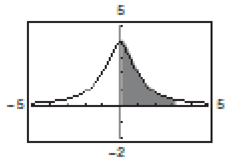
Section 4.4 The Fundamental Theorem of Calculus

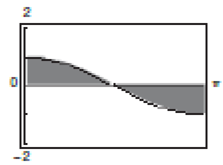
1. $f(x) = \dfrac{4}{x^2 + 1}$

   $\int_0^{\pi} \dfrac{4}{x^2+1}\,dx$ is positive.



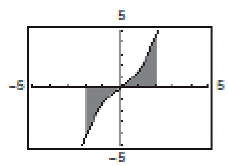
2. $f(x) = \cos x$

   $\int_0^{\pi} \cos x\,dx = 0$



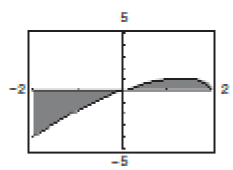
3. $f(x) = x\sqrt{x^2+1}$

   $\int_{-2}^{2} x\sqrt{x^2+1}\,dx = 0$



4. $f(x) = x\sqrt{2-x}$

   $\int_{-2}^{2} x\sqrt{2-x}\,dx$ is negative.



5. $\int_0^2 6x\,dx = \left[3x^2\right]_0^2 = 3(2)^2 - 0 = 12$

6. $\int_{-3}^{1} 8\,dt = \left[8t\right]_{-3}^{1} = 8(1) - 8(-3) = 32$

7. $\int_{-1}^{0} (2x-1)\,dx = \left[x^2 - x\right]_{-1}^{0}$

   $= 0 - \left((-1)^2 - (-1)\right) = -(1+1) = -2$

8. $\int_{-1}^{2} (7-3t)\,dt = \left[7t - \frac{3}{2}t^2\right]_{-1}^{2}$

   $= \left[7(2) - \frac{3}{2}(4)\right] - \left[7(-1) - \frac{3}{2}(-1)^2\right]$

   $= 14 - 6 + 7 + \frac{3}{2} = \frac{33}{2}$

9. $\int_{-1}^{1} \left(t^2 - 2\right)dt = \left[\dfrac{t^3}{3} - 2t\right]_{-1}^{1}$

   $= \left(\dfrac{1}{3} - 2\right) - \left(-\dfrac{1}{3} + 2\right) = -\dfrac{10}{3}$

10. $\int_1^2 \left(6x^2 - 3x\right)dx = \left[2x^3 - \frac{3}{2}x^2\right]_1^2 = \left[2(8) - \frac{3}{2}(4)\right] - \left[2(1) - \frac{3}{2}(1)\right] = (16-6) - \left(2 - \frac{3}{2}\right) = \frac{19}{2}$

11. $\int_0^1 (2t-1)^2\,dt = \int_0^1 \left(4t^2 - 4t + 1\right)dt = \left[\frac{4}{3}t^3 - 2t^2 + t\right]_0^1 = \frac{4}{3} - 2 + 1 = \frac{1}{3}$

12. $\int_1^3 \left(4x^3 - 3x^2\right)dx = \left[x^4 - x^3\right]_1^3 = (81 - 27) - (1 - 1) = 54$

13. $\int_1^2 \left(\dfrac{3}{x^2} - 1\right)dx = \left[-\dfrac{3}{x} - x\right]_1^2 = \left(-\dfrac{3}{2} - 2\right) - (-3 - 1) = \dfrac{1}{2}$

14. $\int_{-2}^{-1}\left(u - \dfrac{1}{u^2}\right)du = \left[\dfrac{u^2}{2} + \dfrac{1}{u}\right]_{-2}^{-1} = \left(\dfrac{1}{2} - 1\right) - \left(2 - \dfrac{1}{2}\right) = -2$

15. $\int_1^4 \dfrac{u-2}{\sqrt{u}}\,du = \int_1^4 \left(u^{1/2} - 2u^{-1/2}\right)du = \left[\dfrac{2}{3}u^{3/2} - 4u^{1/2}\right]_1^4 = \left[\dfrac{2}{3}\left(\sqrt{4}\right)^3 - 4\sqrt{4}\right] - \left[\dfrac{2}{3} - 4\right] = \dfrac{2}{3}$

16. $\int_{-8}^{8} x^{1/3}\,dx = \left[\dfrac{3}{4}x^{4/3}\right]_{-8}^{8} = \dfrac{3}{4}\left[8^{4/3} - (-8)^{4/3}\right] = \dfrac{3}{4}(16 - 16) = 0$

17. $\int_{-1}^{1}\left(\sqrt[3]{t} - 2\right)dt = \left[\frac{3}{4}t^{4/3} - 2t\right]_{-1}^{1} = \left(\frac{3}{4}{}^{2} - 2\right) - \left(\frac{3}{4} + 2\right) = -4$

18. $\int_1^8 \sqrt{\dfrac{2}{x}}\,dx = \sqrt{2}\int_1^8 x^{-1/2}\,dx = \left[\sqrt{2}(2)x^{1/2}\right]_1^8 = \left[2\sqrt{2x}\right]_1^8 = 8 - 2\sqrt{2}$

19. $\int_0^1 \frac{x - \sqrt{x}}{3}\, dx = \frac{1}{3}\int_0^1 \left(x - x^{1/2}\right) dx = \frac{1}{3}\left[\frac{x^2}{2} - \frac{2}{3}x^{3/2}\right]_0^1 = \frac{1}{3}\left(\frac{1}{2} - \frac{2}{3}\right) = -\frac{1}{18}$

20. $\int_0^2 (2 - t)\sqrt{t}\, dt = \int_0^2 \left(2t^{1/2} - t^{3/2}\right) dt = \left[\frac{4}{3}t^{3/2} - \frac{2}{5}t^{5/2}\right]_0^2 = \left[\frac{t\sqrt{t}}{15}(20 - 6t)\right]_0^2 = \frac{2\sqrt{2}}{15}(20 - 12) = \frac{16\sqrt{2}}{15}$

