# Uniform Acceleration - Leaving Cert Applied Maths

Adam Kelly

September 13, 2018

In this section, **linear motion** with **constant acceleration** is studied. *Motion* is a change in the position of a particle. *Linear* motion is motion in one dimension. In the questions these apply to, this means up and down, forwards and backwards and so on.

## Summary

- $v$: final velocity
- $u$: initial velocity
- $a$: acceleration
- $s$: displacement
- $t$: time

These quantities are related through the following equations:

$$v = u + at \tag{1}$$
$$s = \frac{(u+v)}{2}t \tag{2}$$
$$s = ut + \frac{1}{2}at^2 \tag{3}$$
$$v^2 = u^2 + 2as \tag{4}$$
$$\tag{5}$$

## 1 Vector Quantities and Scalar Quantities

In the leaving cert applied maths course, there are two types of quantities to think about, vector quantities and scalar quantities.

DEFINITION

**Scalar Quantity.**

*A quantity with magnitude only.*

DEFINITION

**Vector Quantity.**

*A quantity with both magnitude and direction*

Examples of each of these are

| Scalar Quantity | Vector Quantity |
|---|---|
| Distance | Displacement |

| Scalar Quantity | Vector Quantity |
|---|---|
| Speed | Velocity |
| Length | Acceleration |
| Area | Force |

You should note that sometimes a vector quantity $a$ may be referred to as $\overrightarrow{a}$.

## 2 Displacement, Velocity and Acceleration

DEFINITION

**Displacement.**

*The change in the position of an object,*

$$s = \Delta x = x_f - x_i$$

Where $x_f$ refers to the value of the final position, $x_i$ refers to the initial position, and both $s$ and $\Delta x$ refer to displacement. Displacement is a vector quantity. In the course, displacement is usually measured in metres ($m$).

DEFINITION

**Average Velocity.**

$$\overline{v} = \frac{s}{t} = \frac{\textit{change in position}}{\textit{change in time}} = \frac{\Delta x}{\Delta t}$$

Velocity is a vector quantity. It's unit is usually $ms^{-1}$.
You should know the difference between average velocity and instantaneous velocity. Instantaneous velocity is just the velocity at a specific instant in time, in other words, the average velocity over an infinitesimally small time interval.

DEFINITION

**Instantaneous Velocity.**

*Velocity at a specific instant in time.*

$$v = \lim_{\Delta t \to 0} \frac{s}{\Delta t}$$

DEFINITION

**Acceleration.**

*Accleration is the rate of change of velocity.*

$$a = \frac{\Delta v}{\Delta t} = \frac{v - u}{t}$$

Here, $v$ represents the final velocity and $u$ represents the initial velocity. This convention of $u$ and $v$ are used throughout the course. The unit for acceleration is usually $ms^{-2}$ (metres per second per second).

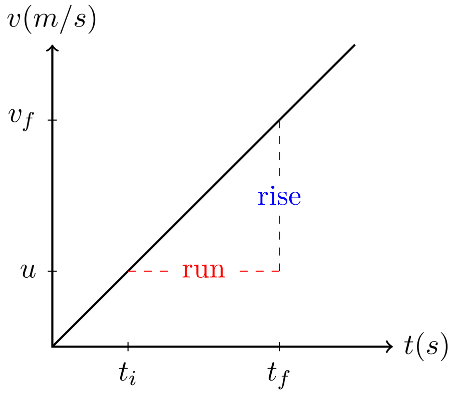
## 3 Velocity-Time Graphs

Velocity-Time graphs, while hard to interpret, are quite useful. They have a number of properties:

- The slope of a velocity-time graphs represents acceleration
- The area under a velocity-time graph represents the displacement of the object

## 3.1 Slope Of A Velocity-Time Graph

Consider the following velocity-time graph:

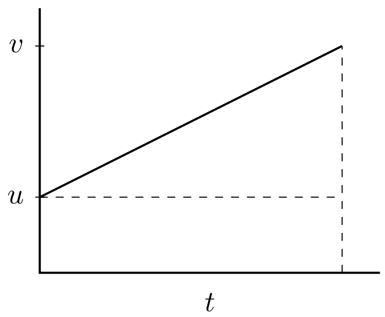



Since $\frac{\Delta v}{\Delta t}$ is the definition of acceleration, the slope of a velocity graph must equal the acceleration of the object.

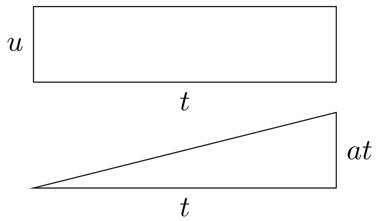
## 3.2 Area Under A Velocity-Time Graph

THEOREM 3.1 **Area Under A Velocity-Time Graph.**

*The area under a velocity-time graph is equal to the distance travelled.*




*Proof.* Since $v = u + at$ (see equation 1), the gap between $u$ and $v$ is of length $at$. The area under the curve is the sum of the area of a rectangle and a triangle.




$$\text{area of rectangle} = ut$$

$$\text{area of triangle} = \frac{a}{2}t^2$$

Adding these,

$$\text{total area} = ut + \frac{a}{2}t^2$$

Which is $s$, the distance covered (see equation 3). QED

## 4 Kinematic Formulae

The kinematic formulae are a set of equations that relate the five kinematic variables, $s$: displacement, $t$: time interval, $u$: initial velocity, $v$: final velocity and $a$: constant acceleration.
The formulae (and their proofs) are stated here.

THEOREM 4.1 **First Kinematic Formula.**

$$v = u + at$$

*Proof.* This formula can be derived by starting with the definition of acceleration, and solving for $v$.

$$\begin{aligned} a &= \frac{v-u}{t} \\ \implies v - u &= at \\ \implies v &= at + u \end{aligned}$$

QED

THEOREM 4.2 **Second Kinematic Formula.**

$$s = (\frac{u+v}{2})t$$

*Proof.* Since the acceleration is constant, the average velocity can be found by averaging the initial and final velocities.

$$\overline{v} = \frac{u+v}{2}$$

by the definition of $v$,

$$v = \frac{s}{t} \implies s = vt = (\frac{u+v}{2})t$$

QED

THEOREM 4.3 **Third Kinematic Formula.**

$$s = ut + \frac{a}{2}t^2$$

*Proof.* This proof can be done in a number of ways. For the geometric proof, see theorem 3.2. I prefer a proof involving a small amount of simple calculus. By integrating the first kinematic equation (for velocity), you will get the displacement:

$$\begin{aligned}\int vdt &= \int u + atdt \\ &= ut + \frac{a}{2}t^2\end{aligned}$$

QED

THEOREM 4.4 **Fourth Kinematic Formula.**

$$v^2 = u^2 + 2as$$

*Proof.* This can be proven by solving the first kinematic formula for $t$.

$$v = u + at \implies t = \frac{v-u}{a}$$

Taking this and using it again with the first formula

$$\begin{aligned}v^2 &= (u+at)^2 \\ &= u^2 + 2uat + a^2t^2 \\ &= u^2 + 2a(ut + \frac{a}{2}t^2) \\ &= u^2 = 2as\end{aligned}$$

QED