac{\cos x}{\sqrt{7}}\right) + C$

$\displaystyle = -\frac{\sqrt{7}}{7}\arctan\left(\frac{\sqrt{7}\cos x}{7}\right) + C$

15. $\displaystyle\int \frac{1}{\sqrt{x}\sqrt{1-x}}\,dx, u = \sqrt{x}, x = u^2, dx = 2u\,du$

$$\int \frac{1}{u\sqrt{1-u^2}}(2u\,du) = 2\int \frac{du}{\sqrt{1-u^2}} = 2\arcsin u + C = 2\arcsin\sqrt{x} + C$$

16. $\displaystyle\int \frac{3}{2\sqrt{x}(1+x)}\,dx, u = \sqrt{x}, du = \frac{1}{2\sqrt{x}}\,dx, dx = 2u\,du$

$$\frac{3}{2}\int \frac{2u\,du}{u\left(1+u^2\right)} = 3\int \frac{du}{1+u^2} = 3\arctan u + C = 3\arctan\sqrt{x} + C$$

17. $\displaystyle\int \frac{x-3}{x^2+1}\,dx = \frac{1}{2}\int \frac{2x}{x^2+1}\,dx - 3\int \frac{1}{x^2+1}\,dx = \frac{1}{2}\ln\left(x^2+1\right) - 3\arctan x + C$

18. $\displaystyle\int \frac{x^2+3}{x\sqrt{x^2-4}}\,dx = \int \frac{x^2}{x\sqrt{x^2-4}}\,dx + \int \frac{3}{x\sqrt{x^2-4}}\,dx$

$\displaystyle = \frac{1}{2}\int \left(x^2-4\right)^{-1/2}2x\,dx + 3\int \frac{1}{x\sqrt{x^2-4}}\,dx$

$\displaystyle = \sqrt{x^2-4} + \frac{3}{2}\operatorname{arcsec}\frac{|x|}{2} + C$

19. $\displaystyle\int \frac{x+5}{\sqrt{9-(x-3)^2}}\,dx = \int \frac{(x-3)}{\sqrt{9-(x-3)^2}}\,dx + \int \frac{8}{\sqrt{9-(x-3)^2}}\,dx$

$\displaystyle = -\sqrt{9-(x-3)^2} + 8\arcsin\frac{x-3}{3} + C = -\sqrt{6x-x^2} + 8\arcsin\frac{x-3}{3} + C$

20. $\displaystyle\int \frac{x-2}{(x+1)^2+4}\,dx = \frac{1}{2}\int \frac{2x+2}{(x+1)^2+4}\,dx - \int \frac{3}{(x+1)^2+4}\,dx$

$\displaystyle = \frac{1}{2}\ln\left(x^2+2x+5\right) - \frac{3}{2}\arctan\frac{x+1}{2} + C$

21. Let $u = 3x$, $du = 3\,dx$.

$$\int_0^{1/6} \frac{3}{\sqrt{1-9x^2}}\,dx = \int_0^{1/6} \frac{1}{\sqrt{1-(3x)^2}}(3)\,dx$$
$$= \left[\arcsin 3x\right]_0^{1/6} = \frac{\pi}{6}$$

22. $$\int_0^{\sqrt{2}} \frac{1}{\sqrt{4-x^2}}\,dx = \left[\arcsin \frac{x}{2}\right]_0^{\sqrt{2}}$$
$$= \arcsin \frac{\sqrt{2}}{2} - \arcsin 0$$
$$= \frac{\pi}{4}$$

23. Let $u = 2x$, $du = 2\,dx$.

$$\int_0^{\sqrt{3}/2} \frac{1}{1+4x^2}\,dx = \frac{1}{2}\int_0^{\sqrt{3}/2} \frac{2}{1+(2x)^2}\,dx$$
$$= \left[\frac{1}{2}\arctan 2x\right]_0^{\sqrt{3}/2} = \frac{\pi}{6}$$

24. $$\int_{\sqrt{3}}^{3} \frac{1}{x\sqrt{4x^2-9}}\,dx = \left[\frac{1}{3}\operatorname{arcsec}\frac{|2x|}{3}\right]_{\sqrt{3}}^{3}$$
$$= \frac{1}{3}\operatorname{arcsec} 2 - \frac{1}{3}\operatorname{arcsec}\frac{2\sqrt{3}}{3}$$
$$= \frac{1}{3}\left(\frac{\pi}{3}\right) - \frac{1}{3}\left(\frac{\pi}{6}\right) = \frac{\pi}{18}$$

