## Section 7.2 Integration by Parts

1. $\int xe^{9x}\,dx$

   $u = x,\ dv = e^{9x}\,dx$

2. $\int x^2e^{2x}\,dx$

   $u = x^2,\ dv = e^{2x}\,dx$

3. $\int (\ln x)^2\,dx$

   $u = (\ln x)^2,\ dv = dx$

4. $\int \ln 4x\,dx$

   $u = \ln 4x,\ dv = dx$

5. $\int x\sec^2 x\,dx$

   $u = x,\ dv = \sec^2 x\,dx$

6. $\int x^2\cos x\,dx$

   $u = x^2,\ dv = \cos x\,dx$

7. $dv = x^3\,dx \Rightarrow v = \int x^3\,dx = \dfrac{x^4}{4}$

   $u = \ln x \quad\Rightarrow du = \dfrac{1}{x}dx$

$$\begin{aligned}\int x^3\ln x\,dx &= uv - \int v\,du\\ &= (\ln x)\frac{x^4}{4} - \int\left(\frac{x^4}{4}\right)\frac{1}{x}\,dx\\ &= \frac{x^4}{4}\ln x - \frac{1}{4}\int x^3\,dx\\ &= \frac{x^4}{4}\ln x - \frac{1}{16}x^4 + C\\ &= \frac{1}{16}x^4(4\ln x - 1) + C\end{aligned}$$

8. $dv = e^x\,dx \quad\Rightarrow v = \int e^x\,dx = e^x$

   $u = 4x + 7 \Rightarrow du = 4dx$

$$\begin{aligned}\int (4x+7)e^x\,dx &= uv - \int v\,du\\ &= (4x+7)e^x - \int e^x\,4\,dx\\ &= (4x+7)e^x - 4e^x + C\\ &= (4x+3)e^x + C\end{aligned}$$

9. $dv = \sin 3x\,dx \Rightarrow v = \int \sin 3x\,dx = -\dfrac{1}{3}\cos 3x$

   $u = x \quad\Rightarrow du = dx$

$$\begin{aligned}\int x\sin 3x\,dx &= uv - \int v\,du\\ &= x\left(-\frac{1}{3}\cos 3x\right) - \int -\frac{1}{3}\cos 3x\,dx\\ &= -\frac{x}{3}\cos 3x + \frac{1}{9}\sin 3x + C\end{aligned}$$

10. $dv = \cos 4x\,dx \Rightarrow v = \int \cos 4x\,dx = \dfrac{1}{4}\sin 4x$

    $u = x \quad\Rightarrow du = dx$

$$\begin{aligned}\int x\cos 4x\,dx &= uv - \int v\,du\\ &= x\left(\frac{1}{4}\sin 4x\right) - \int \frac{1}{4}\sin 4x\,dx\\ &= \frac{x}{4}\sin 4x + \frac{1}{16}\cos 4x + C\end{aligned}$$

**11.** $dv = e^{4x}\,dx \quad\Rightarrow\quad v = \int e^{4x}\,dx = \frac{1}{4}e^{4x}\,dx$

$u = x \quad\Rightarrow\quad du = dx$

$$\int xe^{4x}\,dx = x\left(\frac{1}{4}e^{4x}\right) - \int\left(\frac{1}{4}e^{4x}\right)dx$$
$$= \frac{x}{4}e^{4x} - \frac{1}{16}e^{4x} + C$$
$$= \frac{e^{4x}}{16}(4x - 1) + C$$

**12.** $dv = e^{-2x}\,dx \quad\Rightarrow\quad v = \int e^{-2x}\,dx = -\frac{1}{2}e^{-2x}$

$u = 5x \quad\Rightarrow\quad du = 5\,dx$

$$\int \frac{5x}{e^{2x}}\,dx = \int 5xe^{-2x}\,dx$$
$$= (5x)\left(-\frac{1}{2}e^{-2x}\right) - \int\left(-\frac{1}{2}e^{-2x}\right)5\,dx$$
$$= -\frac{5}{2}xe^{-2x} + \frac{5}{2}\int e^{-2x}\,dx$$
$$= -\frac{5}{2}xe^{-2x} - \frac{5}{4}e^{-2x} + C$$
$$= -\frac{5}{4}e^{-2x}(2x + 1) + C$$

**13.** Use integration by parts three times.

(1) $dv = e^x\,dx \Rightarrow v = \int e^x\,dx = e^x$

$u = x^3 \Rightarrow du = 3x^2\,dx$

(2) $dv = e^x\,dx \Rightarrow v = \int e^x\,dx = e^x$

$u = x^2 \Rightarrow du = 2x\,dx$

(3) $dv = e^x\,dx \Rightarrow v = \int e^x\,dx = e^x$

$u = x \Rightarrow du = dx$

$$\int x^3e^x\,dx = x^3e^x - 3\int x^2e^x\,dx = x^3e^x - 3x^2e^x + 6\int xe^x\,dx$$
$$= x^3e^x - 3x^2e^x + 6xe^x - 6e^x + C = e^x\left(x^3 - 3x^2 + 6x - 6\right) + C$$

**14.** $\displaystyle\int \frac{e^{1/t}}{t^2}\,dt = -\int e^{1/t}\left(\frac{-1}{t^2}\right)dt = -e^{1/t} + C$

**15.** $dv = t\,dt \quad\Rightarrow\quad v = \int t\,dt = \frac{t^2}{2}$

$u = \ln(t + 1) \quad\Rightarrow\quad du = \frac{1}{t + 1}\,dt$

$$\int t\ln(t+1)\,dt = \frac{t^2}{2}\ln(t+1) - \frac{1}{2}\int\frac{t^2}{t+1}\,dt$$
$$= \frac{t^2}{2}\ln(t+1) - \frac{1}{2}\int\left(t - 1 + \frac{1}{t+1}\right)dt$$
$$= \frac{t^2}{2}\ln(t+1) - \frac{1}{2}\left[\frac{t^2}{2} - t + \ln(t+1)\right] + C$$
$$= \frac{1}{4}\left[2\left(t^2 - 1\right)\ln|t+1| - t^2 + 2t\right] + C$$

**16.** $dv = x^5\,dx \quad\Rightarrow\quad v = \int x^5\,dx = \frac{1}{6}x^6$

$u = \ln 3x \quad\Rightarrow\quad du = \frac{1}{x}\,dx$

$$\int x^5\ln 3x\,dx = \frac{x^6}{6}\ln 3x - \int\frac{x^6}{6}\left(\frac{1}{x}\right)dx$$
$$= \frac{x^6}{6}\ln 3x - \frac{x^6}{36} + C$$

**17.** Let $u = \ln x$, $du = \frac{1}{x}\,dx$.

$$\int\frac{(\ln x)^2}{x}\,dx = \int(\ln x)^2\left(\frac{1}{x}\right)dx = \frac{(\ln x)^3}{3} + C$$

**18.** $dv = x^{-3}\,dx \quad \Rightarrow \quad v = \int x^{-3}\,dx = -\frac{1}{2}x^{-2}$

$u = \ln x \quad \Rightarrow \quad du = \frac{1}{x}\,dx$

$$\int \frac{\ln x}{x^3}\,dx = -\frac{1}{2}x^{-2}\ln x - \int\left(-\frac{1}{2}x^{-2}\right)\frac{1}{x}\,dx$$
$$= -\frac{1}{2x^2}\ln x + \frac{1}{2}\int x^{-3}\,dx$$
$$= -\frac{1}{2x^2}\ln x + \left(\frac{1}{2}\right)\frac{x^{-2}}{-2} + C$$
$$= -\frac{1}{2x^2}\ln x - \frac{1}{4x^2} + C$$

**19.** $dv = \frac{1}{(2x+1)^2}\,dx \quad \Rightarrow \quad v = \int (2x+1)^{-2}\,dx$

$$= -\frac{1}{2(2x+1)}$$

$u = xe^{2x} \quad \Rightarrow \quad du = \left(2xe^{2x} + e^{2x}\right)dx$

$$= e^{2x}(2x+1)\,dx$$

$$\int \frac{xe^{2x}}{(2x+1)^2}\,dx = -\frac{xe^{2x}}{2(2x+1)} + \int \frac{e^{2x}}{2}\,dx$$
$$= \frac{-xe^{2x}}{2(2x+1)} + \frac{e^{2x}}{4} + C = \frac{e^{2x}}{4(2x+1)} + C$$

**20.** $dv = \frac{x}{\left(x^2+1\right)^2}\,dx \quad \Rightarrow \quad v = \int \left(x^2+1\right)^{-2} x\,dx = -\frac{1}{2\left(x^2+1\right)}$

$u = x^2e^{x^2} \quad \Rightarrow \quad du = \left(2x^3e^{x^2} + 2xe^{x^2}\right)dx = 2xe^{x^2}\left(x^2+1\right)dx$

$$\int \frac{x^3e^{x^2}}{\left(x^2+1\right)^2}\,dx = -\frac{x^2e^{x^2}}{2\left(x^2+1\right)} + \int xe^{x^2}\,dx = -\frac{x^2e^{x^2}}{2\left(x^2+1\right)} + \frac{e^{x^2}}{2} + C = \frac{e^{x^2}}{2\left(x^2+1\right)} + C$$

**21.** $dv = \sqrt{x-5}\,dx \quad \Rightarrow \quad v = \int (x-5)^{1/2}\,dx = \frac{2}{3}(x-5)^{3/2}$

$u = x \quad \Rightarrow \quad du = dx$

$$\int x\sqrt{x-5}\,dx = \tfrac{2}{3}x(x-5)^{3/2} - \int \tfrac{2}{3}(x-5)^{3/2}\,dx$$
$$= \tfrac{2}{3}x(x-5)^{3/2} - \tfrac{4}{15}(x-5)^{5/2} + C$$
$$= \tfrac{2}{15}(x-5)^{3/2}\left[5x - 2(x-5)\right] + C$$
$$= \tfrac{2}{15}(x-5)^{3/2}(3x+10) + C$$

