Section 2.4 The Chain Rule

	$y = f(g(x))$	$u = g(x)$	$y = f(u)$
1.	$y = (6x - 5)^4$	$u = 6x - 5$	$y = u^4$
2.	$y = \dfrac{1}{\sqrt{x + 1}}$	$u = x + 1$	$y = u^{-1/2}$
3.	$y = \csc^3 x$	$u = \csc x$	$y = u^3$
4.	$y = 3\tan(\pi x^2)$	$u = \pi x^2$	$y = 3\tan u$
5.	$y = e^{-2x}$	$u = -2x$	$y = e^u$
6.	$y = (\ln x)^3$	$u = \ln x$	$y = u^3$

7. $y = (2x - 7)^3$

$$y' = 3(2x - 7)^2(2)$$
$$= 6(2x - 7)^2$$

8. $y = 5(2 - x^3)^4$

$$y' = 5(4)(2 - x^3)^3(-3x^2) = -60x^2(2 - x^3)^3$$
$$= 60x^2(x^3 - 2)^3$$

9. $g(x) = 3(4 - 9x)^4$

$g'(x) = 12(4 - 9x)^3(-9) = -108(4 - 9x)^3$

10. $f(t) = (9t + 2)^{2/3}$

$f'(t) = \frac{2}{3}(9t + 2)^{-1/3}(9) = \frac{6}{\sqrt[3]{9t + 2}}$

11. $f(t) = \sqrt{5 - t} = (5 - t)^{1/2}$

$f'(t) = \frac{1}{2}(5 - t)^{-1/2}(-1) = \frac{-1}{2\sqrt{5 - t}}$

12. $g(x) = \sqrt{4 - 3x^2} = (4 - 3x^2)^{1/2}$

$g'(x) = \frac{1}{2}(4 - 3x^2)^{-1/2}(-6x) = -\frac{3x}{\sqrt{4 - 3x^2}}$

13. $y = \sqrt[3]{6x^2 + 1} = (6x^2 + 1)^{1/3}$

$y' = \frac{1}{3}(6x^2 + 1)^{-2/3}(12x) = \frac{4x}{(6x^2 + 1)^{2/3}} = \frac{4x}{\sqrt[3]{(6x^2 + 1)^2}}$

14. $f(x) = \sqrt{x^2 - 4x + 2} = (x^2 - 4x + 2)^{1/2}$

$f'(x) = \frac{1}{2}(x^2 - 4x + 2)^{-1/2}(2x - 4) = \frac{x - 2}{\sqrt{x^2 - 4x + 2}}$

15. $y = 2\sqrt[4]{9 - x^2} = 2(9 - x^2)^{1/4}$

$y' = 2\left(\frac{1}{4}\right)(9 - x^2)^{-3/4}(-2x)$

$= \frac{-x}{(9 - x^2)^{3/4}} = \frac{-x}{\sqrt[4]{(9 - x^2)^3}}$

16. $f(x) = \sqrt[3]{12x - 5} = (12x - 5)^{1/3}$

$f'(x) = \frac{1}{3}(12x - 5)^{-2/3}(12) = \frac{4}{(12x - 5)^{2/3}}$

17. $y = (x - 2)^{-1}$

$y' = -1(x - 2)^{-2}(1) = \frac{-1}{(x - 2)^2}$

18. $s(t) = \frac{1}{4 - 5t - t^2} = (4 - 5t - t^2)^{-1}$

$s'(t) = -(4 - 5t - t^2)^{-2}(-5 - 2t)$

$= \frac{5 + 2t}{(4 - 5t - t^2)^2} = \frac{2t + 5}{(t^2 + 5t - 4)^2}$ 2

19. $f(t) = (t - 3)^{-2}$

$f'(t) = -2(t - 3)^{-3}(1) = \frac{-2}{(t - 3)^3}$

20. $y = -\frac{3}{(t - 2)^4} = -3(t - 2)^{-4}$

$y' = 12(t - 2)^{-5} = \frac{12}{(t - 2)^5}$

21. $y = \frac{1}{\sqrt{3x + 5}} = (3x + 5)^{-1/2}$

$y' = -\frac{1}{2}(3x + 5)^{-3/2}(3)$

$= \frac{-3}{2(3x + 5)^{3/2}}$

$= -\frac{3}{2\sqrt{(3x + 5)^3}}$

22. $g(t) = \frac{1}{\sqrt{t^2 - 2}} = (t^2 - 2)^{-1/2}$

$g'(t) = -\frac{1}{2}(t^2 - 2)^{-3/2}(2t)$

$= \frac{-t}{(t^2 - 2)^{3/2}}$

$= -\frac{t}{\sqrt{(t^2 - 2)^3}}$

23. $f(x) = x^2(x - 2)^4$

$f'(x) = x^2\left[4(x - 2)^3(1)\right] + (x - 2)^4(2x)$

$= 2x(x - 2)^3\left[2x + (x - 2)\right]$

$= 2x(x - 2)^3(3x - 2)$

24. $f(x) = x(2x - 5)^3$

$f'(x) = x(3)(2x - 5)^2(2) + (2x - 5)^3(1)$

$= (2x - 5)^2\left[6x + (2x - 5)\right]$

$= (2x - 5)^2(8x - 5)$

25. $y = x\sqrt{1-x^2} = x(1-x^2)^{1/2}$

$$y' = x\left[\frac{1}{2}(1-x^2)^{-1/2}(-2x)\right] + (1-x^2)^{1/2}(1)$$
$$= -x^2(1-x^2)^{-1/2} + (1-x^2)^{1/2}$$
$$= (1-x^2)^{-1/2}\left[-x^2 + (1-x^2)\right]$$
$$= \frac{1-2x^2}{\sqrt{1-x^2}}$$

26. $y = \frac{1}{2}x^2\sqrt{16-x^2}$

$$y' = \frac{1}{2}x^2\left(\frac{1}{2}(16-x^2)^{-1/2}(-2x)\right) + x(16-x^2)^{1/2}$$
$$= \frac{-x^3}{2\sqrt{16-x^2}} + x\sqrt{16-x^2} = -\frac{x(3x^2-32)}{2\sqrt{16-x^2}}$$

27. $y = \frac{x}{\sqrt{x^2+1}} = \frac{x}{(x^2+1)^{1/2}}$

$$y' = \frac{(x^2+1)^{1/2}(1) - x\left(\frac{1}{2}\right)(x^2+1)^{-1/2}(2x)}{\left[(x^2+1)^{1/2}\right]^2}$$
$$= \frac{(x^2+1)^{1/2} - x^2(x^2+1)^{-1/2}}{x^2+1}$$
$$= \frac{(x^2+1)^{-1/2}\left[x^2+1-x^2\right]}{x^2+1}$$
$$= \frac{1}{(x^2+1)^{3/2}} = \frac{1}{\sqrt{(x^2+1)^3}}$$

28. $y = \frac{x}{\sqrt{x^4+4}}$

$$y' = \frac{(x^4+4)^{1/2}(1) - x\frac{1}{2}(x^4+4)^{-1/2}(4x^3)}{x^4+4}$$
$$= \frac{x^4+4-2x^4}{(x^4+4)^{3/2}} = \frac{4-x^4}{(x^4+4)^{3/2}} = \frac{4-x^4}{\sqrt{(x^4+4)^3}}$$

29. $g(x) = \left(\frac{x+5}{x^2+2}\right)^2$

$$g'(x) = 2\left(\frac{x+5}{x^2+2}\right)\left(\frac{(x^2+2)-(x+5)(2x)}{(x^2+2)^2}\right)$$
$$= \frac{2(x+5)(2-10x-x^2)}{(x^2+2)^3}$$
$$= \frac{-2(x+5)(x^2+10x-2)}{(x^2+2)^3}$$

30. $h(t) = \left(\frac{t^2}{t^3+2}\right)^2$

$$h'(t) = 2\left(\frac{t^2}{t^3+2}\right)\left(\frac{(t^3+2)(2t)-t^2(3t^2)}{(t^3+2)^2}\right)$$
$$= \frac{2t^2(4t-t^4)}{(t^3+2)^3} = \frac{2t^3(4-t^3)}{(t^3+2)^3}$$

31. $f(v) = \left(\frac{1-2v}{1+v}\right)^3$

$$f'(v) = 3\left(\frac{1-2v}{1+v}\right)^2\left(\frac{(1+v)(-2)-(1-2v)}{(1+v)^2}\right)$$
$$= \frac{-9(1-2v)^2}{(1+v)^4}$$