25. $$\int_3^6 \frac{1}{25+(x-3)^2}\,dx = \left[\frac{1}{5}\arctan\frac{x-3}{5}\right]_3^6$$
$$= \frac{1}{5}\arctan\frac{3}{5}$$
$$\approx 0.108$$

26. $$\int_1^4 \frac{1}{x\sqrt{16x^2-5}}\,dx = \int_1^4 \frac{4\,dx}{(4x)\sqrt{(4x)^2-\left(\sqrt{5}\right)^2}}$$
$$= \left[\left(\frac{1}{\sqrt{5}}\right)\operatorname{arcsec}\frac{|4x|}{\sqrt{5}}\right]_1^4 = \frac{1}{\sqrt{5}}\operatorname{arcsec}\frac{16}{\sqrt{5}} - \frac{1}{\sqrt{5}}\operatorname{arcsec}\frac{4}{\sqrt{5}} \approx 0.091$$

27. Let $u = e^x$, $du = e^x\,dx$

$$\int_0^{\ln 5} \frac{e^x}{1+e^{2x}}\,dx = \left[\arctan e^x\right]_0^{\ln 5} = \arctan 5 - \frac{\pi}{4} \approx 0.588$$

28. Let $u = e^{-x}$, $du = -e^{-x}\,dx$

$$\int_{\ln 2}^{\ln 4} \frac{e^{-x}}{\sqrt{1-e^{-2x}}}\,dx = \left[-\arcsin e^{-x}\right]_{\ln 2}^{\ln 4} = -\arcsin\frac{1}{4} + \arcsin\frac{1}{2} = \frac{\pi}{6} - \arcsin\frac{1}{4} \approx 0.271$$

29. Let $u = \cos x$, $du = -\sin x\,dx$.

$$\int_{\pi/2}^{\pi} \frac{\sin x}{1+\cos^2 x}\,dx = -\int_{\pi/2}^{\pi} \frac{-\sin x}{1+\cos^2 x}\,dx = \left[-\arctan(\cos x)\right]_{\pi/2}^{\pi} = \frac{\pi}{4}$$

30. $$\int_0^{\pi/2} \frac{\cos x}{1+\sin^2 x}\,dx = \left[\arctan(\sin x)\right]_0^{\pi/2} = \frac{\pi}{4}$$

31. Let $u = \arcsin x$, $du = \dfrac{1}{\sqrt{1-x^2}}\,dx$.

$$\int_0^{1/\sqrt{2}} \frac{\arcsin x}{\sqrt{1-x^2}}\,dx = \left[\frac{1}{2}\arcsin^2 x\right]_0^{1/\sqrt{2}} = \frac{\pi^2}{32} \approx 0.308$$

32. Let $u = \arccos x$, $du = -\dfrac{1}{\sqrt{1-x^2}}\,dx$.

$$\int_0^{1/\sqrt{2}} \frac{\arccos x}{\sqrt{1-x^2}}\,dx = -\int_0^{1/\sqrt{2}} \frac{-\arccos x}{\sqrt{1-x^2}}\,dx = \left[-\frac{1}{2}\arccos^2 x\right]_0^{1/\sqrt{2}} = \frac{3\pi^2}{32} \approx 0.925$$

33. $\int_0^2 \frac{dx}{x^2 - 2x + 2} = \int_0^2 \frac{1}{1 + (x-1)^2}\,dx = \left[\arctan(x-1)\right]_0^2 = \frac{\pi}{2}$

Trapezoidal Rule:

$$\int_0^2 \frac{dx}{x^2 - 2x + 2} \approx \frac{2-0}{2(4)}\left[f(0) + 2f\left(\frac{1}{2}\right) + 2f(1) + 2f\left(\frac{3}{2}\right) + f(2)\right]$$

$$= \frac{1}{4}\left[\frac{1}{2} + 2\left(\frac{4}{5}\right) + 2(1) + 2\left(\frac{4}{5}\right) + \frac{1}{2}\right]$$

$$= 1.55$$

34. $\int_{-2}^2 \frac{dx}{x^2 + 4x + 13} = \int_{-2}^2 \frac{dx}{(x+2)^2 + 9} = \left[\frac{1}{3}\arctan\frac{x+2}{3}\right]_{-2}^2 = \frac{1}{3}\arctan\frac{4}{3}$