**22.** $dv = (6x+1)^{-1/2}\,dx \quad \Rightarrow \quad v = \int (6x+1)^{-1/2}\,dx = \frac{1}{3}(6x+1)^{1/2}$

$u = x \quad \Rightarrow \quad du = dx$

$$\int \frac{x}{\sqrt{6x+1}}\,dx = \frac{x\sqrt{6x+1}}{3} - \int \frac{\sqrt{6x+1}}{3}\,dx$$
$$= \frac{x\sqrt{6x+1}}{3} - \frac{(6x+1)^{3/2}}{27} + C$$
$$= \frac{\sqrt{6x+1}}{27}\left[9x - (6x+1)\right] + C$$
$$= \frac{\sqrt{6x+1}}{27}(3x-1) + C$$

**23.** $dv = \cos x\,dx \quad \Rightarrow \quad v = \int \cos x\,dx = \sin x$

$u = x \quad \Rightarrow \quad du = dx$

$$\int x\cos x\,dx = x\sin x - \int \sin x\,dx = x\sin x + \cos x + C$$

**24.** $u = t, du = dt, dv = \csc t \cot t\, dt, v = -\csc t$

$$\int t \csc t \cot t\, dt = -t \csc t + \int \csc t\, dt = -t \csc t - \ln\left|\csc t + \cot t\right| + C$$

**25.** Use integration by parts three times.

(1) $u = x^3, du = 3x^2\, dx, dv = \sin x\, dx, v = -\cos x$

$$\int x^3 \sin dx = -x^3 \cos x + 3\int x^2 \cos x\, dx$$

(2) $u = x^2, du = 2x\, dx, dv = \cos x\, dx, v = \sin x$

$$\int x^3 \sin x\, dx = -x^3 \cos x + 3\left(x^2 \sin x - 2\int x \sin x\, dx\right) = -x^3 \cos x + 3x^2 \sin x - 6\int x \sin x\, dx$$

(3) $u = x, du = dx, dv = \sin x\, dx, v = -\cos x$

$$\begin{aligned}\int x^3 \sin x\, dx &= -x^3 \cos x + 3x^2 \sin x - 6\left(-x \cos x + \int \cos x\, dx\right)\\ &= -x^3 \cos x + 3x^2 \sin x + 6x \cos x - 6 \sin x + C\\ &= \left(6x - x^3\right) \cos x + \left(3x^2 - 6\right) \sin x + C\end{aligned}$$

**26.** Use integration by parts twice.

(1) $u = x^2, du = 2x\, dx, dv = \cos x\, dx, v = \sin x$

$$\int x^2 \cos x\, dx = x^2 \sin x - 2\int x \sin x\, dx$$

(2) $u = x, du = dx, dv = \sin x\, dx, v = -\cos x$

$$\int x^2 \cos x\, dx = x^2 \sin x - 2\left(-x \cos x + \int \cos x\, dx\right) = x^2 \sin x + 2x \cos x - 2 \sin x + C$$

**27.** $dv = dx \quad \Rightarrow \quad v = \int dx = x$

$u = \arctan x \quad \Rightarrow \quad du = \dfrac{1}{1 + x^2}\, dx$

$$\begin{aligned}\int \arctan x\, dx &= x \arctan x - \int \frac{x}{1 + x^2}\, dx\\ &= x \arctan x - \frac{1}{2} \ln\left(1 + x^2\right) + C\end{aligned}$$

**28.** $dv = dx \quad \Rightarrow \quad v = \int dx = x$

$u = \arccos x \quad \Rightarrow \quad du = -\dfrac{1}{\sqrt{1 - x^2}}\, dx$

$$\begin{aligned}4\int \arccos x\, dx &= 4\left(x \arccos x + \int \frac{x}{\sqrt{1 - x^2}}\, dx\right)\\ &= 4\left(x \arccos x - \sqrt{1 - x^2}\right) + C\end{aligned}$$

**29.** Use integration by parts twice.

(1) $dv = e^{-3x}\, dx \quad \Rightarrow \quad v = \int e^{-3x}\, dx = -\frac{1}{3}e^{-3x}$

$u = \sin 5x \quad \Rightarrow \quad du = 5 \cos 5x\, dx$

$$\int e^{-3x} \sin 5x\, dx = \sin 5x\left(-\tfrac{1}{3}e^{-3x}\right) - \int\left(-\tfrac{1}{3}e^{-3x}\right) 5 \cos x\, dx = -\tfrac{1}{3}e^{-3x} \sin 5x + \tfrac{5}{3}\int e^{-3x} \cos 5x\, dx$$

(2) $dv = e^{-3x}\, dx \quad \Rightarrow \quad v = \int e^{-3x}\, dx = -\frac{1}{3}e^{-3x}$

$u = \cos 5x \quad \Rightarrow \quad du = -5 \sin 5x\, dx$

$$\begin{aligned}\int e^{-3x} \sin 5x\, dx &= -\tfrac{1}{3}e^{-3x} \sin 5x + \tfrac{5}{3}\left[\left(-\tfrac{1}{3}e^{-3x} \cos 5x\right) - \int\left(-\tfrac{1}{3}e^{-3x}\right)\left(-5 \sin 5x\right) dx\right]\\ &= -\tfrac{1}{3}e^{-3x} \sin 5x - \tfrac{5}{9}e^{-3x} \cos 5x - \tfrac{25}{9}\int e^{-3x} \sin 5x\, dx\end{aligned}$$

$$\left(1 + \tfrac{25}{9}\right)\int e^{-3x} \sin 5x\, dx = -\tfrac{1}{3}e^{-3x} \sin 5x - \tfrac{5}{9}e^{-3x} \cos 5x$$

$$\int e^{-3x} \sin 5x\, dx = \tfrac{9}{34}\left(-\tfrac{1}{3}e^{-3x} \sin 5x - \tfrac{5}{9}e^{-3x} \cos 5x\right) + C = -\tfrac{3}{34}e^{-3x} \sin 5x - \tfrac{5}{34}e^{-3x} \cos 5x + C$$

**30.** Use integration by parts twice.

(1) $dv = e^{4x}\,dx \quad \Rightarrow \quad v = \int e^{4x}\,dx = \frac{1}{4}e^{4x}$

$u = \cos 2x \quad \Rightarrow \quad du = -2\sin 2x\,dx$

$$\int e^{4x}\cos 2x\,dx = \frac{1}{4}e^{4x}\cos 2x - \int \frac{1}{4}e^{4x}(-2\sin 2x)\,dx = \frac{1}{4}e^{4x}\cos 2x + \frac{1}{2}\int e^{4x}\sin 2x\,dx$$

(2) $dv = e^{4x}\,dx \quad \Rightarrow \quad v = \int e^{4x}\,dx = \frac{1}{4}e^{4x}$

$u = \sin 2x \quad \Rightarrow \quad du = 2\cos 2x\,dx$

$$\int e^{4x}\cos 2x\,dx = \frac{1}{4}e^{4x}\cos 2x + \frac{1}{2}\left[\frac{1}{4}e^{4x}\sin 2x - \int \frac{1}{4}e^{4x}(2\cos 2x)\,dx\right]$$
$$= \frac{1}{4}e^{4x}\cos 2x + \frac{1}{8}e^{4x}\sin 2x - \frac{1}{4}\int e^{4x}\cos 2x\,dx + C$$
$$\left(1 + \frac{1}{4}\right)\int e^{4x}\cos 2x\,dx = \frac{1}{4}e^{4x}\cos 2x + \frac{1}{8}e^{4x}\sin 2x + C$$
$$\int e^{4x}\cos 2x\,dx = \frac{1}{5}e^{4x}\cos 2x + \frac{1}{10}e^{4x}\sin 2x + C$$

**31.** $dv = dx \quad \Rightarrow \quad v = x$

$u = \ln x \quad \Rightarrow \quad du = \frac{1}{x}\,dx$

$y' = \ln x$

$$y = \int \ln x\,dx = x\ln x - \int x\left(\frac{1}{x}\right)dx = x\ln x - x + C = x(-1 + \ln x) + C$$

**32.** $dv = dx \quad \Rightarrow \quad v = \int dx = x$

$u = \arctan\frac{x}{2} \quad \Rightarrow \quad du = \frac{1}{1 + (x/2)^2}\left(\frac{1}{2}\right)dx = \frac{2}{4 + x^2}\,dx$

$$y = \int \arctan\frac{x}{2}\,dx = x\arctan\frac{x}{2} - \int \frac{2x}{4 + x^2}\,dx = x\arctan\frac{x}{2} - \ln\left(4 + x^2\right) + C$$

**33.** Use integration by parts twice.

(1) $dv = \frac{1}{\sqrt{3 + 5t}}\,dt \quad \Rightarrow \quad v = \int (3 + 5t)^{-1/2}\,dt = \frac{2}{5}(3 + 5t)^{1/2}$

$u = t^2 \quad \Rightarrow \quad du = 2t\,dt$

$$\int \frac{t^2}{\sqrt{3 + 5t}}\,dt = \frac{2}{5}t^2(3 + 5t)^{1/2} - \int \frac{2}{5}(3 + 5t)^{1/2}2t\,dt = \frac{2}{5}t^2(3 + 5t)^{1/2} - \frac{4}{5}\int t(3 + 5t)^{1/2}\,dt$$

(2) $dv = (3 + 5t)^{1/2}\,dt \quad \Rightarrow \quad v = \int (3 + 5t)^{1/2}\,dt = \frac{2}{15}(3 + 5t)^{3/2}$

$u = t \quad \Rightarrow \quad du = dt$

$$\int \frac{t^2}{\sqrt{3 + 5t}}\,dt = \frac{2}{5}t^2(3 + 5t)^{1/2} - \frac{4}{5}\left[\frac{2}{15}t(3 + 5t)^{3/2} - \int \frac{2}{15}(3 + 5t)^{3/2}\,dt\right]$$
$$= \frac{2}{5}t^2(3 + 5t)^{1/2} - \frac{8}{75}t(3 + 5t)^{3/2} + \frac{8}{75}\int (3 + 5t)^{3/2}\,dt$$
$$= \frac{2}{5}t^2(3 + 5t)^{1/2} - \frac{8}{75}t(3 + 5t)^{3/2} + \frac{16}{1875}(3 + 5t)^{5/2} + C$$
$$= \frac{2}{1875}\sqrt{3 + 5t}\left(375t^2 - 100t(3 + 5t) + 8(3 + 5t)^2\right) + C$$
$$= \frac{2}{625}\sqrt{3 + 5t}\left(25t^2 - 20t + 24\right) + C$$

**34.** Use integration by parts twice.

(1) $dv = \sqrt{x-3}\,dx \quad \Rightarrow \quad v = \int (x-3)^{1/2}\,dx = \frac{2}{3}(x-3)^{3/2}$