32. $g(x) = \left(\frac{3x^2-2}{2x+3}\right)^3$

$$g'(x) = 3\left(\frac{3x^2-2}{2x+3}\right)^2\left(\frac{(2x+3)(6x)-(3x^2-2)(2)}{(2x+3)^2}\right)$$
$$= \frac{3(3x^2-2)^2(6x^2+18x+4)}{(2x+3)^4}$$
$$= \frac{6(3x^2-2)^2(3x^2+9x+2)}{(2x+3)^4}$$

33. $f(x) = \left((x^2+3)^5 + x\right)^2$

$$f'(x) = 2\left((x^2+3)^5+x\right)\left(5(x^2+3)^4(2x)+1\right)$$
$$= 2\left[10x(x^2+3)^9 + (x^2+3)^5 + 10x^2(x^2+3)^4 + x\right] = 20x(x^2+3)^9 + 2(x^2+3)^5 + 20x^2(x^2+3)^4 + 2x$$

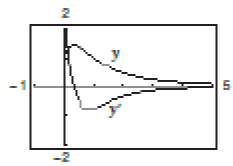
34. $g(x) = \left(2 + (x^2+1)^4\right)^3$

$$g'(x) = 3\left(2+(x^2+1)^4\right)^2\left(4(x^2+1)^3(2x)\right) = 24x(x^2+1)^3\left(2+(x^2+1)^4\right)^2$$

35. $y = \dfrac{\sqrt{x} + 1}{x^2 + 1}$

$y' = \dfrac{1 - 3x^2 - 4x^{3/2}}{2\sqrt{x}(x^2 + 1)^2}$

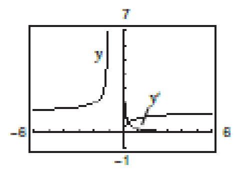
The zero of y' corresponds to the point on the graph of y where the tangent line is horizontal.



36. $y = \sqrt{\dfrac{2x}{x + 1}}$

$y' = \dfrac{1}{\sqrt{2x}(x + 1)^{3/2}}$

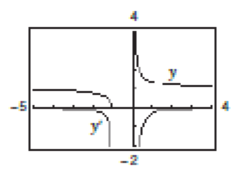
y' has no zeros.



37. $y = \sqrt{\dfrac{x + 1}{x}}$

$y' = -\dfrac{\sqrt{(x + 1)/x}}{2x(x + 1)}$

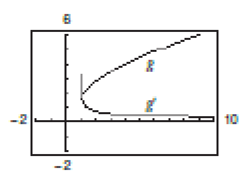
y' has no zeros.



38. $g(x) = \sqrt{x - 1} + \sqrt{x + 1}$

$g'(x) = \dfrac{1}{2\sqrt{x - 1}} + \dfrac{1}{2\sqrt{x + 1}}$

g' has no zeros.

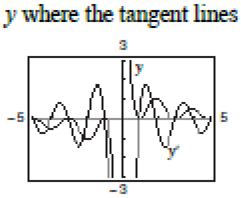


39. $y = \dfrac{\cos \pi x + 1}{x}$

$\dfrac{dy}{dx} = \dfrac{-\pi x \sin \pi x - \cos \pi x - 1}{x^2}$

$= -\dfrac{\pi x \sin \pi x + \cos \pi x + 1}{x^2}$

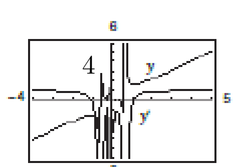
The zeros of y' correspond to the points on the graph of y where the tangent lines are horizontal.



40. $y = x^2 \tan \dfrac{1}{x}$

$\dfrac{dy}{dx} = 2x \tan \dfrac{1}{x} - \sec^2 \dfrac{1}{x}$

The zeros of y' correspond to the points on the graph of y where the tangent lines are horizontal.



41. (a) $y = \sin x$

$y' = \cos x$

$y'(0) = 1$

1 cycle in $[0, 2\pi]$

(b) $y = \sin 2x$

$y' = 2 \cos 2x$

$y'(0) = 2$

2 cycles in $[0, 2\pi]$

The slope of $\sin ax$ at the origin is a.

42. (a) $y = \sin 3x$

$y' = 3 \cos 3x$

$y'(0) = 3$

3 cycles in $[0, 2\pi]$

(b) $y = \sin\left(\dfrac{x}{2}\right)$

$y' = \left(\dfrac{1}{2}\right)\cos\left(\dfrac{x}{2}\right)$

$y'(0) = \dfrac{1}{2}$

Half cycle in $[0, 2\pi]$

So, there are no complete cycles of the graph in the interval $[0, 2\pi]$.

The slope of $\sin ax$ at the origin is a.

43. $y = e^{4x}$

$y' = 4e^{4x}$

At $(0, 1)$, $y' = 4$.

44. $y = e^{-3x}$

$y' = -3e^{-3x}$

At $(0, 1)$, $y' = -3$.

45. $y = \ln x^3 = 3 \ln x$

$y' = \dfrac{3}{x}$

At $(1, 0)$, $y' = 3$.

46. $y = \ln x^{3/2} = \dfrac{3}{2} \ln x$

$y' = \dfrac{3}{2}\left(\dfrac{1}{x}\right) = \dfrac{3}{2x}$

At $(1, 0)$, $y' = \dfrac{3}{2}$.

47. $y = \cos 4x$

$\dfrac{dy}{dx} = -4 \sin 4x$

48. $y = \sin \pi x$

$\frac{dy}{dx} = \pi \cos \pi x$

49. $g(x) = 5 \tan 3x$

$g'(x) = 15 \sec^2 3x$

50. $h(x) = \sec(x^2)$

$h'(x) = 2x \sec(x^2)\tan(x^2)$

51. $y = \sin(\pi x)^2 = \sin(\pi^2 x^2)$

$y' = \cos(\pi^2 x^2)[2\pi^2 x] = 2\pi^2 x \cos(\pi^2 x^2)$

$= 2\pi^2 x \cos(\pi x)^2$

52. $y = \cos(1 - 2x)^2 = \cos((1 - 2x)^2)$

$y' = -\sin(1 - 2x)^2(2(1 - 2x)(-2))$

$= 4(1 - 2x) \sin(1 - 2x)^2$

53. $h(x) = \sin 2x \cos 2x$

$h'(x) = \sin 2x(-2 \sin 2x) + \cos 2x(2 \cos 2x)$

$= 2 \cos^2 2x - 2 \sin^2 2x$

$= 2 \cos 4x$

Alternate solution: $h(x) = \frac{1}{2} \sin 4x$

$h'(x) = \frac{1}{2} \cos 4x(4) = 2 \cos 4x$

54. $g(\theta) = \sec \frac{1}{2}\theta \tan \frac{1}{2}\theta$

$g'(\theta) = \sec(\frac{1}{2}\theta) \sec^2(\frac{1}{2}\theta)\frac{1}{2} + \tan(\frac{1}{2}\theta) \sec(\frac{1}{2}\theta) \tan(\frac{1}{2}\theta)\frac{1}{2}$

$= \frac{1}{2} \sec(\frac{1}{2}\theta)[\sec^2(\frac{1}{2}\theta) + \tan^2(\frac{1}{2}\theta)]$

55. $f(x) = \frac{\cot x}{\sin x} = \frac{\cos x}{\sin^2 x}$

$f'(x) = \frac{\sin^2 x(-\sin x) - \cos x(2 \sin x \cos x)}{\sin^4 x}$

$= \frac{-\sin^2 x - 2 \cos^2 x}{\sin^3 x} = \frac{-1 - \cos^2 x}{\sin^3 x}$

56. $g(v) = \frac{\cos v}{\csc v} = \cos v \cdot \sin v$

$g'(v) = \cos v(\cos v) + \sin v(-\sin v)$

$= \cos^2 v - \sin^2 v = \cos 2v$

57. $y = 4 \sec^2 x$

$y' = 8 \sec x \cdot \sec x \tan x = 8 \sec^2 x \tan x$

58. $g(t) = 5 \cos^2 \pi t = 5(\cos \pi t)^2$

$g'(t) = 10 \cos \pi t(-\sin \pi t)(\pi)$

$= -10\pi(\sin \pi t)(\cos \pi t)$

$= -5\pi \sin 2\pi t$

59. $f(\theta) = \tan^2 5\theta = (\tan 5\theta)^2$

$f'(\theta) = 2(\tan 5\theta)(\sec^2 5\theta)5 = 10 \tan 5\theta \sec^2 5\theta$

60. $g(\theta) = \cos^2 8\theta = (\cos 8\theta)^2$

$g'(\theta) = 2(\cos 8\theta)(-\sin 8\theta)8 = -16 \cos 8\theta \sin 8\theta$

61. $f(\theta) = \frac{1}{4} \sin^2 2\theta = \frac{1}{4}(\sin 2\theta)^2$

$f'(\theta) = 2(\frac{1}{4})(\sin 2\theta)(\cos 2\theta)(2)$

$= \sin 2\theta \cos 2\theta = \frac{1}{2} \sin 4\theta$

62. $h(t) = 2 \cot^2(\pi t + 2)$

$h'(t) = 4 \cot(\pi t + 2)(-\csc^2(\pi t + 2)(\pi))$

$= -4\pi \cot(\pi t + 2) \csc^2(\pi t + 2)$

63. $f(t) = 3\sec^2(\pi t - 1)$

$f'(t) = 6 \sec(\pi t - 1) \sec(\pi t - 1) \tan(\pi t - 1)(\pi)$

$= 6\pi \sec^2(\pi t - 1) \tan(\pi t - 1) = \frac{6\pi \sin(\pi t - 1)}{\cos^3(\pi t - 1)}$