Trapezoidal Rule:

$$\int_{-2}^2 \frac{dx}{x^2 + 4x + 13} \approx \frac{2-(-2)}{2(4)}\left[f(-2) + 2f(-1) + 2f(0) + 2f(1) + f(2)\right]$$

$$= \frac{1}{2}\left[\frac{1}{9} + 2\left(\frac{1}{10}\right) + 2\left(\frac{1}{13}\right) + 2\left(\frac{1}{18}\right) + \frac{1}{25}\right]$$

$$\approx 0.3080$$

35. $\int \frac{2x}{x^2 + 6x + 13}\,dx = \int \frac{2x+6}{x^2 + 6x + 13}\,dx - 6\int \frac{1}{x^2 + 6x + 13}\,dx$

$$= \int \frac{2x+6}{x^2 + 6x + 13}\,dx - 6\int \frac{1}{4 + (x+3)^2}\,dx = \ln\left|x^2 + 6x + 13\right| - 3\arctan\frac{x+3}{2} + C$$

36. $\int \frac{2x-5}{x^2 + 2x + 2}\,dx = \int \frac{2x+2}{x^2 + 2x + 2}\,dx - 7\int \frac{1}{1 + (x+1)^2}\,dx = \ln\left|x^2 + 2x + 2\right| - 7\arctan(x+1) + C$

37. $\int \frac{1}{\sqrt{-x^2 - 4x}}\,dx = \int \frac{1}{\sqrt{4 - (x+2)^2}}\,dx = \arcsin\frac{x+2}{2} + C$

38. $\int \frac{2}{\sqrt{-x^2 + 4x}}\,dx = \int \frac{2}{\sqrt{4 - (x^2 - 4x + 4)}}\,dx = \int \frac{2}{\sqrt{4 - (x-2)^2}}\,dx = 2\arcsin\frac{x-2}{2} + C$

39. $\int_2^3 \frac{2x-3}{\sqrt{4x - x^2}}\,dx = \int_2^3 \frac{2x-4}{\sqrt{4x - x^2}}\,dx + \int_2^3 \frac{1}{\sqrt{4x - x^2}}\,dx$

$$= -\int_2^3 \left(4x - x^2\right)^{-1/2}(4 - 2x)\,dx + \int_2^3 \frac{1}{\sqrt{4 - (x-2)^2}}\,dx$$

$$= \left[-2\sqrt{4x - x^2} + \arcsin\frac{x-2}{2}\right]_2^3 = 4 - 2\sqrt{3} + \frac{\pi}{6} \approx 1.059$$

Trapezoidal Rule:

$$\int_2^3 \frac{2x-3}{\sqrt{4x - x^2}}\,dx \approx \frac{3-2}{2(4)}\left[f(2) + 2f(2.25) + 2f(2.5) + 2f(2.75) + f(3)\right]$$

$$\approx \frac{1}{8}\left[0.50 + 2(0.76) + 2(1.03) + 2(1.35) + 1.73\right]$$

$$\approx 1.0638$$

40. $\int \frac{1}{(x-1)\sqrt{x^2-2x}}\,dx = \int \frac{1}{(x-1)\sqrt{(x-1)^2-1}}\,dx = \operatorname{arcsec}|x-1| + C$

41. Let $u = x^2 + 1$, $du = 2x\,dx$.

$$\int \frac{x}{x^4+2x^2+2}\,dx = \frac{1}{2}\int \frac{2x}{(x^2+1)^2+1}\,dx = \frac{1}{2}\arctan(x^2+1) + C$$

42. Let $u = x^2 - 4$, $du = 2x\,dx$.

$$\int \frac{x}{\sqrt{9+8x^2-x^4}}\,dx = \frac{1}{2}\int \frac{2x}{\sqrt{25-(x^2-4)^2}}\,dx = \frac{1}{2}\arcsin\frac{x^2-4}{5} + C$$

