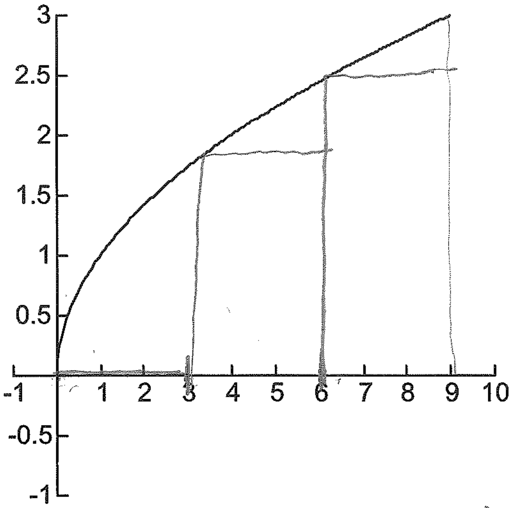
4.2, 4.3 Area and Reimann Sums Day 2

Let's Approximate Areas!

$\int_0^9 \sqrt{x}\,dx \qquad \underline{n = 3}$ # of rectangles.

$\frac{9-0}{3} = 3$

Left or Lower

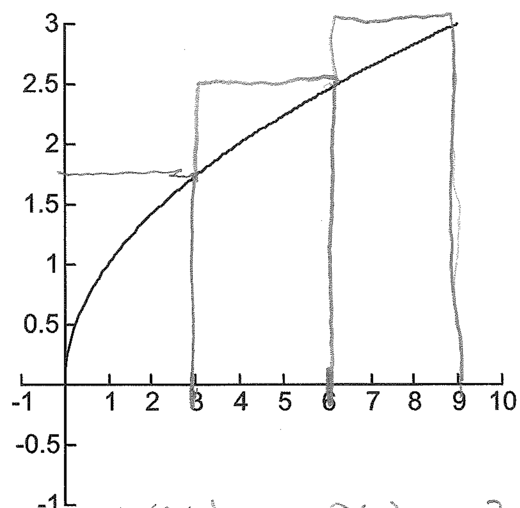


$3(f(0)) + 3f(3) + 3f(6)$

$3(0) + 3\sqrt{3} + 3\sqrt{6}$

$= 3(\sqrt{3}+\sqrt{6}) \approx 12.5446$

Right or Upper

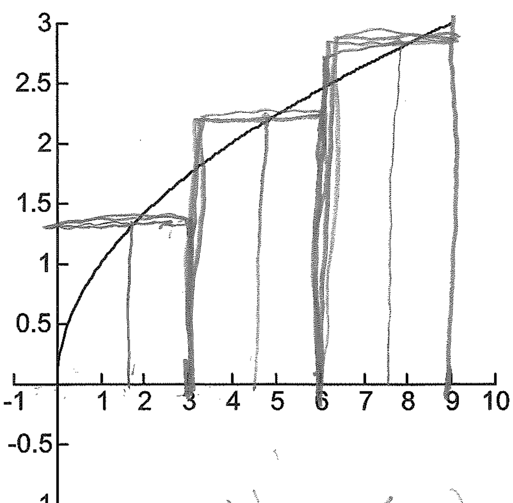


$(3)(f(3) + 3f(6) + 3f(9)$

$3\sqrt{3} + 3\sqrt{6} + (3)(3)$

$3(\sqrt{3}+\sqrt{6}+3) \approx 21.5446$

Midpoint

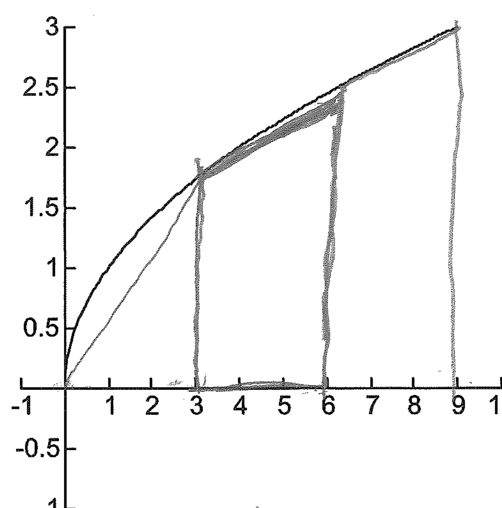