$u = x^2 \quad \Rightarrow \quad du = 2x\,dx$

$$\int x^2\sqrt{x-3}\,dx = \tfrac{2}{3}x^2(x-3)^{3/2} - \int \tfrac{2}{3}(x-3)^{3/2}\,2x\,dx$$

$$= \tfrac{2}{3}x^2(x-3)^{3/2} - \tfrac{4}{3}\int (x-3)^{3/2}\,x\,dx$$

(2) $dv = (x-3)^{3/2}\,dx \quad \Rightarrow \quad v = \int (x-3)^{3/2}\,dx = \frac{2}{5}(x-3)^{5/2}$

$u = x \quad \Rightarrow \quad du = dx$

$$\int x^2\sqrt{x-3}\,dx = \tfrac{2}{3}x^2(x-3)^{3/2} - \tfrac{4}{3}\left[\tfrac{2}{5}x(x-3)^{5/2} - \int \tfrac{2}{5}(x-3)^{5/2}\,dx\right]$$

$$= \tfrac{2}{3}x^2(x-3)^{3/2} - \tfrac{8}{15}x(x-3)^{5/2} + \tfrac{8}{15}\left[\tfrac{2}{7}(x-3)^{7/2}\right] + C$$

$$= \tfrac{2}{35}(x-3)^{3/2}\left(5x^2 + 12x + 24\right) + C$$

**35.** $\dfrac{dy}{dx} = xe^{2x}$

$dv = e^{2x}\,dx \quad \Rightarrow \quad v = \int e^{2x}\,dx = \dfrac{1}{2}e^{2x}$

$u = x \quad \Rightarrow \quad du = dx$

$$y = \int xe^{2x}\,dx = x\left(\frac{1}{2}e^{2x}\right) - \int \frac{1}{2}e^{2x}\,dx$$

$$= \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + C$$

When $y(0) = 4$,

$$4 = \frac{1}{2}(0)e^{2(0)} - \frac{1}{4}e^{2(0)} + C$$

$$4 = -\frac{1}{4} + C$$

$$\frac{17}{4} = C.$$

So, the solution is $y = \dfrac{1}{2}xe^{2x} - \dfrac{1}{4}e^{2x} + \dfrac{17}{4}$.

**36.** $\dfrac{dy}{dx} = (x-4)\cos x$

$dv = \cos x\,dx \quad \Rightarrow \quad v = \int \cos x\,dx = \sin x$

$u = x - 4 \quad \Rightarrow \quad du = dx$

$$y = \int (x-4)\cos x\,dx = (x-4)\sin x - \int \sin x\,dx$$

$$= (x-4)\sin x + \cos x + C$$

When $y(0) = 2$,

$$2 = (0-4)\sin 0 + \cos 0 + C$$

$$2 = 0 + 1 + C$$

$$1 = C.$$

So, the solution is $y = (x-4)\sin x + \cos x + 1$.

**37.** $\dfrac{dy}{dx} = \dfrac{x}{4y}\ln x^3$

$$4y\,dy = x\ln x^3\,dx$$

$$\int 4y\,dy = \int x\ln x^3\,dx$$

$dv = x\,dx \quad \Rightarrow \quad v = \int x\,dx = \frac{1}{2}x^2$

$u = \ln x^3 \quad \Rightarrow \quad du = \dfrac{1}{x^3}(3x^2)dx = \dfrac{3}{x}\,dx$

$$2y^2 = \ln x^3 \cdot \frac{1}{2}x^2 - \int \frac{1}{2}x^2\left(\frac{3}{x}\,dx\right)$$

$$2y^2 = \frac{1}{2}x^2\ln x^3 - \int \frac{3}{2}x\,dx$$

$$2y^2 = \frac{1}{2}x^2\ln x^3 - \frac{3}{4}x^2 + C_1$$

$$y^2 = \frac{1}{4}x^2\ln x^3 - \frac{3}{8}x^2 + C$$

When $y(1) = 2$,

$$2^2 = \frac{1}{4}(1)^2\ln(1)^3 - \frac{3}{8}(1)^2 + C$$

$$4 = 0 - \frac{3}{8} + C$$

$$\frac{35}{8} = C.$$

So, the solution is $y^2 = \dfrac{1}{4}x^2\ln x^3 - \dfrac{3}{8}x^2 + \dfrac{35}{8}$.

**38.** $$\frac{dy}{dx} = e^{-x}e^{y}\sec y$$

$$e^{-y}\cos y\,dy = e^{-x}\,dx$$

$$\int e^{-y}\cos y\,dy = \int e^{-x}\,dx$$

$$dv = e^{-y}\,dy \quad\Rightarrow\quad v = \int e^{-y}\,dy = -e^{-y}$$

$$u = \cos y \quad\Rightarrow\quad du = -\sin y\,dy$$

$$(\cos y)\left(-e^{-y}\right) - \int -e^{-y}(-\sin y)\,dy = -e^{-x} + C$$

$$-e^{-y}\cos y - \int e^{-y}\sin y\,dy = -e^{-x} + C$$

$$dv = e^{-y}\,dy \quad\Rightarrow\quad v = \int e^{-y}\,dy = -e^{-y}$$

$$u = \sin y \quad\Rightarrow\quad du = \cos y\,dy$$

$$-e^{-y}\cos y - (\sin y)\left(-e^{-y}\right) + \int -e^{-y}\cos y\,dy = -e^{-x} + C$$

$$-e^{-y}\cos y + e^{-y}\sin y + e^{-x} = -e^{-x} + C$$

$$e^{-y}\sin y - e^{-y}\cos y = -2e^{-x} + C$$

When $y(1) = 0$,

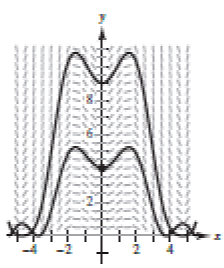
$$e^{0}\sin 0 - e^{0}\cos 0 = -2e^{-1} + C$$

$$-1 = -2e^{-1} + C$$

$$2e^{-1} - 1 = C$$

So, the solution is $e^{-y}\sin y - e^{-y}\cos y = -2e^{-x} + 2e^{-1} - 1$, or $\dfrac{e^{-y}}{2}(\sin y - \cos y) = -e^{-x} + e^{-1} - \dfrac{1}{2}$.

**39.** (a)




(b) $$\frac{dy}{dx} = x\sqrt{y}\cos x, \quad (0, 4)$$

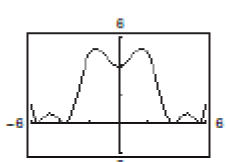
$$\int \frac{dy}{\sqrt{y}} = \int x\cos x\,dx$$

$$\int y^{-1/2}\,dy = \int x\cos x\,dx \qquad (u = x, du = dx, dv = \cos x\,dx, v = \sin x)$$

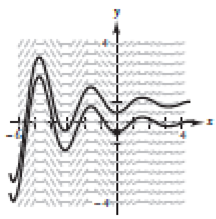
$$2y^{1/2} = x\sin x - \int \sin x\,dx = x\sin x + \cos x + C$$

$$(0, 4):\ 2(4)^{1/2} = 0 + 1 + C \Rightarrow C = 3$$

$$2\sqrt{y} = x\sin x + \cos x + 3$$



**40.** (a)




(b) $\dfrac{dy}{dx} = e^{-x/3}\sin 2x, \quad \left(0, -\dfrac{18}{37}\right)$

$y = \int e^{-x/3}\sin 2x\,dx$

Use integration by parts twice.

(1) $u = \sin 2x,\ du = 2\cos 2x$

$dv = e^{-x/3}\,dx,\ v = -3e^{-x/3}$

$\int e^{-x/3}\sin 2x\,dx = -3e^{-x/3}\sin 2x + \int 6e^{-x/3}\cos 2x\,dx$

(2) $u = \cos 2x,\ du = -2\sin 2x$

$dv = e^{-x/3}\,dx,\ v = -3e^{-x/3}$

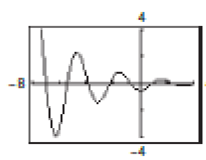
$\int e^{-x/3}\sin 2x\,dx = -3e^{-x/3}\sin 2x + 6\left(-3e^{-x/3}\cos 2x - \int 6e^{-x/3}\sin 2x\,dx\right) + C$

$37\int e^{-x/3}\sin 2x\,dx = -3e^{-x/3}\sin 2x - 18e^{-x/3}\cos 2x + C$

$y = \int e^{-x/3}\sin 2x\,dx = \dfrac{1}{37}\left(-3e^{-x/3}\sin 2x - 18e^{-x/3}\cos 2x\right) + C$

$\left(0, \dfrac{-18}{37}\right)$: $\dfrac{-18}{37} = \dfrac{1}{37}[0 - 18] + C \Rightarrow C = 0$

$y = \dfrac{-1}{37}\left(3e^{-x/3}\sin 2x + 18e^{-x/3}\cos 2x\right)$




**41.** $u = x,\ du = dx,\ dv = e^{x/2}\,dx,\ v = 2e^{x/2}$

$\int xe^{x/2}\,dx = 2xe^{x/2} - \int 2e^{x/2}\,dx = 2xe^{x/2} - 4e^{x/2} + C$

So,

$\int_0^3 xe^{x/2}\,dx = \left[2xe^{x/2} - 4e^{x/2}\right]_0^3 = \left(6e^{3/2} - 4e^{3/2}\right) - (-4) = 4 + 2e^{3/2} \approx 12.963$

**42.** Use integration by parts twice.

(1) $u = x^2,\ du = 2x\,dx,\ dv = e^{-2x}\,dx,$

$v = -\dfrac{1}{2}e^{-2x}$

$\int x^2e^{-2x}\,dx = -\dfrac{1}{2}x^2e^{-2x} - \int\left(-\dfrac{1}{2}e^{-2x}\right)2x\,dx = -\dfrac{1}{2}x^2e^{-2x} + \int xe^{-2x}\,dx$

(2) $u = x,\ du = dx,\ dv = e^{-2x}\,dx,\ v = -\dfrac{1}{2}e^{-2x}$

$\int x^2e^{-2x}\,dx = -\dfrac{1}{2}x^2e^{-2x} + \left(-\dfrac{1}{2}xe^{-2x} - \int -\dfrac{1}{2}e^{-2x}\,dx\right) = -\dfrac{1}{2}x^2e^{-2x} - \dfrac{1}{2}xe^{-2x} - \dfrac{1}{4}e^{-2x} + C = e^{-2x}\left(-\dfrac{1}{2}x^2 - \dfrac{1}{2}x - \dfrac{1}{4}\right)$

