

## Logarithmic Differentiation

1. Take the natural logarithm of both sides of the equation.  
\* Be careful of domain.
2. Use the properties of logarithms
3. Differentiate both sides
4. Solve for  $\frac{dy}{dx}$

### Examples: Logarithmic Differentiation

Find the derivative of  $y = \frac{(3x+5)^2}{(x^2-1)^3}$   $x \neq -\frac{5}{3}$

$$\textcircled{1} \ln y = \ln \frac{(3x+5)^2}{(x^2-1)^3}$$

$$\textcircled{2} \ln y = 2 \ln(3x+5) - 3 \ln(x^2-1)$$

$$\textcircled{3} \frac{1}{y} \cdot \frac{dy}{dx} = 2 \left( \frac{1}{3x+5} \right) (3) - 3 \left( \frac{1}{x^2-1} \right) (2x)$$

$$\textcircled{4} \frac{1}{y} \cdot \frac{dy}{dx} = \frac{6}{3x+5} - \frac{6x}{x^2-1}$$

$$\textcircled{4} \frac{dy}{dx} = \left[ \frac{6}{3x+5} - \frac{6x}{x^2-1} \right] y$$

$$\frac{dy}{dx} = \left[ \frac{6}{3x+5} - \frac{6x}{x^2-1} \right] \frac{(3x+5)^2}{(x^2-1)^3}$$

Find the derivative of  $y = x^{\frac{1}{4}x}$

$$\ln y = \ln x^{\frac{1}{4}x}$$

$$\ln y = \frac{1}{4}x \ln x$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \left( \frac{1}{4}x \right) \cdot \frac{1}{x} + \left( \frac{1}{4} \right) \ln x$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{1}{4} + \frac{1}{4} \ln x$$

$$\frac{dy}{dx} = \frac{1}{4} (1 + \ln x) x^{\frac{1}{4}x}$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{1}{4} x \ln x$$