64. $y = 3x - 5 \cos(\pi x)^2 = 3x - 5 \cos(\pi^2 x^2)$

$\frac{dy}{dx} = 3 + 5 \sin(\pi^2 x^2)(2\pi^2 x) = 3 + 10\pi^2 x \sin(\pi x)^2$

65. $y = \sqrt{x} + \frac{1}{4} \sin(2x)^2 = \sqrt{x} + \frac{1}{4} \sin(4x^2)$

$\frac{dy}{dx} = \frac{1}{2}x^{-1/2} + \frac{1}{4}\cos(4x^2)(8x) = \frac{1}{2\sqrt{x}} + 2x \cos(2x)^2$

66. $y = \sin x^{1/3} + (\sin x)^{1/3}$

$y' = \cos x^{1/3}\left(\frac{1}{3}x^{-2/3}\right) + \frac{1}{3}(\sin x)^{-2/3} \cos x$

$= \frac{1}{3}\left[\frac{\cos x^{1/3}}{x^{2/3}} + \frac{\cos x}{(\sin x)^{2/3}}\right]$

67. $y = \sin(\tan 2x)$

$y' = \cos(\tan 2x)(\sec^2 2x)(2)$

$= 2 \cos(\tan 2x) \sec^2 2x$

68. $y = \cos\sqrt{\sin(\tan \pi x)}$

$$y' = -\sin\sqrt{\sin(\tan \pi x)} \cdot \frac{1}{2}(\sin(\tan \pi x))^{-1/2}\cos(\tan \pi x)\sec^2 \pi x(\pi) = \frac{-\pi \sin\sqrt{\sin(\tan \pi x)}\cos(\tan \pi x)\sec^2 \pi x}{2\sqrt{\sin(\tan \pi x)}}$$

69. $f(x) = e^{2x}$

$f'(x) = 2e^{2x}$

70. $y = e^{-x^2}$

$\dfrac{dy}{dx} = -2xe^{-x^2}$

71. $y = e^{\sqrt{x}}$

$\dfrac{dy}{dx} = \dfrac{e^{\sqrt{x}}}{2\sqrt{x}}$

72. $y = x^2e^{-x}$

$\dfrac{dy}{dx} = -x^2e^{-x} + 2xe^{-x}$

$= xe^{-x}(2 - x)$

73. $g(t) = (e^{-t} + e^t)^3$

$g'(t) = 3(e^{-t} + e^t)^2(e^t - e^{-t})$

74. $g(t) = e^{-3/t^2} = e^{-3t^{-2}}$

$g'(t) = e^{-3/t^2}(6t^{-3}) = \dfrac{6}{t^3e^{3/t^2}} = \dfrac{6e^{-3/t^2}}{t^3}$

75. $y = \ln e^{x^2} = x^2$

$\dfrac{dy}{dx} = 2x$

76. $y = \ln\left(\dfrac{1 + e^x}{1 - e^x}\right)$

$= \ln(1 + e^x) - \ln(1 - e^x)$

$\dfrac{dy}{dx} = \dfrac{e^x}{1 + e^x} + \dfrac{e^x}{1 - e^x}$

$= \dfrac{2e^x}{1 - e^{2x}}$

77. $y = \dfrac{2}{e^x + e^{-x}} = 2(e^x + e^{-x})^{-1}$

$\dfrac{dy}{dx} = -2(e^x + e^{-x})^{-2}(e^x - e^{-x}) = \dfrac{-2(e^x - e^{-x})}{(e^x + e^{-x})^2}$

78. $y = \dfrac{e^x - e^{-x}}{2}$

$\dfrac{dy}{dx} = \dfrac{e^x + e^{-x}}{2}$

79. $y = x^2e^x - 2xe^x + 2e^x = e^x(x^2 - 2x + 2)$

$\dfrac{dy}{dx} = e^x(2x - 2) + e^x(x^2 - 2x + 2) = x^2e^x$

80. $y = xe^x - e^x = e^x(x - 1)$

$\dfrac{dy}{dx} = e^x + e^x(x - 1) = xe^x$

81. $f(x) = e^{-x}\ln x$

$f'(x) = e^{-x}\left(\dfrac{1}{x}\right) - e^{-x}\ln x = e^{-x}\left(\dfrac{1}{x} - \ln x\right)$

82. $f(x) = e^3\ln x$

$f'(x) = \dfrac{e^3}{x}$

83. $y = e^x(\sin x + \cos x)$

$\dfrac{dy}{dx} = e^x(\cos x - \sin x) + (\sin x + \cos x)(e^x)$

$= e^x(2\cos x) = 2e^x\cos x$

84. $y = \ln e^x = x$

$\dfrac{dy}{dx} = 1$

85. $g(x) = \ln x^2 = 2\ln x$

$g'(x) = \dfrac{2}{x}$

86. $h(x) = \ln(2x^2 + 3)$

$h'(x) = \dfrac{4x}{2x^2 + 3}$

87. $y = (\ln x)^4$

$\dfrac{dy}{dx} = 4(\ln x)^3\left(\dfrac{1}{x}\right) = \dfrac{4(\ln x)^3}{x}$

88. $y = x\ln x$

$\dfrac{dy}{dx} = x\left(\dfrac{1}{x}\right) + \ln x = 1 + \ln x$

89. $y = \ln x\sqrt{x^2 - 1} = \ln x + \frac{1}{2}\ln(x^2 - 1)$

$$\frac{dy}{dx} = \frac{1}{x} + \frac{1}{2}\left(\frac{2x}{x^2 - 1}\right) = \frac{2x^2 - 1}{x(x^2 - 1)}$$

90. $y = \ln\sqrt{x^2 - 9} = \frac{1}{2}\ln(x^2 - 9)$

$$y' = \frac{1}{2}\frac{1}{x^2 - 9}(2x) = \frac{x}{x^2 - 9}$$

91. $f(x) = \ln\frac{x}{x^2 + 1} = \ln x - \ln(x^2 + 1)$

$$f'(x) = \frac{1}{x} - \frac{2x}{x^2 + 1} = \frac{1 - x^2}{x(x^2 + 1)}$$

92. $f(x) = \ln\left(\frac{2x}{x + 3}\right) = \ln(2x) - \ln(x + 3)$

$$f'(x) = \frac{1}{2x}(2) - \frac{1}{x + 3} = \frac{1}{x} - \frac{1}{x + 3}$$

93. $g(t) = \frac{\ln t}{t^2}$

$$g'(t) = \frac{t^2(1/t) - 2t\ln t}{t^4} = \frac{1 - 2\ln t}{t^3}$$

94. $h(t) = \frac{\ln t}{t}$

$$h'(t) = \frac{t(1/t) - \ln t}{t^2} = \frac{1 - \ln t}{t^2}$$

95. $y = \ln\sqrt{\frac{x + 1}{x - 1}} = \frac{1}{2}\left[\ln(x + 1) - \ln(x - 1)\right]$

$$\frac{dy}{dx} = \frac{1}{2}\left[\frac{1}{x + 1} - \frac{1}{x - 1}\right] = \frac{1}{1 - x^2}$$

96. $y = \ln\sqrt[3]{\frac{x - 2}{x + 2}} = \frac{1}{3}\left[\ln(x - 2) - \ln(x + 2)\right]$

$$y' = \frac{1}{3}\left[\frac{1}{x - 2} - \frac{1}{x + 2}\right] = \frac{4}{3(x^2 - 4)}$$