So, $\int_0^2 x^2e^{-2x}\,dx = \left[e^{-2x}\left(-\dfrac{1}{2}x^2 - \dfrac{81}{2}x - \dfrac{1}{4}\right)\right]_0^2 = e^{-4}\left(-2 - 1 - \dfrac{1}{4}\right) - \left(-\dfrac{1}{4}\right) = \dfrac{-13}{4e^4} + \dfrac{1}{4} \approx 0.190$

**43.** $u = x,\ du = dx,\ dv = \cos 2x\,dx,\ v = \frac{1}{2}\sin 2x$

$$\int x\cos 2x\,dx = \frac{1}{2}x\sin 2x - \int\frac{1}{2}\sin 2x\,dx = \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x + C$$

So, $\int_0^{\pi/4} x\cos 2x\,dx = \left[\frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x\right]_0^{\pi/4} = \left(\frac{\pi}{8}(1) + 0\right) - \left(0 + \frac{1}{4}\right) = \frac{\pi}{8} - \frac{1}{4} \approx 0.143$

**44.** $dv = \sin 2x\,dx \quad \Rightarrow \quad v = \int \sin 2x\,dx = -\frac{1}{2}\cos 2x$

$u = x \quad \Rightarrow \quad du = dx$

$$\begin{aligned}\int x\sin 2x\,dx &= -\frac{1}{2}x\cos 2x + \frac{1}{2}\int\cos 2x\,dx\\ &= -\frac{1}{2}x\cos 2x + \frac{1}{4}\sin 2x + C\\ &= \frac{1}{4}(\sin 2x - 2x\cos 2x) + C\end{aligned}$$

So,

$$\int_0^{\pi} x\sin 2x\,dx = \left[\frac{1}{4}(\sin 2x - 2x\cos 2x)\right]_0^{\pi} = -\frac{\pi}{2}.$$

**45.** $u = \arccos x,\ du = -\frac{1}{\sqrt{1-x^2}}\,dx,\ dv = dx,\ v = x$

$$\begin{aligned}\int\arccos x\,dx &= x\arccos x + \int\frac{x}{\sqrt{1-x^2}}\,dx\\ &= x\arccos x - \sqrt{1-x^2} + C\end{aligned}$$

So,

$$\begin{aligned}\int_0^{1/2}\arccos x &= \left[x\arccos x - \sqrt{1-x^2}\right]_0^{1/2}\\ &= \frac{1}{2}\arccos\frac{1}{2} - \sqrt{\frac{3}{4}} + 1\\ &= \frac{\pi}{6} - \frac{\sqrt{3}}{2} + 1 \approx 0.658.\end{aligned}$$

**46.** $dv = x\,dx \quad \Rightarrow \quad v = \int x\,dx = \frac{x^2}{2}$

$u = \arcsin x^2 \quad \Rightarrow \quad du = \frac{2x}{\sqrt{1-x^4}}\,dx$

$$\begin{aligned}\int x\arcsin x^2\,dx &= \frac{x^2}{2}\arcsin x^2 - \int\frac{x^3}{\sqrt{1-x^4}}\,dx\\ &= \frac{x^2}{2}\arcsin x^2 + \frac{1}{4}(2)(1-x^4)^{1/2} + C\\ &= \frac{1}{2}\left(x^2\arcsin x^2 + \sqrt{1-x^4}\right) + C\end{aligned}$$

So, $\int_0^1 x\arcsin x^2\,dx = \frac{1}{2}\left[x^2\arcsin x^2 + \sqrt{1-x^4}\right]_0^1 = \frac{1}{4}(\pi - 2).$

**47.** Use integration by parts twice.

(1) $dv = e^x\,dx \quad \Rightarrow \quad v = \int e^x\,dx = e^x$ $\qquad$ (2) $dv = e^x\,dx \quad \Rightarrow \quad v = \int e^x\,dx = e^x$

$u = \sin x \quad \Rightarrow \quad du = \cos x\,dx$ $\qquad$ $u = \cos x \quad \Rightarrow \quad du = -\sin x\,dx$

$$\int e^x\sin x\,dx = e^x\sin x - \int e^x\cos x\,dx = e^x\sin x - e^x\cos x - \int e^x\sin x\,dx$$

$$2\int e^x\sin x\,dx = e^x(\sin x - \cos x)$$

$$\int e^x\sin x\,dx = \frac{e^x}{2}(\sin x - \cos x) + C$$

So, $\int_0^1 e^x\sin x\,dx = \left[\frac{e^x}{2}(\sin x - \cos x)\right]_0^1 = \frac{e}{2}(\sin 1 - \cos 1) + \frac{1}{2} = \frac{e(\sin 1 - \cos 1) + 1}{2} \approx 0.909.$

**48.** $u = \ln(4 + x^2),\ du = \dfrac{2x}{4 + x^2}\,dx,\ dv = dx,\ v = x$

$$\begin{aligned}\int \ln(4 + x^2)\,dx &= x\ln(4 + x^2) - \int \frac{2x^2}{4 + x^2}\,dx \\ &= x\ln(4 + x^2) - 2\int\left(1 - \frac{4}{4 + x^2}\right)dx \\ &= x\ln(4 + x^2) - 2\left(x - \frac{4}{2}\arctan\frac{x}{2}\right) + C \\ &= x\ln(4 + x^2) - 2x + 4\arctan\frac{x}{2} + C\end{aligned}$$

So, $\int_0^1 \ln(4 + x^2)\,dx = \left[x\ln(4 + x^2) - 2x + 4\arctan\dfrac{x}{2}\right]_0^1 = \left(\ln 5 - 2 + 4\arctan\dfrac{1}{2}\right) \approx 1.464.$

**49.** $dv = x\,dx,\ v = \dfrac{x^2}{2},\ u = \operatorname{arcsec} x,\ du = \dfrac{1}{x\sqrt{x^2 - 1}}\,dx$

$$\int x \operatorname{arcsec} x\,dx = \frac{x^2}{2}\operatorname{arcsec} x - \int \frac{x^2/2}{x\sqrt{x^2 - 1}}\,dx = \frac{x^2}{2}\operatorname{arcsec} x - \frac{1}{4}\int \frac{2x}{\sqrt{x^2 - 1}}\,dx = \frac{x^2}{2}\operatorname{arcsec} x - \frac{1}{2}\sqrt{x^2 - 1} + C$$

So,

$$\int_2^4 x \operatorname{arcsec} x\,dx = \left[\frac{x^2}{2}\operatorname{arcsec} x - \frac{1}{2}\sqrt{x^2 - 1}\right]_2^4 = \left(8\operatorname{arcsec} 4 - \frac{\sqrt{15}}{2}\right) - \left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right) = 8\operatorname{arcsec} 4 - \frac{\sqrt{15}}{2} + \frac{\sqrt{3}}{2} - \frac{2\pi}{3} \approx 7.380.$$

**50.** $u = x,\ du = dx,\ dv = \sec^2 2x\,dx,\ v = \dfrac{1}{2}\tan 2x$

$$\int x\sec^2 2x\,dx = \frac{1}{2}x\tan 2x - \int \frac{1}{2}\tan 2x\,dx = \frac{1}{2}x\tan 2x + \frac{1}{4}\ln|\cos 2x| + C$$

So,

$$\int_0^{\pi/8} x\sec^2 2x\,dx = \left[\frac{1}{2}x\tan 2x + \frac{1}{4}\ln|\cos 2x|\right]_0^{\pi/8} = \frac{\pi}{16}(1) + \frac{1}{4}\ln\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{16} - \frac{1}{8}\ln(2) \approx 0.1097.$$

**51.**
$$\begin{aligned}\int x^2e^{2x}\,dx &= x^2\left(\tfrac{1}{2}e^{2x}\right) - (2x)\left(\tfrac{1}{4}e^{2x}\right) + 2\left(\tfrac{1}{8}e^{2x}\right) + C \\ &= \tfrac{1}{2}x^2e^{2x} - \tfrac{1}{2}xe^{2x} + \tfrac{1}{4}e^{2x} + C \\ &= \tfrac{1}{4}e^{2x}(2x^2 - 2x + 1) + C\end{aligned}$$

| Alternate signs | $u$ and its derivatives | $v'$ and its antiderivatives |
|---|---|---|
| + | $x^2$ | $e^{2x}$ |
| − | $2x$ | $\frac{1}{2}e^{2x}$ |
| + | $2$ | $\frac{1}{4}e^{2x}$ |
| − | $0$ | $\frac{1}{8}e^{2x}$ |

52. $\int x^3 e^{-2x} dx = x^3\left(-\frac{1}{2}e^{-2x}\right) - 3x^2\left(\frac{1}{4}e^{-2x}\right) + 6x\left(-\frac{1}{8}e^{-2x}\right) - 6\left(\frac{1}{16}e^{-2x}\right) + C$

$$= -\frac{1}{8}e^{-2x}\left(4x^3 + 6x^2 + 6x + 3\right) + C$$

| Alternate signs | $u$ and its derivatives | $v'$ and its antiderivatives |
|---|---|---|
| + | $x^3$ | $e^{-2x}$ |
| − | $3x^2$ | $-\frac{1}{2}e^{-2x}$ |
| + | $6x$ | $\frac{1}{4}e^{-2x}$ |
| − | $6$ | $-\frac{1}{8}e^{-2x}$ |
| + | $0$ | $\frac{1}{16}e^{-2x}$ |

53. $\int x^3 \sin x \, dx = x^3(-\cos x) - 3x^2(-\sin x) + 6x \cos x - 6 \sin x + C$

$$= -x^3 \cos x + 3x^2 \sin x + 6x \cos x - 6 \sin x + C$$

$$= \left(3x^2 - 6\right)\sin x - \left(x^3 - 6x\right)\cos x + C$$

| Alternate signs | $u$ and its derivatives | $v'$ and its antiderivatives |
|---|---|---|
| + | $x^3$ | $\sin x$ |
| − | $3x^2$ | $-\cos x$ |
| + | $6x$ | $-\sin x$ |
| − | $6$ | $\cos x$ |
| + | $0$ | $\sin x$ |

