5.1 Slope Fields

Recall that integration can be used to solve differential equations:

If $y' = f'(x)$ then $\frac{dy}{dx} = f'(x) \therefore dy = f'(x)dx$

$$\int dy = \int f'(x)dx$$

$$y = f(x) + C$$

Find the general solution for $\frac{dy}{dx} = 10x^4 - 2x^3$.

$\int dy = \int 10x^4 - 2x^3 dx$

$y = 10\left(\frac{x^5}{5}\right) - 2\left(\frac{x^4}{4}\right) + C$

$y = 2x^5 - \frac{1}{2}x^4 + C$

Find the general solution for $\frac{dy}{dx} = tan^2x$

$\int dy = \int tan^2x\ dx$

$y = \int sec^2x - 1\ dx$

$y = tanx - x + C$

Determine if the following ordered pairs are solutions to the equation. Write out the work that leads to your conclusions.

$$3x - 2y = 12$$

(1,4) $3(1) - 2(4) = 12$

$3 - 8 = 12$ no

(6,3) $3(6) - 2(3) = 12$

$18 - 6 = 12$ Yes

$(0, -6)$ $3(0) - 2(-6) = 12$ Yes

Solution of a Differential Equation

A function $y = f(x)$ is called a solution of a differential equation if the equation is satisfied when y and its derivative are replaced by $f(x)$ and its derivatives.

Examples – Determining Solutions to a Differential Equations

Determine if $y = e^{-2x}$ is a solution to the differential equation $y' + 2y = 0$.

$y' = -2e^{-2x}$

$-2e^{-2x} + 2(e^{-2x}) = 0$

$-2e^{-2x} + 2e^{-2x} = 0$ Yes it is a solution

Is $y = e^{2x}$ a solution to $y' + 2y = 0$?

$y' = 2e^{2x}$

$2e^{2x} + 2(e^{2x}) = 0$

$2e^{2x} + 2e^{2x} = 0$ no it is not a solution

Determine whether $y = x^2e^x - 5x^2$ is a solution to the equation $xy' - 2y = x^3e^x$.

$y' = x^2e^x + 2xe^x - 10x$

$x(x^2e^x + 2xe^x - 10x) - 2(x^2e^x - 5x^2) = x^3e^x$

$x^3e^x + \cancel{2x^2e^x} - \cancel{10x^2} - \cancel{2x^2e^x} + \cancel{10x^2} = x^3e^x$

Yes it is a solution.

A general solution of a differential equation represents a family of curves that could represent a solution to the differential equation. The <u>order</u> of a differential equation is determined by the highest-order derivative in the equation.

Determine if $y = \frac{C}{x}$ is a general solution of $xy' + y = 0$.

$y = Cx^{-1}$

$y' = -Cx^{-2}$

$x(-Cx^{-2}) + Cx^{-1} = 0$

$-Cx^{-1} + Cx^{-1} = 0$ Yes it is a solution.

Determine if $y = Csinx$ is a solution to the equation $y'' + y = 0$.

$y' = C(\cos x)$

$y'' = C(-\sin x)$

$-C\sin x + C\sin x = 0$

Yes it is a solution.

Particular solutions of a differential equation can be obtained when initial conditions give the values of dependent variables or one of its derivatives for particular values of the independent variables.

Verify that $y = C_1x + C_2x^3$ is a solution to $x^2y'' - 3xy' + 3y = 0$.

$y' = C_1 + 3C_2x^2$

$y'' = 6C_2x$

$x^2(6C_2x) - 3x(C_1 + 3C_2x^2) + 3(C_1x + C_2x^3) = 0$

$6C_2x^3 - 3C_1x - 9C_2x^3 + 3C_1x + 3C_2x^3 = 0$

Yes it is a solution

For the problem above, find the particular solution if $y = 0$ when $x = 2$ and $y' = 4$ when $x = 2$.