97. $y = \frac{-\sqrt{x^2 + 1}}{x} + \ln\left(x + \sqrt{x^2 + 1}\right)$

$$\frac{dy}{dx} = \frac{-x\left(x/\sqrt{x^2 + 1}\right) + \sqrt{x^2 + 1}}{x^2} + \left(\frac{1}{x + \sqrt{x^2 + 1}}\right)\left(1 + \frac{x}{\sqrt{x^2 + 1}}\right)$$

$$= \frac{1}{x^2\sqrt{x^2 + 1}} + \left(\frac{1}{x + \sqrt{x^2 + 1}}\right)\left(\frac{\sqrt{x^2 + 1} + x}{\sqrt{x^2 + 1}}\right)$$

$$= \frac{1}{x^2\sqrt{x^2 + 1}} + \frac{1}{\sqrt{x^2 + 1}} = \frac{1 + x^2}{x^2\sqrt{x^2 + 1}} = \frac{\sqrt{x^2 + 1}}{x^2}$$

98. $y = \frac{-\sqrt{x^2 + 4}}{2x^2} - \frac{1}{4}\ln\left(\frac{2 + \sqrt{x^2 + 4}}{x}\right) = \frac{-\sqrt{x^2 + 4}}{2x^2} - \frac{1}{4}\ln\left(2 + \sqrt{x^2 + 4}\right) + \frac{1}{4}\ln x$

$$\frac{dy}{dx} = \frac{-2x^2\left(x/\sqrt{x^2 + 4}\right) + 4x\sqrt{x^2 + 4}}{4x^4} - \frac{1}{4}\left(\frac{1}{2 + \sqrt{x^2 + 4}}\right)\left(\frac{x}{\sqrt{x^2 + 4}}\right) + \frac{1}{4x}$$

Note that $\frac{1}{2 + \sqrt{x^2 + 4}} = \frac{1}{2 + \sqrt{x^2 + 4}} \cdot \frac{2 - \sqrt{x^2 + 4}}{2 - \sqrt{x^2 + 4}} = \frac{2 - \sqrt{x^2 + 4}}{-x^2}.$

So, $\frac{dy}{dx} = \frac{-1}{2x\sqrt{x^2 + 4}} + \frac{\sqrt{x^2 + 4}}{x^3} - \frac{1}{4}\frac{\left(2 - \sqrt{x^2 + 4}\right)}{-x^2}\left(\frac{x}{\sqrt{x^2 + 4}}\right) + \frac{1}{4x}$

$$= \frac{-1 + (1/2)\left(2 - \sqrt{x^2 + 4}\right)}{2x\sqrt{x^2 + 4}} + \frac{\sqrt{x^2 + 4}}{x^3} + \frac{1}{4x}$$

$$= \frac{-\sqrt{x^2 + 4}}{4x\sqrt{x^2 + 4}} + \frac{\sqrt{x^2 + 4}}{x^3} + \frac{1}{4x} = \frac{\sqrt{x^2 + 4}}{x^3}.$$

99. $y = \ln|\sin x|$

$$\frac{dy}{dx} = \frac{\cos x}{\sin x} = \cot x$$

100. $y = \ln|\csc x|$

$$y' = \frac{1}{\csc x}(-\csc x\cot x) = -\cot x$$

101. $y = \ln\left|\dfrac{\cos x}{\cos x - 1}\right|$

$= \ln|\cos x| - \ln|\cos x - 1|$

$$\frac{dy}{dx} = \frac{-\sin x}{\cos x} - \frac{-\sin x}{\cos x - 1} = -\tan x + \frac{\sin x}{\cos x - 1}$$

102. $y = \ln|\sec x + \tan x|$

$$\frac{dy}{dx} = \frac{\sec x \tan x + \sec^2 x}{\sec x + \tan x}$$

$$= \frac{\sec x(\sec x + \tan x)}{\sec x + \tan x} = \sec x$$

103. $y = \ln\left|\dfrac{-1 + \sin x}{2 + \sin x}\right| = \ln|-1 + \sin x| - \ln|2 + \sin x|$

$$\frac{dy}{dx} = \frac{\cos x}{-1 + \sin x} - \frac{\cos x}{2 + \sin x} = \frac{3\cos x}{(\sin x - 1)(\sin x + 2)}$$

104. $y = \ln\sqrt{1 + \sin^2 x} = \dfrac{1}{2}\ln(1 + \sin^2 x)$

$$\frac{dy}{dx} = \left(\frac{1}{2}\right)\frac{2\sin x \cos x}{1 + \sin^2 x} = \frac{\sin x \cos x}{1 + \sin^2 x}$$

105. The Chain Rule was not used for $1 - x$.

If $y = (1 - x)^{1/2}$, then

$y' = \frac{1}{2}(1 - x)^{-1/2}(-1) = -\frac{1}{2}(1 - x)^{-1/2}$.

106. The Chain Rule was not used for $2x$.

If $f(x) = \sin^2 2x$, then

$$f'(x) = 2(\sin 2x)(\cos 2x)(2)$$

$$= 4(\sin 2x)(\cos 2x).$$

107. The Chain Rule was not used for $3x$.

If $y = \dfrac{4^{3x}}{x}$, then $y' = \dfrac{x(\ln 4)4^{3x}(3) - 4^{3x}}{x^2}$

$$= \frac{4^{3x}(3x\ln 4 - 1)}{x^2}.$$

108. The Chain Rule was not used for $-2x$.

If $g(x) = x^4 e^{-2x}$, then

$$g'(x) = x^4 e^{-2x}(-2) + e^{-2x}(4x^3)$$

$$= -2x^3 e^{-2x}(x - 2).$$

109. $y = \sqrt{x^2 + 8x} = (x^2 + 8x)^{1/2}$, $(1, 3)$

$$y' = \frac{1}{2}(x^2 + 8x)^{-1/2}(2x + 8) = \frac{2(x + 4)}{2(x^2 + 8x)^{1/2}} = \frac{x + 4}{\sqrt{x^2 + 8x}}$$

$$y'(1) = \frac{1 + 4}{\sqrt{1^2 + 8(1)}} = \frac{5}{\sqrt{9}} = \frac{5}{3}$$

110. $y = (3x^3 + 4x)^{1/5}$, $(2, 2)$

$$y' = \frac{1}{5}(3x^3 + 4x)^{-4/5}(9x^2 + 4) = \frac{9x^2 + 4}{5(3x^3 + 4x)^{4/5}}$$

$$y'(2) = \frac{1}{2}$$

111. $f(x) = \dfrac{5}{x^3 - 2} = 5(x^3 - 2)^{-1}$, $\left(-2, -\dfrac{1}{2}\right)$

$$f'(x) = -5(x^3 - 2)^{-2}(3x^2) = \frac{-15x^2}{(x^3 - 2)^2}$$

$$f'(-2) = -\frac{60}{100} = -\frac{3}{5}$$

112. $f(x) = \dfrac{1}{(x^2 - 3x)^2} = (x^2 - 3x)^{-2}$, $\left(4, \dfrac{1}{16}\right)$

$$f'(x) = -2(x^2 - 3x)^{-3}(2x - 3) = \frac{-2(2x - 3)}{(x^2 - 3x)^3}$$

$$f'(4) = -\frac{5}{32}$$

113. $f(t) = \dfrac{3t + 2}{t - 1}$, $(0, -2)$

$$f'(t) = \frac{(t - 1)(3) - (3t + 2)(1)}{(t - 1)^2}$$

$$= \frac{3t - 3 - 3t - 2}{(t - 1)^2}$$

$$= \frac{-5}{(t - 1)^2}$$

$$f'(0) = -5$$

114. $f(x) = \dfrac{x + 4}{2x - 5}$, $(9, 1)$

$$f'(x) = \frac{(2x - 5)(1) - (x + 4)(2)}{(2x - 5)^2}$$

$$= \frac{2x - 5 - 2x - 8}{(2x - 5)^2}$$

$$= -\frac{13}{(2x - 5)^2}$$

$$f'(9) = -\frac{13}{(18 - 5)^2} = -\frac{1}{13}$$

115. $y = 26 - \sec^3 4x, \quad (0, 25)$

$$y' = -3\sec^2 4x \sec 4x \tan 4x\, 4$$
$$= -12\sec^3 4x \tan 4x$$
$$y'(0) = 0$$

116. $y = \dfrac{1}{x} + \sqrt{\cos x} = x^{-1} + (\cos x)^{1/2}, \ \left(\dfrac{\pi}{2}, \dfrac{2}{\pi}\right)$

$$y' = -x^{-2} + \frac{1}{2}(\cos x)^{-1/2}(-\sin x) = -\frac{1}{x^2} - \frac{\sin x}{2\sqrt{\cos x}}$$

$y'(\pi/2)$ is undefined.