43. Let $u = \sqrt{e^t - 3}$. Then $u^2 + 3 = e^t$, $2u\,du = e^t\,dt$, and $\frac{2u\,du}{u^2+3} = dt$.

$$\int \sqrt{e^t-3}\,dt = \int \frac{2u^2}{u^2+3}\,du = \int 2\,du - \int 6\frac{1}{u^2+3}\,du$$

$$= 2u - 2\sqrt{3}\arctan\frac{u}{\sqrt{3}} + C = 2\sqrt{e^t-3} - 2\sqrt{3}\arctan\sqrt{\frac{e^t-3}{3}} + C$$

44. Let $u = \sqrt{x-2}$, $u^2 + 2 = x$, $2u\,du = dx$.

$$\int \frac{\sqrt{x-2}}{x+1}\,dx = \int \frac{2u^2}{u^2+3}\,du = \int \frac{2u^2+6-6}{u^2+3}\,du = 2\int du - 6\int \frac{1}{u^2+3}\,du$$

$$= 2u - \frac{6}{\sqrt{3}}\arctan\frac{u}{\sqrt{3}} + C = 2\sqrt{x-2} - 2\sqrt{3}\arctan\sqrt{\frac{x-2}{3}} + C$$

45. $\int_1^3 \frac{dx}{\sqrt{x}(1+x)}$

Let $u = \sqrt{x}$, $u^2 = x$, $2u\,du = dx$, $1 + x = 1 + u^2$.

$$\int_1^{\sqrt{3}} \frac{2u\,du}{u(1+u^2)} = \int_1^{\sqrt{3}} \frac{2}{1+u^2}\,du$$

$$= \left[2\arctan u\right]_1^{\sqrt{3}}$$

$$= 2\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \frac{\pi}{6}$$

46. $\int_0^1 \frac{dx}{2\sqrt{3-x}\sqrt{x+1}}$

Let $u = \sqrt{x+1}$, $u^2 = x + 1$, $2u\,du = dx$,

$\sqrt{3-x} = \sqrt{4-u^2}$.

$$\int_1^{\sqrt{2}} \frac{2u\,du}{2\sqrt{4-u^2}u} = \int_1^{\sqrt{2}} \frac{du}{\sqrt{4-u^2}}$$

$$= \left[\arcsin\frac{u}{2}\right]_1^{\sqrt{2}}$$

$$= \arcsin\frac{\sqrt{2}}{2} - \arcsin\frac{1}{2}$$

$$= \frac{\pi}{4} - \frac{\pi}{6} = \frac{\pi}{12}$$

47. (a) $\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin x + C, \quad u = x$

(b) $\int \frac{x}{\sqrt{1-x^2}}\,dx = -\sqrt{1-x^2} + C, \quad u = 1 - x^2$

(c) $\int \frac{1}{x\sqrt{1-x^2}}\,dx$ cannot be evaluated using the basic integration rules.

48. (a) $\int e^{x^2}\,dx$ cannot be evaluated using the basic integration rules.

(b) $\int xe^{x^2}\,dx = \frac{1}{2}e^{x^2} + C, \quad u = x^2$

(c) $\int \frac{1}{x^2}e^{1/x}\,dx = -e^{1/x} + C, \quad u = \frac{1}{x}$

49. (a) $\int \sqrt{x-1}\,dx = \frac{2}{3}(x-1)^{3/2} + C, \quad u = x - 1$

(b) Let $u = \sqrt{x-1}$. Then $x = u^2 + 1$ and $dx = 2u\,du$.

$$\begin{aligned}
\int x\sqrt{x-1}\,dx &= \int \left(u^2+1\right)(u)(2u)\,du \\
&= 2\int \left(u^4 + u^2\right) du \\
&= 2\left(\frac{u^5}{5} + \frac{u^3}{3}\right) + C \\
&= \frac{2}{15}u^3\left(3u^2 + 5\right) + C \\
&= \frac{2}{15}(x-1)^{3/2}\left[3(x-1) + 5\right] + C \\
&= \frac{2}{15}(x-1)^{3/2}(3x+2) + C
\end{aligned}$$

(c) Let $u = \sqrt{x-1}$. Then $x = u^2 + 1$ and $dx = 2u\,du$.

$$\begin{aligned}
\int \frac{x}{\sqrt{x-1}}\,dx &= \int \frac{u^2+1}{u}(2u)\,du \\
&= 2\int \left(u^2 + 1\right) du \\
&= 2\left(\frac{u^3}{3} + u\right) + C \\
&= \frac{2}{3}u\left(u^2 + 3\right) + C \\
&= \frac{2}{3}\sqrt{x-1}(x+2) + C
\end{aligned}$$

Note: In (b) and (c), substitution was necessary *before* the basic integration rules could be used.

