7.1 Basic Integration Rules

Try to fill in the basic integration rules WITHOUT using your text!

$\int kf(u)du = KF(u) + C$

$\int [f(u) \pm g(u)]du = F(u) + G(u) + C$

$\int du = u + C$

$\int u^n du \quad \frac{u^{n+1}}{n+1} + C \quad n \neq 1$

$\int \frac{du}{u} = \ln|u| + C$

$\int e^u du = e^u + C$

$\int a^u du = \left(\frac{1}{\ln a}\right) \cdot a^u + C$

$\int sinu\, du = -\cos u + C$

$\int cosu\, du = \sin u + C$

$\int tanu\, du = -\ln|\cos u| + C$

$\int cotu\, du = \ln|\sin u| + C$

$\int secu\, du = \ln|\sec u + \tan u| + C$

$\int cscu\, du = -\ln|\csc u + \cot u| + C$

$\int sec^2u\, du = \tan u + C$

$\int csc^2u\, du = -\cot u + C$

$\int secutanu\, du = \sec u + C$

$\int cscucotu\, du = -\csc u + C$

$\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin \frac{u}{a} + C$

$\int \frac{du}{a^2+u^2} = \frac{1}{a} \arctan \frac{u}{a} + C$

$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a} \operatorname{arcsec} \frac{|u|}{a} + C$

Procedures for Fitting Integrands to Basic Integration Rules

Expand (numerator)

Separate the numerator

Complete the Square

Divide improper rational function

Add and subtract terms in the numerator

Use trigonometric identities

Multiply and Divide by Pythagorean Conjugate

Examples: Integrating

$\int \frac{2}{\sqrt{1-x^2}}\,dx$ $\quad u^2 = x^2 \quad u = x \quad du = dx \quad a = 1$

$2\int \frac{dx}{\sqrt{1-x^2}}\,dx$

$\boxed{2\arcsin\frac{x}{1} + C}$

$\int \frac{2x}{1-x^2}\,dx$ $\quad u = 1-x^2 \quad du = -2x\,dx \quad dx = \frac{du}{-2x}$

$\int 2x \cdot \frac{1}{u} \cdot \frac{du}{-2x}$

$-1\int \frac{1}{u}\,du = -1\ln|u| + C = \boxed{-\ln|1-x^2| + C}$

$\int \frac{2x}{\sqrt{1-x^2}}\,dx$ $\quad u = 1-x^2 \quad du = -2x\,dx \quad dx = \frac{du}{-2x}$

$\int 2x \cdot u^{-\frac{1}{2}} \cdot \frac{du}{-2x}$

$-1\int u^{-1/2}\,du = -1\left(\frac{u^{1/2}}{1/2}\right) + C = \boxed{-2(1-x^2)^{\frac{1}{2}} + C}$

$\int \frac{1}{x\sqrt{x^4-4}}\,dx$ $\quad u^2 = x^4 \quad u = x^2 \quad du = 2x\,dx \quad dx = \frac{du}{2x} \quad a^2 = 4 \quad a = 2$

$\frac{1}{2}\int \frac{1}{u\sqrt{u^2-2^2}}\,du = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\operatorname{arcsec}\frac{|x^2|}{2} + C$

$= \boxed{\frac{1}{4}\operatorname{arcsec}\frac{x^2}{2} + C}$

$\int (\sin x)e^{-3\cos x}\,dx$ $\quad u = -3\cos x \quad du = -3(-\sin x)\,dx \quad dx = \frac{du}{3\sin x}$

$\int \sin x \cdot e^u \cdot \frac{du}{3\sin x} = \frac{1}{3}e^u + C = \boxed{\frac{1}{3}e^{-3\cos x} + C}$

$\int \frac{6}{e^x+6}\,dx$

$\int \frac{6+e^x-e^x}{e^x+6}\,dx = \int \frac{6+e^x}{e^x+6}\,dx - \int \frac{e^x}{e^x+6}\,dx$

$= \int 1\,dx - \int \frac{e^x}{e^x+6}\,dx \quad u = e^x+6 \quad du = e^x\,dx \quad dx = \frac{du}{e^x}$

$= x - \int e^x \cdot \frac{1}{u}\,\frac{du}{e^x}$

$= x - \ln|e^x+6| + C$

$\int$ [illegible] dx

$= \int \tan^2 x\,dx$

$= \int \sec^2 x - 1\,dx$

$= \tan x - x + C$

$\int_0^4 \frac{x-7}{x^2+16}\,dx$

(other sheet)

$$\int \frac{sece^{-2x}tane^{-2x}}{e^{2x}}\,dx$$

$u = e^{-2x}$

$du = -2e^{-2x}dx$

$dx = \dfrac{du}{-2e^{-2x}}$

$$\int \frac{\sec u \tan u}{e^{2x}} \cdot \frac{du}{-2e^{-2x}}$$

$$= -\frac{1}{2}\int \sec u \tan u \, du = \boxed{-\frac{1}{2}\sec e^{-2x} + C}$$

$$\int \frac{1}{\sqrt{9-4x^2}}\,dx$$

$u^2 = 4x^2$ $\quad a = 3$

$u = 2x$

$du = 2dx$

$$\int \frac{1}{\sqrt{3^2-u^2}} \cdot \frac{du}{2}$$

$$= \boxed{\left(\frac{1}{2}\right)\arcsin\frac{2x}{3} + C}$$

$$\int \frac{2(lnx)^2}{3x}\,dx$$

$u = \ln x$

$du = \frac{1}{x}dx$

$dx = x\,du$

$$\frac{2}{3}\int \frac{u^2}{x} \cdot x\,du$$

$$\frac{2}{3}\left(\frac{u^3}{3}\right) + C$$

$$\boxed{\frac{2}{9}(\ln x)^3 + C}$$

$$\int_0^4 \frac{x-7}{x^2+16}\,dx = \int_0^4 \frac{x}{x^2+16}\,dx - \int_0^4 \frac{7}{x^2+16}\,dx$$

$u = x^2+16$
$du = 2x\,dx$
$dx = \frac{du}{2x}$
$u(4) = 32$
$u(0) = 16$

$u^2 = x^2$ $\quad a^2 = 16$
$u = x$ $\quad a = 4$
$du = dx$

$$= \int_{16}^{32} x \cdot \frac{1}{u} \cdot \frac{du}{2x} - \frac{7}{4}\arctan\frac{x}{4}\Big|_0^4$$

$$= \frac{1}{2}\ln|u|\Big|_{16}^{32} - \frac{7}{4}\arctan\frac{x}{4}\Big|_0^4$$

$$= \frac{1}{2}\ln 32 - \frac{1}{2}\ln 16 - \left(\frac{7}{4}\arctan 1 - \frac{7}{4}\arctan 0\right)$$

$$= \frac{1}{2}\ln 32 - \frac{1}{2}\ln 16 - \frac{7}{4}\left(\frac{\pi}{4}\right) + \frac{7}{4}(0)$$

$$= \frac{1}{2}\ln 32 - \frac{1}{2}\ln 16 - \frac{7\pi}{16}$$