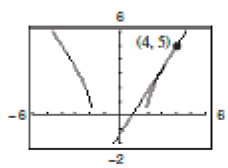
117. (a) $f(x) = \left(2x^2 - 7\right)^{1/2}, \ (4, 5)$

$$f'(x) = \frac{1}{2}\left(2x^2 - 7\right)^{-1/2}(4x) = \frac{2x}{\sqrt{2x^2 - 7}}$$
$$f'(4) = \frac{8}{5}$$

Tangent line:

$$y - 5 = \frac{8}{5}(x - 4) \Rightarrow y = \frac{8}{5}x - \frac{7}{5}$$

(b)



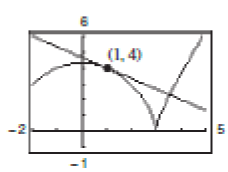
118. (a) $f(x) = \left(9 - x^2\right)^{2/3}, \ (1, 4)$

$$f'(x) = \frac{2}{3}\left(9 - x^2\right)^{-1/3}(-2x) = \frac{-4x}{3\left(9 - x^2\right)^{1/3}}$$
$$f'(1) = \frac{-4}{3(8)^{1/3}} = -\frac{2}{3}$$

Tangent line:

$$y - 4 = -\frac{2}{3}(x - 1) \Rightarrow y = -\frac{2}{3}x + \frac{14}{3}$$

(b)

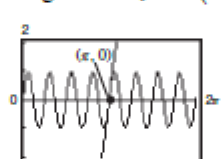


119. (a) $f(x) = \sin 8x, \quad (\pi, 0)$

$$f'(x) = 8\cos 8x$$
$$f'(\pi) = 8$$

Tangent line: $y = 8(x - \pi) = 8x - 8\pi$

(b)



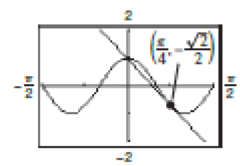
120. (a) $y = \cos 3x, \ \left(\dfrac{\pi}{4}, -\dfrac{\sqrt{2}}{2}\right)$

$$y' = -3\sin 3x$$
$$y'\left(\frac{\pi}{4}\right) = -3\sin\left(\frac{3\pi}{4}\right) = \frac{-3\sqrt{2}}{2}$$

Tangent line: $y + \dfrac{\sqrt{2}}{2} = \dfrac{-3\sqrt{2}}{2}\left(x - \dfrac{\pi}{4}\right)$

$$y = \frac{-3\sqrt{2}}{2}x + \frac{3\sqrt{2}\pi}{8} - \frac{\sqrt{2}}{2}$$

(b)



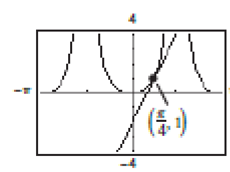
121. (a) $f(x) = \tan^2 x, \ \left(\dfrac{\pi}{4}, 1\right)$

$$f'(x) = 2\tan x \sec^2 x$$
$$f'\left(\frac{\pi}{4}\right) = 2(1)(2) = 4$$

Tangent line:

$$y - 1 = 4\left(x - \frac{\pi}{4}\right) \Rightarrow 4x - y + (1 - \pi) = 0$$

(b)



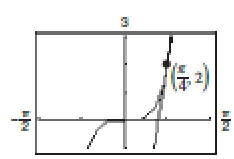
122. (a) $y = 2\tan^3 x, \ \left(\dfrac{\pi}{4}, 2\right)$

$$y' = 6\tan^2 x \cdot \sec^2 x$$
$$y'\left(\frac{\pi}{4}\right) = 6(1)(2) = 12$$

Tangent line:

$$y - 2 = 12\left(x - \frac{\pi}{4}\right) \Rightarrow 12x - y + (2 - 3\pi) = 0$$

(b)



123. (a) $y = 4 - x^2 - \ln\left(\frac{1}{2}x + 1\right), \quad (0, 4)$

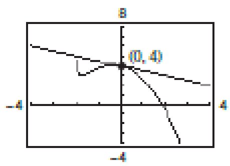
$$\frac{dy}{dx} = -2x - \frac{1}{(1/2)x + 1}\left(\frac{1}{2}\right)$$
$$= -2x - \frac{1}{x + 2}$$

When $x = 0$, $\frac{dy}{dx} = -\frac{1}{2}$.

Tangent line: $y - 4 = -\frac{1}{2}(x - 0)$

$$y = -\frac{1}{2}x + 4$$

(b)



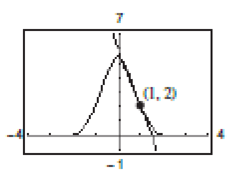
124. (a) $y = 2e^{1-x^2}, \quad (1, 2)$

$$y' = 2e^{1-x^2}(-2x) = -4xe^{1-x^2}$$
$$y'(1) = -4$$

Tangent line: $y - 2 = -4(x - 1)$

$$y = -4x + 6$$

(b)

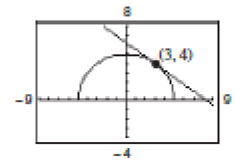


125. $f(x) = \sqrt{25 - x^2} = (25 - x^2)^{1/2}, \quad (3, 4)$

$$f'(x) = \frac{1}{2}(25 - x^2)(-2x) = \frac{-x}{\sqrt{25 - x^2}}$$
$$f'(3) = -\frac{3}{4}$$

Tangent line:

$$y - 4 = -\frac{3}{4}(x - 3) \Rightarrow 3x + 4y - 25 = 0$$

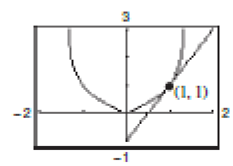


126. $f(x) = \frac{|x|}{\sqrt{2 - x^2}} = |x|(2 - x^2)^{-1/2}, \quad (1, 1)$

$$f'(x) = \frac{2}{(2 - x^2)^{3/2}} \text{ for } x > 0$$
$$f'(1) = 2$$

Tangent line:

$$y - 1 = 2(x - 1) \Rightarrow 2x - y - 1 = 0$$



127.

$$f(x) = 2\cos x + \sin 2x, \quad 0 < x < 2\pi$$
$$f'(x) = -2\sin x + 2\cos 2x$$
$$= -2\sin x + 2 - 4\sin^2 x = 0$$
$$2\sin^2 x + \sin x - 1 = 0$$
$$(\sin x + 1)(2\sin x - 1) = 0$$
$$\sin x = -1 \Rightarrow x = \frac{3\pi}{2}$$
$$\sin x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}$$

Horizontal tangents at $x = \frac{\pi}{6}, \frac{3\pi}{2}, \frac{5\pi}{6}$

Horizontal tangent at the points $\left(\frac{\pi}{6}, \frac{3\sqrt{3}}{2}\right)$, $\left(\frac{3\pi}{2}, 0\right)$, and $\left(\frac{5\pi}{6}, -\frac{3\sqrt{3}}{2}\right)$

128. $f(x) = \dfrac{x}{\sqrt{2x-1}}$

$$f'(x) = \frac{(2x-1)^{1/2} - x(2x-1)^{-1/2}}{2x-1} = \frac{2x-1-x}{(2x-1)^{3/2}} = \frac{x-1}{(2x-1)^{3/2}}$$

$$\frac{x-1}{(2x-1)^{3/2}} = 0 \Rightarrow x = 1$$

Horizontal tangent at $(1, 1)$

129. $f(x) = 5(2-7x)^4$

$f'(x) = 20(2-7x)^3(-7) = -140(2-7x)^3$

$f''(x) = -420(2-7x)^2(-7) = 2940(2-7x)^2$

130. $f(x) = 6(x^3+4)^3$

$f'(x) = 18(x^3+4)^2(3x^2) = 54x^2(x^3+4)^2$

$$f''(x) = 54x^2(2)(x^3+4)(3x^2) + 108x(x^3+4)^2 = 108x(x^3+4)[3x^3+x^3+4] = 432x(x^3+4)(x^3+1)$$

131. $f(x) = \dfrac{1}{x-6} = (x-6)^{-1}$

$f'(x) = -(x-6)^{-2}$

$f''(x) = 2(x-6)^{-3} = \dfrac{2}{(x-6)^3}$

132. $f(x) = \dfrac{8}{(x-2)^2} = 8(x-2)^{-2}$

$f'(x) = -16(x-2)^{-3}$

$f''(x) = 48(x-2)^{-4} = \dfrac{48}{(x-2)^4}$

133. $f(x) = \sin x^2$

$f'(x) = 2x\cos x^2$

$$f''(x) = 2x\left[2x\left(-\sin x^2\right)\right] + 2\cos x^2 = 2\left(\cos x^2 - 2x^2\sin x^2\right)$$

