4.1 Antiderivatives and Indefinite Integrals, Day 2

Solving Differential Equations

Find the particular solution to the differential equation

$$f''(x) = 2, f'(2) = 5, f(2) = 10$$

$f'(x) = \int 2\,dx$

$f'(x) = 2x + C$

$5 = 2(2) + C$

$1 = C$

$f'(x) = 2x + 1$

$f(x) = \int 2x + 1\,dx$

$f(x) = 2\left(\frac{x^2}{2}\right) + x + C$

$f(x) = x^2 + x + C$

$10 = 4 + 2 + C$

$4 = C$

$\boxed{f(x) = x^2 + x + 4}$

A baseball is thrown upward from ground level with a velocity of 10 meters per second. Determine its maximum height using calculus. (Use $a(t) = -9.8\ m/sec^2$)

$a(t) = -9.8$

$v(t) = \int -9.8\,dt$

$v(t) = -9.8t + C$

$10 = -9.8(0) + C$

$10 = C$

$v(t) = -9.8t + 10$

$s(t) = \int -9.8t + 10\,dt$

$s(t) = -9.8\left(\frac{t^2}{2}\right) + 10t + C$

$s(t) = -4.9t^2 + 10t + C$

$s(t) = -4.9t^2 + 10t$

$-9.8t + 10 = 0$

$-9.8t = -10$

$t = \frac{-10}{-9.8}$

$s\left(\frac{10}{9.8}\right) = -4.9\left(\frac{10}{9.8}\right)^2 + 10\left(\frac{10}{9.8}\right)$

≈ 5.102 meters

Examples – Solving differential equations and applications

Find the general solution of $F'(x) = 9x^2 - x$.

$F(x) = \int 9x^2 - x\,dx$

$F(x) = 9\left(\frac{x^3}{3}\right) - \frac{1}{2}x^2 + C$

$F(x) = 3x^3 - \frac{1}{2}x^2 + C$

Find the particular solution for the equation above given the initial condition $F(2) = 16$.

$16 = 3(8) - \frac{1}{2}(4) + C$

$16 = 24 - 2 + C$

$16 = 22 + C$

$-6 = C$

$F(x) = 3x^3 - \frac{1}{2}x^2 - 6$

Find the particular solution for $f''(x) = sinx, f'(0) = 1, f(0) = 6$

$F'(x) = \int \sin x \, dx$

$F'(x) = -\cos x + C$

$1 = -\cos(0) + C$

$1 = -1 + C$

$2 = C$

$F'(x) = -\cos x + 2$

$F(x) = \int -\cos x + 2 \, dx$

$F(x) = -(\sin x) + 2x + C$

$F(x) = -\sin x + 2x + C$

$6 = -\sin 0 + 2(0) + C$

$6 = C$

$F(x) = -\sin x + 2x + 6$

A car traveling at 45 miles per hour is brought to a stop, at constant deceleration, 132 feet from where the brakes are applied. How far has the car moved when its speed has been reduced to 30 miles per hour?

acceleration = a

$\frac{45 \text{ mile}}{\text{hour}} \cdot \frac{1 \text{ hour}}{3600 \text{ sec}} \cdot \frac{5280 \text{ ft}}{1 \text{ mi}} = 66 \text{ ft/sec}$

$v(t) = \int -a \, dt$

$v(t) = -at + C$

$66 = -a(0) + C$

$66 = C$

$v(t) = -at + 66$

$0 = -at + 66$

$-66 = -at$

$\frac{66}{a} = t$

$s(t) = \int -at + 66 \, dt$

$s(t) = -a\left(\frac{t^2}{2}\right) + 66t + C$

$s(t) = -\frac{1}{2}at^2 + 66t$

$132 = -\frac{1}{2}a\left(\frac{66}{a}\right)^2 + 66\left(\frac{66}{a}\right)$

$132 = \left(-\frac{1}{2}\right)\left(\frac{66^2}{a}\right) + \frac{66^2}{a}$

$132 = \frac{-2178}{a} + \frac{66^2}{a}$

$132 = \frac{2178}{a}$

$a = 16.5$

$v(t) = -16.5t + 66$

$s(t) = -8.25t^2 + 66t$

$\frac{30 \text{ mi}}{\text{hour}} \cdot \frac{1 \text{ hour}}{3600 \text{ sec}} \cdot \frac{5280 \text{ ft.}}{1 \text{ mi}} = 44$

$44 = -16.5t + 66$

$-22 = -16.5t$

$\frac{4}{3} = t$

$s\left(\frac{4}{3}\right) = -8.25\left(\frac{4}{3}\right)^2 + 66\left(\frac{4}{3}\right)$

≈ 73.3 feet