21. $\int_{-1}^0 \left(t^{1/3} - t^{2/3}\right) dt = \left[\frac{3}{4}t^{4/3} - \frac{3}{5}t^{5/3}\right]_{-1}^0 = 0 - \left(\frac{3}{4} + \frac{3}{5}\right) = -\frac{27}{20}$

22. $\int_{-8}^{-1} \frac{x - x^2}{2\sqrt[3]{x}}\, dx = \frac{1}{2}\int_{-8}^{-1} \left(x^{2/3} - x^{5/3}\right) dx$

$= \frac{1}{2}\left[\frac{3}{5}x^{5/3} - \frac{3}{8}x^{8/3}\right]_{-8}^{-1} = \left[\frac{x^{5/3}}{80}(24 - 15x)\right]_{-8}^{-1} = -\frac{1}{80}(39) + \frac{32}{80}(144) = \frac{4569}{80}$

23. $\int_0^5 |2x - 5|\, dx = \int_0^{5/2} (5 - 2x)\, dx + \int_{5/2}^5 (2x - 5)\, dx$ (Split up the integral at the zero $x = \frac{5}{2}$.)

$= \left[5x - x^2\right]_0^{5/2} + \left[x^2 - 5x\right]_{5/2}^5 = \left(\frac{25}{2} - \frac{25}{4}\right) - 0 + (25 - 25) - \left(\frac{25}{4} - \frac{25}{2}\right) = 2\left(\frac{25}{2} - \frac{25}{4}\right) = \frac{25}{2}$

Note: By symmetry, $\int_0^5 |2x - 5|\, dx = 2\int_{5/2}^5 (2x - 5)\, dx$.

24. $\int_1^4 (3 - 1x - 31)\, dx = \int_1^3 \left[3 + (x - 3)\right] dx + \int_3^4 \left[3 - (x - 3)\right] dx$

$= \int_1^3 x\, dx + \int_3^4 (6 - x)\,dx$

$= \left[\frac{x^2}{2}\right]_1^3 + \left[6x - \frac{x^2}{2}\right]_3^4$

$= \left(\frac{9}{2} - \frac{1}{2}\right) + \left[(24 - 8) - \left(18 - \frac{9}{2}\right)\right]$

$= 4 + 16 - 18 + \frac{9}{2} = \frac{13}{2}$

25. $\int_0^4 \left|x^2 - 9\right| dx = \int_0^3 \left(9 - x^2\right) dx + \int_3^4 \left(x^2 - 9\right) dx$ (split up integral at the zero $x = 3$)

$= \left[9x - \frac{x^3}{3}\right]_0^3 + \left[\frac{x^3}{3} - 9x\right]_3^4 = (27 - 9) + \left(\frac{64}{3} - 36\right) - (9 - 27) = \frac{64}{3}$

26. $\int_0^4 \left|x^2 - 4x + 3\right| dx = \int_0^1 \left(x^2 - 4x + 3\right) dx - \int_1^3 \left(x^2 - 4x + 3\right) dx + \int_3^4 \left(x^2 - 4x + 3\right) dx$ (split up the integral at the zeros $x = 1, 3$)

$= \left[\frac{x^3}{3} - 2x^2 + 3x\right]_0^1 - \left[\frac{x^3}{3} - 2x^2 + 3x\right]_1^3 + \left[\frac{x^3}{3} - 2x^2 + 3x\right]_3^4$

$= \left(\frac{1}{3} - 2 + 3\right) - (9 - 18 + 9) + \left(\frac{1}{3} - 2 + 3\right) + \left(\frac{64}{3} - 32 + 12\right) - (9 - 18 + 9)$

$= \frac{4}{3} - 0 + \frac{4}{3} + \frac{4}{3} - 0 = 4$

27. $\int_0^\pi (1 + \sin x)\, dx = [x - \cos x]_0^\pi = (\pi + 1) - (0 - 1) = 2 + \pi$

28. $\int_0^\pi (2 + \cos x)\, dx = [2x + \sin x]_0^\pi = (2\pi + 0) - 0 = 2\pi$

29. $\int_0^{\pi/4} \frac{1 - \sin^2\theta}{\cos^2\theta}\,d\theta = \int_0^{\pi/4} d\theta = \left[\theta\right]_0^{\pi/4} = \frac{\pi}{4}$

30. $\int_0^{\pi/4} \frac{\sec^2\theta}{\tan^2\theta + 1}\,d\theta = \int_0^{\pi/4} \frac{\sec^2\theta}{\sec^2\theta}\,d\theta = \int_0^{\pi/4} d\theta = \left[\theta\right]_0^{\pi/4} = \frac{\pi}{4}$

31. $\int_{-\pi/6}^{\pi/6} \sec^2 x\,dx = \left[\tan x\right]_{-\pi/6}^{\pi/6} = \frac{\sqrt{3}}{3} - \left(-\frac{\sqrt{3}}{3}\right) = \frac{2\sqrt{3}}{3}$

32. $\int_{\pi/4}^{\pi/2} \left(2 - \csc^2 x\right) dx = \left[2x + \cot x\right]_{\pi/4}^{\pi/2} = (\pi + 0) - \left(\frac{\pi}{2} + 1\right) = \frac{\pi}{2} - 1 = \frac{\pi - 2}{2}$

33. $\int_{-\pi/3}^{\pi/3} 4\sec\theta\tan\theta\,d\theta = \left[4\sec\theta\right]_{-\pi/3}^{\pi/3} = 4(2) - 4(2) = 0$

34. $\int_{-\pi/2}^{\pi/2} (2t + \cos t)\,dt = \left[t^2 + \sin t\right]_{-\pi/2}^{\pi/2} = \left(\frac{\pi^2}{4} + 1\right) - \left(\frac{\pi^2}{4} - 1\right) = 2$

35. $\int_0^2 \left(2^x + 6\right) dx = \left[\frac{2^x}{\ln 2} + 6x\right]_0^2 = \left(\frac{4}{\ln 2} + 12\right) - \left(\frac{1}{\ln 2} + 0\right) = \frac{3}{\ln 2} + 12$