134. $f(x) = \sec^2 \pi x$

$f'(x) = 2\sec \pi x(\pi \sec \pi x \tan \pi x) = 2\pi \sec^2 \pi x \tan \pi x$

$$f''(x) = 2\pi \sec^2 \pi x(\sec^2 \pi x)(\pi) + 2\pi \tan \pi x(2\pi \sec^2 \pi x \tan \pi x) = 2\pi^2 \sec^4 \pi x + 4\pi^2 \sec^2 \pi x \tan^2 \pi x = 2\pi^2 \sec^2 \pi x(\sec^2 \pi x + 2\tan^2 \pi x) = 2\pi^2 \sec^2 \pi x(3\sec^2 \pi x - 2)$$

135. $f(x) = (3+2x)e^{-3x}$

$f'(x) = (3+2x)\left(-3e^{-3x}\right) + 2e^{-3x} = (-7-6x)e^{-3x}$

$f''(x) = (-7-6x)\left(-3e^{-3x}\right) - 6e^{-3x} = 3(6x+5)e^{-3x}$

136. $g(x) = \sqrt{x} + e^x \ln x$

$g'(x) = \dfrac{1}{2\sqrt{x}} + \dfrac{e^x}{x} + e^x \ln x$

$$g''(x) = -\frac{1}{4x^{3/2}} + \frac{xe^x - e^x}{x^2} + \frac{e^x}{x} + e^x \ln x = -\frac{1}{4x\sqrt{x}} + \frac{e^x(2x-1)}{x^2} + e^x \ln x$$

137. $h(x) = \frac{1}{9}(3x+1)^3, \quad \left(1, \frac{64}{9}\right)$

$h'(x) = \frac{1}{9}3(3x+1)^2(3) = (3x+1)^2$

$h''(x) = 2(3x+1)(3) = 18x + 6$

$h''(1) = 24$

138. $f(x) = \dfrac{1}{\sqrt{x+4}} = (x+4)^{-1/2}, \quad \left(0, \dfrac{1}{2}\right)$

$f'(x) = -\dfrac{1}{2}(x+4)^{-3/2}$

$f''(x) = \dfrac{3}{4}(x+4)^{-5/2} = \dfrac{3}{4(x+4)^{5/2}}$

$f''(0) = \dfrac{3}{128}$

139. $f(x) = \cos x^2, \quad (0, 1)$

$$f'(x) = -\sin(x^2)(2x) = -2x\sin(x^2)$$

$$f''(x) = -2x\cos(x^2)(2x) - 2\sin(x^2)$$

$$= -4x^2\cos(x^2) - 2\sin(x^2)$$

$$f''(0) = 0$$

140. $g(t) = \tan 2t, \quad \left(\frac{\pi}{6}, \sqrt{3}\right)$

$$g'(t) = 2\sec^2(2t)$$

$$g''(t) = 4\sec(2t)\cdot\sec(2t)\tan(2t)2$$

$$= 8\sec^2(2t)\tan(2t)$$

$$g''\left(\frac{\pi}{6}\right) = 32\sqrt{3}$$

141. $f(x) = 3^x$

$$f'(x) = (\ln 3)3^x$$

142. $g(x) = 5^{-x}$

$$g'(x) = -(\ln 5)5^{-x}$$

143. $y = 4^{2x-3}$

$$y' = (\ln 4)(4^{2x-3})(2)$$

$$= (2\ln 4)4^{2x-3}$$

144. $y = x(6^{-2x})$

$$y' = x(-2\ln 6)6^{-2x} + 6^{-2x}$$

$$= 6^{-2x}(-2x\ln 6 + 1)$$

145. $g(t) = t^2 2^t$

$$g'(t) = t^2(\ln 2)2^t + (2t)2^t$$

$$= t2^t(t\ln 2 + 2)$$

$$= 2^t t(2 + t\ln 2)$$

146. $f(t) = \dfrac{3^{2t}}{t}$

$$f'(t) = \frac{t(2\ln 3)3^{2t} - 3^{2t}}{t^2} = \frac{3^{2t}(2t\ln 3 - 1)}{t^2}$$

147. $h(\theta) = 2^{-\theta}\cos\pi\theta$

$$h'(\theta) = 2^{-\theta}(-\pi\sin\pi\theta) - (\ln 2)2^{-\theta}\cos\pi\theta$$

$$= -2^{-\theta}[(\ln 2)\cos\pi\theta + \pi\sin\pi\theta]$$

148. $g(\alpha) = 5^{-\alpha/2}\sin 2\alpha$

$$g'(\alpha) = 5^{-\alpha/2}2\cos 2\alpha - \tfrac{1}{2}(\ln 5)5^{-\alpha/2}\sin 2\alpha$$

149. $y = \log_3 x$

$$\frac{dy}{dx} = \frac{1}{x\ln 3}$$

150. $h(x) = \log_3\dfrac{x\sqrt{x-1}}{2}$

$$= \log_3 x + \frac{1}{2}\log_3(x-1) - \log_3 2$$

$$h'(x) = \frac{1}{x\ln 3} + \frac{1}{2}\cdot\frac{1}{(x-1)\ln 3} - 0$$

$$= \frac{1}{\ln 3}\left[\frac{1}{x} + \frac{1}{2(x-1)}\right]$$

$$= \frac{1}{\ln 3}\left[\frac{3x-2}{2x(x-1)}\right]$$

151. $y = \log_5\sqrt{x^2-1} = \frac{1}{2}\log_5(x^2-1)$

$$\frac{dy}{dx} = \frac{1}{2}\cdot\frac{2x}{(x^2-1)\ln 5} = \frac{x}{(x^2-1)\ln 5}$$

152. $y = \log_{10}\dfrac{x^2-1}{x}$

$$= \log_{10}(x^2-1) - \log_{10}x$$

$$\frac{dy}{dx} = \frac{2x}{(x^2-1)\ln 10} - \frac{1}{x\ln 10}$$

$$= \frac{1}{\ln 10}\left[\frac{2x}{x^2-1} - \frac{1}{x}\right] = \frac{1}{\ln 10}\left[\frac{x^2+1}{x(x^2-1)}\right]$$

153. $g(t) = \dfrac{10\log_4 t}{t} = \dfrac{10}{\ln 4}\left(\dfrac{\ln t}{t}\right)$

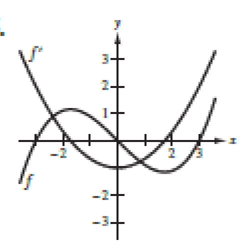
$$g'(t) = \frac{10}{\ln 4}\left[\frac{t(1/t) - \ln t}{t^2}\right]$$

$$= \frac{10}{t^2\ln 4}[1 - \ln t] = \frac{5}{t^2\ln 2}(1 - \ln t)$$

154. $f(t) = t^{3/2}\log_2\sqrt{t+1} = t^{3/2}\dfrac{1}{2}\dfrac{\ln(t+1)}{\ln 2}$

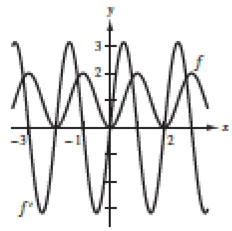
$$f'(t) = \frac{1}{2\ln 2}\left[t^{3/2}\frac{1}{t+1} + \frac{3}{2}t^{1/2}\ln(t+1)\right]$$

155.



The zeros of f' correspond to the points where the graph of f has horizontal tangents.

156.



The zeros of f' correspond to the points where the graph of f has horizontal tangents.

157. $g(x) = f(3x)$

$g'(x) = f'(3x)(3) \Rightarrow g'(x) = 3f'(3x)$

158. $g(x) = f(x^2)$

$g'(x) = f'(x^2)(2x) \Rightarrow g'(x) = 2xf'(x^2)$

159. $f(x) = g(x)h(x)$

$f'(x) = g(x)h'(x) + g'(x)h(x)$

$f'(5) = (-3)(-2) + (6)(3) = 24$

160. $f(x) = g(h(x))$

$f'(x) = g'(h(x))h'(x)$

$f'(5) = g'(3)(-2) = -2g'(3)$

Not possible, you need $g'(3)$ to find $f'(5)$.

