7.7 Indeterminate Forms and L'Hopital's Rule

Indeterminate Forms

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{0}{0} = \frac{\infty}{\infty} = \frac{}{} = \frac{}{} = \frac{}{}$$

L'Hopital's Rule

Let f and g be functions that are differentiable on an open interval (a, b) containing c except possibly at c itself. Assume that $g'(x) \neq 0$ for all x in (a, b), except possibly at c itself. If the limit as x approaches c produces an indeterminate form then $\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)}$.

Examples:

$$\lim_{x \to (-1)} \frac{2x^2-2}{x+1} \quad \frac{0}{0}$$

$$\lim_{x \to (-1)} \frac{4x}{1} = -4$$

$$\lim_{x \to \infty} \frac{3x^2-1}{2x^2+1} \quad \frac{\infty}{\infty}$$

$$\lim_{x \to \infty} \frac{6x}{4x} \quad \frac{\infty}{\infty}$$

$$\lim_{x \to \infty} \frac{6}{4} = \frac{3}{2}$$

$$\lim_{x \to 0} \frac{e^{2x}-1}{e^x-1} \quad \frac{0}{0}$$

$$\lim_{x \to 0} \frac{2e^{2x}}{e^x} = \lim_{x \to 0} 2e^x = 2$$

$$\lim_{x \to 0} \frac{e^{2x}-1}{x} \quad \frac{0}{0}$$

$$\lim_{x \to 0} \frac{2e^{2x}}{1} = 2$$

$$\lim_{x \to (-1)} \frac{2x^2-x-3}{x+1} \quad \frac{0}{0}$$

$$\lim_{x \to (-1)} = \frac{4x-1}{1} = -5$$

$$\lim_{x \to \infty} \frac{2x+1}{4x^2+x} \quad \frac{\infty}{\infty}$$

$$\lim_{x \to \infty} \frac{2}{8x+1} = 0$$

$$\lim_{x\to 0^+} \frac{e^x-(1+x)}{x^3} \quad \frac{0}{0}$$

$$\lim_{x\to 0^+} \frac{e^x-1}{3x^2} \quad \frac{0}{0}$$

$$\lim_{x\to 0^+} \frac{e^x}{6x} \quad \frac{1}{0}$$

$$+\infty$$

$$\lim_{x\to\infty} \frac{\ln x}{x} \quad \frac{\infty}{\infty}$$

$$\lim_{x\to\infty} \frac{\frac{1}{x}}{1} = \lim_{x\to\infty} \frac{1}{x} = 0$$

$$\lim_{x\to(-\infty)} \frac{x^2}{e^{-x}} \quad \frac{\infty}{\infty}$$

$$\lim_{x\to(-\infty)} \frac{2x}{-e^{-x}} \quad \frac{\infty}{\infty}$$

$$\lim_{x\to(-\infty)} \frac{2}{e^{-x}} = 0$$

$$\lim_{x\to 0} \frac{\sin 5x}{2x} \quad \frac{0}{0}$$

$$\lim_{x\to 0} \frac{5\cos 5x}{2} \quad \frac{5}{2}$$

$$\lim_{x\to\infty} \frac{x(\ln x-1)}{e^x} \quad \frac{\infty}{\infty}$$

$$\lim_{x\to\infty} \frac{x\left(\frac{1}{x}\right)+(1)(\ln x)}{e^x} = \frac{1+\ln x}{e^x} \quad \frac{\infty}{\infty}$$

$$\lim_{x\to\infty} \frac{\frac{1}{x}}{e^x} = \frac{1}{xe^x} \quad 0$$

$$\lim_{x\to\infty} xe^{-\frac{x}{2}} = \lim_{x\to\infty} \frac{x}{e^{\frac{x}{2}}} \quad \frac{\infty}{\infty}$$

$$\lim_{x\to\infty} \frac{1}{\frac{1}{2}e^{\frac{x}{2}}} \quad 0$$

$$\lim_{x\to 1^-} \left(1-\frac{1}{x}\right)^{1-x} \quad 0^0$$

I'm leaving this one for you to try! See Example 6 in the book to help you! You can come see me for a solution.

Methods for Finding Limits

(1) Direct Substitution
(2) Factor
(3) Rationalize
(4) Trig
(5) Special Limits
(6) L'Hopital
(7) Squeeze Theorem