$3(f(1.5)) + 3f(4.5) + 3f(7.5)$

$3\sqrt{1.5} + 3\sqrt{4.5} + 3\sqrt{7.5}$

≈ 18.2540

Trapezoid



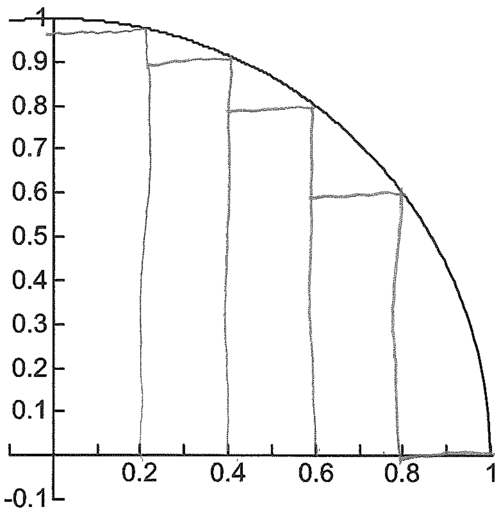
$\left(\frac{1}{2}\right)(3)(0+\sqrt{3}) + \left(\frac{1}{2}\right)(3)(\sqrt{3}+\sqrt{6})$

$+ \frac{1}{2}(3)(\sqrt{6}+3)$

≈ 17.0446

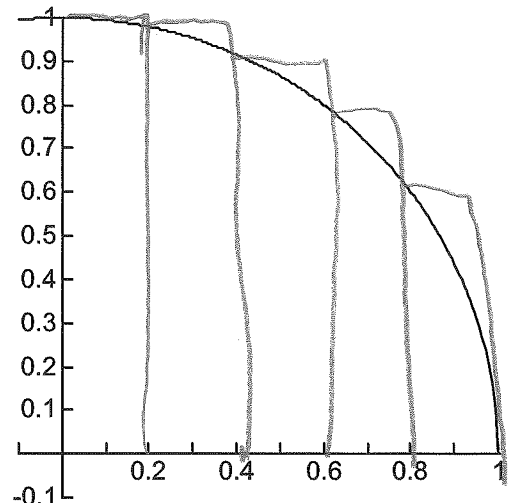
$\int_0^1 \sqrt{1-x^2}\,dx \quad n = 5$

Right or Lower



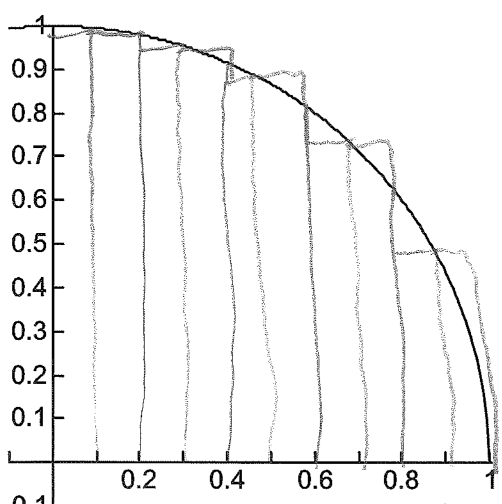
$(.2)(\sqrt{1-.2^2}) + (.2)(\sqrt{1-.4^2}) + (.2)(\sqrt{1-.6^2})$
$+ (.2)(\sqrt{1-.8^2}) + (.2)(\sqrt{1-1^2})$
$\approx .6592622072$