50. (a) $\int \frac{1}{1+x^4}\,dx$ cannot be evaluated using the basic integration rules.

(b) $$\begin{aligned}\int \frac{x}{1+x^4}\,dx &= \frac{1}{2}\int \frac{2x}{1+\left(x^2\right)^2}\,dx \\ &= \frac{1}{2}\arctan x^2 + C, \quad u = x^2\end{aligned}$$

(c) $$\begin{aligned}\int \frac{x^3}{1+x^4}\,dx &= \frac{1}{4}\int \frac{4x^3}{1+x^4}\,dx \\ &= \frac{1}{4}\ln\left(1+x^4\right) + C, \quad u = 1 + x^4\end{aligned}$$

51. The integrals should be subtracted in the first step.

$$\begin{aligned}
\int \frac{x-5}{\sqrt{1-x^2}}\,dx &= \int \frac{x}{\sqrt{1-x^2}}\,dx - \int \frac{5}{\sqrt{1-x^2}}\,dx \\
&= -\frac{1}{2}\int \left(1-x^2\right)^{-1/2}(-2x)\,dx - 5\int \frac{1}{\sqrt{1-x^2}}\,dx \\
&= -\frac{1}{2}\left[\frac{\left(1-x^2\right)^{1/2}}{1/2}\right] - 5\arcsin x + C \\
&= -\sqrt{1-x^2} - 5\arcsin x + C
\end{aligned}$$

52. The expression $x^2 + 2x + 3 = \left(x^2 + 2x + 1\right) - 1 + 3 = (x+1)^2 + 2$, so $a = \sqrt{2}$.

$$\begin{aligned}
\int_{-1}^{3} \frac{dx}{x^2 + 2x + 3} = \int_{-1}^{3} \frac{dx}{(x+1)^2 + 2} &= \left[6\frac{1}{\sqrt{2}}\arctan\frac{x+1}{\sqrt{2}}\right]_{-1}^{3} \\
&= \frac{\sqrt{2}}{2}\arctan 2\sqrt{2}
\end{aligned}$$

62. The area is approximately the area of a square of side 1. So, (c) best approximates the area.

63. $F(x) = \frac{1}{2}\int_x^{x+2} \frac{2}{t^2+1}\,dt$

(a) $F(x)$ represents the average value of $f(x)$ over the interval $[x, x+2]$. Maximum at $x = -1$, because the graph is greatest on $[-1, 1]$.

(b) $F(x) = \left[\arctan t\right]_x^{x+2} = \arctan(x+2) - \arctan x$

$$F'(x) = \frac{1}{1+(x+2)^2} - \frac{1}{1+x^2} = \frac{(1+x^2)-(x^2+4x+5)}{(x^2+1)(x^2+4x+5)} = \frac{-4(x+1)}{(x^2+1)(x^2+4x+5)} = 0 \text{ when } x = -1.$$

64. $\int \frac{1}{\sqrt{6x-x^2}}\,dx$

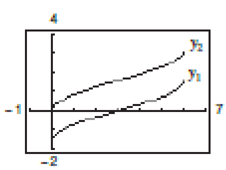
(a) $6x - x^2 = 9 - (x^2 - 6x + 9) = 9 - (x-3)^2$

$$\int \frac{1}{\sqrt{6x-x^2}}\,dx = \int \frac{dx}{\sqrt{9-(x-3)^2}} = \arcsin\frac{x-3}{3} + C$$

(b) $u = \sqrt{x}, u^2 = x, 2u\,du = dx$

$$\int \frac{1}{\sqrt{6u^2-u^4}}(2u\,du) = \int \frac{2}{\sqrt{6-u^2}}\,du = 2\arcsin\frac{u}{\sqrt{6}} + C = 2\arcsin\frac{\sqrt{x}}{\sqrt{6}} + C$$

(c)



The antiderivatives differ by a constant, $\pi/2$.