54. $\int x^3 \cos 2x \, dx = x^3\left(\frac{1}{2}\sin 2x\right) - 3x^2\left(-\frac{1}{4}\cos 2x\right) + 6x\left(-\frac{1}{8}\sin 2x\right) - 6\left(\frac{1}{16}\cos 2x\right) + C$

$$= \frac{1}{2}x^3 \sin 2x + \frac{3}{4}x^2 \cos 2x - \frac{3}{4}x \sin 2x - \frac{3}{8}\cos 2x + C$$

$$= \frac{1}{8}\left(4x^3 \sin 2x + 6x^2 \cos 2x - 6x \sin 2x - 3 \cos 2x\right) + C$$

| Alternate signs | $u$ and its derivatives | $v'$ and its antiderivatives |
|---|---|---|
| + | $x^3$ | $\cos 2x$ |
| − | $3x^2$ | $\frac{1}{2}\sin 2x$ |
| + | $6x$ | $-\frac{1}{4}\cos 2x$ |
| − | $6$ | $-\frac{1}{8}\sin 2x$ |
| + | $0$ | $\frac{1}{16}\cos 2x$ |

55. $\int x \sec^2 x \, dx = x \tan x + \ln\left|\cos x\right| + C$

| Alternate signs | $u$ and its derivatives | $v'$ and its antiderivatives |
|---|---|---|
| + | $x$ | $\sec^2 x$ |
| − | $1$ | $\tan x$ |
| + | $0$ | $-\ln\left|\cos x\right|$ |

**56.** $\int x^2(x-2)^{3/2}\,dx = \frac{2}{5}x^2(x-2)^{5/2} - \frac{8}{35}x(x-2)^{7/2} + \frac{16}{315}(x-2)^{9/2} + C = \frac{2}{315}(x-2)^{5/2}(35x^2 + 40x + 32) + C$

| Alternate signs | $u$ and its derivatives | $v'$ and its antiderivatives |
|---|---|---|
| + | $x^2$ | $(x-2)^{3/2}$ |
| − | $2x$ | $\frac{2}{5}(x-2)^{5/2}$ |
| + | 2 | $\frac{4}{35}(x-2)^{7/2}$ |
| − | 0 | $\frac{8}{315}(x-2)^{9/2}$ |

**57.** $u = \sqrt{x} \Rightarrow u^2 = x \Rightarrow 2u\,du = dx$

$\int \sin\sqrt{x}\,dx = \int \sin u(2u\,du) = 2\int u\sin u\,du$

Integration by parts:

$w = u,\ dw = du,\ dv = \sin u\,du,\ v = -\cos u$

$2\int u\sin u\,du = 2\left(-u\cos u + \int \cos u\,du\right) = 2(-u\cos u + \sin u) + C = 2\left(-\sqrt{x}\cos\sqrt{x} + \sin\sqrt{x}\right) + C$

**58.** $u = x^2,\ du = 2x\,dx$

$\int 2x^3\cos(x^2)\,dx = \int x^2(\cos x^2)(2x)\,dx = \int u\cos u\,du$

Integration by parts:

$w = u,\ dw = du,\ dv = \cos u\,du,\ v = \sin u$

$\int u\cos u\,du = u\sin u - \int \sin u\,du = u\sin u + \cos u + C = x^2\sin x^2 + \cos x^2 + C$

**59.** $u = x^2,\ du = 2x\,dx$

$\int x^5 e^{x^2}\,dx = \frac{1}{2}\int e^{x^2}x^4\,2x\,dx = \frac{1}{2}\int e^u u^2\,du$

Integration by parts twice.

(1) $w = u^2,\ dw = 2u\,du,\ dv = e^u\,du,\ v = e^u$

$$\frac{1}{2}\int e^u u^2\,du = \frac{1}{2}\left[u^2e^u - \int 2ue^u\,du\right] = \frac{1}{2}u^2e^u - \int ue^u\,du$$

(2) $w = u,\ dw = du,\ dv = e^u\,du,\ v = e^u$

$$\frac{1}{2}\int e^u u^2\,du = \frac{1}{2}u^2e^u - \left(ue^u - \int e^u du\right) = \frac{1}{2}u^2e^u - ue^u + e^u + C = \frac{1}{2}x^4e^{x^2} - x^2e^{x^2} + e^{x^2} + C = \frac{e^{x^2}}{2}(x^4 - 2x^2 + 2) + C$$

**60.** Let $u = \sqrt{2x},\ u^2 = 2x,\ 2u\,du = 2dx$.

$\int e^{\sqrt{2x}}\,dx = \int e^u(u\,du)$

Now use integration by parts.

$dv = e^u\,du \Rightarrow v = \int e^u\,du = e^u$

$w = u \Rightarrow dw = du$

$$\int e^{\sqrt{2x}}\,dx = ue^u - \int e^u\,du = ue^u - e^u + C = \sqrt{2x}\,e^{\sqrt{2x}} - e^{\sqrt{2x}} + C$$

**61.** The expression $\left(\frac{x^5}{5}\right)\left(\frac{1}{x}\right)$ should be simplified to $\frac{x^4}{5}$.

$$\int x^4\ln x\,dx = \frac{x^5}{5}\ln x - \int\left(\frac{x^5}{5}\right)\left(\frac{1}{x}\right)dx = \frac{x^5}{5}\ln x - \frac{1}{5}\int x^4\,dx = \frac{x^5}{5}\ln x - \frac{1}{25}x^5 + C$$

**62.** The negative sign was not carried through in the third step.

$$\begin{aligned}\int 2x \arctan x\, dx &= x^2 \arctan x - \int \frac{x^2}{1+x^2}\, dx \\ &= x^2 \arctan x - \int \frac{x^2+1-1}{1+x^2}\, dx \\ &= x^2 \arctan x - \int \frac{x^2+1}{1+x^2}\, dx + \int \frac{1}{1+x^2}\, dx \\ &= x^2 \arctan x - \int dx + \int \frac{1}{1+x^2}\, dx \\ &= x^2 \arctan x - x + \arctan x + C\end{aligned}$$

**63.** (a) Integration by parts is based on the Product Rule.

(b) Answers will vary. Sample answer: You want $dv$ to be the most complicated portion of the integrand.

**64.** In order for the integration by parts technique to be efficient, you want $dv$ to be the most complicated portion of the integrand, and you want $u$ to be the portion of the integrand whose derivative is a function simpler than $u$. Suppose you let $u = \sin x$ and $dv = x\, dx$. Then $du = \cos x\, dx$ and $v = x^2/2$. So

$$\int x \sin x\, dx = uv - \int v\, du = \frac{x^2}{2}\sin x - \int \frac{x^2}{2}\cos x\, dx,$$

which is a more complicated integral than the original one.

**65.** (a) No; Substitution

(b) Yes; $u = \ln x$, $dv = x\, dx$

(c) Yes; $u = x^2$, $dv = e^{-3x}\, dx$

(d) No; Substitution

(e) Yes; Let $u = x$ and

$$dv = \frac{1}{\sqrt{x+1}}\, dx.$$

(Substitution also works. Let $u = \sqrt{x+1}$.)

(f) No; Substitution

**66.** (a) The slope of $f$ at $x = 2$ is approximately 1.4 because $f'(2) \approx 1.4$.

(b) $f' < 0$ on $(0, 1) \Rightarrow f$ is decreasing on $(0, 1)$.

$f' > 0$ on $(1, \infty) \Rightarrow f$ is increasing on $(1, \infty)$.

**67.** (a) $dv = \dfrac{x}{\sqrt{4+x^2}}\, dx \quad \Rightarrow \quad v = \int (4+x^2)^{-1/2} x\, dx = \sqrt{4+x^2}$

$u = x^2 \quad \Rightarrow \quad du = 2x\, dx$

$$\int \frac{x^3}{\sqrt{4+x^2}}\, dx = x^2\sqrt{4+x^2} - 2\int x\sqrt{4+x^2}\, dx = x^2\sqrt{4+x^2} - \frac{2}{3}(4+x^2)^{3/2} + C = \frac{1}{3}\sqrt{4+x^2}(x^2-8) + C$$

(b) $u = 4 + x^2 \Rightarrow x^2 = u - 4$ and $2x\, dx = du \Rightarrow x\, dx = \frac{1}{2}du$

$$\begin{aligned}\int \frac{x^3}{\sqrt{4+x^2}}\, dx &= \int \frac{x^2}{\sqrt{4+x^2}} x\, dx = \int \left(\frac{u-4}{\sqrt{u}}\right)\frac{1}{2}\, du \\ &= \frac{1}{2}\int \left(u^{1/2} - 4u^{-1/2}\right) du = \frac{1}{2}\left(\frac{2}{3}u^{3/2} - 8u^{1/2}\right) + C \\ &= \frac{1}{3}u^{1/2}(u - 12) + C \\ &= \frac{1}{3}\sqrt{4+x^2}\left[(4+x^2) - 12\right] + C = \frac{1}{3}\sqrt{4+x^2}(x^2-8) + C\end{aligned}$$

**68.** (a) $dv = \sqrt{4-x}\,dx \quad \Rightarrow \quad v = \int (4-x)^{1/2}\,dx$

$$= -\tfrac{2}{3}(4-x)^{3/2}$$

$u = x \quad \Rightarrow \quad du = dx$

$$\int x\sqrt{4-x}\,dx = -\tfrac{2}{3}x(4-x)^{3/2} + \tfrac{2}{3}\int (4-x)^{3/2}\,dx$$
$$= -\tfrac{2}{3}x(4-x)^{3/2} - \tfrac{4}{15}(4-x)^{5/2} + C$$
$$= -\tfrac{2}{15}(4-x)^{3/2}\left[5x + 2(4-x)\right] + C = -\tfrac{2}{15}(4-x)^{3/2}(3x+8) + C$$

(b) $u = 4 - x \Rightarrow x = 4 - u$ and $dx = -du$

$$\int x\sqrt{4-x}\,dx = -\int (4-u)\sqrt{u}\,du$$
$$= -\int \left(4u^{1/2} - u^{3/2}\right) du$$
$$= -\tfrac{8}{3}u^{3/2} + \tfrac{2}{5}u^{5/2} + C$$
$$= -\tfrac{2}{15}u^{3/2}(20 - 3u) + C$$
$$= -\tfrac{2}{15}(4-x)^{3/2}\left[20 - 3(4-x)\right] + C$$
$$= -\tfrac{2}{15}(4-x)^{3/2}(3x+8) + C$$