$0 = C_1 \cdot (2) + C_2(8)$

$4 = C_1 + 3C_2(4)$

$0 = 2C_1 + 8C_2$

$4 = C_1 + 12C_2$

$0 = 2C_1 + 8C_2$

$-8 = -2C_1 - 24C_2$

$-8 = -16C_2$

$\frac{1}{2} = C_2$

$0 = 2C_1 + 8(\frac{1}{2})$

$0 = 2C_1 + 4$

$-4 = 2C_1$

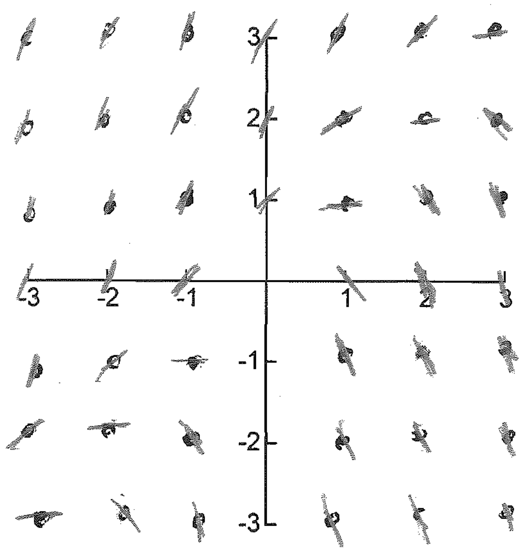
$-2 = C_1$

$\boxed{y = -2x + \frac{1}{2}x^3}$

Slope Fields are a graphical representation of the solutions to a differential equation. A slope field is created by sketching short line segments that represent the slope at a point (x, y). These short line segments represent the slope of the solution curve through that point.

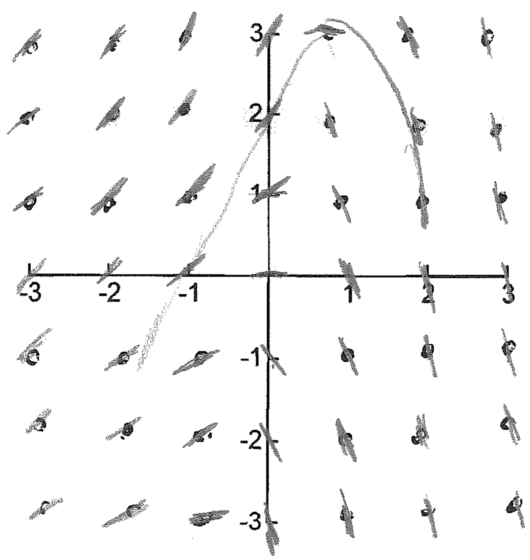
Examples – Slope Fields

Sketch the slope field for $y' = y - x$.



(x,y)	y'
(1,1)	1-1=0
(1,2)	2-1=1
(1,3)	3-1=2
(1,0)	0-1=-1
(1,-1)	-1-1=-2
(1,-2)	-2-1=-3
(2,0)	0-2=-2
(2,1)	1-2=-1

Sketch the slope field for the differential equation $y' = y - 3x$.



(x,y)	y'
(1,1)	1-3=-2
(1,2)	1-6=-5
(1,0)	0-3=-3
(1,-1)	-1-3=-4
(1,-2)	-2-3=-5
(1,-3)	-3-3=-6
(0,1)	1-0=
(0,2)	2-0=2
(0,-1)	-1-0
(2,3)	3-6=-3
(2,2)	2-6=-4
(2,1)	1-6=-5
(2,0)	0-6=-6
(2,-1)	-1-6=-7

(x,y)	y'
(2,-2)	-2-6
(3,2)	2-9=-7
(-1,3)	3+3=6
(-1,2)	2+3=5
(-1,1)	1+3=4
(-1,0)	0+3

Use the slope field to sketch the solution that passes through the point (2,1).