36. $\int_0^3 \left(t - 5^t\right) dt = \left[\frac{t^2}{2} - \frac{5^t}{\ln 5}\right]_0^3 = \left(\frac{9}{2} - \frac{125}{\ln 5}\right) - \left(0 - \frac{1}{\ln 5}\right) = \frac{9}{2} - \frac{124}{\ln 5} \approx -72.546$

37. $\int_{-1}^1 \left(e^\theta + \sin\theta\right) d\theta = \left[e^\theta - \cos\theta\right]_{-1}^1 = (e - \cos 1) - \left[e^{-1} - \cos(-1)\right] = e - \frac{1}{e}$

38. $\int_e^{2e} \left(\cos x - \frac{1}{x}\right) dx = \left[\sin x - \ln x\right]_e^{2e} = (\sin 2e - \ln 2e) - (\sin e - \ln e) = \sin 2e - \sin e - \ln 2$

39. $A = \int_0^1 \left(x - x^2\right) dx = \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = \frac{1}{6}$

40. $A = \int_1^2 \frac{1}{x^2}\,dx = \left[-\frac{1}{x}\right]_1^2 = \frac{1}{2}$

41. $A = \int_0^{\pi/2} \cos x\,dx = \left[\sin x\right]_0^{\pi/2} = 1$

42. $A = \int_0^{\pi} (x + \sin x)\,dx = \left[\frac{x^2}{2} - \cos x\right]_0^{\pi} = \frac{\pi^2}{2} + 2 = \frac{\pi^2 + 4}{2}$

43. Because $y > 0$ on $[0, 2]$,

Area $= \int_0^2 \left(5x^2 + 2\right) dx = \left[\frac{5}{3}x^3 + 2x\right]_0^2 = \frac{40}{3} + 4 = \frac{52}{3}$.

44. Because $y > 0$ on $[0, 2]$,

Area $= \int_0^2 \left(x^3 + x\right) dx = \left[\frac{x^4}{4} + \frac{x^2}{2}\right]_0^2 = 4 + 2 = 6.$

45. Because $y > 0$ on $[0, 8]$,

$$\text{Area} = \int_0^8 \left(1 + x^{1/3}\right) dx = \left[x + \frac{3}{4}x^{4/3}\right]_0^8 = 8 + \frac{3}{4}(16) = 20.$$

46. Because $y > 0$ on $[0, 4]$,

$$\text{Area} = \int_0^4 \left(-x^2 + 4x\right) dx = \left[-\frac{x^3}{3} + 2x^2\right]_0^4 = -\frac{64}{3} + 32 = \frac{32}{3}.$$

47. Because $y > 0$ on $[1, e]$,

$$\text{Area} = \int_1^e \frac{4}{x}\, dx = \left[4 \ln x\right]_1^e = 4 \ln e - 4 \ln 1 = 4.$$

48. Because $y > 0$ on $[0, 2]$,

$$\text{Area} = \int_0^2 e^x\, dx = \left[e^x\right]_0^2 = e^2 - e^0 = e^2 - 1.$$

49. $\displaystyle\int_0^3 x^3\, dx = \left[\frac{x^4}{4}\right]_0^3 = \frac{81}{4}$

$$\begin{aligned} f(c)(3 - 0) &= \frac{81}{4} \\ f(c) &= \frac{27}{4} \\ c^3 &= \frac{27}{4} \\ c &= \frac{3}{\sqrt[3]{4}} = \frac{3}{2}\sqrt[3]{2} \approx 1.8899 \end{aligned}$$

50. $\displaystyle\int_4^9 \sqrt{x}\, dx = \left[\tfrac{2}{3}x^{3/2}\right]_4^9 = \tfrac{2}{3}(27 - 8) = \tfrac{38}{3}$

$$\begin{aligned} f(c)(9 - 4) &= \tfrac{38}{3} \\ f(c) &= \tfrac{38}{15} \\ \sqrt{c} &= \tfrac{38}{15} \\ c &= \tfrac{1444}{225} \approx 6.4178 \end{aligned}$$

51. $\displaystyle\int_1^4 \left(5 - \frac{1}{x}\right) dx = \left[5x - \ln x\right]_1^4$

$$\begin{aligned} &= (20 - \ln 4) - (5 - 0) = 15 - \ln 4 \\ f(c)(4 - 1) &= 15 - \ln 4 \\ \left(5 - \frac{1}{c}\right)(3) &= 15 - \ln 4 \\ 15 - \frac{3}{c} &= 15 - \ln 4 \\ \frac{3}{c} &= \ln 4 \\ c &= \frac{3}{\ln 4} \approx 2.1640 \end{aligned}$$

52. $\displaystyle\int_0^3 \left(10 - 2^x\right) dx = \left[10x - \frac{2^x}{\ln 2}\right]_0^3$

$$\begin{aligned} &= \left(30 - \frac{8}{\ln 2}\right) - \left(0 - \frac{1}{\ln 2}\right) \\ &= 30 - \frac{7}{\ln 2} \\ f(c)(3 - 0) &= 30 - \frac{7}{\ln 2} \\ \left(10 - 2^c\right)(3) &= 30 - \frac{7}{\ln 2} \\ 3\left(2^c\right) &= \frac{7}{\ln 2} \\ 2^c &= \frac{7}{3 \ln 2} \\ c &= \log_2\left(\frac{7}{3 \ln 2}\right) \approx 1.7512 \end{aligned}$$

53. $\displaystyle\int_{-\pi/4}^{\pi/4} 2 \sec^2 x\, dx = \left[2 \tan x\right]_{-\pi/4}^{\pi/4} = 2(1) - 2(-1) = 4$

$$\begin{aligned} f(c)\left[\frac{\pi}{4} - \left(-\frac{\pi}{4}\right)\right] &= 4 \\ 2 \sec^2 c &= \frac{8}{\pi} \\ \sec^2 c &= \frac{4}{\pi} \\ \sec c &= \pm\frac{2}{\sqrt{\pi}} \\ c &= \pm\operatorname{arcsec}\left(\frac{2}{\sqrt{\pi}}\right) \\ &= \pm\arccos\frac{\sqrt{\pi}}{2} \approx \pm 0.4817 \end{aligned}$$