**69.** $n = 0$: $\int \ln x\,dx = x(\ln x - 1) + C$

$n = 1$: $\int x \ln x\,dx = \dfrac{x^2}{4}(2\ln x - 1) + C$

$n = 2$: $\int x^2 \ln x\,dx = \dfrac{x^3}{9}(3\ln x - 1) + C$

$n = 3$: $\int x^3 \ln x\,dx = \dfrac{x^4}{16}(4\ln x - 1) + C$

$n = 4$: $\int x^4 \ln x\,dx = \dfrac{x^5}{25}(5\ln x - 1) + C$

In general, $\int x^n \ln x\,dx = \dfrac{x^{n+1}}{(n+1)^2}\left[(n+1)\ln x - 1\right] + C.$

**70.** $n = 0$: $\int e^x\,dx = e^x + C$

$n = 1$: $\int xe^x\,dx = xe^x - e^x + C = xe^x - \int e^x\,dx$

$n = 2$: $\int x^2e^x\,dx = x^2e^x - 2xe^x + 2e^x + C = x^2e^x - 2\int xe^x\,dx$

$n = 3$: $\int x^3e^x\,dx = x^3e^x - 3x^2e^x + 6xe^x - 6e^x + C = x^3e^x - 3\int x^2e^x\,dx$

$n = 4$: $\int x^4e^x\,dx = x^4e^x - 4x^3e^x + 12x^2e^x - 24xe^x + 24e^x + C = x^4e^x - 4\int x^3e^x\,dx$

In general, $\int x^ne^x\,dx = x^ne^x - n\int x^{n-1}e^x\,dx.$

**71** $dv = \sin x\,dx \quad \Rightarrow \quad v = -\cos x$

$u = x^n \quad \Rightarrow \quad du = nx^{n-1}\,dx$

$\int x^n \sin x\,dx = -x^n \cos x + n\int x^{n-1}\cos x\,dx$

**72.** $dv = \cos x\,dx \quad \Rightarrow \quad v = \sin x$

$u = x^n \quad \Rightarrow \quad du = nx^{n-1}\,dx$

$\int x^n \cos x\,dx = x^n \sin x - n\int x^{n-1}\sin x\,dx$

73. $dv = x^n\,dx \;\Rightarrow\; v = \dfrac{x^{n+1}}{n+1}$

$u = \ln x \;\Rightarrow\; du = \dfrac{1}{x}\,dx$

$$\int x^n \ln x\,dx = \frac{x^{n+1}}{n+1}\ln x - \int \frac{x^n}{n+1}\,dx = \frac{x^{n+1}}{n+1}\ln x - \frac{x^{n+1}}{(n+1)^2} + C = \frac{x^{n+1}}{(n+1)^2}\left[(n+1)\ln x - 1\right] + C$$

74. $dv = e^{ax}\,dx \;\Rightarrow\; v = \dfrac{1}{a}e^{ax}$

$u = x^n \;\Rightarrow\; du = nx^{n-1}dx$

$$\int x^n e^{ax}\,dx = \frac{x^n e^{ax}}{a} - \frac{n}{a}\int x^{n-1}e^{ax}\,dx$$

75. Use integration by parts twice.

(1) $dv = e^{ax}\,dx \;\Rightarrow\; v = \dfrac{1}{a}e^{ax}$ (2) $dv = e^{ax}\,dx \;\Rightarrow\; v = \dfrac{1}{a}e^{ax}$

$u = \sin bx \;\Rightarrow\; du = b\cos bx\,dx$ $u = \cos bx \;\Rightarrow\; du = -b\sin bx\,dx$

$$\int e^{ax}\sin bx\,dx = \frac{e^{ax}\sin bx}{a} - \frac{b}{a}\int e^{ax}\cos bx\,dx$$

$$= \frac{e^{ax}\sin bx}{a} - \frac{b}{a}\left(\frac{e^{ax}\cos bx}{a} + \frac{b}{a}\int e^{ax}\sin bx\,dx\right) = \frac{e^{ax}\sin bx}{a} - \frac{b}{a^2}e^{ax}\cos bx - \frac{b^2}{a^2}\int e^{ax}\sin bx\,dx$$

Therefore, $\left(1 + \dfrac{b^2}{a^2}\right)\displaystyle\int e^{ax}\sin bx\,dx = \dfrac{e^{ax}(a\sin bx - b\cos bx)}{a^2}$

$$\int e^{ax}\sin bx\,dx = \frac{e^{ax}(a\sin bx - b\cos bx)}{a^2+b^2} + C.$$

76. Use integration by parts twice.

(1) $dv = e^{ax}\,dx \;\Rightarrow\; v = \dfrac{1}{a}e^{ax}$ (2) $dv = e^{ax}\,dx \;\Rightarrow\; v = \dfrac{1}{a}e^{ax}$

$u = \cos bx \;\Rightarrow\; du = -b\sin bx$ $u = \sin bx \;\Rightarrow\; du = b\cos bx$

$$\int e^{ax}\cos bx\,dx = \frac{e^{ax}\cos bx}{a} + \frac{b}{a}\int e^{ax}\sin bx\,dx = \frac{e^{ax}\cos bx}{a} + \frac{b}{a}\left(\frac{e^{ax}\sin bx}{a} - \frac{b}{a}\int e^{ax}\cos bx\,dx\right)$$

$$= \frac{e^{ax}\cos bx}{a} + \frac{be^{ax}\sin bx}{a^2} - \frac{b^2}{a^2}\int e^{ax}\cos bx\,dx$$

Therefore, $\left(1 + \dfrac{b^2}{a^2}\right)\displaystyle\int e^{ax}\cos bx\,dx = \dfrac{e^{ax}(a\cos bx + b\sin bx)}{a^2}$

$$\int e^{ax}\cos bx\,dx = \frac{e^{ax}(a\cos bx + b\sin bx)}{a^2+b^2} + C.$$

77. $n = 2$ (Use formula in Exercise 71.)

$\displaystyle\int x^2\sin x\,dx = -x^2\cos x + 2\int x\cos x\,dx$ (Use formula in Exercise 72 with $n = 1$.)

$$= -x^2\cos x + 2\left[x\sin x - \int \sin x\,dx\right]$$

$$= -x^2\cos x + 2x\sin x + 2\cos x + C$$

**78.** $n = 2$ (Use formula in Exercise 72.)

$$\int x^2 \cos x\,dx = x^2 \sin x - 2\int x \sin x\,dx \qquad \text{(Use formula in Exercise 71 with } n = 1.)$$

$$= x^2 \sin x - 2\left(-x\cos x + \int \cos x\,dx\right) = x^2 \sin x + 2x\cos x - 2\sin x + C$$

**79.** $n = 5$ (Use formula in Exercise 73.)

$$\int x^5 \ln x\,dx = \frac{x^6}{6^2}(-1 + 6\ln x) + C = \frac{x^6}{36}(-1 + 6\ln x) + C$$

**80.** $n = 3, a = 2$ (Use formula in Exercise 74 three times.)

$$\int x^3 e^{2x}\,dx = \frac{x^3e^{2x}}{2} - \frac{3}{2}\int x^2 e^{2x}\,dx \qquad (n = 3, a = 2)$$

$$= \frac{x^3e^{2x}}{2} - \frac{3}{2}\left[\frac{x^2e^{2x}}{2} - \int xe^{2x}\,dx\right] \qquad (n = 2, a = 2)$$

$$= \frac{x^3e^{2x}}{2} - \frac{3x^2e^{2x}}{4} + \frac{3}{2}\left[\frac{xe^{2x}}{2} - \frac{1}{2}\int e^{2x}\,dx\right]$$

$$= \frac{x^3e^{2x}}{2} - \frac{3x^2e^{2x}}{4} + \frac{3xe^{2x}}{4} - \frac{3e^{2x}}{8} + C \qquad (n = 1, a = 2)$$

$$= \frac{e^{2x}}{8}\left(4x^3 - 6x^2 + 6x - 3\right) + C$$

**81.** $a = -3, b = 4$ (Use formula in Exercise 75.)

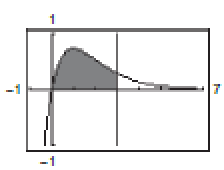
$$\int e^{-3x}\sin 4x\,dx = \frac{e^{-3x}(-3\sin 4x - 4\cos 4x)}{(-3)^2 + 4^2} + C$$

$$= \frac{-e^{-3x}(3\sin 4x + 4\cos 4x)}{25} + C$$

**82.** $a = 2, b = 3$ (Use formula in Exercise 76.)

$$\int e^{2x}\cos 3x\,dx = \frac{e^{2x}(2\cos 3x + 3\sin 3x)}{13} + C$$

**83.**




$$dv = e^{-x}\,dx \implies v = \int e^{-x}\,dx = -e^{-x}$$

$$u = 2x \implies du = 2\,dx$$

$$\int 2xe^{-x}\,dx = 2x\left(-e^{-x}\right) - \int -2e^{-x}\,dx$$

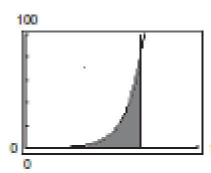
$$= -2xe^{-x} - 2e^{-x} + C$$

$$A = \int_0^3 2xe^{-x}\,dx = \left[-2xe^{-x} - 2e^{-x}\right]_0^3$$

$$= \left(-6e^{-3} - 2e^{-3}\right) - (-2)$$

$$= 2 - 8e^{-3} \approx 1.602$$

**84.**




$$A = \int_0^2 \frac{1}{10}xe^{3x}\,dx = \frac{1}{10}\int_0^2 xe^{3x}\,dx$$

$$dv = e^{3x}\,dx \implies v = \int e^{3x}\,dx = \frac{1}{3}e^{3x}$$

$$u = x \implies du = dx$$

$$\frac{1}{10}\int xe^{3x}\,dx = \frac{1}{10}\left[\frac{x}{3}e^{3x} - \int \frac{1}{3}e^{3x}\,dx\right]$$