Left or Upper



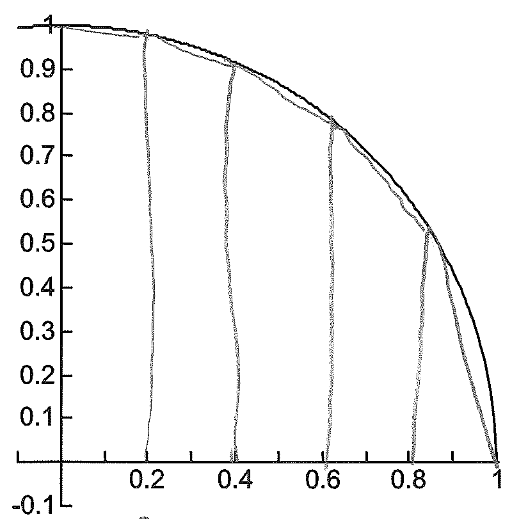
$(.2)(\sqrt{1-0^2}) + (.2)(\sqrt{1-.2^2}) + (.2)(\sqrt{1-.4^2})$
$+ (.2)(\sqrt{1-.6^2}) + (.2)(\sqrt{1-.8^2})$
$\approx .8592622072$

Midpoint



$(.2)(\sqrt{1-.1^2}) + (.2)(\sqrt{1-.3^2}) + (.2)(\sqrt{1-.5^2})$
$(.2)(\sqrt{1-.7^2}) + (.2)(\sqrt{1-.9^2})$
$\approx .7929969559$

Trapezoid



$(.2)(\tfrac{1}{2})\big[\sqrt{1-0^2} + 2\sqrt{1-.2^2} + 2\sqrt{1-.4^2}$
$+ 2\sqrt{1-.6^2} + 2\sqrt{1-.8^2} + \sqrt{1-1^2}\big]$
$\approx .7592622072$

Examples – Approximating from tables

Consumer demand for a certain product is changing over time, and the rate of change of this demand $f'(t)$, in units/week, is given, in week t in the following table. Approximate the total products demanded using the description.

Find a right side approximation for the total products demanded over the 13 weeks.

t	0	1	4	5	8	9	10	12	13
$f'(t)$	12	10	4	-2	-3	-1	3	7	11

$(1)(10) + (3)(4) + (1)(-2) + (3)(-3) + (1)(-1) + (1)(3) + (2)(7) + (1)(11)$

$= 38$ units

Find a lower approximation for the total products demanded over the 13 weeks.

t	0	1	4	5	8	9	10	12	13
$f'(t)$	12	10	4	-2	-3	-1	3	7	11

$(1)(10) + (3)(4) + (1)(-2) + (3)(-3) + (1)(-3) + (1)(-1) + (2)(3) + (1)(7)$

$= 20$ units

Find a midpoint approximation for the total products demanded over the 8 week period. using 4 equal subintervals

t	0	1	2	3	4	5	6	7	8
$f'(t)$	12	10	4	-2	-3	-1	3	7	11

$(2)(10) + (2)(-2) + (2)(-1) + (2)(7) = 28$ units

Approximate the total products demanded using 4 equal subintervals.

t	0	1	2	3	4	5	6	7	8
$f'(t)$	12	10	4	-2	-3	-1	3	7	11

$\left(\frac{1}{2}\right)(2)\left[12 + 2(4) + 2(-3) + 2(3) + 11\right] = 31$ units

★ Trapezoid Approximation vs. Trapezoid Rule.

Examples – Approximating Integrals

Approximate the following integral using (A) a right Reimann sum and (B) a midpoint sum

$\int_1^4 x^2\,dx$ $\quad n = 3 \quad \frac{4-1}{3} = 1.$

Right $(1)\left[2^2 + 3^2 + 4^2\right] = 29$

Midpoint $(1)\left[1.5^2 + 2.5^2 + 3.5^2\right] = 20.75$

Approximate the following integral using (A) a left Reimann sum and (B) a trapezoid sum

$\int_1^2 \frac{1}{x}\,dx$ $\quad n = 5 \quad \frac{2-1}{5} = \frac{1}{5}$

Left : $\frac{1}{5}\left[\frac{1}{1} + \frac{1}{1.2} + \frac{1}{1.4} + \frac{1}{1.6} + \frac{1}{1.8}\right] \approx .7456349206$

Trapezoid: $\left(\frac{1}{5}\right)\left(\frac{1}{2}\right)\left[\frac{1}{1} + 2\left(\frac{1}{1.2}\right) + 2\left(\frac{1}{1.4}\right) + 2\left(\frac{1}{1.6}\right) + 2\left(\frac{1}{1.8}\right) + \frac{1}{2}\right]$

$\approx .6956349206$