54. $\displaystyle\int_{-\pi/3}^{\pi/3} \cos x\, dx = \left[\sin x\right]_{-\pi/3}^{\pi/3} = \sqrt{3}$

$$\begin{aligned} f(c)\left[\frac{\pi}{3} - \left(-\frac{\pi}{3}\right)\right] &= \sqrt{3} \\ \cos c &= \frac{3\sqrt{3}}{2\pi} \\ c &\approx \pm 0.5971 \end{aligned}$$

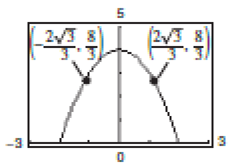
55. $f(x) = 4 - x^2, [-2, 2]$

$$\frac{1}{2-(-2)}\int_{-2}^{2}(4 - x^2)\,dx = \frac{1}{4}\left[4x - \frac{1}{3}x^3\right]_{-2}^{2}$$
$$= \frac{1}{4}\left[\left(8 - \frac{8}{3}\right) - \left(-8 + \frac{8}{3}\right)\right]$$
$$= \frac{8}{3}$$

Average value $= \frac{8}{3}$

$4 - x^2 = \frac{8}{3}$ when $x^2 = 4 - \frac{8}{3}$ or

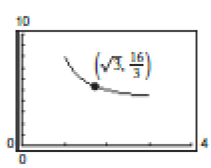
$x = \pm\sqrt{\frac{4}{3}} = \pm\frac{2\sqrt{3}}{3} \approx \pm 1.1547$



56. $$\frac{1}{3-1}\int_{1}^{3}\frac{4(x^2+1)}{x^2}\,dx = 2\int_{1}^{3}(1 + x^{-2})\,dx$$
$$= 2\left[x - \frac{1}{x}\right]_{1}^{3}$$
$$= 2\left(3 - \frac{1}{3}\right) = \frac{16}{3}$$

Average value $= \frac{16}{3}$

$\frac{4(x^2+1)}{x^2} = \frac{16}{3} \Rightarrow x = \sqrt{3}$ (on [1, 3])



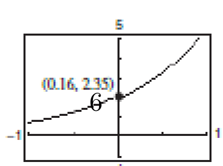
57. $$\frac{1}{1-(-1)}\int_{-1}^{1}2e^x\,dx = \int_{-1}^{1}e^x\,dx$$
$$= \left[e^x\right]_{-1}^{1} = e - e^{-1} \approx 2.3504$$

Average value $= e - e^{-1} \approx 2.3504$

$2e^x = e - e^{-1}$

$e^x = \frac{1}{2}(e - e^{-1})$

$x = \ln\left(\frac{e - e^{-1}}{2}\right) \approx 0.1614$



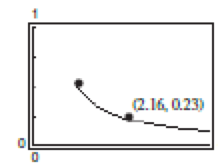
58. $$\frac{1}{4-1}\int_{1}^{4}\frac{1}{2x}\,dx = \left[\frac{1}{6}\ln x\right]_{1}^{4} = \frac{1}{6}\ln 4 \approx 0.2310$$

Average value $= \frac{1}{6}\ln 4 \approx 0.2310$

$\frac{1}{2x} = \frac{1}{6}\ln 4$

$2x = \frac{6}{\ln 4}$

$x = \frac{3}{\ln 4} \approx 2.1640$

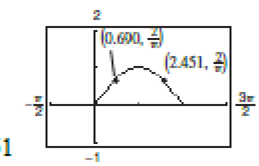


59. $$\frac{1}{\pi - 0}\int_{0}^{\pi}\sin x\,dx = \left[-\frac{1}{\pi}\cos x\right]_{0}^{\pi} = \frac{2}{\pi}$$

Average value $= \frac{2}{\pi}$

$\sin x = \frac{2}{\pi}$

$x \approx 0.690, 2.451$

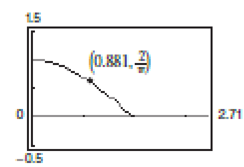


60. $$\frac{1}{(\pi/2) - 0}\int_{0}^{\pi/2}\cos x\,dx = \left[\frac{2}{\pi}\sin x\right]_{0}^{\pi/2} = \frac{2}{\pi}$$

Average value $= \frac{2}{\pi}$

$\cos x = \frac{2}{\pi}$

$x \approx 0.881$

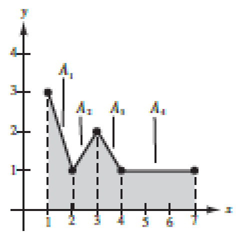


61. The distance traveled is $\int_0^8 v(t)\,dt$. The area under the curve from $0 \le t \le 8$ is approximately (18 squares) $(30) \approx 540$ ft.

62. The distance traveled is $\int_0^5 v(t)\,dt$. The area under the curve from $0 \le t \le 5$ is approximately (29 squares) $(5) = 145$ ft.

63. (a) $\int_1^7 f(x)\,dx$ = Sum of the areas

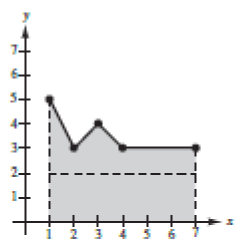
$$= A_1 + A_2 + A_3 + A_4$$
$$= \tfrac{1}{2}(3+1) + \tfrac{1}{2}(1+2) + \tfrac{1}{2}(2+1) + (3)(1)$$
$$= 8$$



(b) Average value $= \dfrac{\int_1^7 f(x)\,dx}{7-1} = \dfrac{8}{6} = \dfrac{4}{3}$

(c) $A = 8 + (6)(2) = 20$

Average value $= \dfrac{20}{6} = \dfrac{10}{3}$



64. $r(t)$ represents the weight in pounds of the dog at time t.

$\int_2^6 r'(t)\,dt$ represents the net change in the weight of the dog from year 2 to year 6.