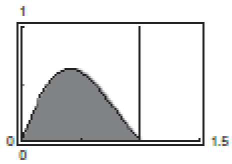
$$= \frac{x}{30}e^{3x} - \frac{1}{90}e^{3x} + C$$

$$A = \left[\frac{x}{30}e^{3x} - \frac{1}{90}e^{3x}\right]_0^2$$

$$= \left(\frac{1}{15}e^6 - \frac{1}{90}e^6\right) + \frac{1}{90}$$

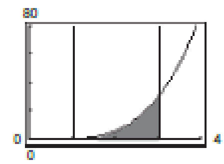
$$= \frac{1}{90}\left(5e^6 + 1\right) \approx 22.424$$

**85.**




$$A = \int_0^1 e^{-x} \sin \pi x \, dx$$
$$= \left[ \frac{e^{-x}(-\sin \pi x - \pi \cos \pi x)}{1 + \pi^2} \right]_0^1$$
$$= \frac{1}{1 + \pi^2}\left( \frac{\pi}{e} + \pi \right)$$
$$= \frac{\pi}{1 + \pi^2}\left( \frac{1}{e} + 1 \right)$$
$$\approx 0.395 \quad \text{(See Exercise 71.)}$$

**86.**




$$dv = x^3\, dx \quad \Rightarrow \quad v = \int x^3\, dx = \frac{x^4}{4}$$
$$u = \ln x \quad \Rightarrow \quad du = \frac{1}{x}\, dx$$
$$\int x^3 \ln x\, dx = \frac{x^4}{4} \ln x - \int \frac{x^4}{4}\left( \frac{1}{x}\, dx \right)$$
$$= \frac{x^4}{4} \ln x - \int \frac{x^3}{4}\, dx$$
$$= \frac{x^4}{4} \ln x - \frac{x^4}{16} + C$$

$$A = \int_1^3 x^3 \ln x\, dx = \left[ \frac{x^4}{4} \ln x - \frac{x^4}{16} \right]_1^3$$
$$= \left( \frac{81}{4} \ln 3 - \frac{81}{16} \right) + \frac{1}{16}$$
$$= \frac{81}{4} \ln 3 - 5 \approx 17.247$$

**87.** Average value $= \dfrac{1}{\pi}\displaystyle\int_0^{\pi} e^{-4t}(\cos 2t + 5 \sin 2t)\, dt$

$$= \frac{1}{\pi}\left[ e^{-4t}\left( \frac{-4 \cos 2t + 2 \sin 2t}{20} \right) + 5e^{-4t}\left( \frac{-4 \sin 2t - 2 \cos 2t}{20} \right) \right]_0^{\pi} \quad \text{(From Exercises 71 and 72)}$$
$$= \frac{7}{10\pi}\left(1 - e^{-4\pi}\right) \approx 0.223$$

**88.** (a) Average $= \displaystyle\int_1^2 (1.6t \ln t + 1)\, dt = \left[ 0.8t^2 \ln t - 0.4t^2 + t \right]_1^2 = 3.2(\ln 2) - 0.2 \approx 2.018$

(b) Average $= \displaystyle\int_3^4 (1.6t \ln t + 1)\, dt = \left[ 0.8t^2 \ln t - 0.4t^2 + t \right]_3^4 = 12.8(\ln 4) - 7.2(\ln 3) - 1.8 \approx 8.035$

**89.** $c(t) = 100{,}000 + 4000t, r = 5\%, t_1 = 10$

$$P = \int_0^{10} (100{,}000 + 4000t)e^{-0.05t}\, dt = 4000\int_0^{10} (25 + t)e^{-0.05t}\, dt$$

Let $u = 25 + t, dv = e^{-0.05t}\, dt, du = dt, v = -\dfrac{100}{5}e^{-0.05t}$.

$$P = 4000\left\{ \left[ (25 + t)\left( -\frac{100}{5}e^{-0.05t} \right) \right]_0^{10} + \frac{100}{5}\int_0^{10} e^{-0.05t} dt \right\} = 4000\left\{ \left[ (25 + t)\left( -\frac{100}{5}e^{-0.05t} \right) \right]_0^{10} - \left[ \frac{10{,}000}{25}e^{-0.05t} \right]_0^{10} \right\} \approx \$931,$$

**90.** $c(t) = 30{,}000 + 500t, r = 7\%, t_1 = 5$

$$P\int_0^5 (30{,}000 + 500t)e^{-0.07t}\,dt = 500\int_0^5 (60 + t)e^{-0.07t}\,dt$$

Let $u = 60 + t, dv = e^{-0.07t}\,dt, du = dt, v = -\dfrac{100}{7}e^{-0.07t}.$

$$P = 500\left\{\left[(60+t)\left(-\frac{100}{7}e^{-0.07t}\right)\right]_0^5 + \frac{100}{7}\int_0^5 e^{-0.07t}\,dt\right\} = 500\left\{\left[(60+t)\left(-\frac{100}{7}e^{-0.07t}\right)\right]_0^5 - \left[\frac{10{,}000}{49}e^{-0.07t}\right]_0^5\right\} \approx \$131{,}528.68$$

**91.** $$\int_{-\pi}^{\pi} x \sin nx\,dx = \left[-\frac{x}{n}\cos nx + \frac{1}{n^2}\sin nx\right]_{-\pi}^{\pi} = -\frac{\pi}{n}\cos \pi n - \frac{\pi}{n}\cos(-\pi n) = -\frac{2\pi}{n}\cos \pi n = \begin{cases} -2\pi/n, & \text{if } n \text{ is even} \\ 2\pi/n, & \text{if } n \text{ is odd} \end{cases}$$

**92.** $$\begin{aligned}\int_{-\pi}^{\pi} x^2 \cos nx\,dx &= \left[\frac{x^2}{n}\sin nx + \frac{2x}{n^2}\cos nx - \frac{2}{n^3}\sin nx\right]_{-\pi}^{\pi} \\ &= \frac{2\pi}{n^2}\cos n\pi + \frac{2\pi}{n^2}\cos(-n\pi) \\ &= \frac{4\pi}{n^2}\cos n\pi \\ &= \begin{cases} (4\pi/n^2), & \text{if } n \text{ is even} \\ -(4\pi/n^2), & \text{if } n \text{ is odd} \end{cases} \\ &= \frac{(-1)^n 4\pi}{n^2}\end{aligned}$$

**93.** Let $u = x, dv = \sin\dfrac{n\pi x}{2}\,dx, du = dx, v = -\dfrac{2}{n\pi}\cos\dfrac{n\pi x}{2}.$

$$\begin{aligned} I_1 = \int_0^1 x\sin\frac{n\pi x}{2}\,dx &= \left[\frac{-2x}{n\pi}\cos\frac{n\pi x}{2}\right]_0^1 + \frac{2}{n\pi}\int_0^1 \cos\frac{n\pi x}{2}\,dx \\ &= -\frac{2}{n\pi}\cos\frac{n\pi}{2} + \left[\left(\frac{2}{n\pi}\right)^2 \sin\frac{n\pi x}{2}\right]_0^1 \\ &= -\frac{2}{n\pi}\cos\frac{n\pi}{2} + \left(\frac{2}{n\pi}\right)^2 \sin\frac{n\pi}{2}\end{aligned}$$

Let $u = (-x + 2), dv = \sin\dfrac{n\pi x}{2}\,dx, du = -dx, v = -\dfrac{2}{n\pi}\cos\dfrac{n\pi x}{2}.$

$$\begin{aligned} I_2 = \int_1^2 (-x+2)\sin\frac{n\pi x}{2}\,dx &= \left[\frac{-2(-x+2)}{n\pi}\cos\frac{n\pi x}{2}\right]_1^2 - \frac{2}{n\pi}\int_1^2 \cos\frac{n\pi x}{2}\,dx \\ &= \frac{2}{n\pi}\cos\frac{n\pi}{2} - \left[\left(\frac{2}{n\pi}\right)^2 \sin\left(\frac{n\pi x}{2}\right)\right]_1^2 \\ &= \frac{2}{n\pi}\cos\frac{n\pi}{2} + \left(\frac{2}{n\pi}\right)^2 \sin\frac{n\pi}{2}\end{aligned}$$

$$h(I_1 + I_2) = b_n = h\left[\left(\frac{2}{n\pi}\right)^2 \sin\frac{n\pi}{2} + \left(\frac{2}{n\pi}\right)^2 \sin\frac{n\pi}{2}\right] = \frac{8h}{(n\pi)^2}\sin\frac{n\pi}{2}$$

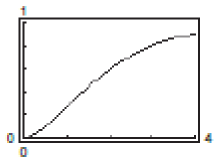
**94.** $f'(x) = xe^{-x}$

(a) $f(x) = \int xe^{-x}\,dx = -xe^{-x} - e^{-x} + C$

(Parts: $u = x, dv = e^{-x}\,dx$)

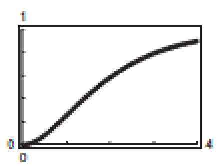
$f(0) = 0 = -1 + C \Rightarrow C = 1$

$f(x) = -xe^{-x} - e^{-x} + 1$

(b) 

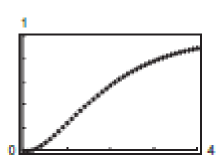

(c) Using $h = 0.05$, you obtain the points:

| $n$ | $x_n$ | $y_n$ |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0.05 | 0 |
| 2 | 0.10 | $2.378 \times 10^{-3}$ |
| 3 | 0.15 | 0.0069 |
| 4 | 0.20 | 0.0134 |
| ⋮ | ⋮ | ⋮ |
| 80 | 4.0 | 0.9064 |




(d) Using $h = 0.1$, you obtain the points:

| $n$ | $x_n$ | $y_n$ |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0.1 | 0 |
| 2 | 0.2 | 0.0090484 |
| 3 | 0.3 | 0.025423 |
| 4 | 0.4 | 0.047648 |
| ⋮ | ⋮ | ⋮ |
| 40 | 4.0 | 0.9039 |




(e) The result in part (c) is better because $h$ is smaller.