Domain: $[0, 6]$

65. False. $\int \frac{dx}{3x\sqrt{9x^2-16}} = \frac{1}{12}\operatorname{arcsec}\frac{|3x|}{4} + C$

66. False. $\int \frac{dx}{25+x^2}\,dx = \frac{1}{5}\arctan\frac{x}{5} + C$

67. True.

$$\frac{d}{dx}\left[-\arccos\frac{x}{2} + C\right] = \frac{1/2}{\sqrt{1-(x/2)^2}} = \frac{1}{\sqrt{4-x^2}}$$

68. False. Use substitution: $u = 9 - e^{2x}, du = -2e^{2x}dx$

69.
$$\frac{d}{dx}\left[\arcsin\frac{u}{a} + C\right] = \frac{1}{\sqrt{1-(u^2/a^2)}}\left(\frac{u'}{a}\right) = \frac{u'}{\sqrt{a^2-u^2}}$$

So, $\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a} + C.$

70.
$$\frac{d}{dx}\left[\frac{1}{a}\arctan\frac{u}{a} + C\right] = \frac{1}{a}\left[\frac{u'/a}{1+(u/a)^2}\right]$$
$$= \frac{1}{a^2}\left[\frac{u'}{(a^2+u^2)/a^2}\right] = \frac{u'}{a^2+u^2}$$

So, $\int \frac{du}{a^2+u^2} = \int \frac{u'}{a^2+u^2}\,dx = \frac{1}{a}\arctan\frac{u}{a} + C.$

71. Assume $u > 0$.

$$\frac{d}{dx}\left[\frac{1}{a}\operatorname{arcsec}\frac{u}{a} + C\right] = \frac{1}{a}\left[\frac{u'/a}{(u/a)\sqrt{(u/a)^2 - 1}}\right] = \frac{1}{a}\left[\frac{u'}{u\sqrt{(u^2 - a^2)/a^2}}\right] = \frac{u'}{u\sqrt{u^2 - a^2}}.$$

The case $u < 0$ is handled in a similar manner.

So, $\displaystyle\int \frac{du}{u\sqrt{u^2 - a^2}} = \int \frac{u'}{u\sqrt{u^2 - a^2}}\,dx = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a} + C.$

72. (a) Area $= \displaystyle\int_0^1 \frac{1}{1 + x^2}\,dx$

(b) Trapezoidal Rule: $n = 8, b - a = 1 - 0 = 1$

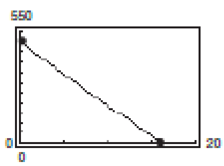
Area ≈ 0.7847

(c) Because

$$\int_0^1 \frac{1}{1 + x^2}\,dx = \left[\arctan x\right]_0^1 = \frac{\pi}{4},$$

you can use the Trapezoidal Rule to approximate $\pi/4$, and therefore, π. For example, using $n = 200$, you obtain $\pi \approx 4(0.785397) = 3.141588$.

73. (a) $v(t) = -32t + 500$



(b) $s(t) = \int v(t)\,dt = \int(-32t + 500)\,dt$

$= -16t^2 + 500t + C$

$s(0) = -16(0) + 500(0) + C = 0 \Rightarrow C = 0$

$s(t) = -16t^2 + 500t$

When the object reaches its maximum height, $v(t) = 0$.

$$\begin{aligned} v(t) &= -32t + 500 = 0 \\ -32t &= -500 \\ t &= 15.625 \end{aligned}$$

$$\begin{aligned} s(15.625) &= -16(15.625)^2 + 500(15.625) \\ &= 3906.25 \text{ ft (Maximum height)} \end{aligned}$$

(c) $$\int \frac{1}{32 + kv^2}\,dv = -\int dt$$

$$\frac{1}{\sqrt{32k}} \arctan\left(\sqrt{\frac{k}{32}}v\right) = -t + C_1$$

$$\arctan\left(\sqrt{\frac{k}{32}}v\right) = -\sqrt{32k}t + C$$

$$\sqrt{\frac{k}{32}}v = \tan\left(C - \sqrt{32k}t\right)$$

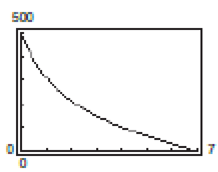
$$v = \sqrt{\frac{32}{k}}\tan\left(C - \sqrt{32k}t\right)$$

When $t = 0$, $v = 500$, $C = \arctan\left(500\sqrt{k/32}\right)$, and you have

$$v(t) = \sqrt{\frac{32}{k}}\tan\left[\arctan\left(500\sqrt{\frac{k}{32}}\right) - \sqrt{32k}t\right].$$

(d) When $k = 0.001$:

$$v(t) = \sqrt{32{,}000}\ \tan\left[\arctan\left(500\sqrt{0.00003125}\right) - \sqrt{0.032}t\right]$$



$v(t) = 0$ when $t_0 \approx 6.86$ sec.