65. (a) $F(x) = k\sec^2 x$

$F(0) = k = 500$

$F(x) = 500\sec^2 x$

(b) $\dfrac{1}{\pi/3 - 0}\displaystyle\int_0^{\pi/3} 500\sec^2 x\,dx = \dfrac{1500}{\pi}\Big[\tan x\Big]_0^{\pi/3}$

$$= \frac{1500}{\pi}\left(\sqrt{3} - 0\right)$$
$$\approx 826.99 \text{ N}$$
$$\approx 827 \text{ N}$$

66. $\dfrac{1}{R-0}\displaystyle\int_0^R k\left(R^2 - r^2\right)dr = \dfrac{k}{R}\left[R^2 r - \dfrac{r^3}{3}\right]_0^R = \dfrac{2kR^2}{3}$

67. $\dfrac{1}{5-0}\displaystyle\int_0^5 \left(0.1729t + 0.1522t^2 - 0.0374t^3\right)dt \approx \dfrac{1}{5}\Big[0.08645t^2 + 0.05073t^3 - 0.00935t^4\Big]_0^5 \approx 0.5318$ L

68. (a) Because $y < 0$ on $[0, 2]$, $\int_0^2 f(x)\,dx = -(\text{area of region } A) = -1.5$.

(b) $\int_2^6 f(x)\,dx = (\text{area of region } B) = \int_0^6 f(x)\,dx - \int_0^2 f(x)\,dx = 3.5 - (-1.5) = 5.0$

(c) $\int_0^6 |f(x)|\,dx = -\int_0^2 f(x)\,dx + \int_2^6 f(x)\,dx = 1.5 + 5.0 = 6.5$

(d) $\int_0^2 -2f(x)\,dx = -2\int_0^2 f(x)\,dx = -2(-1.5) = 3.0$

(e) $\int_0^6 \left[2 + f(x)\right]dx = \int_0^6 2\,dx + \int_0^6 f(x)\,dx = 12 + 3.5 = 15.5$

(f) Average value $= \tfrac{1}{6}\int_0^6 f(x)\,dx = \tfrac{1}{6}(3.5) = 0.5833$

69. $F(x) = \int_0^x (4t - 7)\,dt = \left[2t^2 - 7t\right]_0^x = 2x^2 - 7x$

$F(2) = 2\left(2^2\right) - 7(2) = -6$

$F(5) = 2\left(5^2\right) - 7(5) = 15$

$F(8) = 2\left(8^2\right) - 7(8) = 72$

70. $F(x) = \int_2^x (t^3 + 2t - 2)\,dt = \left[\dfrac{t^4}{4} + t^2 - 2t\right]_2^x = \left(\dfrac{x^4}{4} + x^2 - 2x\right) - (4 + 4 - 4) = \dfrac{x^4}{4} + x^2 - 2x - 4$

$F(2) = 4 + 4 - 4 - 4 = 0 \quad \left[\textbf{Note: } F(2) = \int_2^2 (t^3 + 2t - 2)\,dt = 0\right]$

$F(5) = \dfrac{625}{4} + 25 - 10 - 4 = 167.25$

$F(8) = \dfrac{8^4}{4} + 64 - 16 - 4 = 1068$

71. $F(x) = \int_1^x \dfrac{20}{v^2}\,dv = \int_1^x 20v^{-2}\,dv = -\dfrac{20}{v}\Big]_1^x$

$= -\dfrac{20}{x} + 20 = 20\left(1 - \dfrac{1}{x}\right)$

$F(2) = 20\left(\frac{1}{2}\right) = 10$

$F(5) = 20\left(\frac{4}{5}\right) = 16$

$F(8) = 20\left(\frac{7}{8}\right) = \frac{35}{2}$

72. $F(x) = \int_2^x -\dfrac{2}{t^3}\,dt = -\int_2^x 2t^{-3}\,dt = \dfrac{1}{t^2}\Big]_2^x = \dfrac{1}{x^2} - \dfrac{1}{4}$

$F(2) = \dfrac{1}{4} - \dfrac{1}{4} = 0$

$F(5) = \dfrac{1}{25} - \dfrac{1}{4} = -\dfrac{21}{100} = -0.21$

$F(8) = \dfrac{1}{64} - \dfrac{1}{4} = -\dfrac{15}{64}$

73. $F(x) = \int_1^x \cos\theta\,d\theta = \sin\theta\Big]_1^x = \sin x - \sin 1$

$F(2) = \sin 2 - \sin 1 \approx 0.0678$

$F(5) = \sin 5 - \sin 1 \approx -1.8004$

$F(8) = \sin 8 - \sin 1 \approx 0.1479$

74. $F(x) = \int_0^x \sin\theta\,d\theta = -\cos\theta\Big]_0^x$

$= -\cos x + \cos 0$

$= 1 - \cos x$

$F(2) = 1 - \cos 2 \approx 1.4161$

$F(5) = 1 - \cos 5 \approx 0.7163$

$F(8) = 1 - \cos 8 \approx 1.1455$

75. $g(x) = \int_0^x f(t)\,dt$

(a) $g(0) = \int_0^0 f(t)\,dt = 0$

$g(2) = \int_0^2 f(t)\,dt \approx 4 + 2 + 1 = 7$

$g(4) = \int_0^4 f(t)\,dt \approx 7 + 2 = 9$

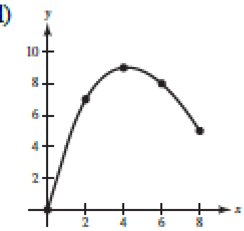
$g(6) = \int_0^6 f(t)\,dt \approx 9 + (-1) = 8$

$g(8) = \int_0^8 f(t)\,dt \approx 8 - 3 = 5$

(b) g increasing on $(0, 4)$ and decreasing on $(4, 8)$

(c) g is a maximum of 9 at $x = 4$.

(d)



76. $g(x) = \int_0^x f(t)\,dt$

(a) $g(0) = \int_0^0 f(t)\,dt = 0$

$g(2) = \int_0^2 f(t)\,dt = -\frac{1}{2}(2)(4) = -4$

$g(4) = \int_0^4 f(t)\,dt = -\frac{1}{2}(4)(4) = -8$

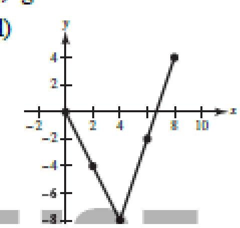
$g(6) = \int_0^6 f(t)\,dt = -8 + 2 + 4 = -2$

$g(8) = \int_0^8 f(t)\,dt = -2 + 6 = 4$

(b) g decreasing on $(0, 4)$ and increasing on $(4, 8)$

(c) g is a minimum of -8 at $x = 4$.