**95.** $f'(x) = 3x \sin 2x,\ f(0) = 0$

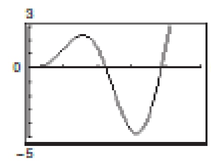
(a) $f(x) = \int 3x \sin 2x\,dx$

$= -\frac{3}{4}(2x \cos 2x - \sin 2x) + C$

(Parts: $u = 3x, dv = \sin 2x\,dx$)

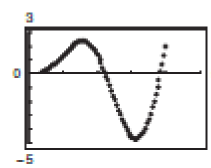
$f(0) = 0 = -\frac{3}{4}(0) + C \Rightarrow C = 0$

$f(x) = -\frac{3}{4}(2x \cos 2x - \sin 2x)$

(b) 

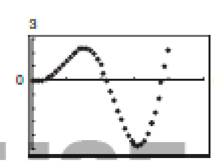

(c) Using $h = 0.05$, you obtain the points:

| $n$ | $x_n$ | $y_n$ |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0.05 | 0 |
| 2 | 0.10 | $7.4875 \times 10^{-4}$ |
| 3 | 0.15 | 0.0037 |
| 4 | 0.20 | 0.0104 |
| ⋮ | ⋮ | ⋮ |
| 80 | 4.0 | 1.3181 |




(d) Using $h = 0.1$, you obtain the points:

| $n$ | $x_n$ | $y_n$ |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0.1 | 0 |
| 2 | 0.2 | 0.0060 |
| 3 | 0.3 | 0.0293 |
| 4 | 0.4 | 0.0801 |
| ⋮ | ⋮ | ⋮ |
| 40 | 4.0 | 1.0210 |



**96.** $f'(x) = \cos\sqrt{x},\ f(0) = 1$

(a) Let $w = \sqrt{x},\ w^2 = x,\ 2w\,dw = dx.$

$\int \cos\sqrt{x}\,dx = \int \cos w(2w\,dw)$

Now use parts: $u = 2w,\ dv = \cos w\,dw.$

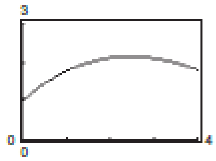
$\int \cos\sqrt{x}\,dx = 2w\sin w + 2\cos w + C$

$= 2\sqrt{x}\sin\sqrt{x} + 2\cos\sqrt{x} + C$

$f(0) = 1 = 2 + C \Rightarrow C = -1$

$f(x) = 2\sqrt{x}\sin\sqrt{x} + 2\cos\sqrt{x} - 1$

(b)




(c) Using $h = 0.05$, you obtain the points:

| $n$ | $x_n$ | $y_n$ |
|---|---|---|
| 0 | 0 | 1 |
| 1 | 0.05 | 1.05 |
| 2 | 0.1 | 1.0988 |
| 3 | 0.15 | 1.1463 |
| 4 | 0.2 | 1.1926 |
| ⋮ | ⋮ | ⋮ |
| 80 | 4.0 | 1.8404 |

(d) Using $h = 0.1$, you obtain the points:

| $n$ | $x_n$ | $y_n$ |
|---|---|---|
| 0 | 0 | 1 |
| 1 | 0.1 | 1.1 |
| 2 | 0.2 | 1.1950 |
| 3 | 0.3 | 1.2852 |
| 4 | 0.4 | 1.3706 |
| ⋮ | ⋮ | ⋮ |
| 80 | 4.0 | 1.8759 |

**97.** (a) $A = \int_0^{\pi} x\sin x\,dx = \left[\sin x - x\cos x\right]_0^{\pi} = \pi$

(b) $\int_{\pi}^{2\pi} x\sin x\,dx = \left[\sin x - x\cos x\right]_{\pi}^{2\pi} = -2\pi - \pi = -3\pi$

$A = 3\pi$

(c) $\int_{2\pi}^{3\pi} x\sin x\,dx = \left[\sin x - x\cos x\right]_{2\pi}^{3\pi} = 3\pi + 2\pi = 5\pi$

$A = 5\pi$

The area between $y = x\sin x$ and $y = 0$ on $\left[n\pi, (n+1)\pi\right]$ is $(2n+1)\pi$:

$\int_{n\pi}^{(n+1)\pi} x\sin x\,dx = \left[\sin x - x\cos x\right]_{n\pi}^{(n+1)\pi} = \pm(n+1)\pi \pm n\pi = \pm(2n+1)\pi$

$A = \left|\pm(2n+1)\pi\right| = (2n+1)\pi$

**98.** On $\left[0, \frac{\pi}{2}\right]$, $\sin x \le 1 \Rightarrow x\sin x \le x \Rightarrow \int_0^{\pi/2} x\sin x\,dx \le \int_0^{\pi/2} x\,dx.$

**99.** For any integrable function, $\int f(x)\,dx = C + \int f(x)\,dx$, but this cannot be used to imply that $C = 0$.

**100.** $\int x\sin 8x\,dx$

$dv = \sin 8x\,dx \quad \Rightarrow \quad v = \frac{1}{8}\int \sin 8x(8)\,dx = -\frac{1}{8}\cos 8x$

$u = x \quad \Rightarrow \quad du = dx$

$\int x\sin 8x\,dx = -\frac{x}{8}\cos 8x + \frac{1}{8}\int \cos 8x\,dx = -\frac{x}{8}\cos 8x + \frac{1}{64}\sin 8x + C$

So, the answer is A.

**101.** (a) $f'(x) = x \ln x$

$f'(e) = e \ln e = e$

So, the tangent line is $y - 4 = e(x - e)$

$y = ex - e^2 + 4.$

(b) $f'(x) = x \ln x = 0$ when $x = 0$ and $x = 1$.

Because $x = 0$ is not in the domain of $f'(x)$ and $f'(x)$ goes from negative to positive at $x = 1$, the graph of $f(x)$ has a relative minimum at $x = 1$.

(c) $f'(x) = x \ln x$

$f''(x) = x\left(\frac{1}{x}\right) + (\ln x)(1) = 1 + \ln x = 0$

$\ln x = -1$

$x = e^{-1}$

$x = \frac{1}{e}$

So, the graph of $f(x)$ is concave upward on $\left(\frac{1}{e}, \infty\right)$ and concave downward on $\left(0, \frac{1}{e}\right)$.

(d) $f(x) = \int f'(x)\, dx = \int x \ln x\, dx$

$u = \ln x \Rightarrow du = \frac{1}{x}\, dx$

$dv = x\, dx \Rightarrow v = \frac{1}{2}x^2$

$$\int x \ln x\, dx = \frac{1}{2}x^2 \ln x - \int \frac{1}{2}x^2\left(\frac{1}{x}\right) dx$$

$$= \frac{1}{2}x^2 \ln x - \int \frac{1}{2}x\, dx$$

$$= \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C$$

Use $f(e) = 4$ to find $C$.

$$f(e) = \frac{1}{2}(e)^2 \ln e - \frac{1}{4}(e)^2 + C_1$$

$$4 = \frac{1}{2}e^2 - \frac{1}{4}e^2 + C_1$$

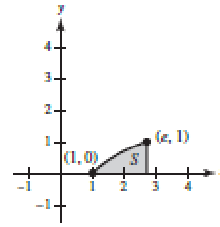
$$4 = \frac{1}{4}e^2 + C_1$$

$$16 = e^2 + C \quad (C = 4C_1)$$

$$C = 16 - e^2$$

So, $f(x) = \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + 16 - e^2$.

**102.** (a)




$A = \int_1^e \ln x\, dx$

$u = \ln x \Rightarrow du = \frac{1}{x}\, dx$

$dv = dx \Rightarrow v = x$

$$A = \left[x \ln x\right]_1^e - \int_1^e x \cdot \frac{1}{x}\, dx$$

$$= \left[x \ln x\right]_1^e - \int_1^e dx$$

$$= \left[x \ln x - x\right]_1^e$$

$$= (e - e) - (0 - 1)$$

$$= 1$$

(b) $V = \pi \int_1^e (\ln x)^2\, dx$

$u = (\ln x)^2 \Rightarrow du = \frac{2}{x} \ln x\, dx$

$dv = dx \Rightarrow v = x$

$$= \pi\left(\left[x(\ln x)^2\right]_1^e - \int_1^e x \cdot \frac{2}{x} \ln x\, dx\right)$$

$$= \pi\left(\left[x(\ln x)^2\right]_1^e - 2\int_1^e \ln x\, dx\right)$$

$u = \ln x \Rightarrow du = \frac{1}{x}\, dx$

$dv = dx \Rightarrow v = x$

$$= \pi\left(\left[x(\ln x)^2\right]_1^e - 2\left[x \ln x\right]_1^e - \int_1^e x \cdot \frac{1}{x}\, dx\right)$$

$$= \pi\left(\left[x(\ln x)^2\right]_1^e - 2\left[x \ln x - x\right]_1^e\right)$$

$$= \pi\left[(e - 0) - (0 + 2)\right]$$

$$= \pi(e - 2) \approx 2.257$$

(c) $V = \pi \int_0^1 \left[e^2 - \left(e^y\right)^2\right] dy$

$$= \pi \int_0^1 \left(e^2 - e^{2y}\right) dy$$

$$= \pi\left[e^2 y - \frac{1}{2}e^{2y}\right]_0^1$$

$$= \pi\left[\left(e^2 - \frac{1}{2}e^2\right) - \left(0 - \frac{1}{2}\right)\right]$$

$$= \frac{\pi}{2}\left(e^2 + 1\right) \approx 13.177$$

**103.** (a) $A = \int_0^1 \left(xe^{-x} + x\right) dx = \int_0^1 xe^{-x}\, dx + \int_0^1 x\, dx$

$u = x \quad \Rightarrow \quad du = dx$

$dv = e^{-x}\, dx \quad \Rightarrow \quad v = -e^{-x}$

$A = \left[-xe^{-x}\right]_0^1 - \int_0^1 -e^{-x}\, dx + \left[\frac{1}{2}x^2\right]_0^1 = \left[-xe^{-x} - e^{-x}\right]_0^1 + \frac{1}{2}x^2\Big]_0^1 = \left(-\frac{1}{e} - \frac{1}{e}\right) - (-1) + \frac{1}{2} = \frac{3}{2} - \frac{2}{e} \approx 0.764$

(b) $V = \pi \int_0^1 \left[\left(-1 - xe^{-x}\right)^2 - (-1 + x)^2\right] dx \approx 4.009$

(c) $y = xe^{-x}$

$\frac{dy}{dx} = x\left(-e^{-x}\right) + e^{-x} = e^{-x}(1 - x)$

$\frac{d^2y}{dx^2} = e^{-2x}(1 - x)^2$

Perimeter $= \sqrt{2} + \frac{1}{e} + 1 + \int_0^1 \sqrt{1 + e^{-2x}(x - 1)^2}\, dx$