(e) $$h = \int_0^{6.86} \sqrt{32{,}000}\ \tan\left[\arctan\left(500\sqrt{0.00003125}\right) - \sqrt{0.032}\ t\right] dt$$

Graphing utility: $n = 10$; $h \approx 1088$ ft

(f) Air resistance lowers the maximum height.

74. Evaluate each integral.

I: $$\int_{5/2}^{5} \frac{2}{\sqrt{5x - x^2}}\,dx = 2\left[\arcsin\frac{\sqrt{5x}}{x}\right]_{5/2}^{5} = 2\left(\frac{\pi}{2}\right) = \pi$$

II: $$\int_{\pi}^{2\pi} \frac{\pi}{2}\sin\frac{x}{2}\,dx = \pi\int_{\pi}^{2\pi}\sin\frac{x}{2}\left(\frac{1}{2}\right)dx = \pi\left[-\cos\frac{x}{2}\right]_{\pi}^{2\pi} = \pi(1 - 0) = \pi$$

III: $$\int_3^8 \frac{\pi}{2\sqrt{x+1}}\,dx = \frac{\pi}{2}\int_3^8 (x+1)^{-1/2}\,dx = \frac{\pi}{2}\left[2(x+1)^{1/2}\right]_3^8 = \pi(3 - 2) = \pi$$

So, the answer is D.

75. $u = x$, $du = dx$, $a = 4$

$$\int \frac{4}{\sqrt{16 - x^2}}\,dx = 4\int \frac{1}{\sqrt{a^2 - u^2}}\,du = 4\left[\arcsin\frac{u}{a} + C\right] = 4\arcsin\frac{x}{4} + C$$

So, the answer is B.

76. Let $f(x) = \arccos x$ and $g(x) = x^2$.

$$\begin{aligned} h(x) &= f(g(x)) \\ &= f(x^2) \\ &= \arccos x^2 \end{aligned}$$

(a) $h(x) = \arccos x^2$

$$h'(x) = \frac{-2x}{\sqrt{1 - x^2}}$$

When $h'(x) = 0$, $x = 0$. So, the graph of $h(x)$ has a relative maximum at $x = 0$.

(b) $\int_1^{\pi/3} \arccos x^2 \, dx$

(c) Because $f(x) = \arccos x$, $f^{-1}(x) = \cos x$.

$$\begin{aligned} \frac{d}{dx}\left[f^{-1}(x)\right] &= \frac{d}{dx}[\cos x] \\ &= -\sin x \end{aligned}$$

So, $\frac{d}{dx}\left[f^{-1}\left(\frac{\pi}{3}\right)\right] = -\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2}$.

77. (a) Yes; Because $\lim_{x \to 0} g(x) = \frac{1}{2}$ and $g(0) = \frac{1}{2}$, g is continuous at $x = 0$.

(b)
$$\begin{aligned} \int_0^1 f(x) &= \int_0^1 \frac{1}{\sqrt{4 - x^2}}\,dx \\ &= \left[\arcsin\frac{x}{2}\right]_0^1 \\ &= \frac{\pi}{6} - 0 = \frac{\pi}{6} \end{aligned}$$

(c)
$$\begin{aligned} \int_{-1}^1 g(x) &= \int_{-1}^0 \frac{1}{\sqrt{4 - x^2}}\,dx + \int_0^1 \left(x + \frac{1}{2}\right)dx \\ &= \left[\arcsin\frac{x}{2}\right]_{-1}^0 + \left[\frac{1}{2}x^2 + \frac{1}{2}x\right]_0^1 \\ &= 0 - \left(-\frac{\pi}{6}\right) + 1 - 0 \\ &= \frac{\pi + 6}{6} \end{aligned}$$