1.4 Continuity and One-Sided Limits

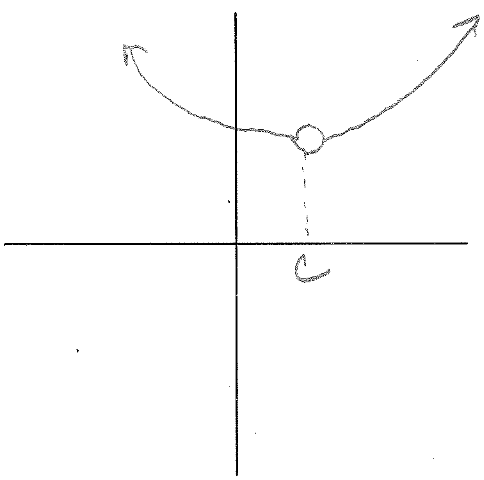
Definition of Continuity:

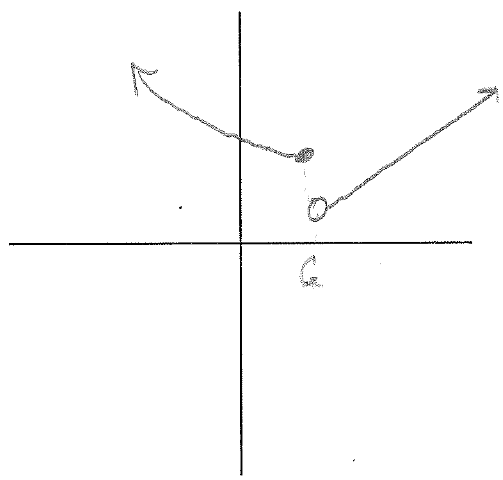
A function f is continuous at c when three conditions are met:

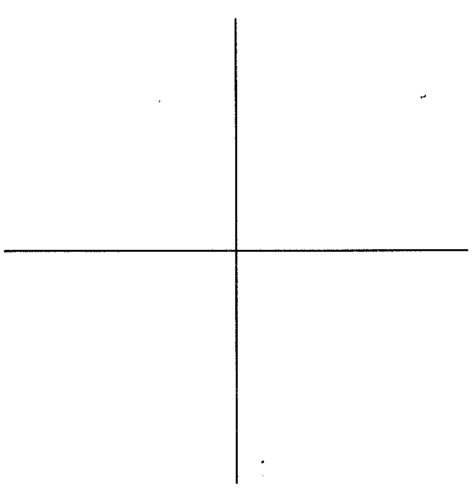
1. $f(c)$ is defined
2. $\lim_{x \to c} f(x)$ exists
3. $f(c) = \lim_{x \to c} f(x)$

☆ Must meet all three!

Sketch an example of each, where it would NOT be continuous at c.







Removable Discontinuity: f can be made continuous at c by appropriately redefining $f(c)$.

$\lim_{x \to c} f(x)$ exists

Non-removable Discontinuity: f cannot be made continuous

$\lim_{x \to c} f(x)$ does not exist

Examples: Determining Continuity

Discuss the continuity of each function. Identify any discontinuities as removable or non-removable

$$f(x) = \frac{2x+1}{x+1}$$

$x = -1$

Continuous $(-\infty, -1) \cup (-1, \infty)$

$x = -1$ non removable, $\lim_{x \to -1} f(x)$ DNE

$f(-1)$ DNE

$$f(x) = \frac{4}{x^2+1}$$

Continuous $(-\infty, \infty)$

$$f(x) = \frac{x^2+5x+6}{x+3} = \frac{\cancel{(x+3)}(x+2)}{\cancel{x+3}}$$

Continuous $(-\infty, -3) \cup (-3, \infty)$

Discontinuous at $x = -3$ Removable

$f(-3)$ DNE

$$g(x) = \begin{cases} 3x-5, & x < 1 \\ -2x^2, & x \geq 1 \end{cases}$$

$\lim_{x \to 1^-} f(x) = -2$

$\lim_{x \to 1^+} f(x) = -2$

Continuous $(-\infty, \infty)$

One – Sided Limit

Limit from the right: $\lim_{x \to c^+} f(x) = L$

Limit from the left: $\lim_{x \to c^-} f(x) = L$

Examples: One-Sided Limits

$\lim_{x\to 0^-} \frac{|x|}{4x} = -\frac{1}{4}$

$\lim_{x\to 0^+} \frac{|x|}{4x} = \frac{1}{4}$

$\begin{cases} \frac{x}{4x} & x > 0 \\ \frac{-x}{4x} & x < 0 \end{cases}$

$\lim_{x\to 3^-} [x - 2] = 0$ Step function Greatest Integer

$\lim_{x\to 3^+} [x - 2] = 1$

Continuity on a Closed Interval

A function f is continuous on the closed interval $[a, b]$ when f is continuous on (a, b) and

$\lim_{x\to a^+} f(x) = f(a)$ and $\lim_{x\to b^-} f(x) = f(b)$

Properties of Continuity

If b is a real number and f and g are continuous at $x = c$, then the functions listed below are also continuous at c.

1. Scalar multiple $b \cdot f$
2. Sum or Difference $f \pm g$
3. Product $f \cdot g$
4. Quotient $\frac{f}{g}$ $g(c) \neq 0$

Continuous Functions

The following functions are continuous at every point on their domain:

1. Polynomial

2. Rational

3. Radical

4. Trigonometric

5. Exponential & Logarithmic

Continuity of Composite Functions

If g is continuous at c and f is continuous at $g(c)$, then the composite function given by

$(f \circ g)(x)$ OR $f(g(x))$ is continuous at c.

Examples: Determining Continuity

Determine the continuity of $f(x) = tan\frac{\pi x}{6}$.

$$\frac{\sin\frac{\pi x}{6}}{\cos\frac{\pi x}{6}}$$

$\cos\frac{\pi x}{6} = 0$

$\frac{\pi x}{6} = \frac{\pi}{2} + \pi n$

$x = \left(\frac{\pi}{2} + \pi n\right)\frac{6}{\pi}$

$x = 3 + 6n$

Continuous: $(-9,-3) \cup (-3,3) \cup (3,9) \ldots$

Determine the continuity of $g(x) = cosx^2$.

Continuous $(-\infty, \infty)$

Intermediate Value Theorem

If f is continuous on $[a, b]$, $f(a) \neq f(b)$, and k is any number between $f(a)$ and $f(b)$, then there is at least one number c in $[a, b]$ such that $f(c) = k$.

Now, restate it in your own words....what conditions have to be met and what does it tell you?

Examples: Applying the Intermediate Value Theorem

Determine if $f(x) = -x^5 + 3x^2 - 2x + 2$ has a zero on the interval [1,2].

f(x) is continuous [1,2]

$f(1) = -1 + 3 - 2 + 2 = 2$

$f(2) = -32 + 12 - 4 + 2 = -22$

$-22 < 0 < 2$

there exists a c such that $1 < c < 2$ and $f(c) = 0$

What other values are guaranteed?

any value between -22 and 2

Use your graphing calculator to find the zero. $x \approx 1.375$ or 1.376

Use your graphing calculator to find one of the other values that is guaranteed.

$\underset{y_1}{-x^5 + 3x^2 - 2x + 2} = \underset{y_2}{-5}$

Calc

Intersect