2.2 Basic Differentiation and Rates of Change

Constant Rule	$\frac{d}{dx}[c] = 0$
Power Rule	$\frac{d}{dx}[x^n] = nx^{n-1}$
Constant Multiple Rule	$\frac{d}{dx}[cf(x)] = cf'(x)$
Sum and Difference Rules	$\frac{d}{dx}[f(x)+g(x)] = f'(x)+g'(x)$
Derivative of Sine	$\frac{d}{dx}[\sin x] = \cos x$
Derivative of Cosine	$\frac{d}{dx}[\cos x] = -\sin x$
Derivative of Natural Exponential	$\frac{d}{dx}[e^x] = e^x$

Examples: Using Derivative Rules ☆ - Do together

☆ $\frac{d}{dx}\left[\frac{3}{7}\right] = 0$

☆ If $f(x) = x^5, f'(x) =$ $5x^4$

☆ If $f(x) = 2x^7, f'(x) =$ $14x^6$

$f'(x) = (2)(7x^6) =$

If $y = \frac{3}{8}x^4, y' =$ $\frac{3}{2}x^3$

$\frac{d}{dx}[-x^4 + 3x - 9] =$

$= -4x^3 + 3$

$\frac{d}{dx}[3cosx] =$ $-3\sin x$

$3(-\sin x)$

If $f(x) = 5e^x, f'(x) =$ $5e^x$

If $g(t) = e^{0.12},\ g'(t) =$ 0

☆ If $g(x) = \frac{3}{x^2}, g'(x) =$ $-6x^{-3}$

OR

$-\frac{6}{x^3}$

$g(x) = 3x^{-2}$

If $y = \sqrt[4]{x^3}, \frac{dy}{dx} =$ $\frac{3}{4}x^{-\frac{1}{4}}$

$y = x^{3/4}$

OR

$\frac{3}{4x^{1/4}}$

If $g(x) = \sqrt{x} + 3x^2, g'(x) =$ $\frac{1}{2}x^{-\frac{1}{2}} + 6x$

$g(x) = x^{\frac{1}{2}} + 3x^2$

OR

$\frac{1}{2x^{1/2}} + 6x$

Differentiate $y = \frac{(3x)^4}{8}$

$y = \frac{81x^4}{8}$ $\quad y' = \frac{81}{2}x^3$

If $g(x) = \frac{2sinx}{3},\ g'(x) =$ $\frac{2}{3}\cos x$

Differentiate $g(x) = e^x - 4x$

$g'(x) = e^x - 4$

If $y = \frac{1}{x^3}, find\ \frac{dy}{dx}$

$y = x^{-3}$

$y' = -3x^{-4}$ OR $\frac{-3}{x^4}$

Differentiate $y = \frac{\sqrt[6]{x^5}}{8}$

$y = \frac{1}{8}x^{5/6}$

$y' = \left(\frac{1}{8}\right)\left(\frac{5}{6}\right)x^{-\frac{1}{6}}$

$= \frac{5}{48}x^{-1/6}$ OR $\frac{5}{48x^{1/6}}$

If $y = \frac{cosx}{2} + cos\frac{\pi}{2},\ y' =$ $\frac{1}{2}(-\sin x)$

$= -\frac{1}{2}\sin x$

If $g(x) = 7e^x - cosx, find\ g'(x)$

$g'(x) = 7e^x + \sin x$

Find $\frac{dy}{dx}\ if\ y = \frac{9}{7x^2}$

$y = \frac{9}{7}x^{-2}$

$y' = -\frac{18}{7}x^{-3}$ OR $\frac{-18}{7x^3}$

Differentiate $y = \frac{x^3 - 3x^2 - 5}{x^2}$

$y = x - 3 - 5x^{-2}$

$y' = 1 - (5)(-2x^{-3})$

$= 1 + 10x^{-3}$ OR $1 + \frac{10}{x^3}$

If $f(x) = \frac{9}{(7x)^2}$ $= \frac{9}{49x^2} = \frac{9}{49}x^{-2}$

$f'(x) = \left(\frac{9}{49}\right)\left(-2x^{-3}\right)$

$= \frac{-18}{49}x^{-3}$ OR $\frac{-18}{49x^3}$

Do together

Find the slope of the graph of $f(x) = \frac{1}{x^4}$ at $x = 2$.

$f(x) = x^{-4}$

$f'(x) = -4x^{-5}$

$f'(2) = -4(2)^{-5} = -\frac{4}{32} = -\frac{1}{8}$

Students Try

Find the equation of the tangent line to the graph of $f(x) = \sqrt[3]{x}$ when $x = 1$.

$f(x) = x^{1/3}$ $\quad f(1) = \sqrt[3]{1} = 1$

$f'(x) = \frac{1}{3}x^{-2/3}$

$f'(1) = \frac{1}{3}(1)^{-2/3} = \frac{1}{3}$ $\quad y - 1 = \frac{1}{3}(x-1)$

Average Velocity: $\frac{\text{Change in Distance}}{\text{Change in Time}} = \frac{\Delta s}{\Delta t} = \frac{s(b)-s(a)}{b-a}$

Examples: Average Velocity

A tennis ball is dropped from a height of 150 feet. The ball's height, s at time t is the position function $s(t) = -16t^2 + 150$, where s is measured in feet and t is measured in seconds. Find the average velocity over each of the following intervals.

a. [2,3] $\frac{(-16(9)+150) - (-16(4)+150)}{3-2} = -80$ ft/sec

b. [2, 2.5] $\frac{(-16(2.5)^2+150) - (-16(4)+150)}{2.5-2} = -72$ ft/sec

c. [2, 2.1] $\frac{(-16(2.1)^2+150) - (-16(2)^2+150)}{2.1-2} = -65.6$ ft/sec

Velocity: (Instantaneous Velocity) $= s'(t) =$ derivative of the position function

Example: Velocity Applications

A water balloon is thrown upward from the top of an 80 foot building with an initial velocity of 64 feet per second. The height s (in feet) of the balloon can be modeled by the position function

$s(t) = -16t^2 + 64t + 80$ where t is the time in seconds since it was thrown.

a. How long is the water balloon in the air?

$0 = -16t^2 + 64t + 80$

$0 = -16(t^2 - 4t - 5)$

$0 = -16(t-5)(t+1)$

5 seconds

b. What is the velocity of the water balloon when it hits the ground?

$s'(t) = v(t) = -32t + 64$

$v(5) = -32(5) + 64$

$= -96$ ft/sec