(d)



77. (a) $\int_0^x (t + 2)\,dt = \left[\frac{t^2}{2} + 2t\right]_0^x = \frac{1}{2}x^2 + 2x$

(b) $\frac{d}{dx}\left[\frac{1}{2}x^2 + 2x\right] = x + 2$

78. (a) $\int_0^x t(t^2 + 1)dt = \int_0^x (t^3 + t)\,dt$

$$= \left[\frac{1}{4}t^4 + \frac{1}{2}t^2\right]_0^x$$

$$= \frac{1}{4}x^4 + \frac{1}{2}x^2 = \frac{x^2}{4}(x^2 + 2)$$

(b) $\frac{d}{dx}\left[\frac{1}{4}x^4 + \frac{1}{2}x^2\right] = x^3 + x = x(x^2 + 1)$

79. (a) $\int_8^x \sqrt[3]{t}\,dt = \left[\frac{3}{4}t^{4/3}\right]_8^x = \frac{3}{4}(x^{4/3} - 16) = \frac{3}{4}x^{4/3} - 12$

(b) $\frac{d}{dx}\left[\frac{3}{4}x^{4/3} - 12\right] = x^{1/3} = \sqrt[3]{x}$

80. (a) $\int_4^x \sqrt{t}\,dt = \left[\frac{2}{3}t^{3/2}\right]_4^x$

$$= \frac{2}{3}x^{3/2} - \frac{16}{3}$$

$$= \frac{2}{3}(x^{3/2} - 8)$$

(b) $\frac{d}{dx}\left[\frac{2}{3}x^{3/2} - \frac{16}{3}\right] = x^{1/2} = \sqrt{x}$

81. (a) $\int_{\pi/4}^x \sec^2 t\,dt = [\tan t]_{\pi/4}^x = \tan x - 1$

(b) $\frac{d}{dx}[\tan x - 1] = \sec^2 x$

82. (a) $\int_{\pi/3}^x \sec t \tan t\,dt = [\sec t]_{\pi/3}^x = \sec x - 2$

(b) $\frac{d}{dx}[\sec x - 2] = \sec x \tan x$

83. (a) $F(x) = \int_{-1}^x e^t\,dt = e^t\Big]_{-1}^x = e^x - e^{-1}$

(b) $\frac{d}{dx}(e^x - e^{-1}) = e^x$

84. (a) $F(x) = \int_1^x \frac{1}{t}dt = \ln t\Big]_1^x = \ln x$

(b) $\frac{d}{dx}(\ln x) = \frac{1}{x}$

85. $F(x) = \int_{-2}^x (t^2 - 2t)\,dt$

$F'(x) = x^2 - 2x$

86. $F(x) = \int_1^x \frac{t^2}{t^2 + 1}\,dt$

$F'(x) = \frac{x^2}{x^2 + 1}$

87. $F(x) = \int_{-1}^x \sqrt{t^4 + 1}\,dt$

$F'(x) = \sqrt{x^4 + 1}$

88. $F(x) = \int_1^x \sqrt[4]{t}\,dt$

$F'(x) = \sqrt[4]{x}$

89. $F(x) = \int_0^x t \cos t\,dt$

$F'(x) = x \cos x$

90. $F(x) = \int_0^x \sec^3 t\,dt$

$F'(x) = \sec^3 x$

91. $F(x) = \int_x^{x+2} (4t + 1)\,dt$

$$= \left[2t^2 + t\right]_x^{x+2}$$

$$= \left[2(x + 2)^2 + (x + 2)\right] - \left[2x^2 + x\right]$$

$$= 8x + 10$$

$F'(x) = 8$

Alternate solution:

$F(x) = \int_x^{x+2} (4t + 1)\,dt$

$$= \int_x^0 (4t + 1)\,dt + \int_0^{x+2} (4t + 1)\,dt$$

$$= -\int_0^x (4t + 1)\,dt + \int_0^{x+2} (4t + 1)\,dt$$

$F'(x) = -(4x + 1) + 4(x + 2) + 1 = 8$

92. $F(x) = \int_{-x}^x t^3\,dt = \left[\frac{t^4}{4}\right]_{-x}^x = 0$

$F'(x) = 0$

Alternate solution:

$F(x) = \int_{-x}^x t^3\,dt$

$$= \int_{-x}^0 t^3\,dt + \int_0^x t^3\,dt$$

$$= -\int_0^{-x} t^3\,dt + \int_0^x t^3\,dt$$

$F'(x) = -(-x)^3(-1) + (x^3) = 0$

93. $F(x) = \int_0^{\sin x} \sqrt{t}\, dt = \left[\frac{2}{3}t^{3/2}\right]_0^{\sin x} = \frac{2}{3}(\sin x)^{3/2}$

$F'(x) = (\sin x)^{1/2}\cos x = (\cos x)\sqrt{\sin x}$

Alternate solution:

$F(x) = \int_0^{\sin x} \sqrt{t}\, dt$

$F'(x) = \sqrt{\sin x}\frac{d}{dx}(\sin x) = \sqrt{\sin x}(\cos x)$

94. $F(x) = \int_2^{x^2} t^{-3}\, dt = \left[\frac{t^{-2}}{-2}\right]_2^{x^2} = \left[-\frac{1}{2t^2}\right]_2^{x^2} = \frac{-1}{2x^4} + \frac{1}{8}$

$F'(x) = 2x^{-5}$

Alternate solution:

$F(x) = \int_2^{x^2} t^{-3}\, dt$

$F'(x) = \left(x^2\right)^{-3}(2x) = 2x^{-5}$

95. $F(x) = \int_0^{x^3} \sin t^2\, dt$

$F'(x) = \sin\left(x^3\right)^2 \cdot 3x^2 = 3x^2 \sin x^6$

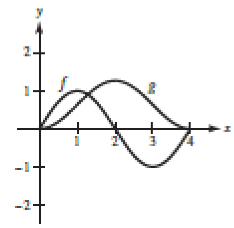
96. $F(x) = \int_0^{x^2} \sin \theta^2\, d\theta$

$F'(x) = \sin\left(x^2\right)^2(2x) = 2x \sin x^4$

97. $g(x) = \int_0^x f(t)\, dt$

$g(0) = 0, g(1) \approx \frac{1}{2}, g(2) \approx 1, g(3) \approx \frac{1}{2}, g(4) = 0$

g has a relative maximum at $x = 2$.