161. $f(x) = \dfrac{g(x)}{h(x)}$

$f'(x) = \dfrac{h(x)g'(x) - g(x)h'(x)}{[h(x)]^2}$

$f'(5) = \dfrac{(3)(6) - (-3)(-2)}{(3)^2} = \dfrac{12}{9} = \dfrac{4}{3}$

162. $f(x) = [g(x)]^3$

$f'(x) = 3[g(x)]^2 g'(x)$

$f'(5) = 3(-3)^2(6) = 162$

163. (a) $h(x) = f(g(x)),\ g(1) = 4,\ g'(1) = -\frac{1}{2},\ f'(4) = -1$

$h'(x) = f'(g(x))g'(x)$

$h'(1) = f'(g(1))g'(1) = f'(4)g'(1) = (-1)\left(-\frac{1}{2}\right) = \frac{1}{2}$

(b) $s(x) = g(f(x)),\ f(5) = 6,\ f'(5) = -1,\ g'(6)$ does not exist.

$s'(x) = g'(f(x))\,f'(x)$

$s'(5) = g'(f(5))f'(5) = g'(6)(-1)$

$s'(5)$ does not exist because g is not differentiable at 6.

164. (a) $h(x) = f(g(x))$

$h'(x) = f'(g(x))g'(x)$

$h'(3) = f'(g(3))g'(3) = f'(5)(1) = \frac{1}{2}$

(b) $s(x) = g(f(x))$

$s'(x) = g'(f(x))f'(x)$

$s'(9) = g'(f(9))f'(9) = g'(8)(2) = (-1)(2) = -2$

165. (a) $F = 132{,}400(331 - v)^{-1}$

$F' = (-1)(132{,}400)(331 - v)^{-2}(-1) = \dfrac{132{,}400}{(331 - v)^2}$

When $v = 30$, $F' \approx 1.461$.

(b) $F = 132{,}400(331 + v)^{-1}$

$F' = (-1)(132{,}400)(331 + v)^{-2}(-1) = \dfrac{-132{,}400}{(331 + v)^2}$

When $v = 30$, $F' \approx -1.016$.

166. $y = \frac{1}{3}\cos 12t - \frac{1}{4}\sin 12t$

$v = y' = \frac{1}{3}[-12\sin 12t] - \frac{1}{4}[12\cos 12t]$

$= -4\sin 12t - 3\cos 12t$

When $t = \pi/8$, $y = 0.25$ ft and $v = 4$ ft/sec.

167. $\theta = 0.2\cos 8t$

The maximum angular displacement is $\theta = 0.2$ (because $-1 \le \cos 8t \le 1$).

$\dfrac{d\theta}{dt} = 0.2[-8\sin 8t] = -1.6\sin 8t$

When $t = 3$, $d\theta/dt = -1.6\sin 24 \approx 1.4489$ rad/sec.

168. $y = A\cos\omega t$

(a) Amplitude: $A = \dfrac{3.5}{2} = 1.75$

$y = 1.75\cos\omega t$

Period: $10 \Rightarrow \omega = \dfrac{2\pi}{10} = \dfrac{\pi}{5}$

$y = 1.75\cos\dfrac{\pi t}{5}$

(b) $v = y' = 1.75\left[-\dfrac{\pi}{5}\sin\dfrac{\pi t}{5}\right] = -0.35\pi\sin\dfrac{\pi t}{5}$

169. (a) $T'(35) \approx \dfrac{T(40) - T(30)}{40 - 30} = \dfrac{267.25 - 250.33}{10} = 1.692$

$T'(70) \approx \dfrac{T(80) - T(60)}{80 - 60} = \dfrac{312.03 - 292.71}{20} = 0.966$

At a pressure of 35 pounds per square inch, the rate of change of the temperature is about 1.692 degrees Fahrenheit per pound per square inch. At a pressure of 70 pounds per square inch, the rate of change of the temperature is about 0.966 degree Fahrenheit per pound per square inch.

(b) $T(p) = 87.97 + 34.96\ln p + 7.91\sqrt{p}$

$T'(p) = 0 + 34.96\left(\dfrac{1}{p}\right) + 7.91\left(\dfrac{1}{2}p^{-1/2}\right) = \dfrac{34.96}{p} + \dfrac{3.955}{\sqrt{p}}$

$T'(35) = \dfrac{34.96}{35} + \dfrac{3.955}{\sqrt{35}} \approx 1.667°\text{F}/(\text{lb/in.}^2)$

$T'(70) = \dfrac{34.96}{70} + \dfrac{3.955}{\sqrt{70}} \approx 0.972°\text{F}/(\text{lb/in.}^2)$

The approximations in part (a) are close to the actual rates of change.

170. (a) According to the graph $C'(4) > C'(1)$.

(b) Answers will vary.

171. (a) $g(x) = f(x) - 2 \Rightarrow g'(x) = f'(x)$

(b) $h(x) = 2f(x) \Rightarrow h'(x) = 2f'(x)$

(c) $r(x) = f(-3x) \Rightarrow r'(x) = f'(-3x)(-3) = -3f'(-3x)$

So, you need to know $f'(-3x)$.

$r'(0) = -3f'(0) = (-3)\left(-\frac{1}{3}\right) = 1$

$r'(-1) = -3f'(3) = (-3)(-4) = 12$

(d) $s(x) = f(x + 2) \Rightarrow s'(x) = f'(x + 2)$

So, you need to know $f'(x + 2)$.

$s'(-2) = f'(0) = -\frac{1}{3}$, etc.

x	-2	-1	0	1	2	3
$f'(x)$	4	$\frac{2}{3}$	$-\frac{1}{3}$	-1	-2	-4
$g'(x)$	4	$\frac{2}{3}$	$-\frac{1}{3}$	-1	-2	-4
$h'(x)$	8	$\frac{4}{3}$	$-\frac{2}{3}$	-2	-4	-8
$r'(x)$		12	1			
$s'(x)$	$-\frac{1}{3}$	-1	-2	-4		

172. $C(t) = P(1.05)^t$

(a) $C(10) = 29.95(1.05)^{10} \approx \$48.79$

(b) $\dfrac{dC}{dt} = P\ln(1.05)(1.05)^t$

When $t = 1$, $\dfrac{dC}{dt} \approx 0.051P$.

When $t = 8$, $\dfrac{dC}{dt} \approx 0.072P$.

(c) $\dfrac{dC}{dt} = \ln(1.05)\left[P(1.05)^t\right]$

$= \ln(1.05)C(t)$

The constant of proportionality is $\ln 1.05$.

173. $N = 400\left[1 - \dfrac{3}{(t^2+2)^2}\right] = 400 - 1200(t^2+2)^{-2}$

$N'(t) = 2400(t^2+2)^{-3}(2t) = \dfrac{4800t}{(t^2+2)^3}$

(a) $N'(0) = 0$ bacteria/day

(b) $N'(1) = \dfrac{4800(1)}{(1+2)^3} = \dfrac{4800}{27} \approx 177.8$ bacteria/day

(c) $N'(2) = \dfrac{4800(2)}{(4+2)^3} = \dfrac{9600}{216} \approx 44.4$ bacteria/day

(d) $N'(3) = \dfrac{4800(3)}{(9+2)^3} = \dfrac{14{,}400}{1331} \approx 10.8$ bacteria/day

(e) $N'(4) = \dfrac{4800(4)}{(16+2)^3} = \dfrac{19{,}200}{5832} \approx 3.3$ bacteria/day

(f) The rate of change of the population is decreasing as $t \to \infty$.

174. (a) $V = \dfrac{k}{\sqrt{t+1}}$

$V(0) = 10{,}000 = \dfrac{k}{\sqrt{0+1}} = k$

$V = \dfrac{10{,}000}{\sqrt{t+1}} = 10{,}000(t+1)^{-1/2}$

(b) $\dfrac{dV}{dt} = 10{,}000\left(-\dfrac{1}{2}\right)(t+1)^{-3/2} = \dfrac{-5000}{(t+1)^{3/2}}$

$V'(1) = \dfrac{-5000}{2^{3/2}} \approx -1767.77$ dollars/year

(c) $V'(3) = \dfrac{-5000}{4^{3/2}} = \dfrac{-5000}{8} = -625$ dollars/year

175. $f(x) = \sin\beta x$

(a) $f'(x) = \beta\cos\beta x$

$f''(x) = -\beta^2\sin\beta x$

$f'''(x) = -\beta^3\cos\beta x$

$f^{(4)} = \beta^4\sin\beta x$

(b) $f''(x) + \beta^2 f(x) = -\beta^2\sin\beta x + \beta^2(\sin\beta x) = 0$

(c) $f^{(2k)}(x) = (-1)^k\beta^{2k}\sin\beta x$

$f^{(2k-1)}(x) = (-1)^{k+1}\beta^{2k-1}\cos\beta x$

176. (a) Yes, if $f(x+p) = f(x)$ for all x, then $f'(x+p) = f'(x)$, which shows that f' is periodic as well.

