Differentiate $y = e^{\frac{x}{4}}$

$$y' = e^{\frac{x}{4}} \cdot \frac{1}{4}$$

$$= \frac{e^{\frac{x}{4}}}{4}$$

Find $f'(x)$ if $f(x) = cscx^2$

$$f'(x) = (-cscx^2 cotx^2)(2x)$$

$$= -2x\, cscx^2 cotx^2$$

$\frac{d}{dx}[csc^2x]$ $= (2\csc x)(-\csc x \cot x)$

$$= -2\csc^2 x \cot x$$

Find the derivative of $f(t) = cos^2t^3$ $= (\cos t^3)^2$

$$f'(t) = 2(\cos t^3)(-\sin t^3)(3t^2)$$

$$= -6t^2 \cos t^3 \sin t^3$$

Derivative of the Natural Logarithmic Function

Let u be a differentiable function of x.

$$\frac{d}{dx}[lnx] = \frac{1}{x}$$

$$\frac{d}{dx}[lnu] = \frac{1}{u} \cdot u'$$

Examples: Differentiating the Natural Logarithm Function

Find y' if $y = ln(2x^3)$

$$y' = \frac{1}{2x^3} \cdot 6x^2 = \frac{3}{x}$$

Find $\frac{dy}{dx}$ if $y = x^2 lnx$

$$\frac{dy}{dx} = x^2 \cdot \frac{1}{x} + 2x\ln x = x + 2x\ln x$$

$$= x(1 + 2\ln x)$$

Differentiate $y = [\ln(x+5)]^2$

$$y' = 2[\ln(x+5)] \cdot \frac{1}{x+5} \cdot 1 = \frac{2\ln(x+5)}{x+5}$$

Differentiate $f(x) = ln(x-3)^2$

$$f'(x) = \frac{1}{(x-3)^2} \cdot 2(x-3) \cdot 1$$

$$= \frac{2}{x-3}$$

$$f(x) = 2\ln(x-3)$$

$$f'(x) = 2 \cdot \frac{1}{x-3} \cdot 1$$

$$= \frac{2}{x-3}$$

Find $f'(x)$ if $f(x) = ln\frac{x^3(x-3)}{\sqrt{x^2-9}}$ $= 3\ln x + \ln(x-3) - \frac{1}{2}\ln(x^2-9)$

$$f'(x) = \frac{3}{x} + \frac{1}{x-3} - \frac{x}{x^2-9}$$

Definition of Exponential Function to Base a

If a is a positive real number ($a \neq 1$) and x is any real number then:

$$a^x = e^{(\ln a)x}$$

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Definition of Logarithmic Function to Base a

If a is a positive real number ($a \neq 1$) and x is any positive real number then:

$$\log_a x = \frac{1}{\ln a} \cdot \ln x = \frac{\ln x}{\ln a}$$

Derivatives for Bases other than e

$$\frac{d}{dx}[a^x] = \frac{d}{dx}\left[e^{(\ln a)x}\right] = e^{(\ln a)x} \cdot \ln a = a^x \cdot \ln a$$

$$\therefore \frac{d}{dx}[a^x] = (\ln a)a^x$$

$$\therefore \frac{d}{dx}[a^u] = (\ln a) \cdot a^u \cdot u'$$

$$\frac{d}{dx}[\log_a x] = \frac{d}{dx}\left[\frac{1}{\ln a} \cdot \ln x\right] = \frac{1}{\ln a} \cdot \frac{1}{x}$$

$$\therefore \frac{d}{dx}[\log_a x] = \frac{1}{\ln a} \cdot \frac{1}{x}$$

$$\therefore \frac{d}{dx}[\log_a u] = \frac{1}{\ln a} \cdot \frac{1}{u} \cdot u'$$

Examples: Differentiating Functions with Bases Other than *e*

Find the derivative of each function

$y = 7^x$

$$y' = (\ln 7) \cdot 7^x$$

$y = 7^{\frac{x}{2}}$

$$y' = (\ln 7)\left(7^{\frac{x}{2}}\right)\left(\frac{1}{2}\right)$$

$y = \log_4 cscx$

$$y' = \frac{1}{\ln 4} \cdot \frac{1}{\csc x} \cdot -\csc x \cot x = \frac{-\cot x}{\ln 4}$$

$y = \log_5 \frac{x+3}{(2x-1)^2}$

$$= \log_5 (x+3) - 2\log_5 (2x-1)$$

$$= \left(\frac{1}{\ln 5}\right)\left(\frac{1}{x+3}\right) - \left(\frac{2}{\ln 5}\right)\left(\frac{1}{2x-1}\right)(2)$$

$$= \frac{1}{\ln 5}\left(\frac{1}{x+3} - \frac{4}{2x-1}\right)$$

$$= \frac{1}{\ln 5}\left[\frac{2x-1-4(x+3)}{(x+3)(2x-1)}\right]$$

$$= \frac{1}{\ln 5}\left[\frac{-2x-13}{(x+3)(2x-1)}\right]$$