98. (a) $g(t) = 4 - \frac{4}{t^2}$

$\lim_{t \to \infty} g(t) = 4$

Horizontal asymptote: $y = 4$

(b) $A(x) = \int_1^x \left(4 - \frac{4}{t^2}\right) dt = \left[4t + \frac{4}{t}\right]_1^x = 4x + \frac{4}{x} - 8 = \frac{4x^2 - 8x + 4}{x} = \frac{4(x-1)^2}{x}$

$\lim_{x \to \infty} A(x) = \lim_{x \to \infty}\left(4x + \frac{4}{x} - 8\right) = \infty + 0 - 8 = \infty$

The graph of $A(x)$ does not have a horizontal asymptote.

99. (a) $v(t) = 5t - 7,\ 0 \le t \le 3$

Displacement $= \int_0^3 (5t - 7)\, dt = \left[\frac{5t^2}{2} - 7t\right]_0^3 = \frac{45}{2} - 21 = \frac{3}{2}$ ft to the right

(b) Total distance traveled $= \int_0^3 |5t - 7|\, dt$

$$= \int_0^{7/5} (7 - 5t)\, dt + \int_{7/5}^3 (5t - 7)\, dt$$

$$= \left[7t - \frac{5t^2}{2}\right]_0^{7/5} + \left[\frac{5t^2}{2} - 7t\right]_{7/5}^3$$

$$= 7\left(\frac{7}{5}\right) - \frac{5}{2}\left(\frac{7}{2}\right)^2 + \left(\frac{5}{2}(9) - 21\right) - \left(\frac{5}{2}\left(\frac{7}{5}\right)^2 - 7\left(\frac{7}{5}\right)\right)$$

$$= \frac{49}{5} - \frac{49}{10} + \frac{45}{2} - 21 - \frac{49}{10} + \frac{49}{5} = \frac{113}{10} \text{ ft}$$

100. (a) $v(t) = t^2 - t - 12 = (t-4)(t+3),\ 1 \le t \le 5$

Displacement $= \int_1^5 \left(t^2 - t - 12\right) dt$

$$= \left[\frac{t^3}{3} - \frac{t^2}{2} - 12t\right]_1^5 = \left(\frac{125}{3} - \frac{25}{2} - 60\right) - \left(\frac{1}{3} - \frac{1}{2} - 12\right) = -\frac{56}{3}\ \left(\frac{56}{3}\text{ ft to the left}\right)$$

(b) Total distance traveled $= \int_1^4 \left(-t^2 + t + 12\right) dt + \int_4^5 \left(t^2 - t - 12\right) dt$

$$= \left[-\frac{t^3}{3} + \frac{t^2}{2} + 12t\right]_1^4 + \left[\frac{t^3}{3} - \frac{t^2}{2} - 12t\right]_4^5$$

$$= \left(-\frac{64}{3} + 8 + 48\right) - \left(-\frac{1}{3} + \frac{1}{2} + 12\right) + \left(\frac{125}{3} - \frac{25}{2} - 60\right) - \left(\frac{64}{3} - 8 - 48\right)$$

$$= \frac{104}{3} - \frac{73}{6} + \left(-\frac{185}{6}\right) - \left(-\frac{104}{3}\right) = \frac{79}{3}\text{ ft}$$

101. (a) $v(t) = t^3 - 10t^2 + 27t - 18 = (t-1)(t-3)(t-6),\ 1 \le t \le 7$

Displacement $= \int_1^7 \left(t^3 - 10t^2 + 27t - 18\right) dt$

$$= \left[\frac{t^4}{4} - \frac{10t^3}{3} + \frac{27t^2}{2} - 18t\right]_1^7$$

$$= \left[\frac{7^4}{4} - \frac{10(7^3)}{3} + \frac{27(7^2)}{2} - 18(7)\right] - \left[\frac{1}{4} - \frac{10}{3} + \frac{27}{2} - 18\right]$$

$$= -\frac{91}{12} - \left(-\frac{91}{12}\right) = 0\text{ ft}$$

(b) Total distance traveled $= \int_1^7 \left|v(t)\right| dt$

$$= \int_1^3 \left(t^3 - 10t^2 + 27t - 18\right) dt - \int_3^6 \left(t^3 - 10t^2 + 27t - 18\right) dt + \int_6^7 \left(t^3 - 10t^2 + 27t - 18\right) dt$$

Evaluating each of these integrals, you obtain

Total distance $= \frac{16}{3} - \left(-\frac{63}{4}\right) + \frac{125}{12} = \frac{63}{2}$ ft

102. (a) $v(t) = t^3 - 8t^2 + 15t = t(t-3)(t-5),\ 0 \le t \le 5$

Displacement $= \int_0^5 \left(t^3 - 8t^2 + 15t\right) dt$

$$= \left[\frac{t^4}{4} - \frac{8t^3}{3} + \frac{15t^2}{2}\right]_0^5$$

$$= \frac{625}{4} - \frac{8(125)}{3} + \frac{375}{2} = \frac{125}{12}\text{ ft to the right}$$

(b) Total distance traveled $= \int_0^5 \left|v(t)\right| dt$

$$= \int_0^3 \left(t^3 - 8t^2 + 15t\right) dt - \int_3^5 \left(t^3 - 8t^2 + 15t\right) dt$$

Evaluating each of these integrals, you obtain

Total distance $= \frac{63}{4} - \left(-\frac{16}{3}\right) = \frac{253}{12} \approx 21.08$ ft