(b) Yes, if $g(x) = f(2x)$, then $g'(x) = 2f'(2x)$. Because f' is periodic, so is g'.

177. (a) $r'(x) = f'(g(x))g'(x)$

$r'(1) = f'(g(1))g'(1)$

Note that $g(1) = 4$ and $f'(4) = \dfrac{5-0}{6-2} = \dfrac{5}{4}$.

Also, $g'(1) = 0$. So, $r'(1) = 0$.

(b) $s'(x) = g'(f(x))f'(x)$

$s'(4) = g'(f(4))f'(4)$

Note that $f(4) = \dfrac{5}{2}, g'\left(\dfrac{5}{2}\right) = \dfrac{6-4}{6-2} = \dfrac{1}{2}$ and

$f'(4) = \dfrac{5}{4}$. So, $s'(4) = \dfrac{1}{2}\left(\dfrac{5}{4}\right) = \dfrac{5}{8}$.

178. (a) $g(x) = \sin^2 x + \cos^2 x = 1 \Rightarrow g'(x) = 0$

$g'(x) = 2\sin x\cos x + 2\cos x(-\sin x) = 0$

(b) $\tan^2 x + 1 = \sec^2 x$

$g(x) + 1 = f(x)$

Taking derivatives of both sides, $g'(x) = f'(x)$. Equivalently,

$f'(x) = 2\sec x \cdot \sec x\tan x = 2\sec^2 x\tan x$ and

$g'(x) = 2\tan x \cdot \sec^2 x = 2\sec^2 x\tan x$, which are the same.

179. (a) If $f(-x) = -f(x)$, then

$$\frac{d}{dx}[f(-x)] = \frac{d}{dx}[-f(x)]$$
$$f'(-x)(-1) = -f'(x)$$
$$f'(-x) = f'(x).$$

So, $f'(x)$ is even.

(b) If $f(-x) = f(x)$, then

$$\frac{d}{dx}[f(-x)] = \frac{d}{dx}[f(x)]$$
$$f'(-x)(-1) = f'(x)$$
$$f'(-x) = -f'(x).$$

So, f' is odd.

180. $$|u| = \sqrt{u^2}$$
$$\frac{d}{dx}[|u|] = \frac{d}{dx}\left[\sqrt{u^2}\right] = \frac{1}{2}(u^2)^{-1/2}(2uu')$$
$$= \frac{uu'}{\sqrt{u^2}} = u'\frac{u}{|u|}, \quad u \neq 0$$

181. $g(x) = |3x - 5|$

$$g'(x) = 3\left(\frac{3x-5}{|3x-5|}\right), \quad x \neq \frac{5}{3}$$

182. $f(x) = |x^2 - 9|$

$$f'(x) = 2x\left(\frac{x^2-9}{|x^2-9|}\right), \quad x \neq \pm 3$$

183. $h(x) = |x|\cos x$

$$h'(x) = -|x|\sin x + \frac{x}{|x|}\cos x, \quad x \neq 0$$

184. $f(x) = |\sin x|$

$$f'(x) = \cos x\left(\frac{\sin x}{|\sin x|}\right), x \neq k\pi$$

185. True

186. True

187. (a)

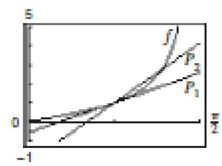
$f(x) = \tan x \qquad f(\pi/4) = 1$

$f'(x) = \sec^2 x \qquad f'(\pi/4) = 2$

$f''(x) = 2\sec^2 x \tan x \qquad f''(\pi/4) = 4$

$$P_1(x) = 2(x - \pi/4) + 1$$
$$P_2(x) = \frac{1}{2}(4)(x - \pi/4)^2 + 2(x - \pi/4) + 1$$
$$= 2(x - \pi/4)^2 + 2(x - \pi/4) + 1$$

(b)



(c) P_2 is a better approximation than P_1.

(d) The accuracy worsens as you move away from $x = \pi/4$.

188. (a)

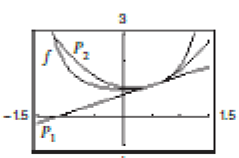
$f(x) = \sec x \qquad f(\pi/6) = \frac{2}{\sqrt{3}}$

$f'(x) = \sec x \tan x \qquad f'(\pi/6) = \frac{2}{3}$

$f''(x) = \sec x(\sec^2 x) + \tan x(\sec x \tan x) \qquad f''(\pi/6) = \frac{10\sqrt{3}}{9}$

$$= \sec^3 x + \sec x \tan^2 x$$
$$P_1(x) = \frac{2}{3}(x - \pi/6) + \frac{2}{\sqrt{3}}$$
$$P_2(x) = \frac{1}{2}\cdot\left(\frac{10}{3\sqrt{3}}\right)\left(x - \frac{\pi}{6}\right)^2 + \frac{2}{3}\left(x - \frac{\pi}{6}\right) + \frac{2}{\sqrt{3}} = \left(\frac{5}{3\sqrt{3}}\right)\left(x - \frac{\pi}{6}\right)^2 + \frac{2}{3}\left(x - \frac{\pi}{6}\right) + \frac{2}{\sqrt{3}}$$

(b)



(c) P_2 is a better approximation than P_1.

(d) The accuracy worsens as you move away from $x = \pi/6$.

189. (a) $f(x) = e^x \qquad f(0) = 1$

$f'(x) = e^x \qquad f'(0) = 1$

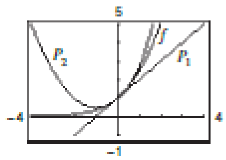
$f''(x) = e^x \qquad f''(0) = 1$

$P_1(x) = 1(x - 0) + 1 = x + 1$

$P_2(x) = \frac{1}{2}(1)(x - 0)^2 + 1(x - 0) + 1$

$= \frac{1}{2}x^2 + x + 1$

(b)



(c) P_2 is a better approximation than P_1.

(d) The accuracy worsens as you move away from $x = 0$.

190. (a) $f(x) = \ln x \qquad f(1) = \ln(1) = 0$

$f'(x) = \frac{1}{x} \qquad f'(1) = 1$

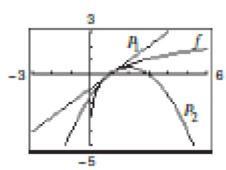
$f''(x) = -1/x^2 \qquad f''(1) = -1$

$P_1(x) = 1(x - 1) + 0 = x - 1$

$P_2(x) = \frac{1}{2}(-1)(x - 1)^2 + 1(x - 1) + 0$

$= -\frac{1}{2}(x + 1)^2 + x - 1$

(b)



(c) P_2 is a better approximation than P_1.

(d) The accuracy worsens as you move away from $x = 0$.

191. $h(x) = xf(x) + g(2x - 5)$

$h'(x) = \left[xf'(x) + f(x)(1)\right] + g'(2x - 5)(2)$

$= xf'(x) = f(x) + 2g'(2x - 5)$

$h'(3) = (3)(4) + (-4) + 2g'(1)$

$= 12 + (-4) + 2(-2)$

$= 4$

So, the answer is A.

192. $h(\theta) = \cos^3 8\theta$

$h'(\theta) = 3(\cos^2 8\theta)(-\sin 8\theta)(8) = -24\cos^2 8\theta \sin 8\theta$

So, the answer is C.

193. (a) $g(x) = (2x^2 + 1)^3$

$g'(x) = 3(2x^2 + 1)^2(4x) = 12x(2x^2 + 1)^2$

$g'(1) = 12(1)\left[2(1)^2 + 1\right]^2 = 108$

So, the slope of the tangent line is 108.

(b) Use $g(1) = 27$ and $m = 108$ to write the equation of the tangent line.

$y - 27 = 108(x - 1)$

$y = 108x - 81$

(c) $g'(x) = 12x(2x^2 + 1)^2 = 0$ when $x = 0$. Because $g(0) = \left[2(0)^2 + 1\right]^3 = 1$, the graph of f has a horizontal tangent at $(0, 1)$.

(d) $g'(x) = 12x(2x^2 + 1)^2$

$g''(x) = 12\left[2(2x^2 + 1)(4x)\right] + (2x^2 + 1)^2(12)$

$= 12(2x^2 + 1)\left[x(2)(4x) + (2x^2 + 1)\right]$

$= 12(2x^2 + 1)(10x^2 + 1)$