103. (a) $v(t) = \dfrac{1}{\sqrt{t}}, 1 \le t \le 4$

Because $v(t) > 0$,

Displacement = Total Distance

Displacement $= \int_1^4 t^{-1/2}\,dt = \left[2t^{1/2}\right]_1^4 = 4 - 2 = 2$ ft to the right

(b) Total distance $= 2$ ft

104. (a) $v(t) = \cos t, 0 \le t \le 3\pi$

Displacement $= \int_0^{3\pi} \cos t\,dt = \left[\sin t\right]_0^{3\pi} = 0$ ft

(b) Total distance $= \int_0^{\pi/2} \cos t\,dt - \int_{\pi/2}^{3\pi/2} \cos t\,dt + \int_{3\pi/2}^{5\pi/2} \cos t\,dt - \int_{5\pi/2}^{3\pi} \cos t\,dt$

$= \left[\sin t\right]_0^{\pi/2} - \left[\sin t\right]_{\pi/2}^{3\pi/2} + \left[\sin t\right]_{3\pi/2}^{5\pi/2} - \left[\sin t\right]_{5\pi/2}^{3\pi} = 1 - (-2) + 2 - (-1) = 6$ ft

105. $x(t) = t^3 - 6t^2 + 9t - 2$

$x'(t) = 3t^2 - 12t + 9 = 3(t^2 - 4t + 3) = 3(t-3)(t-1)$

Total distance $= \int_0^5 \left|x'(t)\right| dt$

$= \int_0^5 3\left|(t-3)(t-1)\right| dt$

$= 3\int_0^1 (t^2 - 4t + 3)\,dt - 3\int_1^3 (t^2 - 4t + 3)\,dt + 3\int_3^5 (t^2 - 4t + 3)\,dt = 4 + 4 + 20 = 28$ units

106. $x(t) = (t-1)(t-3)^2 = t^3 - 7t^2 + 15t - 9$

$x'(t) = 3t^2 - 14t + 15$

Using a graphing utility,

Total distance $= \int_0^5 \left|x'(t)\right| dt \approx 27.37$ units.

107. Let $c(t)$ be the amount of water that is flowing out of the tank. Then $c'(t) = 500 - 5t$ L/min is the rate of flow.

$\int_0^{18} c'(t)dt = \int_0^{18} (500 - 5t)\,dt = \left[500t - \dfrac{5t^2}{2}\right]_0^{18} = 9000 - 810 = 8190$ L

108. Let $c(t)$ be the amount of oil leaking and $t = 0$ represent 1 P.M. Then $c'(t) = 4 + 0.75t$ gal/min is the rate of flow.

(a) From 1 P.M. to 4 P.M. (3 hours):

$\int_0^3 (4 + 0.75t)\,dt = \left[4t + \dfrac{0.75}{2}t^2\right]_0^3 = \dfrac{123}{8} = 15.375$ gal

(b) From 4 P.M. to 7 P.M. (3 hours)

$\int_3^6 (4 + 0.75t)\,dt = \left[4t + \dfrac{0.75}{2}t^2\right]_3^6 = 22.125$ gal

(c) The second answer is larger because the rate of flow is increasing.

109. The function $f(x) = \dfrac{2}{x^3}$ is not continuous on $[-2, 1]$.

$$\int_{-2}^{1} \frac{2}{x^3}\, dx = \int_{-2}^{0} \frac{2}{x^3}\, dx + \int_{0}^{1} \frac{2}{x^3}\, dx$$

Each of these integrals is infinite. $f(x) = \dfrac{2}{x^3}$ has a nonremovable discontinuity at $x = 0$.

110. The function $f(x) = \sec^2 x$ is not continuous on $\left[\dfrac{\pi}{4}, \dfrac{3\pi}{4}\right]$.

$$\int_{\pi/4}^{3\pi/4} \sec^2 x\, dx = \int_{\pi/4}^{\pi/2} \sec^2 x\, dx + \int_{\pi/2}^{3\pi/4} \sec^2 x\, dx$$

Each of these integrals is infinite. $f(x) = \sec^2 x$ has a nonremovable discontinuity at $x = \dfrac{\pi}{2}$.

111. True

112. True

113. $f(x) = \displaystyle\int_0^{1/x} \frac{1}{t^2 + 1}\, dt + \int_0^{x} \frac{1}{t^2 + 1}\, dt$

By the Second Fundamental Theorem of Calculus, you have $f'(x) = \dfrac{1}{(1/x)^2 + 1}\left(-\dfrac{1}{x^2}\right) + \dfrac{1}{x^2 + 1} = -\dfrac{1}{1 + x^2} + \dfrac{1}{x^2 + 1} = 0.$

Because $f'(x) = 0$, $f(x)$ must be constant.

114. $\displaystyle\int_c^x f(t)\, dt = x^2 + x - 2$

Let $f(t) = 2t + 1$. Then

$$\int_c^x f(t)dt = \int_c^x (2t + 1)dt = \left[t^2 + t\right]_c^x =$$

$$x^2 + x - c^2 - c = x^2 + x - 2$$
$$-c^2 - c = -2$$
$$c^2 + c - 2 = 0$$
$$(c + 2)(c - 1) = 0 \Rightarrow c = 1, -2.$$

So, $f(x) = 2x + 1$, and $c = 1$ or $c = -2$.

115. Average value $= \dfrac{1}{b - a}\displaystyle\int_a^b f(x)\, dx$

$$12 = \frac{1}{k}\int_o^k x^3\, dx$$
$$12 = \frac{1}{k}\left[\frac{1}{4}x^4\right]_0^k$$
$$12 = \frac{1}{k}\left(\frac{1}{4}k^4 - 0\right)$$
$$12 = \frac{k^3}{4}$$
$$48 = k^3$$
$$48^{1/3} = k$$

So, the answer is D.

116. $\dfrac{d}{dx}\left[\displaystyle\int_0^{x^2} e^{t^2}\, dt\right]$

Let $u = x^2$ and $du = 2x\, dx$. By the Second Fundamental Theorem of Calculus,

$$\frac{d}{du}\left[\int_0^4 e^{t^2}\, dt\right]\frac{du}{dx} = e^{u^2}(2x)$$
$$= 2xe^{x^4}.$$

So, the answer is C.